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Integrated Model Development for Liquid Fueled Rocket Propulsion Systems

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Aspects of Model-Based Rocket Engine Condition Monitoring and Control

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INTEGRATED MODEL DEVELOPMENT FOR LIQUID FUELED ROCKET PROPULSION SYSTEMS

INTERIM REPORT

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1.0 BACKGROUND

As detailed in the original Statement of Work, the objective of Phase I of this research effort was to develop an advanced mathematical-empirical model of SSME steady-state performance. Specific Phase I development tasks are defined as follows:

Task 1) Examine SSME test and flight program data base. Identify performance control variables and performance prediction variables to be used in gains model. Identify baseline case for model indexing.

Task 2) Construct data base access software to isolate and accumulate pertinent performance parameters from SSME data base.

Task 3) Construct software to determine first order and possibly higher order system influence coefficients for SSME baseline case.

Task 4) Construct a mathematically rigorous and structured, engine system gains model based on SSME program test data.

Task 5) Perform integrity tests on new gains model to determine statistical distributions of predicted performance characteristics.

Task 6) Develop component specific modification strategy for baseline case influence coefficient matrices.

Task 7) Write interim report documenting software developed.

SSME steady-state performance is defined by the fluid, flow, and hardware characteristics that exist throughout the engine during steady-state operation. The minimum amount of information required to specify a SSME operating state is provided by the set of performance control, or independent variables. The set of performance control variables includes system external fluid properties, component hardware characteristics, and hardware control settings. Any performance characteristic of an operating engine should be expressible as a function of the control variables as indicated below:

$$Y_i \quad = \quad F_i \; (\; \mathbf{X} \;) \qquad (1)$$

where

Y_i = performance dependent variable i

F_i = functional form of performance variable i in terms of performance control variables.

$\mathbf{X}$ = vector of performance control variables x_j where the x_j's include system external fluid properties, component hardware characteristics, and hardware control settings, i.e. $\mathbf{X} = [\; x_1 \; x_2 \; . \; . \; . \; x_n \;]^T$.

Theoretical analysis based on fundamental physics would indicate that the form of F_i for a typical engine performance variable Y_i is in general to complex for explicit analytical expression.

The first goal of the Phase I effort was to develop a mathematical strategy for efficiently and reliably approximating the functional form of the various performance variable relations F_i. Although any number of truncated orthogonal series expansions could have been used to approximate the form of F_i, the multivariate Taylor series was selected for simplicity, and in the absence of information suggesting the use of other expansions. The Taylor series expansion for F_i takes the form shown below:

$F_i(\mathbf{X})$ = (F_i at baseline control variable values $\mathbf{X}_o$)
+ (1st order variations in F_i due to $\mathbf{X}$ changes)
+ (2nd order variations in F_i due to $\mathbf{X}$ changes)
+ (higher order variations in F_i)

which is expressed mathematically as

$$\begin{aligned} F_i(\mathbf{X}) \quad = \quad & F_i(\mathbf{X}_o) \qquad (2) \\ + \quad & \nabla[F_i]_{\mathbf{X}o} * (\mathbf{X}-\mathbf{X}_o) \\ + \quad & (1/2) * (\mathbf{X}-\mathbf{X}_o)^T * H_{\mathbf{X}o} * (\mathbf{X}-\mathbf{X}_o) \\ + \quad & O(\; |\mathbf{X}-\mathbf{X}_o|^3 \;) \end{aligned}$$

where

$$\nabla[F_i]_{\mathbf{X}o} = \text{gradient of } F_i \text{ at } \mathbf{X}_o$$

$$H_{\mathbf{X}o} = \text{Hessian matrix of } F_i \text{ at } \mathbf{X}_o .$$

It is convenient to express the equation (2) expansion in influence coefficient form as indicated below:

$$F_i(\mathbf{X}) = F_i(\ x_{o1}\ x_{o2}\ .\ .\ .\ x_{on}\ ) \tag{3}$$

$$+ \sum_{j=1}^{n} [\ I_{ji} * (x_j - x_{oj})\ ]$$

$$+ \sum_{k=1}^{n} \sum_{l=k}^{n} [\ I_{kli} * (x_k - x_{ok}) * (x_l - x_{ol})\ ]$$

$$+ \text{higher order terms}$$

where

I_{ji} = performance variable Y_i first order influence coefficient associated with changes in control variable x_j

I_{kli} = performance variable Y_i second order influence coefficient associated with the product of changes in control variables x_k and x_l

n = number of performance control variables .

A complete gains model requires specification of the following information:

1. Baseline control variable ($\mathbf{X}_o$) settings
2. Values of the desired performance variables Y_i at the baseline control variable settings (i.e. $Y_{oi} = F_i(\mathbf{X}_o)$)
3. Influence coefficient values for each performance variable Y_i and for the order model chosen
   1st order models - (I_{ji} $j = 1\ 2\ .\ .\ .\ n$)
   2nd order models - (I_{ji} $j = 1\ 2\ .\ .\ .\ n$ and
   I_{kli} $k = 1\ 2\ .\ .\ .\ n,$ $l = k\ k{+}1\ .\ .\ .\ n$)

Influence coefficient values for a given performance variable Y_i can be obtained in a deterministic procedure if the behavior of

Y_i is known at a sufficient number of control variable states. Let Y_{mi} be the value of performance variable Y_i at control variable state $\mathbf{X}_m$. First order influence coefficients associated with Y_i can be determined by solving the linear system described below:

$$[\ x_{mj} - x_{oj}\ ]^{(nxn)} \ * \ [\ I_{ji}\ ]^{(nx1)} \ = \ [\ Y_{mi}\ ]^{(nx1)}. \qquad (4)$$

Equation (4) assumes that the number of states m at which $\mathbf{X}_m$ and the corresponding Y_{mi} are known is equal to the number of control variables n.

A similar procedure can be followed to determine influence coefficients associated with a second order model. The matrix equation form is described below:

$$[\ (x_{mj}-x_{oj}) \ \vdots\ (x_{mk}-x_{ok})*(x_{ml}-x_{ol})\ ]^{(pxp)} \ * \ \begin{bmatrix} I_{ji} \\ \text{----} \\ I_{kli} \end{bmatrix}^{(px1)}$$

$$= \ [\ Y_{mi}\ ]^{(px1)} \qquad (5)$$

where

p = number of 1st order influence coefficients
+ number of 2nd order influence coefficients
= n + [n*(n+1)/2]

Equation (5) assumes that the number of states m at which $\mathbf{X}_m$ and the corresponding Y_{mi} are known is equal to the number of second order model terms p.

After discussions with John Butas of NASA/MSFC/EP52 it was determined that the test and flight data base incorporated within Rocketdyne's power balance model (PBM) [1], a FORTRAN based SSME performance prediction package, was the best available archive of SSME test program experience. With this in mind, various control states $\mathbf{X}_m$ were specified as inputs to PBM analyses. The performance variable values established by PBM analyses were used as the corresponding Y_{mi}. Equations (4) and (5) were then applied to determine influence coefficient values, associated with both first and second order models respectively, for several SSME performance characteristics.

Section 2 of this report describes the control variable basis of three different gains models. The procedure used to establish influence coefficients for each of these three models is also described in Section 2. Gains model analysis results are compared to PBM predictions in Section 3.

2.0 GAINS ANALYSIS PROCEDURE

The following procedure was followed in order to construct the SSME performance gains models developed in this research effort:

Step 1. Select a baseline steady-state SSME operating case.

Step 2. Define the performance control variables, i.e., the gains model independent variables.

Step 3. Select the performance variables to be predicted, i.e., the gains model dependent variables.

Step 4. Select model definition states $\mathbf{X}_m$, i.e., select values of the performance control variables at a number of independent states m equal to the number of model influence coefficients.

Step 5. Perform a PBM analysis at each of the model definition states, i.e., perform a PBM analysis with performance control variables set at the values selected in step 4 for each definition state.

Step 6. Extract values of the performance dependent variables selected in step 2 from the output of each PBM analysis performed in step 5.

Step 7. Solve for gains model influence coefficients I_{ji} (and I_{kli} for second order models) associated with each selected dependent variable Y_i, i.e., solve matrix equation (4) (equation (5) for second order models) for the influence coefficients associated with each selected dependent variable Y_i.

Step 8. Identify model test cases, i.e., identify values of the model performance control variables for states other than model definition states.

Step 9. Compare gains model predictions with PBM results, i.e., compare values of the selected dependent variables derived from equation (3) (appropriately truncated) using the influence coefficients determined in step 7 with PBM predictions for the test cases defined in step 8.

The SSME performance state at a nominal 104% of rated power level (RPL) was selected as the baseline steady-state operating case. This nominal performance state was selected because it is central to the 100% to 109% RPL range used in flight. A detailed listing of SSME performance variables at baseline state conditions in presented in Table 1. Definitions of the performance variables associated with the A-array addresses presented in Table 1 can be found in Rocketdyne's Power Balance Model documentation [1].

Three separate gains models were investigated in this study. A number of assumptions were made in selecting the performance control variables for these models. Component hardware characteristics were assumed to be invariant and hence the impact of hardware modification was not considered for any of the three models. Propellant and oxidizer inlet temperatures were also assumed invariant and not considered as control variables. The specific independent variables selected for each of the three models are presented in Table 2. The associated A-array address for each variable is also indicated

Model 1 is a five independent variable model with the fuel preburner oxidizer valve (FPOV) resistance, oxygen preburner oxidizer valve (OPOV) resistance, thrust level (PERTHR), and suction pressures (P1FP1 and P1OP1) specified as engine control variables. Model 1 presents a primitive variable approach with engine performance defined at least partially by control valve settings. By contrast, model 2 is a four independent variable feedback model. Engine hardware settings (including valve position and resistance) are adjusted by feedback from the engine controller to match desired conditions. Hence, in model 2, the desired oxidizer-fuel mixture ratio (RMEP) is considered a control variable together with PERTHR, P1FP1 and P1OP1. Model 3 is a reduced version of model 2, with overall system pressure differences rather than point pressures, employed as control variables. Such an approach effectively reduces the number of control pressure inputs and simplifies the model to a three independent variable form.

In order to test the reliability and accuracy of the three models described above, predictions for six SSME performance variables were examined. These six dependent variables, and their A-array locations, are also defined in Table 2. They include two pressures, P2FP2 P2OP3, two temperatures, T2FT2 T2OT2, and two flow rates, WFT1 WTCJBY.

Model definition states were selected to reasonably span the power range of interest (100% to 109% RPL), and provide symmetric (directionally unbiased) variation of the control variables over prescribed ranges about base case conditions. Definition case values of the independent variables for each of models 1, 2, and 3 are specified in Tables 3, 4, and 5 respectively. Power Balance Model predicted values of the selected dependent variables for each definition case were extracted from the PBM output A-array using the EXTRACT routine listed in Appendix C1. Definition case dependent variable values are also presented in Tables 3-5.

The FORTRAN computer routine INFLUENCE was developed to determine gains model influence coefficients from normalized definition case information. This routine is the heart of the gains model approach. A listing of routine INFLUENCE is presented in Appendix C2 and the function of routine INFLUENCE is described as follows. For each model, values of the normalized variables (both independent and dependent) for the base case and first n definition cases, where n is the number of model control variables, are used to construct equation (4). First order model influence coefficients are then determined by solving equation (4). For each model, values of the normalized variables for the base case and all definition cases are used to construct equation (5). Both first order and second order influence coefficients required by the second order model are then determined by solving equation (5). The influence coefficients so derived for each of models 1, 2, and 3 are presented in Tables 7, 8, and 9 respectively.

For each of the three models, once the influence coefficients for a specific dependent variable are determined, gains model predictions for that variable can be made at any operating state by specifying the model control variables at that state and using equation (3). For first order models, equation (3) is truncated to retain only first order terms in $(x_j - x_{oj})$. For second order models, equation (3) is truncated to retain terms through order two.

FORTRAN routine COMPARE (see Appendix C3) was constructed in order to compare Power Balance Model results with gains model predictions. Five test cases were defined for comparison purposes. These test cases are prescribed by the control (independent) variable settings in Table 6. PBM predicted values of the six specified performance dependent variables for each test case are also presented in Table 6. Detailed comparisons with gains model predictions are presented in the next section of this report.

3.0 GAINS ANALYSIS RESULTS

PBM results were compared to predictions derived from each of the three gains models described in the previous section. Predictions from both first and second order forms of each gains model were examined for each of the six selected dependent variables in each of the five test cases. The integrity of the various gains model predictions was measured in terms of the percent deviation from PBM prediction as defined below:

$$\%\ \text{deviation} = 100 * \frac{|\ \text{PBM prediction} - \text{gains prediction}\ |}{\text{PBM prediction}} \tag{6}$$

Appendix B1 contains percent deviation results for gains model 1 predictions of each order (first and second) for each of six dependent variables in each of five test cases. Appendices B2 and B3 contain similar results for gains models 2 and 3 respectively.

To facilitate evaluation of results, prediction acceptability criteria were adopted as prescribed below:

gains prediction	-	acceptable	-	% deviation < 0.5%
		marginal	-	0.5% <= % deviation <= 1.0%
		unacceptable	-	% deviation > 1.0%

The information detailed in Appendices B1 through B3 has been summarized according to the above criteria in Table 10. A number of observations are suggested upon examination of Table 10.

1. Both the first and second order forms of each of the three gains models were acceptable for pressure prediction.
2. The second order forms of models 2 and 3 provided more accurate predictions in general than the first order forms of the same models.
3. The first order form of model 1 provided more accurate predictions in general than the second order form of the same model. The cause of this anomalous behavior is unknown although it may be related to the feedback character of PBM

relative to the primitive variable approach of model 1.

4. Of the dependent variables considered, the high pressure oxidizer turbine discharge temperature was in general the most difficult performance variable to predict accurately.
5. The second order form of gains model 2 provided acceptable prediction accuracy for all variables considered for all test cases.
6. The second order form of gains model 3 required a smaller number (9) of definition cases yet provided acceptable prediction accuracy for all variables except T2OT2. In four of the five test cases, prediction accuracy was only marginal for T2OT2. In no case was prediction accuracy unacceptable.
7. The first order form of model 1 provided acceptable accuracy for all variables except T2OT2.

No unacceptable predictions were returned by the second order forms of either model 2 or model 3. Direct comparisons of the prediction accuracy provided by these two second order gains models are presented in Appendix B4. Examination of the deviation plots in Appendix B4 indicates that the second order form of gains model 2 is exceptionally accurate, with the majority of deviations from PBM prediction under 0.1%.

In order to better understand the second order form of model 2, the four normalized control variables of this model were uniformly varied +1.0% in one case and +5.0% in a second case. The contribution to dependent variable variation from first order terms was then calculated. The contribution from second order terms was then computed separately. The magnitude of the contribution from second order terms was divided by the magnitude of the contribution from first order terms for each of the performance dependent variables. The results are displayed in Appendix B5. The ratio of second order contributions to first order contributions is observed to be negligible for pressure, but significant for both temperature and flow. This suggests that a first order gains model is sufficient for pressure prediction while a second order gains model is needed to accurately predict SSME temperatures and flow rates.

4.0 OBSERVATIONS AND RECOMMENDATIONS

Specific observations and recommendations that follow from the results reported in the previous section are presented below.

1. Gains model influence coefficients provide an explicit and well defined measure of control variable contributions to performance conditions. This facilitates understanding of performance dependencies, efficient and reliable code modification, and model documentation.
2. The second order form of gains model 2 is an efficient, reliable, and accurate SSME performance prediction method based on comparisons with PBM results.
3. The control variable range limits of gains model 2 should be established.
4. A multiple base form of gains model 2 should be developed to extend the control variable range limits to cover potential flight conditions.
5. A statistical strategy for incorporating ongoing TTB program test results in the computation of gains model influence coefficients should be developed. Such a strategy would provide a continuous and self-documenting model updating procedure as well as an explicit method of incorporating test results in analytical models.
6. An expanded gains model should be developed for use as a standard SSME performance prediction tool. Component hardware operating characteristics should be incorporated as control variables in an expanded gains model.

5.0 REFERENCES

1. "SSME Engine Model User's Guide," Version PBM90A, Rockwell International, Rocketdyne Division, SSME Performance and Decision Analysis Unit, April, 1992.

2. Kreyszig, Erwin, Advanced Engineering Mathematics, Third Edition, Wiley, New York, 1972.

APPENDICES

APPENDIX A

TABLES

TABLE 1. Baseline Case A-Array

I	A(I)	A(I+1)	A(I+2)	A(I+3)	A(I+4)
1	0.104000E+01	0.601100E+01	0.000000E+00	0.300000E+02	0.100000E+03
6	0.370000E+02	0.164000E+03	0.723521E+00	0.161023E+01	0.312624E+04
11	0.000000E+00	0.100000E+01	0.100000E+01	0.100000E+01	0.265000E+02
16	0.153000E+02	0.423000E-01	0.133300E-01	0.750000E+04	0.997870E+00
21	0.000000E+00	0.540000E+03	0.102897E+02	0.903324E+02	0.000000E+00
26	0.100000E+02	0.330000E+02	0.196200E+02	0.397200E+01	0.831563E+02
31	0.109220E+01	0.108000E+02	0.138600E+01	0.289600E+01	0.100854E+01
36	0.120000E+02	0.120000E+02	0.740000E+01	0.101900E+02	0.117250E+02
41	0.685000E+01	0.500000E+01	0.600000E+01	0.100900E+02	0.149200E+02
46	0.169200E+02	0.500000E+00	0.000000E+00	0.600000E-01	0.000000E+00
51	0.000000E+00	0.215000E+00	0.383000E+00	0.120200E+02	0.520000E+01
56	0.520000E+01	0.340000E+01	0.100000E+01	0.101420E+01	0.991510E+00
61	0.100575E+01	0.100000E+01	0.100000E+01	0.105000E+01	0.100570E+01
66	0.104193E+01	0.380000E+05	0.768832E+02	0.470000E+06	0.639332E+04
71	0.200296E+04	0.144545E+04	0.996604E+00	0.978000E+00	0.825000E+02
76	0.825000E+02	0.255000E+01	0.258000E+02	0.312624E+04	0.100000E+01
81	0.100000E+01	0.100000E+01	0.100000E+01	0.100000E+01	0.412000E+03
86	0.000000E+00	0.100270E+01	0.102480E+01	0.103000E+01	0.102300E+01
91	0.986300E+00	0.190920E+03	0.125000E+05	0.423400E-01	0.508700E+04
96	0.187000E+00	0.163100E+04	0.215100E-02	0.259624E+00	0.267000E+00
101	0.000000E+00	0.194600E-02	0.115798E+02	0.712000E-03	0.102900E-01
106	0.324700E-02	0.651700E-02	0.160000E+05	0.545200E-01	0.270000E-02
111	0.323800E-02	0.275400E-01	0.263200E-02	0.104100E+00	0.210000E-02
116	0.985580E+05	0.139000E+04	0.500000E-01	0.113200E+02	0.488100E+04
121	0.800000E-02	0.998653E+00	0.000000E+00	0.000000E+00	0.000000E+00
126	0.406000E+01	0.380000E+05	0.600000E+04	0.171000E-01	0.910000E-01
131	0.900000E-02	0.579000E-01	0.762000E-01	0.107000E-01	0.981430E+00
136	0.590800E-01	0.481500E+00	0.570000E-02	0.103100E+01	0.120000E+01
141	0.473000E-01	0.187900E+01	0.685000E+00	0.166000E+03	0.384000E-01
146	0.186100E+01	0.155000E+00	0.300000E+02	0.568700E-02	0.523000E-01
151	0.174600E+06	0.685900E-03	0.250800E+00	0.766000E+03	0.134000E+00
156	0.112500E+00	0.192000E+00	0.126000E-01	0.500000E-02	0.300000E+00
161	0.618500E+00	0.120000E+01	0.173400E+00	0.529000E+00	0.570000E-01
166	0.208100E-01	0.118000E-01	0.136100E+01	0.153000E+00	0.138850E+06
171	0.745680E+05	0.339270E+06	0.899650E+05	0.451380E+05	0.535500E+06
176	0.300000E+02	0.130000E+03	0.245000E+06	0.194000E+06	0.712000E-02
181	0.557000E+00	0.150000E+01	0.168000E+00	0.000000E+00	0.650000E+05
186	0.101115E+01	0.125000E-01	0.265000E-01	0.100000E+01	0.100000E+01
191	0.100000E+01	0.100000E+01	0.884000E+00	0.795310E+00	0.102237E+01
196	0.109811E+01	0.100000E+01	0.100000E+01	0.000000E+00	0.100000E+01
201	0.823313E+02	0.188072E+01	0.958090E+00	0.196299E+01	0.270995E+04
206	0.430309E+04	0.286853E+03	0.422280E+04	0.000000E+00	0.520000E+05
211	0.147233E+04	0.147223E+04	0.774980E+04	0.768418E+04	0.173603E+04
216	0.156557E+04	0.157119E+04	-.804234E+03	-.839968E+03	0.000000E+00
221	0.000000E+00	-.814218E+03	-.858387E+03	0.000000E+00	0.000000E+00
226	0.000000E+00	0.485008E+02	0.229927E+02	0.824779E+00	0.410176E+03
231	0.698208E+02	0.598345E+02	0.355514E+02	0.957059E+02	0.306040E+04
236	0.104659E+01	0.125936E+02	0.619040E+01	0.197953E+03	0.125674E+02
241	0.265653E+03	0.128314E+04	0.367260E+02	0.188328E+02	0.621658E+01
246	0.374097E+04	0.876353E+02	0.948230E+01	0.214469E+04	0.100000E+01

TABLE 1. Baseline Case A-Array

I	A(I)	A(I+1)	A(I+2)	A(I+3)	A(I+4)
1	0.104000E+01	0.601100E+01	0.000000E+00	0.300000E+02	0.100000E+03
6	0.370000E+02	0.164000E+03	0.723521E+00	0.161023E+01	0.312624E+04
11	0.000000E+00	0.100000E+01	0.100000E+01	0.100000E+01	0.265000E+02
16	0.153000E+02	0.423000E-01	0.133300E-01	0.750000E+04	0.997870E+00
21	0.000000E+00	0.540000E+03	0.102897E+02	0.903324E+02	0.000000E+00
26	0.100000E+02	0.330000E+02	0.196200E+02	0.397200E+01	0.831563E+02
31	0.109220E+01	0.108000E+02	0.138600E+01	0.289600E+01	0.100854E+01
36	0.120000E+02	0.120000E+02	0.740000E+01	0.101900E+02	0.117250E+02
41	0.685000E+01	0.500000E+01	0.600000E+01	0.100900E+02	0.149200E+02
46	0.169200E+02	0.500000E+00	0.000000E+00	0.600000E-01	0.000000E+00
51	0.000000E+00	0.215000E+00	0.383000E+00	0.120200E+02	0.520000E+01
56	0.520000E+01	0.340000E+01	0.100000E+01	0.101420E+01	0.991510E+00
61	0.100575E+01	0.100000E+01	0.100000E+01	0.105000E+01	0.100570E+01
66	0.104193E+01	0.380000E+05	0.768832E+02	0.470000E+06	0.639332E+04
71	0.200296E+04	0.144545E+04	0.996604E+00	0.978000E+00	0.825000E+02
76	0.825000E+02	0.255000E+01	0.258000E+02	0.312624E+04	0.100000E+01
81	0.100000E+01	0.100000E+01	0.100000E+01	0.100000E+01	0.412000E+03
86	0.000000E+00	0.100270E+01	0.102480E+01	0.103000E+01	0.102300E+01
91	0.986300E+00	0.190920E+03	0.125000E+05	0.423400E-01	0.508700E+04
96	0.187000E+00	0.163100E+04	0.215100E-02	0.259624E+00	0.267000E+00
101	0.000000E+00	0.194600E-02	0.115798E+02	0.712000E-03	0.102900E-01
106	0.324700E-02	0.651700E-02	0.160000E+05	0.545200E-01	0.270000E-02
111	0.323800E-02	0.275400E-01	0.263200E-02	0.104100E+00	0.210000E-02
116	0.985580E+05	0.139000E+04	0.500000E-01	0.113200E+02	0.488100E+04
121	0.800000E-02	0.998653E+00	0.000000E+00	0.000000E+00	0.000000E+00
126	0.406000E+01	0.380000E+05	0.600000E+04	0.171000E-01	0.910000E-01
131	0.900000E-02	0.579000E-01	0.762000E-01	0.107000E-01	0.981430E+00
136	0.590800E-01	0.481500E+00	0.570000E-02	0.103100E+01	0.120000E+01
141	0.473000E-01	0.187900E+01	0.685000E+00	0.166000E+03	0.384000E-01
146	0.186100E+01	0.155000E+00	0.300000E+02	0.568700E-02	0.523000E-01
151	0.174600E+06	0.685900E-03	0.250800E+00	0.766000E+03	0.134000E+00
156	0.112500E+00	0.192000E+00	0.126000E-01	0.500000E-02	0.300000E+00
161	0.618500E+00	0.120000E+01	0.173400E+00	0.529000E+00	0.570000E-01
166	0.208100E-01	0.118000E-01	0.136100E+01	0.153000E+00	0.138850E+06
171	0.745680E+05	0.339270E+06	0.899650E+05	0.451380E+05	0.535500E+06
176	0.300000E+02	0.130000E+03	0.245000E+06	0.194000E+06	0.712000E-02
181	0.557000E+00	0.150000E+01	0.168000E+00	0.000000E+00	0.650000E+05
186	0.101115E+01	0.125000E-01	0.265000E-01	0.100000E+01	0.100000E+01
191	0.100000E+01	0.100000E+01	0.884000E+00	0.795310E+00	0.102237E+01
196	0.109811E+01	0.100000E+01	0.100000E+01	0.000000E+00	0.100000E+01
201	0.823313E+02	0.188072E+01	0.958090E+00	0.196299E+01	0.270995E+04
206	0.430309E+04	0.286853E+03	0.422280E+04	0.000000E+00	0.520000E+05
211	0.147233E+04	0.147223E+04	0.774980E+04	0.768418E+04	0.173603E+04
216	0.156557E+04	0.157119E+04	-.804234E+03	-.839968E+03	0.000000E+00
221	0.000000E+00	-.814218E+03	-.858387E+03	0.000000E+00	0.000000E+00
226	0.000000E+00	0.485008E+02	0.229927E+02	0.824779E+00	0.410176E+03
231	0.698208E+02	0.598345E+02	0.355514E+02	0.957059E+02	0.306040E+04
236	0.104659E+01	0.125936E+02	0.619040E+01	0.197953E+03	0.125674E+02
241	0.265653E+03	0.128314E+04	0.367260E+02	0.188328E+02	0.621658E+01
246	0.374097E+04	0.876353E+02	0.948230E+01	0.214469E+04	0.100000E+01

TABLE 1. Baseline Case A-Array
(continued)

I	A(I)	A(I+1)	A(I+2)	A(I+3)	A(I+4)
251	0.244087E+02	0.000000E+00	0.187950E+02	0.775845E+02	0.150861E+02
256	0.409447E+02	0.805659E+01	0.117090E+03	0.103741E+03	0.101009E+04
261	0.572348E+03	0.360492E+03	0.709764E+02	0.999966E+03	0.118244E+02
266	0.268979E+02	0.990746E+03	0.389787E+03	0.109709E+04	0.891969E+02
271	0.775883E+03	0.376965E+03	0.126316E+04	0.229591E+02	0.350002E+02
276	0.702079E+03	0.170408E+02	0.149823E+03	0.281347E+04	0.263520E+03
281	0.649220E+02	0.899785E+02	0.224735E+02	0.850000E+05	0.224831E+02
286	0.113757E+03	0.114594E+02	0.110104E+03	0.394673E+02	0.433085E+02
291	0.158364E+02	0.271302E+02	0.655675E+02	0.867357E+02	0.485623E+02
296	0.444076E+03	0.115984E+04	0.984612E+02	0.316506E+03	0.137406E+04
301	0.188328E+02	0.692180E+02	0.118838E+03	0.332315E+03	0.362979E+02
306	0.602735E+03	0.141939E+02	0.124408E+02	0.313943E+03	0.384617E+04
311	0.691897E+04	0.538718E+02	-.814900E+03	-.859072E+03	0.000000E+00
316	0.401250E+00	0.532535E+01	0.513420E+02	0.163271E+02	0.665690E+02
321	0.670093E+01	0.144828E+03	0.121901E+03	-.388654E+02	-.451436E+02
326	-.901641E+03	0.000000E+00	-.860322E+03	0.000000E+00	0.135638E+04
331	0.000000E+00	0.000000E+00	0.000000E+00	0.362170E+03	0.358722E+03
336	0.230553E+02	0.121278E+02	0.218000E+01	0.280000E+00	0.706948E+00
341	0.772329E+00	0.558912E+00	0.806903E+00	0.683003E+00	0.668349E+00
346	0.255366E+00	0.144405E+03	0.776890E+04	0.534667E+04	0.100000E+01
351	0.343340E+02	0.828289E+00	0.636835E+00	0.792320E+00	0.452862E+03
356	0.453012E+03	0.453012E+03	0.488368E+06	0.159871E+05	0.348972E+05
361	0.000000E+00	0.394261E+06	0.000000E+00	0.510709E+04	0.279377E+05
366	0.488368E+06	0.171822E+02	0.488800E+06	0.191944E+03	0.000000E+00
371	0.000000E+00	0.199109E+03	0.159165E+04	0.198422E+03	0.137670E+04
376	-.132922E+01	-.105534E+01	0.884053E+04	0.176840E+06	-.803995E+03
381	-.778600E+03	0.000000E+00	0.000000E+00	0.653126E+03	0.799390E+04
386	0.622969E+04	0.350485E+04	0.643473E+05	0.350485E+04	0.643494E+05
391	0.161148E+04	0.240976E+05	0.148158E+04	0.161148E+04	0.255792E+05
396	0.132875E+03	-.839753E+03	0.202023E+03	0.831096E+03	-.108601E+03
401	-.887326E+02	0.155520E+04	-.531897E+03	-.578197E+03	-.925326E+02
406	0.205475E+03	0.140856E+04	0.147324E+04	-.830245E+03	-.983608E+02
411	0.133652E+03	-.290912E+02	-.385794E+02	0.340278E+04	0.339280E+04
416	0.424678E+04	0.572504E+04	0.339844E+04	0.341112E+04	0.519545E+04
421	-.555784E+02	-.568071E+02	-.539479E+02	-.387553E+02	-.934272E+03
426	0.339853E+04	0.227052E+00	0.184457E+00	0.217927E+00	0.138644E+00
431	0.843459E-01	0.313501E+04	0.349849E+04	0.417756E+04	0.333296E+04
436	0.341731E+04	0.341163E+04	0.584213E+04	0.479852E+04	0.315397E+04
441	0.357483E+04	0.340474E+04	0.518532E+04	0.132774E+01	0.146993E+01
446	0.356813E+04	0.577578E+04	0.571234E+04	0.409905E+05	0.147000E-02
451	0.150488E+01	0.308459E+04	0.405925E+00	0.170415E+01	0.307819E+00
456	0.368865E+00	0.539531E+00	0.625878E+04	0.613561E+04	0.267180E+03
461	0.458377E+04	0.517840E+04	0.423170E+04	0.385527E+03	0.416322E+04
466	0.516236E+04	0.290630E+02	0.239714E+03	0.426977E+04	0.138896E+04
471	-.559679E+02	-.436764E+02	-.307506E+02	0.000000E+00	0.395592E+04
476	0.728462E+04	0.557511E+04	0.628241E+04	0.678924E+04	0.688602E+04
481	0.557241E+04	0.645861E+04	0.416940E+04	0.424678E+04	0.302732E+03
486	0.633637E+04	0.422253E+03	0.434075E+04	0.730450E+04	0.276024E+03
491	0.620611E+04	0.356535E+04	0.422127E+04	0.629660E+04	0.647105E+04
496	0.621789E+04	0.406468E+04	0.000000E+00	0.000000E+00	0.000000E+00

TABLE 1. Baseline Case A-Array
(continued)

I	A(I)	A(I+1)	A(I+2)	A(I+3)	A(I+4)
501	0.345231E+04	0.352290E+04	0.345271E+03	0.343042E+04	0.583492E+04
506	0.409292E+04	0.727295E+04	0.383709E+04	0.000000E+00	0.000000E+00
511	0.000000E+00	0.000000E+00	0.473360E+04	0.159973E+01	0.621784E+04
516	0.000000E+00	0.000000E+00	0.727396E+04	0.608165E+04	0.629387E+04
521	0.610778E+04	0.649807E+04	0.579161E+04	0.000000E+00	0.566161E+04
526	0.729644E+04	0.114000E+01	0.560201E+04	0.634517E+04	0.252571E+01
531	0.252244E+01	0.656951E+04	0.690862E+02	0.689869E+02	0.411753E+04
536	0.370736E+04	0.619541E+04	0.000000E+00	0.568874E+04	0.568874E+04
541	0.568875E+04	0.156987E+05	0.160740E+05	0.585701E+04	0.708024E+04
546	0.701979E+02	0.699644E+02	0.706071E+02	0.601100E+01	0.118590E+01
551	0.801693E+00	0.873890E+00	0.779733E+00	0.685961E+00	0.636501E+03
556	0.603380E+01	0.601100E+01	0.920864E+00	0.681329E+00	0.667166E+00
561	0.440777E+01	0.710262E+02	0.440720E+01	0.431591E+01	0.702449E+02
566	0.435586E+01	0.497911E+01	0.496575E+01	0.702119E+02	0.707238E+02
571	0.568437E+00	0.435493E+00	0.489568E+00	0.181980E+01	0.269083E+01
576	0.712497E+02	0.496553E+01	0.105654E+03	0.105554E+02	0.000000E+00
581	0.647114E+03	0.488519E+01	0.937215E+02	0.490061E+01	0.278447E+03
586	0.944863E+02	0.460892E+03	0.461519E+03	0.468825E+03	0.167770E+04
591	0.155159E+04	0.133859E+04	0.000000E+00	0.456239E+03	0.864250E+03
596	0.169919E+03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
601	0.365596E+03	0.365717E+03	0.462375E+03	0.115142E+04	0.968472E+04
606	0.165724E+04	0.480872E+04	0.188455E+04	0.148137E+04	0.676912E+03
611	0.506710E+03	0.000000E+00	0.278447E+03	0.000000E+00	0.423254E+02
616	0.452845E+03	0.479419E+00	0.577082E+00	0.164529E+00	0.134521E+04
621	0.404566E+02	0.164817E+03	0.179753E+03	0.200663E+03	0.000000E+00
626	0.936674E+02	0.436518E+03	0.173972E+04	0.169964E+04	0.384530E+02
631	0.662825E+02	0.133358E+04	0.165778E+03	0.192974E+03	0.205102E+03
636	0.191079E+03	0.135947E+04	0.134700E+04	0.940686E+02	0.190484E+00
641	0.360580E+00	0.466103E+00	0.291272E+00	0.837084E+03	0.182721E+04
646	0.150505E+00	0.641705E+00	0.432620E-01	0.208274E+03	0.208935E+03
651	0.516202E+03	0.155161E+04	0.261278E+03	0.835023E+03	0.609506E+03
656	0.133703E+03	0.122998E+04	0.452862E+03	0.186591E+02	0.426395E+01
661	0.154149E+03	0.934747E+00	0.302206E+02	0.000000E+00	0.150302E+03
666	0.125795E+01	0.108074E+04	0.233552E+01	0.000000E+00	0.728716E+02
671	0.180851E+03	0.640557E+02	0.000000E+00	0.978194E+00	0.425997E+00
676	0.167800E+02	0.793530E+02	0.202684E+03	0.730733E+02	0.154149E+03
681	0.154566E+03	0.155906E+04	0.153267E+03	0.156692E+03	0.158852E+00
686	0.302206E+02	0.152426E+03	0.227137E+03	0.290774E+02	0.377714E+01
691	0.336056E+01	0.824921E+03	0.387179E+01	0.338913E+01	0.358741E+01
696	0.121458E+01	0.655277E+00	0.184544E+00	0.346800E+00	0.186985E+01
701	0.926590E+03	0.531345E+00	0.824736E+03	0.141000E+00	0.122974E+02
706	0.159165E+04	0.388896E+02	0.397885E+02	0.264966E+02	0.000000E+00
711	0.926857E+03	0.110825E+04	0.110810E+04	0.111642E+04	0.107888E+03
716	0.000000E+00	0.924780E+03	0.660689E+02	0.571477E-01	0.181394E+03
721	0.100355E+01	0.491305E+00	0.109761E+03	0.825268E+03	0.107888E+03
726	0.428104E+04	0.627996E+01	0.203827E+01	0.640741E+02	0.416882E+00
731	0.995698E+02	0.478407E+01	0.437628E+01	0.395122E+00	0.623195E+00
736	0.898989E+00	0.107805E+04	0.117124E+03	0.609723E+02	0.561513E+02
741	0.108074E+04	0.146609E+01	0.216000E+00	0.935029E+03	0.260053E+02
746	0.726473E+02	0.000000E+00	0.258481E+02	0.723690E+02	-.157145E+00

TABLE 1. Baseline Case A-Array
(continued)

I	A(I)	A(I+1)	A(I+2)	A(I+3)	A(I+4)
751	0.648450E+00	-.278214E+00	0.704211E+00	0.434960E+00	0.536886E+00
756	0.123068E+01	0.106006E+01	0.108219E+01	0.714194E+00	0.726839E+00
761	0.754776E+00	0.753917E+01	0.223655E+02	0.804420E+01	0.831824E+01
766	0.387850E+02	0.783748E+02	0.258481E+02	0.723690E+02	0.378860E+02
771	0.783748E+02	0.941093E+02	0.744471E+03	0.116631E+05	0.998075E+00
776	0.200793E+03	0.437701E+01	0.410008E+03	0.000000E+00	0.342981E+03
781	0.000000E+00	0.911124E+01	0.419833E+03	-.105003E+01	-.104242E+01
786	0.838248E+02	0.140186E+02	0.363162E+03	0.232846E+03	0.443856E+02
791	0.242833E+03	0.478772E+03	0.100000E+02	0.000000E+00	0.100196E+01
796	0.100000E+01	0.100000E+01	0.665903E+00	0.804651E+00	-.577778E+03
801	0.192974E+03	0.111000E+00	0.121400E-01	0.121400E-01	0.200000E+01
806	0.200000E+01	0.561513E+02	0.117124E+03	0.360000E-01	0.185105E+01
811	0.155358E+03	0.168228E+04	0.653861E+02	0.175490E+04	0.875000E+03
816	0.131180E+01	0.657000E+00	0.888052E+00	0.281347E+04	0.283788E+04
821	0.163072E+04	0.795206E+04	0.293094E+01	0.133301E+01	0.000000E+00
826	0.100000E+01	-.812037E+03	0.380568E+01	0.493845E+03	0.000000E+00
831	0.276650E+03	0.615800E+01	0.697119E+02	0.354958E+04	0.000000E+00
836	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
841	0.120000E+04	0.000000E+00	0.365000E+03	0.385000E+03	0.330000E+03
846	0.350000E+00	0.630000E+03	0.000000E+00	0.926590E+03	0.154149E+03
851	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
856	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
861	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
866	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
871	0.137113E+01	0.545956E+04	0.151262E+01	0.944528E+00	0.261858E+02
876	0.135202E+01	0.654898E+04	0.148176E+01	0.235093E+01	0.283945E+02
881	0.100000E+01	0.100000E+01	0.100000E+01	0.100000E+01	0.100000E+01
886	0.100000E+01	0.100000E+01	0.993987E+00	0.991207E+00	0.300600E+04
891	0.253677E+04	0.278528E+03	0.161752E+02	0.169948E+03	0.830375E+04
896	0.100000E+01	0.159814E+02	0.457981E+03	0.202333E+05	0.100000E+01
901	0.312624E+04	0.613594E+04	0.406468E+04	0.154149E+03	0.926590E+03
906	0.247852E+03	0.423254E+02	0.159871E+05	0.348972E+05	0.169964E+04
911	0.519545E+04	0.345271E+03	0.510709E+04	0.192974E+03	0.727295E+04
916	0.279377E+05	0.134700E+04	0.518532E+04	0.333296E+04	0.452845E+03
921	0.000000E+00	0.458743E+04	0.926590E+03	0.154149E+03	0.488368E+06
926	0.601100E+01	0.104000E+01	0.100196E+01	0.100000E+01	0.100000E+01
931	0.665903E+00	0.804651E+00	0.000000E+00	0.000000E+00	0.000000E+00
936	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.700000E+02
941	0.000000E+00	0.102930E+02	0.903530E+02	0.475000E+02	0.475000E+02
946	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
951	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
956	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
961	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
966	0.628241E+04	0.645861E+04	0.725696E+04	0.557511E+04	0.557241E+04
971	0.444622E+04	0.339853E+04	0.584213E+04	0.616706E+04	0.000000E+00
976	0.576528E+04	0.370736E+04	0.381997E+03	0.410247E+04	0.000000E+00
981	0.561493E+04	0.000000E+00	0.340374E+04	0.100000E+01	0.000000E+00
986	0.000000E+00	0.864250E+03	0.357483E+04	0.936674E+02	0.000000E+00
991	0.169919E+03	0.205102E+03	0.278447E+03	0.000000E+00	0.000000E+00
996	0.723521E+00	0.000000E+00	0.000000E+00	0.161023E+01	0.000000E+00

TABLE 1. Baseline Case A-Array
(continued)

I	A(I)	A(I+1)	A(I+2)	A(I+3)	A(I+4)
1001	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1006	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1011	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1016	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1021	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1026	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1031	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1036	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1041	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1046	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1051	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1056	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1061	0.000000E+00	0.000000E+00	0.510000E+02	0.510000E+02	0.510000E+02
1066	0.150000E+02	0.450000E+02	0.150000E+02	0.150000E+02	0.450000E+02
1071	0.150000E+02	0.305000E+02	0.360000E+02	0.250000E+02	0.250000E+02
1076	0.335138E+02	0.335138E+02	0.000000E+00	0.000000E+00	0.000000E+00
1081	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1086	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1091	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1096	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1101	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1106	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1111	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1116	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1121	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1126	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1131	0.000000E+00	0.000000E+00	0.000000E+00	0.100000E+01	0.000000E+00
1136	0.000000E+00	0.000000E+00	0.290610E+01	0.000000E+00	0.100000E+01
1141	0.100000E+01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1146	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1151	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1156	0.000000E+00	0.570580E+02	0.256380E+02	0.000000E+00	0.000000E+00
1161	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1166	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1171	0.000000E+00	0.272443E+02	0.499937E+03	0.203459E+02	0.563186E+03
1176	0.240351E+02	0.489838E+03	0.196169E+02	0.000000E+00	0.315501E+02
1181	0.145752E+05	0.129419E+08	0.561362E+06	0.188135E+06	0.130260E+03
1186	0.247832E+03	0.382629E+03	0.399063E+02	0.414959E+03	0.468741E+02
1191	0.600142E+03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1196	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1201	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1206	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1211	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1216	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1221	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1226	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1231	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1236	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1241	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1246	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.697200E+00

TABLE 1. Baseline Case A-Array (continued)

I	A(I)	A(I+1)	A(I+2)	A(I+3)	A(I+4)
1251	-.118200E+01	0.520800E+00	0.000000E+00	0.500000E-01	0.000000E+00
1256	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1261	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1266	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1271	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1276	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1281	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1286	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1291	0.100000E+01	0.100000E+01	0.100800E+01	0.100000E+01	0.100000E+01
1296	0.100000E+01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1301	0.000000E+00	0.000000E+00	0.000000E+00	0.100000E+01	0.000000E+00
1306	0.000000E+00	0.000000E+00	0.100000E+01	0.100000E+01	0.000000E+00
1311	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.280000E+03
1316	0.000000E+00	0.000000E+00	0.100000E+01	0.000000E+00	0.000000E+00
1321	0.000000E+00	0.000000E+00	0.000000E+00	0.778260E+03	0.321740E+02
1326	0.314159E+01	0.201600E+01	0.154543E+04	0.144000E+03	0.000000E+00
1331	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1336	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1341	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1346	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.104025E+05

TABLE 2. Gains Analysis Variables

Independent Variables

Model 1 A-pos	Model 1 Name	Model 2 A-pos	Model 2 Name	Model 3 A-pos	Model 3 Name
1	PERTHR	1	PERTHR	10-4	PCCOM-P1FP1
4	P1FP1	2	RMEP	10-5	PCCOM-P1OP1
5	P1OP1	4	P1FP1	2	RMEP
578	R14	5	P1OP1		
579	R18				

Dependent Variables

Model 1 A-pos	Model 1 Name	Model 2 A-pos	Model 2 Name	Model 3 A-pos	Model 3 Name
486	P2FP2	486	P2FP2	10-486	PCCOM-P2FP2
489	P2OP3	489	P2OP3	10-489	PCCOM-P2OP3
628	T2FT2	628	T2FT2	628	T2FT2
637	T2OT2	637	T2OT2	637	T2OT2
686	WFT1	686	WFT1	686	WFT1
739	WTCJBY	739	WTCJBY	739	WTCJBY

A-pos	Name	Variable Descriptions
1	PERTHR	fraction of RPL vacuum thrust
2	RMEP	engine mixture ratio (O2/H2)
4	P1FP1	low-pressure fuel pump inlet pressure
5	P1OP1	low-pressure oxidizer pump inlet pressure
10	PCCOM	commanded chamber pressure
578	R14	oxidizer preburner oxidizer valve resistance
579	R18	fuel preburner oxidizer valve resistance
486	P2FP2	high-pressure fuel pump discharge pressure
489	P2OP3	high-pressure oxidizer pump boost stage discharge pressure
628	T2FT2	high-pressure fuel turbine discharge temp
637	T2OT2	high-pressure oxidizer turbine discharge temp
686	WFT1	low-pressure fuel turbine flowrate
739	WTCJBY	nozzle jacket bypass flowrate

TABLE 3. Model 1 - Definition Cases

Independent Variables

Case #	A(1)	A(4)	A(5)	A(578)	A(579)
base	.1040E+01	.3000E+02	.1000E+03	.1073E+03	.1051E+02
1	.1080E+01	.3000E+02	.1000E+03	.7774E+02	.8054E+01
2	.1040E+01	.3000E+02	.1000E+03	.9338E+02	.1402E+02
3	.1040E+01	.3600E+02	.1000E+03	.1073E+03	.1062E+02
4	.1040E+01	.3000E+02	.1200E+03	.1086E+03	.1027E+02
5	.1000E+01	.3000E+02	.1000E+03	.1310E+03	.1270E+02
6	.1040E+01	.3000E+02	.1000E+03	.1228E+03	.7041E+01
7	.1040E+01	.2400E+02	.1000E+03	.1074E+03	.1040E+02
8	.1040E+01	.3000E+02	.8000E+02	.1044E+03	.1075E+02
9	.1080E+01	.3000E+02	.1000E+03	.9045E+02	.4733E+01
10	.1040E+01	.2400E+02	.1000E+03	.9350E+02	.1389E+02
11	.1040E+01	.3600E+02	.8000E+02	.1044E+03	.1086E+02
12	.1000E+01	.3000E+02	.1200E+03	.1326E+03	.1250E+02
13	.1080E+01	.2400E+02	.1000E+03	.7778E+02	.7953E+01
14	.1040E+01	.3000E+02	.8000E+02	.9030E+02	.1427E+02
15	.1000E+01	.3600E+02	.1000E+03	.1309E+03	.1283E+02
16	.1040E+01	.3000E+02	.1200E+03	.1232E+03	.6802E+01
17	.1080E+01	.3000E+02	.8000E+02	.7443E+02	.8307E+01
18	.1000E+01	.3000E+02	.1000E+03	.1163E+03	.1630E+02
19	.1040E+01	.3600E+02	.1000E+03	.1227E+03	.7144E+01
20	.1040E+01	.2400E+02	.1200E+03	.1087E+03	.1016E+02

Dependent Variables

Case #	A(486)	A(489)	A(628)	A(637)	A(686)	A(739)
base	.6348E+04	.7306E+04	.1734E+04	.1348E+04	.2965E+02	.6122E+02
1	.6657E+04	.7644E+04	.1768E+04	.1416E+04	.3112E+02	.6188E+02
2	.6263E+04	.7407E+04	.1717E+04	.1490E+04	.2900E+02	.5994E+02
3	.6347E+04	.7308E+04	.1731E+04	.1349E+04	.2965E+02	.6122E+02
4	.6346E+04	.7288E+04	.1736E+04	.1338E+04	.2964E+02	.6123E+02
5	.6029E+04	.6963E+04	.1699E+04	.1315E+04	.2823E+02	.5999E+02
6	.6447E+04	.7205E+04	.1752E+04	.1218E+04	.3040E+02	.6258E+02
7	.6350E+04	.7305E+04	.1736E+04	.1346E+04	.2966E+02	.6121E+02
8	.6352E+04	.7324E+04	.1730E+04	.1362E+04	.2968E+02	.6120E+02
9	.6762E+04	.7530E+04	.1791E+04	.1275E+04	.3189E+02	.6328E+02
10	.6265E+04	.7406E+04	.1719E+04	.1488E+04	.2901E+02	.5993E+02
11	.6350E+04	.7325E+04	.1727E+04	.1363E+04	.2967E+02	.6121E+02
12	.6027E+04	.6950E+04	.1701E+04	.1307E+04	.2821E+02	.6000E+02
13	.6659E+04	.7642E+04	.1771E+04	.1415E+04	.3113E+02	.6187E+02
14	.6267E+04	.7425E+04	.1712E+04	.1508E+04	.2903E+02	.5992E+02
15	.6027E+04	.6965E+04	.1696E+04	.1317E+04	.2822E+02	.6000E+02
16	.6445E+04	.7186E+04	.1754E+04	.1212E+04	.3038E+02	.6259E+02
17	.6662E+04	.7666E+04	.1763E+04	.1436E+04	.3116E+02	.6186E+02
18	.5950E+04	.7054E+04	.1686E+04	.1448E+04	.2760E+02	.5875E+02
19	.6445E+04	.7206E+04	.1750E+04	.1219E+04	.3039E+02	.6259E+02
20	.6347E+04	.7287E+04	.1739E+04	.1337E+04	.2964E+02	.6122E+02

TABLE 5. Model 3 - Definition Cases

Independent Variables

Case #	A(2)	A(10)-A(4)	A(10)-A(5)
base	.6011E+01	.3096E+04	.3026E+04
1	.6239E+01	.3096E+04	.3026E+04
2	.6011E+01	.2971E+04	.2907E+04
3	.6011E+01	.2977E+04	.2887E+04
4	.5783E+01	.3096E+04	.3026E+04
5	.6011E+01	.3221E+04	.3145E+04
6	.6011E+01	.3215E+04	.3165E+04
7	.6239E+01	.3221E+04	.3145E+04
8	.6011E+01	.3090E+04	.3046E+04
9	.5783E+01	.2977E+04	.2887E+04

Dependent Variables

Case #	A(10) -A(486)	A(10) -A(489)	A(628)	A(637)	A(686)	A(739)
base	-.3222E+04	-.4180E+04	.1734E+04	.1348E+04	.2965E+02	.6122E+02
1	-.3137E+04	-.4281E+04	.1717E+04	.1490E+04	.2900E+02	.5994E+02
2	-.3020E+04	-.3957E+04	.1696E+04	.1317E+04	.2822E+02	.6000E+02
3	-.3019E+04	-.3943E+04	.1701E+04	.1307E+04	.2821E+02	.6000E+02
4	-.3320E+04	-.4078E+04	.1752E+04	.1218E+04	.3040E+02	.6258E+02
5	-.3414E+04	-.4397E+04	.1771E+04	.1415E+04	.3113E+02	.6187E+02
6	-.3417E+04	-.4421E+04	.1763E+04	.1436E+04	.3116E+02	.6186E+02
7	-.3324E+04	-.4507E+04	.1749E+04	.1577E+04	.3047E+02	.6056E+02
8	-.3224E+04	-.4199E+04	.1727E+04	.1363E+04	.2967E+02	.6121E+02
9	-.3110E+04	-.3852E+04	.1716E+04	.1184E+04	.2892E+02	.6133E+02

TABLE 4. Model 2 - Definition Cases

Independent Variables

Case #	A(1)	A(2)	A(4)	A(5)
base	.1040E+01	.6011E+01	.3000E+02	.1000E+03
1	.1080E+01	.6011E+01	.3000E+02	.1000E+03
2	.1040E+01	.6239E+01	.3000E+02	.1000E+03
3	.1040E+01	.6011E+01	.3600E+02	.1000E+03
4	.1040E+01	.6011E+01	.3000E+02	.1200E+03
5	.1000E+01	.6011E+01	.3000E+02	.1000E+03
6	.1040E+01	.5783E+01	.3000E+02	.1000E+03
7	.1040E+01	.6011E+01	.2400E+02	.1000E+03
8	.1040E+01	.6011E+01	.3000E+02	.8000E+02
9	.1080E+01	.5783E+01	.3000E+02	.1000E+03
10	.1040E+01	.6239E+01	.2400E+02	.1000E+03
11	.1040E+01	.6011E+01	.3600E+02	.8000E+02
12	.1000E+01	.6011E+01	.3000E+02	.1200E+03
13	.1080E+01	.6011E+01	.2400E+02	.1000E+03
14	.1040E+01	.6239E+01	.3000E+02	.8000E+02

Dependent Variables

Case #	A(486)	A(489)	A(628)	A(637)	A(686)	A(739)
base	.6348E+04	.7306E+04	.1734E+04	.1348E+04	.2965E+02	.6122E+02
1	.6657E+04	.7644E+04	.1768E+04	.1416E+04	.3112E+02	.6188E+02
2	.6263E+04	.7407E+04	.1717E+04	.1490E+04	.2900E+02	.5994E+02
3	.6347E+04	.7308E+04	.1731E+04	.1349E+04	.2965E+02	.6122E+02
4	.6346E+04	.7288E+04	.1736E+04	.1338E+04	.2964E+02	.6123E+02
5	.6029E+04	.6963E+04	.1699E+04	.1315E+04	.2823E+02	.5999E+02
6	.6447E+04	.7205E+04	.1752E+04	.1218E+04	.3040E+02	.6258E+02
7	.6350E+04	.7305E+04	.1736E+04	.1346E+04	.2966E+02	.6121E+02
8	.6352E+04	.7324E+04	.1730E+04	.1362E+04	.2968E+02	.6120E+02
9	.6762E+04	.7530E+04	.1791E+04	.1275E+04	.3189E+02	.6328E+02
10	.6265E+04	.7406E+04	.1719E+04	.1488E+04	.2901E+02	.5993E+02
11	.6350E+04	.7325E+04	.1727E+04	.1363E+04	.2967E+02	.6121E+02
12	.6027E+04	.6950E+04	.1701E+04	.1307E+04	.2821E+02	.6000E+02
13	.6659E+04	.7642E+04	.1771E+04	.1415E+04	.3113E+02	.6187E+02
14	.6267E+04	.7425E+04	.1712E+04	.1508E+04	.2903E+02	.5992E+02

TABLE 6. Test Case Variable Values

Independent Variables
(various models)

Case #	A(1)	A(2)	A(4)	A(5)	A(578)	A(579)
1	.1090E+01	.6011E+01	.3000E+02	.1000E+03	.7001E+02	.7408E+01
2	.1061E+01	.6206E+01	.2686E+02	.8295E+02	.7765E+02	.1231E+02
3	.1006E+01	.6130E+01	.3512E+02	.8955E+02	.1185E+03	.1444E+02
4	.1019E+01	.5816E+01	.3313E+02	.1171E+03	.1339E+03	.8489E+01
5	.1074E+01	.5892E+01	.2488E+02	.1104E+03	.9045E+02	.6468E+01

PBM Predicted Dependent Variables

Case #	A(486)	A(489)	A(628)	A(637)	A(686)	A(739)
1	.6739E+04	.7733E+04	.1778E+04	.1438E+04	.3154E+02	.6197E+02
2	.6440E+04	.7590E+04	.1732E+04	.1525E+04	.2987E+02	.6050E+02
3	.6032E+04	.7070E+04	.1693E+04	.1395E+04	.2811E+02	.5950E+02
4	.6257E+04	.7031E+04	.1730E+04	.1217E+04	.2950E+02	.6173E+02
5	.6663E+04	.7524E+04	.1779E+04	.1321E+04	.3127E+02	.6254E+02

TABLE 7. Model 1 Influence Coefficients

MODEL 1 — 1st ORDER MODEL INFLUENCE COEFFICIENTS

IND VAR #	DEPENDENT VARIABLE NUMBER 486	489	628	637	686	739
1	.1441E+01	.1427E+01	.4591E+00	-.1942E+01	.9287E+00	.1493E+01
4	.2707E-03	-.1136E-02	-.5790E-02	-.1899E-02	.2290E-02	.1166E-02
5	-.7152E-02	-.7590E-02	.3853E-02	.1022E-01	-.1076E-01	-.8773E-02
578	.4226E-01	-.4429E-02	.1147E-01	-.5424E+00	.3207E-02	.1648E+00
579	-.2370E-01	.3969E-01	-.2483E-01	.1058E+00	-.6467E-01	.1775E-02

MODEL 1 — 2nd ORDER MODEL INFLUENCE COEFFICIENTS

IND VAR #		DEPENDENT VARIABLE NUMBER 486	489	628	637	686	739
1		.1681E+01	.1390E+01	-.4176E+01	.1523E+01	-.1188E+02	.1260E+00
4		-.8039E-03	-.1105E-02	.8067E-02	-.9778E-02	.3965E-01	.3956E-02
5		-.1433E-01	-.7005E-02	.5186E-01	-.9786E-02	.1148E+00	-.6868E-02
578		.1080E+00	-.8967E-02	-.7402E+00	-.3933E-01	-.2035E+01	.1009E-01
579		-.1406E-03	.3820E-01	-.3287E+00	.2873E+00	-.8902E+00	-.6090E-01
1	1	-.4463E+03	-.2953E+02	.3115E+04	-.1356E+04	.8009E+04	.4438E+02
1	4	.1696E+01	.1184E+00	-.1204E+02	.5231E+01	-.3096E+02	-.1971E+00
1	5	.3054E+01	.3560E+00	-.2478E+02	.1102E+02	-.6288E+02	-.7241E+00
1	578	-.9193E+02	-.6779E+01	.6507E+03	-.2841E+03	.1671E+04	.1062E+02
1	579	-.3655E+02	-.2736E+01	.2591E+03	-.1142E+03	.6652E+03	.4342E+01
4	4	-.1316E-02	.7458E-04	.9481E-02	-.4696E-02	.2553E-01	.1241E-03
4	5	-.5194E-02	-.6141E-03	.4471E-01	-.2116E-01	.1144E+00	.1520E-02
4	578	.1703E+00	.1330E-01	-.1223E+01	.5341E+00	-.3139E+01	-.2094E-01
4	579	.6783E-01	.5151E-02	-.4903E+00	.2130E+00	-.1256E+01	-.8577E-02
5	5	.2603E-01	-.2649E-02	-.1583E+00	.4017E-01	-.4133E+00	.1759E-02
5	578	.3094E+00	.4684E-01	-.2574E+01	.1144E+01	-.6498E+01	-.7970E-01
5	579	.1224E+00	.2201E-01	-.1016E+01	.4698E+00	-.2562E+01	-.3268E-01
578	578	-.4640E+01	-.3750E+00	.3326E+02	-.1442E+02	.8536E+02	.5782E+00
578	579	-.3674E+01	-.3201E+00	.2640E+02	-.1177E+02	.6765E+02	.4765E+00
579	579	-.7207E+00	-.6861E-01	.5267E+01	-.2352E+01	.1349E+02	.1048E+00

TABLE 8. Model 2 Influence Coefficients

MODEL 2 — 1st ORDER MODEL INFLUENCE COEFFICIENTS

IND VAR #		DEPENDENT VARIABLE NUMBER 486	489	628	637	686	739
1		.1280E+01	.1215E+01	.5287E+00	.1343E+01	.1303E+01	.2863E+00
2		-.3525E+00	.3633E+00	-.2570E+00	.2784E+01	-.5781E+00	-.5484E+00
4		-.1142E-02	.9921E-03	-.7153E-02	.5677E-02	-.1163E-02	.6696E-03
5		-.1827E-02	-.1248E-01	.7441E-02	-.3502E-01	-.3035E-02	.1021E-02

MODEL 2 — 2nd ORDER MODEL INFLUENCE COEFFICIENTS

IND VAR #		DEPENDENT VARIABLE NUMBER 486	489	628	637	686	739
1		.1302E+01	.1225E+01	.5286E+00	.9868E+00	.1284E+01	.4060E+00
2		-.3801E+00	.3650E+00	-.2701E+00	.2659E+01	-.6188E+00	-.5679E+00
4		-.1154E-02	.9613E-03	-.7124E-02	.5677E-02	-.1399E-02	.6738E-03
5		-.2260E-02	-.1232E-01	.9114E-02	-.4373E-01	-.3819E-02	.1233E-02
1	1	-.5919E+00	-.2707E+00	.3953E-02	.9371E+01	.5173E+00	-.3150E+01
1	2	-.7626E+00	.1140E+01	-.1758E+01	.5864E+01	-.6469E+00	-.4039E+00
1	4	.6175E-03	.3605E-02	.1510E-02	.1172E-01	.4443E-02	-.1074E-02
1	5	-.9321E-02	-.9383E-01	.3035E-01	-.1435E+00	-.1154E-01	.4731E-02
2	2	.7261E+00	-.4644E-01	.3436E+00	.3300E+01	.1070E+01	.5120E+00
2	4	.2692E-02	.8680E-05	-.1529E-02	.4589E-01	.9319E-02	-.8612E-03
2	5	-.8502E-02	.2709E-02	.3264E-01	-.3525E+00	-.1509E-01	.6019E-02
4	4	.5865E-04	.1537E-03	-.1444E-03	-.9834E-18	.1181E-02	-.2103E-04
4	5	.2750E-03	.2389E-03	-.4313E-03	-.1112E-02	.5902E-03	.2041E-03
5	5	.2166E-02	-.8045E-03	-.8365E-02	.4350E-01	.3920E-02	-.1062E-02

TABLE 9. Model 3 Influence Coefficients

MODEL 3 — 1st ORDER MODEL INFLUENCE COEFFICIENTS

IND VAR #	DEPENDENT VARIABLE NUMBER 10-486	10-489	628	637	686	739
10-4	.1199E+01	.6359E+00	.7235E+00	-.3832E+00	.9078E+00	.3909E+00
10-5	.3676E+00	.7055E+00	-.1946E+00	.9734E+00	.2983E+00	.1042E+00
2	-.6944E+00	.6356E+00	-.2570E+00	.2785E+01	-.5772E+00	-.5485E+00

MODEL 3 — 2nd ORDER MODEL INFLUENCE COEFFICIENTS

IND VAR #		DEPENDENT VARIABLE NUMBER 10-486	10-489	628	637	686	739
10-4		.1116E+01	.4896E+00	.8123E+00	-.6107E+00	.8803E+00	.3147E+00
10-5		.4100E+00	.8375E+00	-.2860E+00	.1551E+01	.3456E+00	.6658E-01
2		-.7488E+00	.6384E+00	-.2701E+00	.2659E+01	-.6182E+00	-.5679E+00
10-4	10-4	-.6906E+00	-.2528E+01	-.1278E-01	-.1749E+00	.7372E+00	-.1394E+01
10-4	10-5	-.1335E+01	.1171E+01	.1955E+01	.2009E+01	-.1607E+01	-.1297E+01
10-4	2	.3149E+01	-.2701E+00	-.4169E+01	.3829E+02	.2502E+01	-.3502E+00
10-5	10-5	.1018E+01	.9993E+00	-.2000E+01	.6890E+01	.1357E+01	-.1486E+00
10-5	2	-.4082E+01	.1778E+01	.2292E+01	-.2935E+02	-.2852E+01	-.7636E-01
2	2	.1432E+01	-.7286E-01	.3436E+00	.3307E+01	.1081E+01	.5114E+00

TABLE 10. Model Performance Summary for Five Test Cases

1st Order Model Results

Variable A-loc	Variable Name	Model 1 Case 12345	Model 2 Case 12345	Model 3 Case 12345
486	P2FP2	AAAAA	AAAAA	AAAAA
489	P2OP3	AAAAA	AAAAA	AAAAA
628	T2FT2	AAAAA	AAAAA	AAAAA
637	T2OT2	AAMUA	AMUUA	UUAUU
686	WFT1	AAAAA	AAAAA	AAAAA
739	WTCJBY	AAAAA	AAMAA	UAAAM

2nd Order Model Results

Variable A-loc	Variable Name	Model 1 Case 12345	Model 2 Case 12345	Model 3 Case 12345
486	P2FP2	AAAAA	AAAAA	AAAAA
489	P2OP3	AAAAA	AAAAA	AAAAA
628	T2FT2	UUUUM	AAAAA	AAAAA
637	T2OT2	UUMMA	AAAAA	AMMMM
686	WFT1	UUUUU	AAAAA	AAAAA
739	WTCJBY	AAAAA	AAAAA	AAAAA

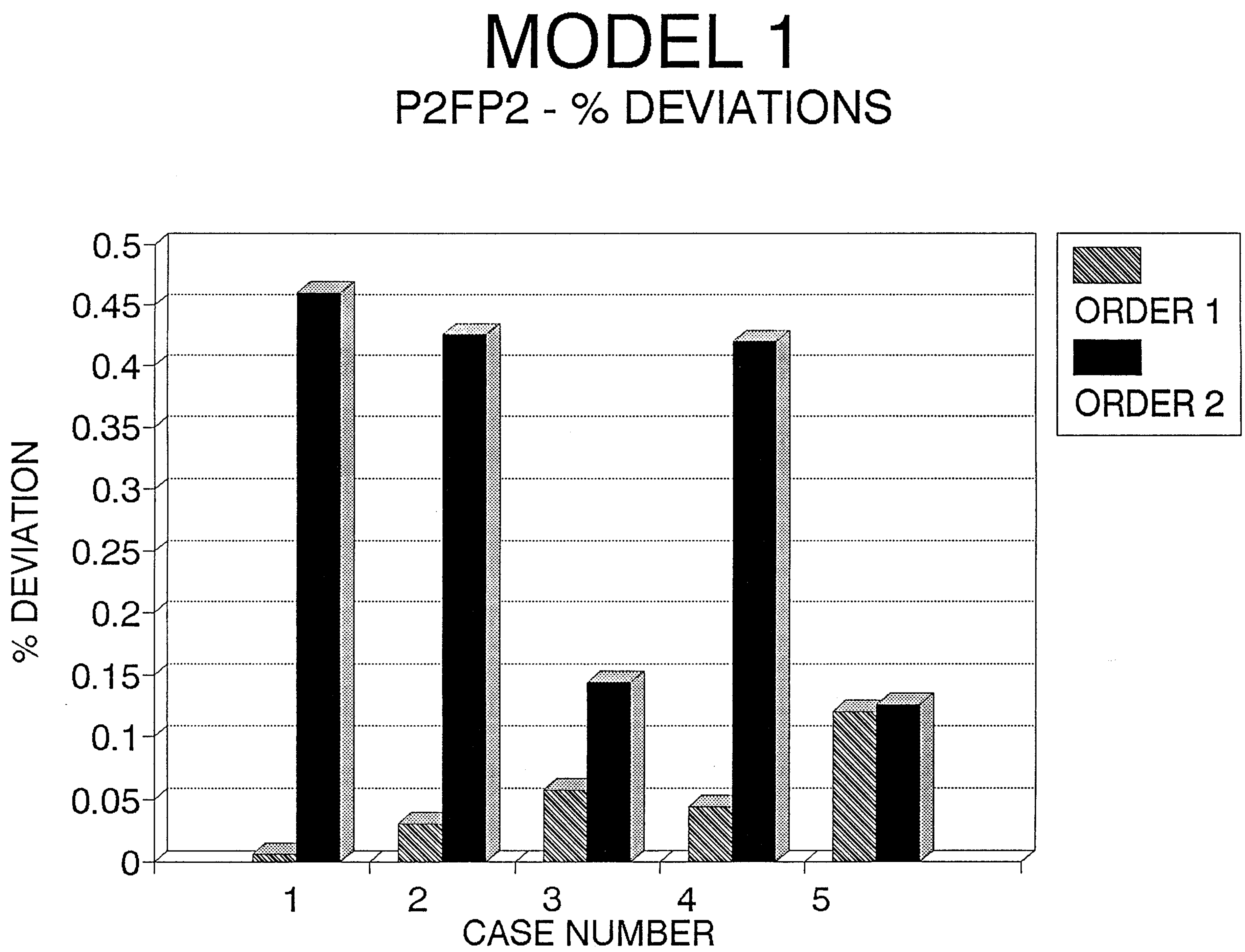
A - Acceptable (% deviation from PBM prediction < 0.5%)

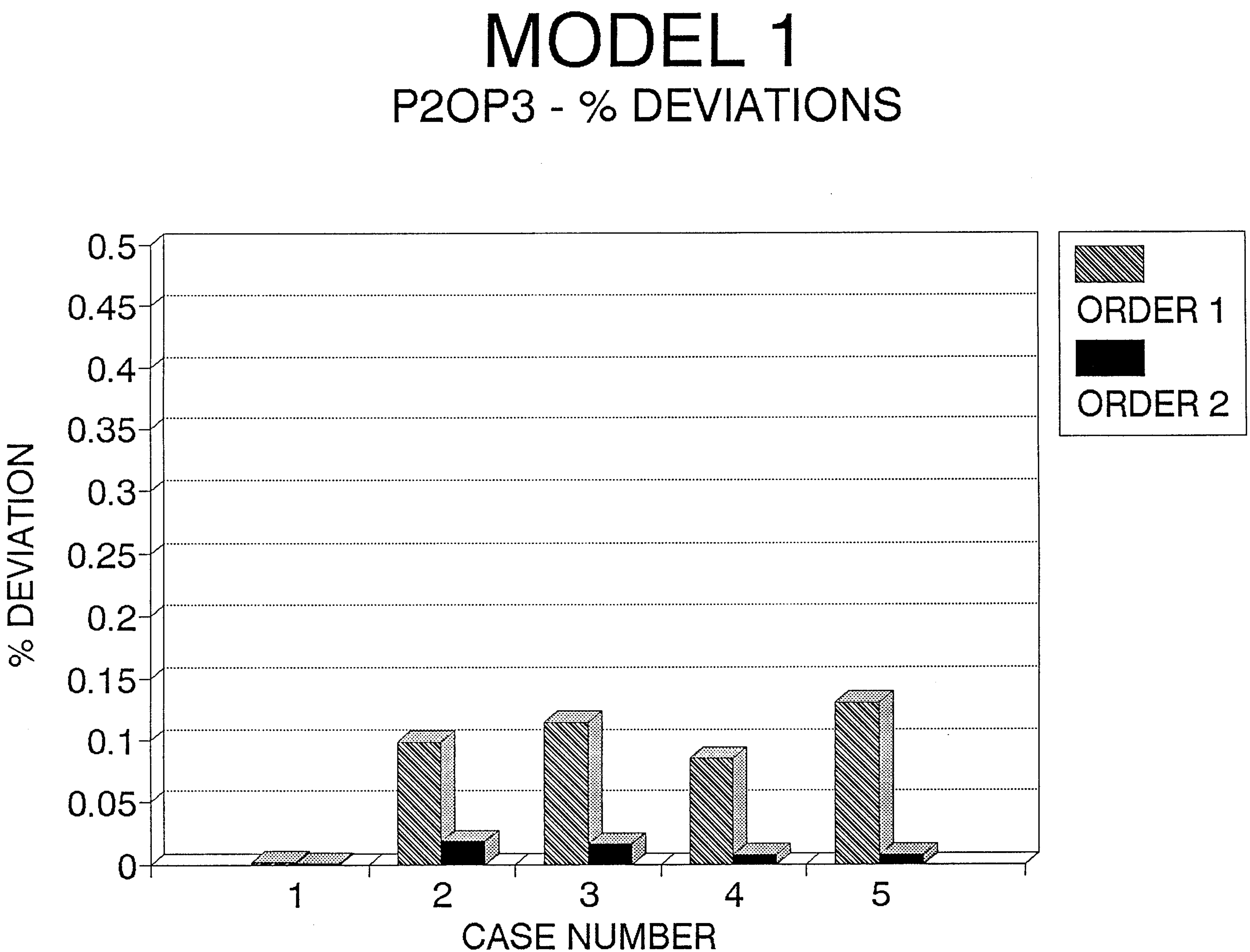
M - Marginal (0.5% <= % deviation from PBM prediction <= 1.0%)

U - Unacceptable (% deviation from PBM prediction > 1.0%)

APPENDIX B1

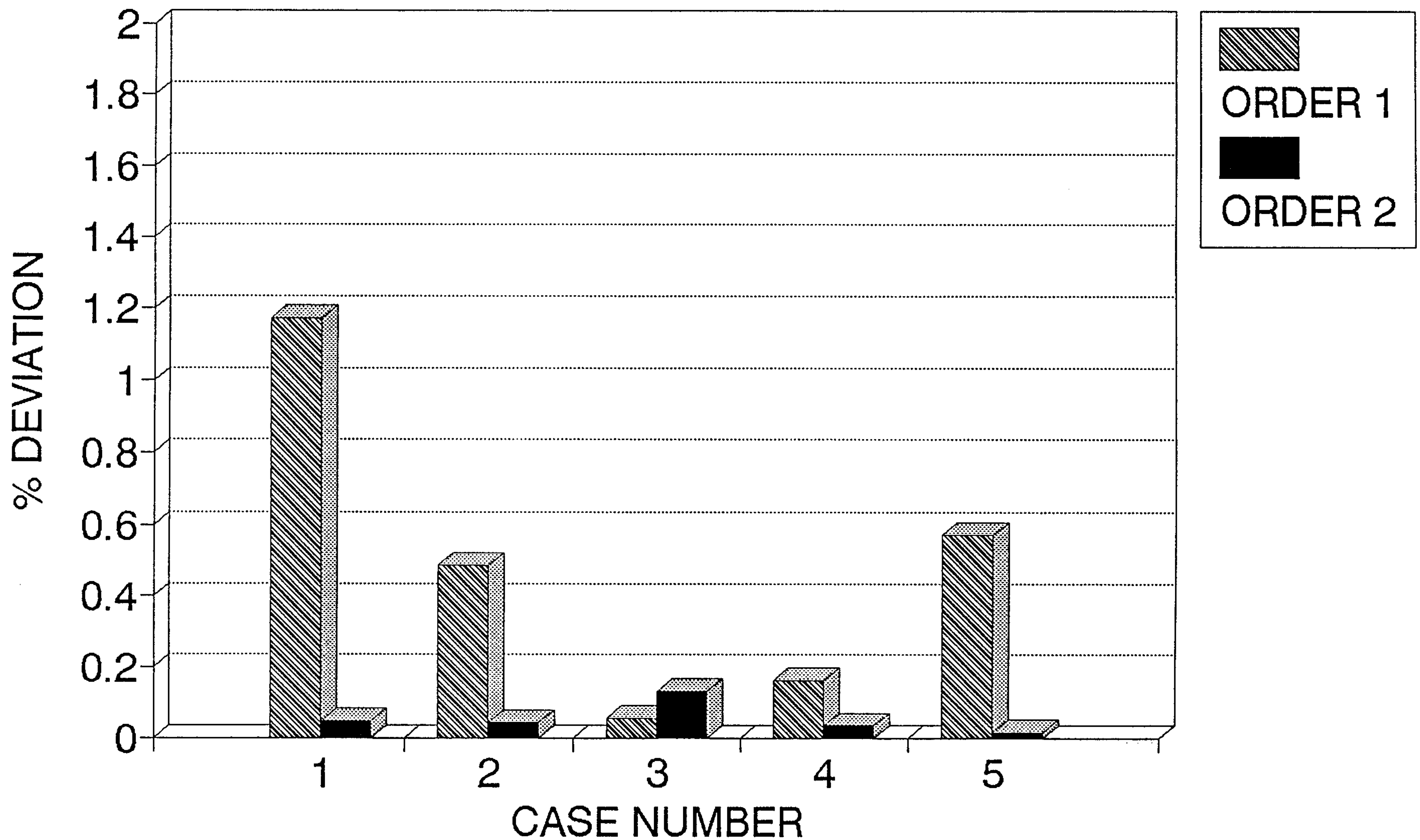
MODEL 1 DEVIATIONS


MODEL 1
P2FP2 - % DEVIATIONS
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
% DEVIATION
1
2
3
4
5
CASE NUMBER
ORDER 1
ORDER 2


MODEL 1
P2OP3 - % DEVIATIONS
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
% DEVIATION
1
2
3
4
5
CASE NUMBER
ORDER 1
ORDER 2

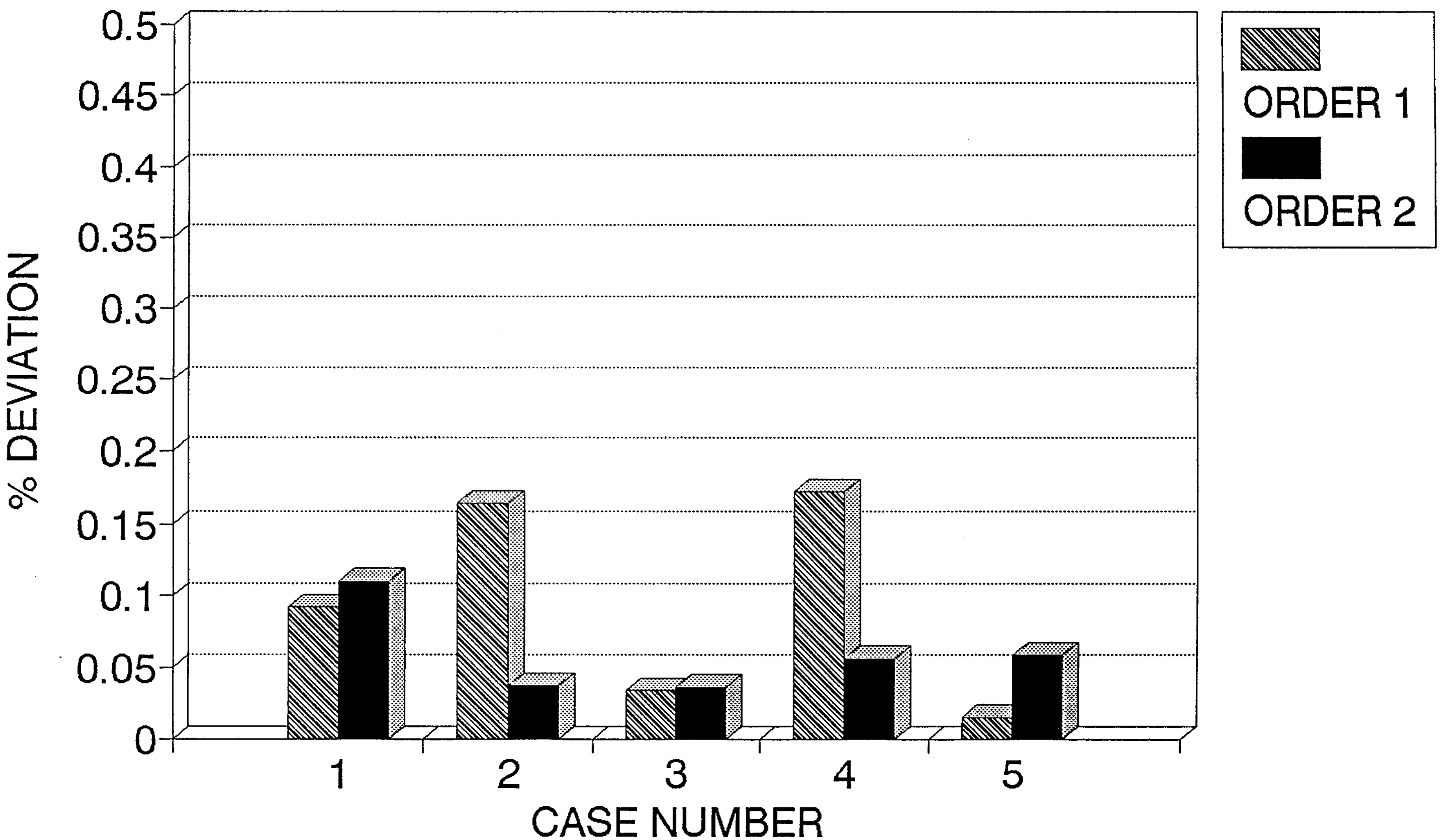
MODEL 3

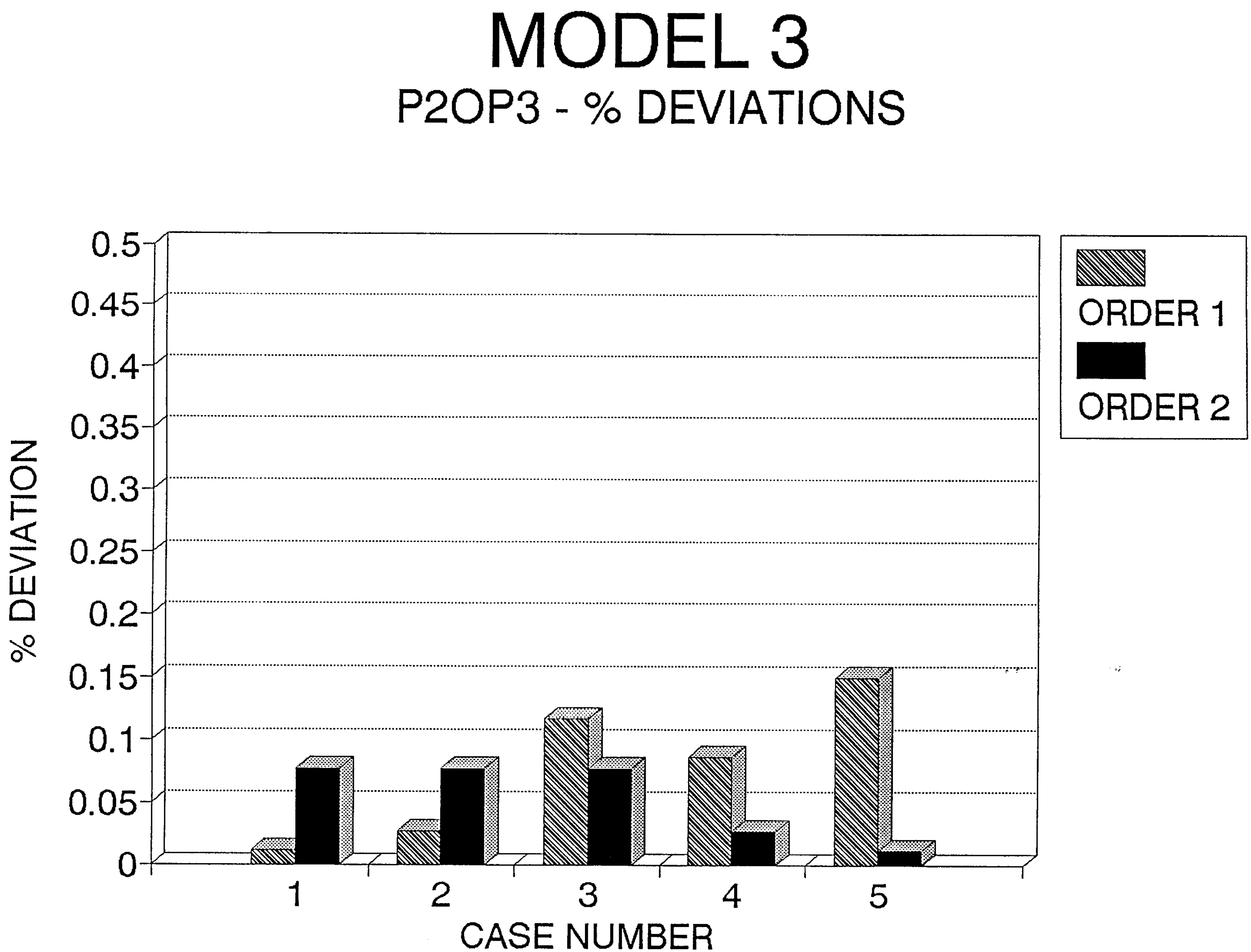
WTCJBY - % DEVIATIONS

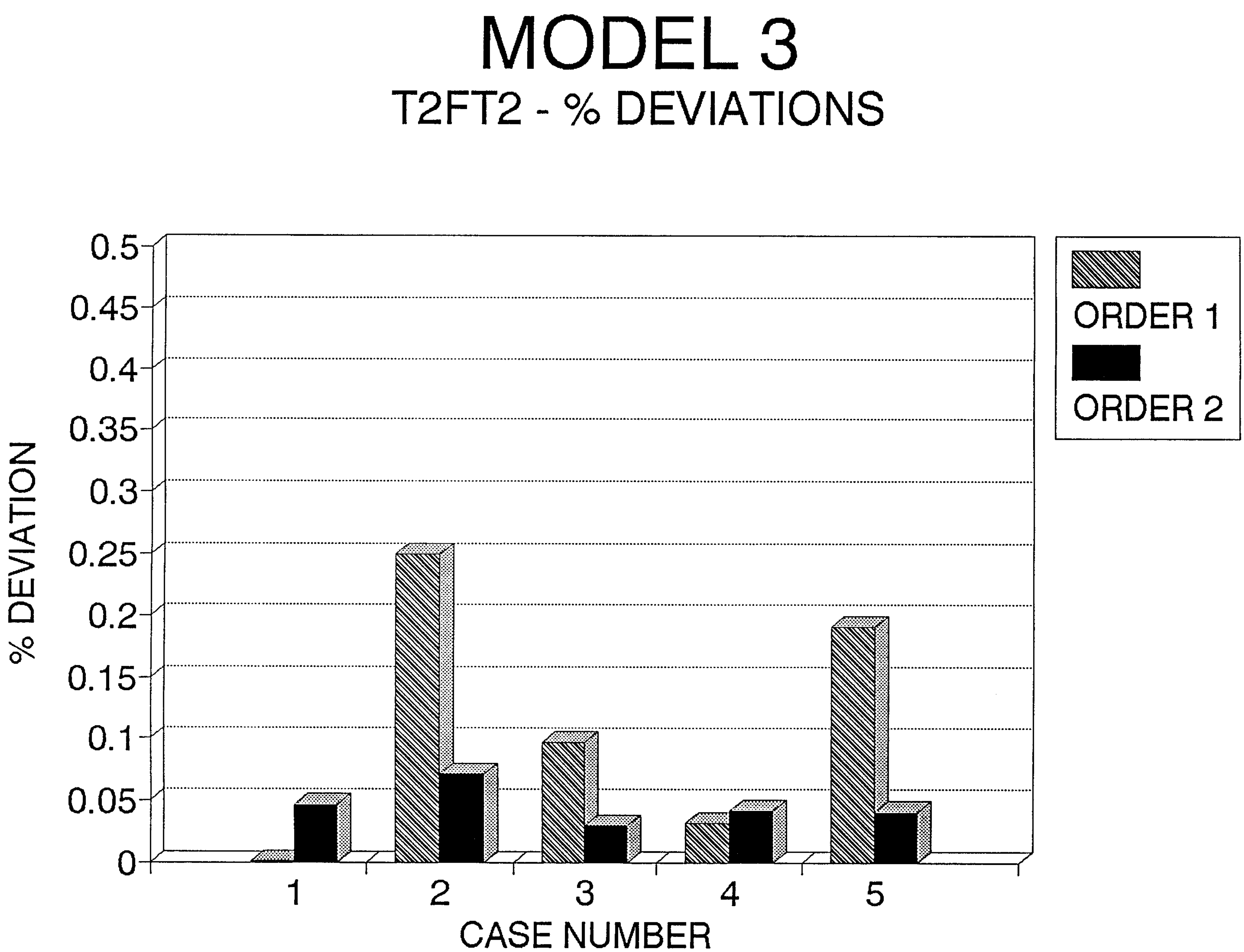


MODEL 3

P2FP2 - % DEVIATIONS

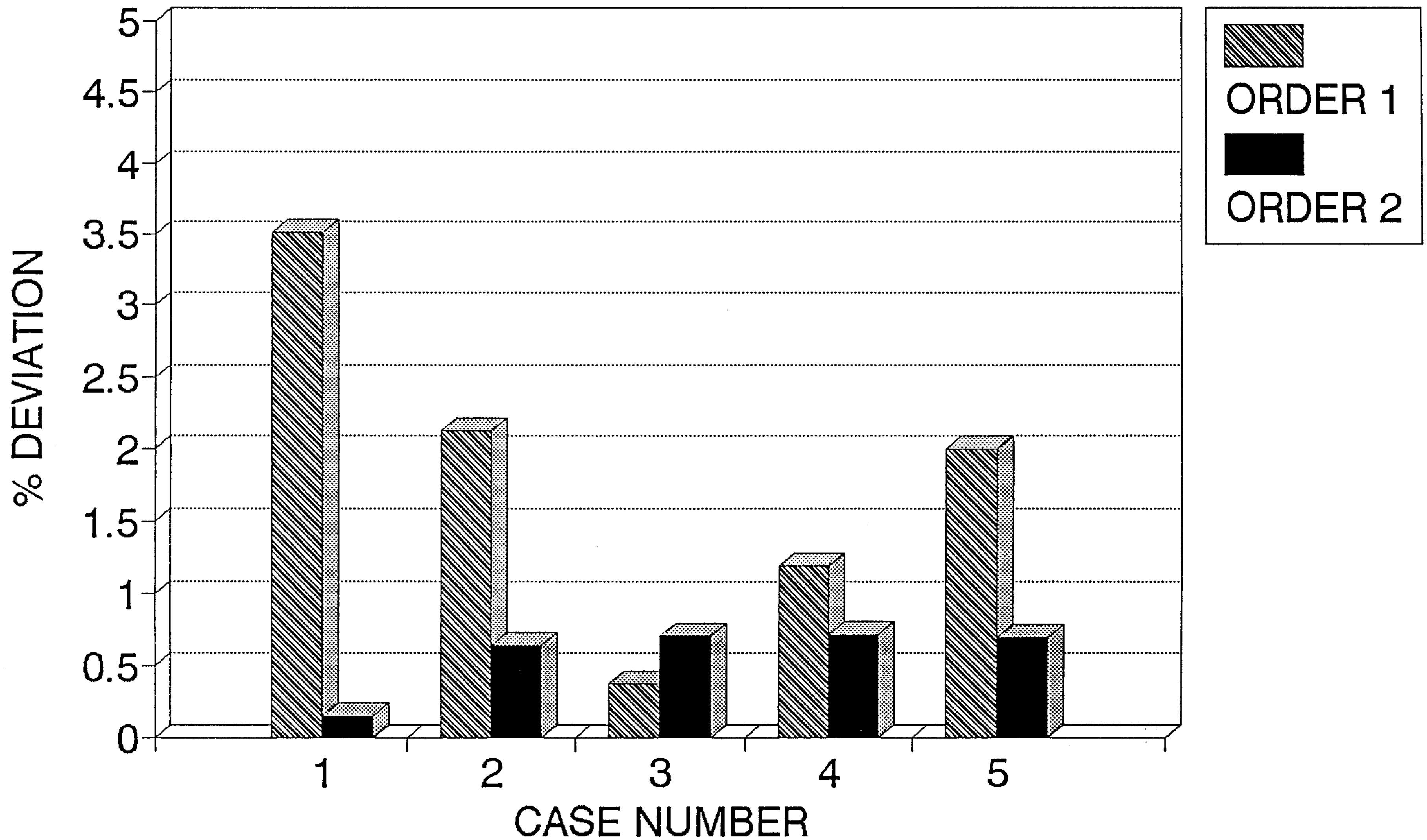



MODEL 3
P2OP3 - % DEVIATIONS
% DEVIATION
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5
CASE NUMBER
ORDER 1
ORDER 2


MODEL 3
T2FT2 - % DEVIATIONS
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
% DEVIATION
1
2
3
4
5
CASE NUMBER
ORDER 1
ORDER 2

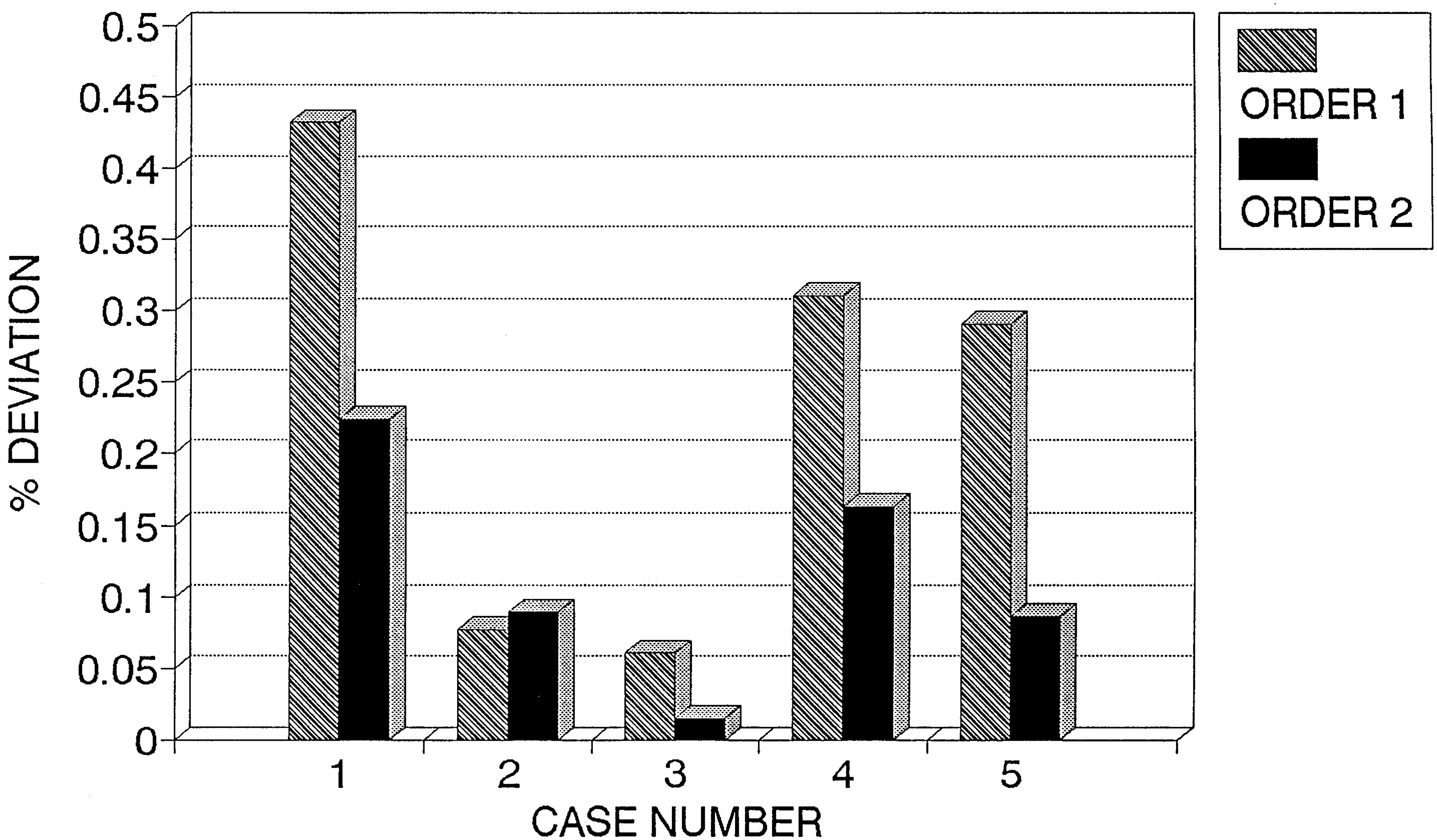
MODEL 3

T2OT2 - % DEVIATIONS



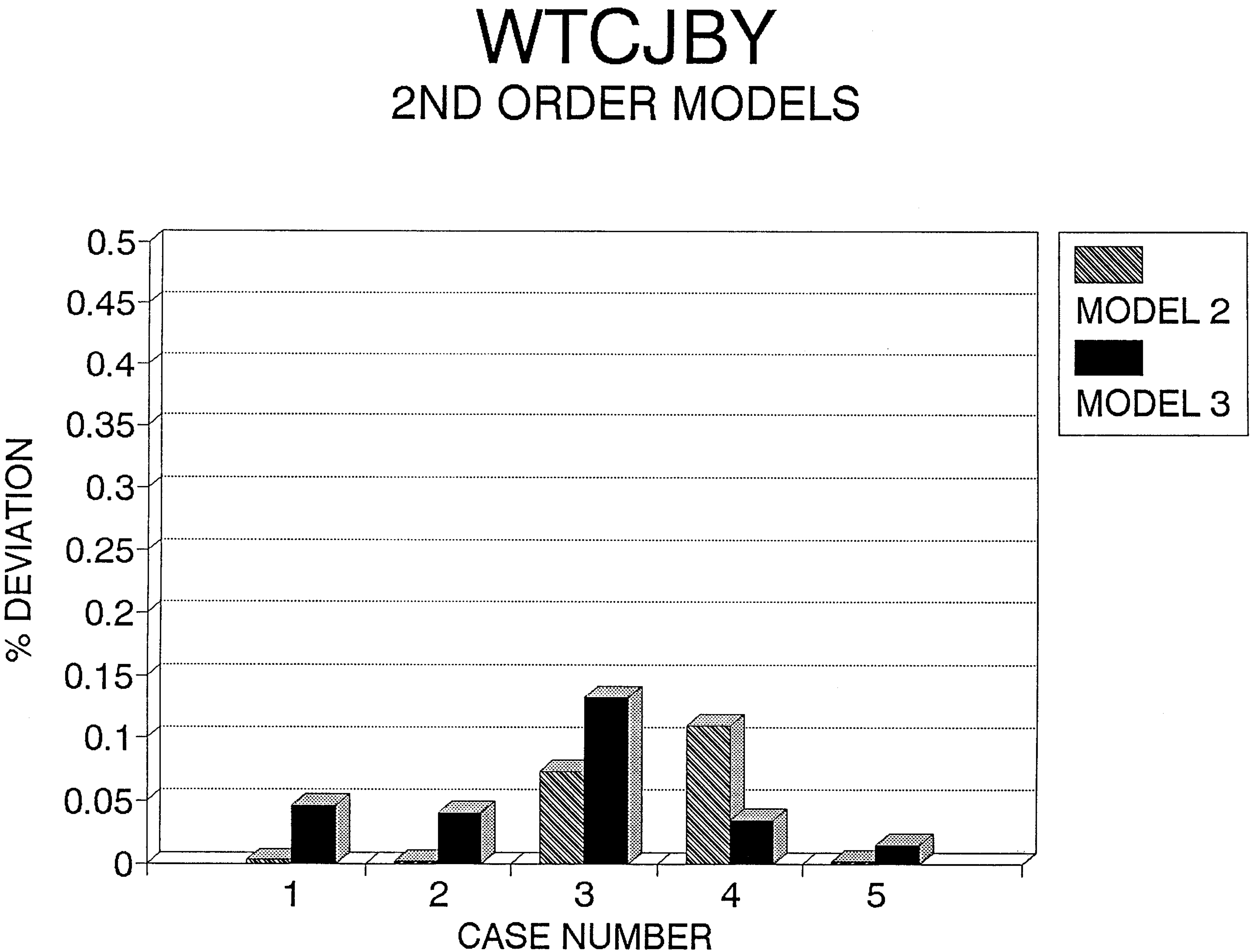
MODEL 3

WFT1 - % DEVIATIONS



APPENDIX B4

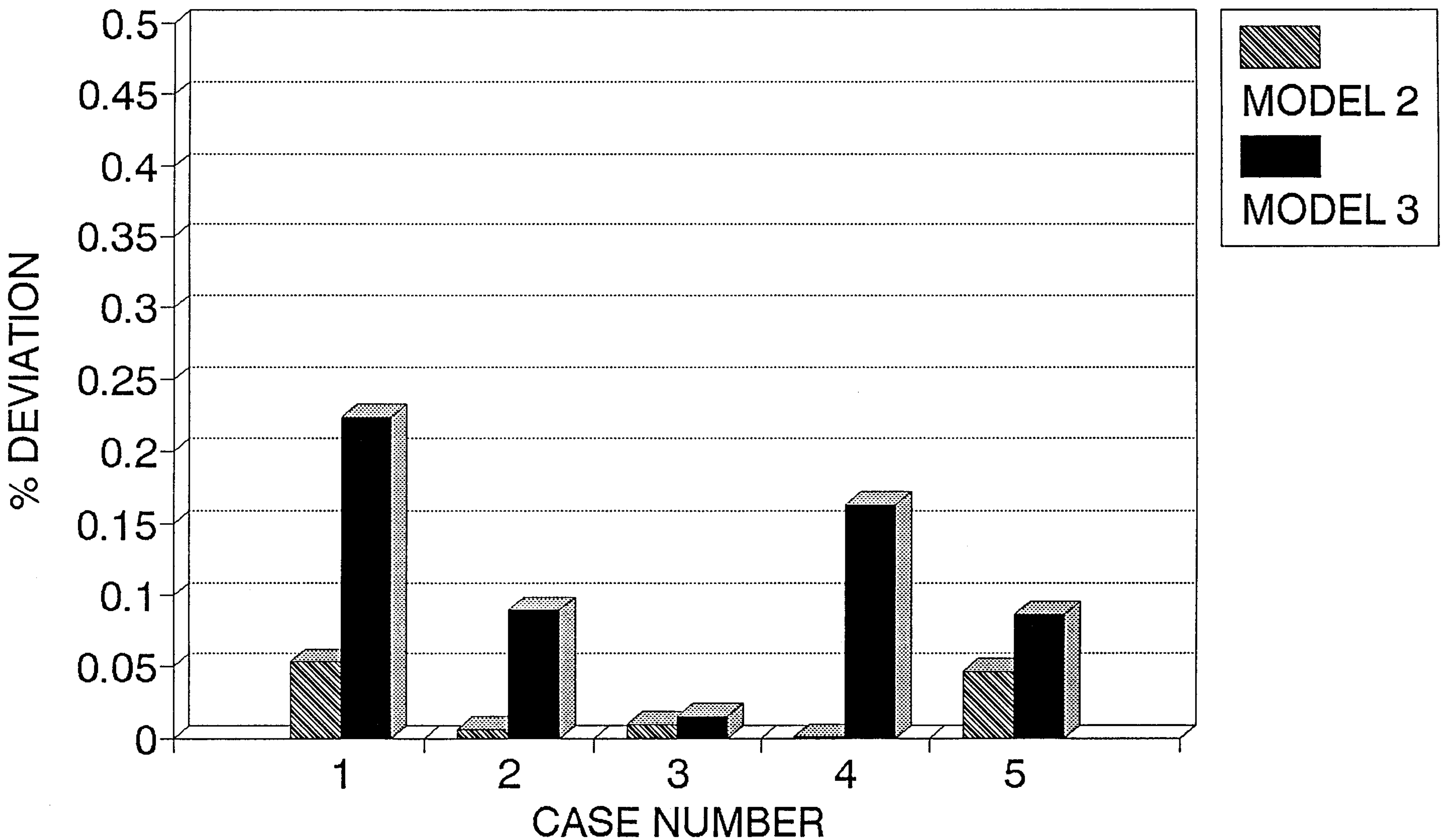
SECOND ORDER MODEL COMPARISONS

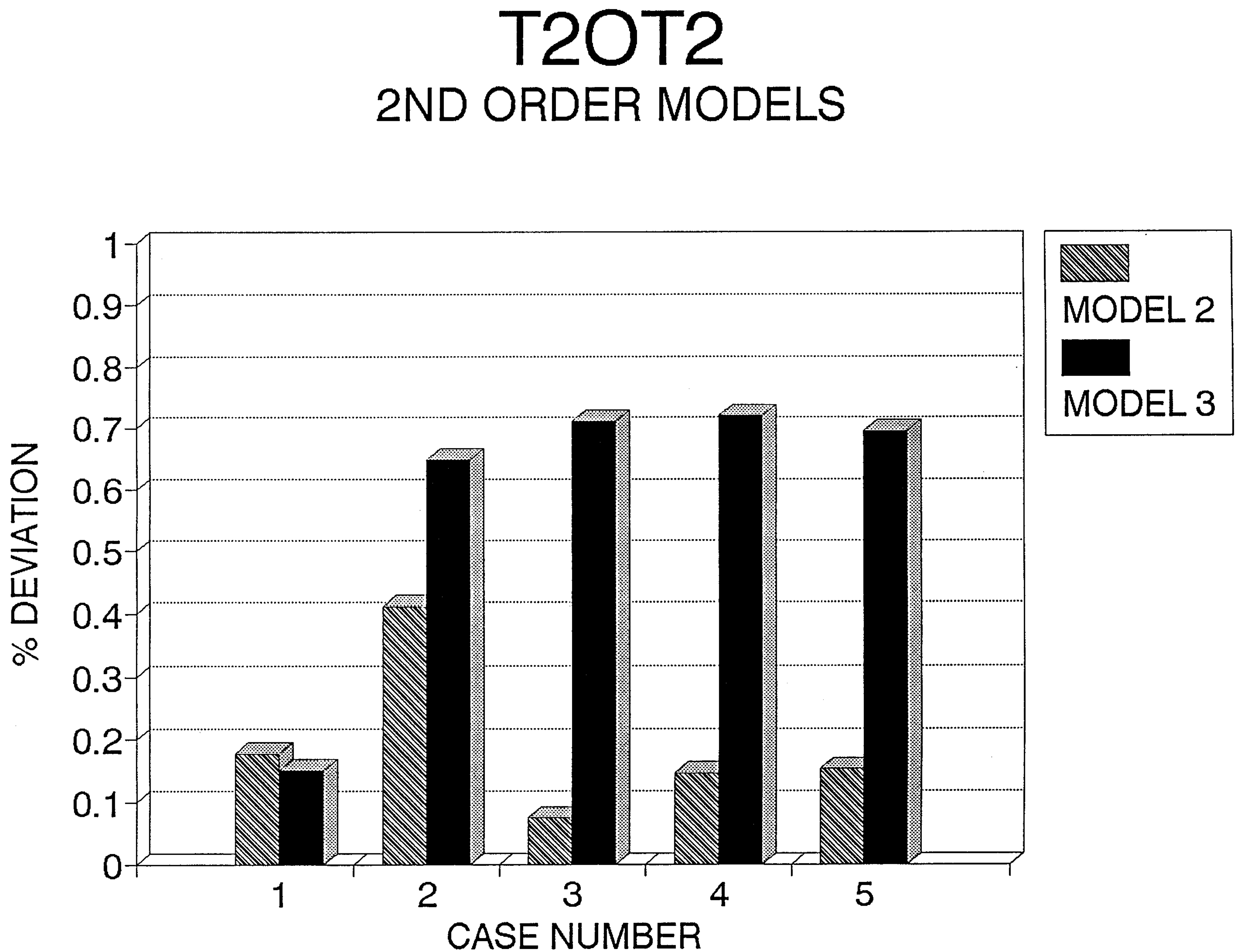


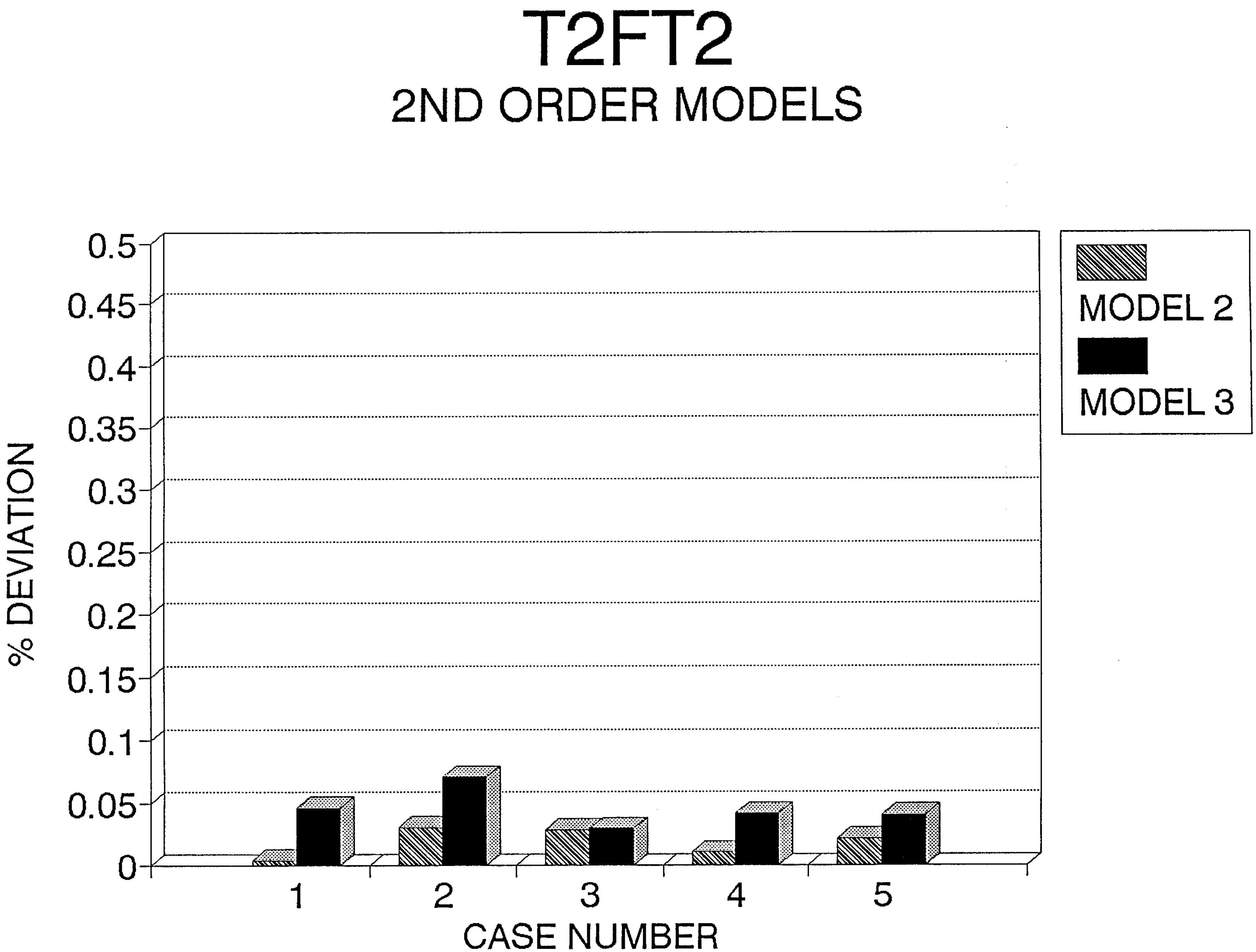
WTCJBY
2ND ORDER MODELS
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
% DEVIATION
1
2
3
4
5
CASE NUMBER
MODEL 2
MODEL 3

WFT1

2ND ORDER MODELS

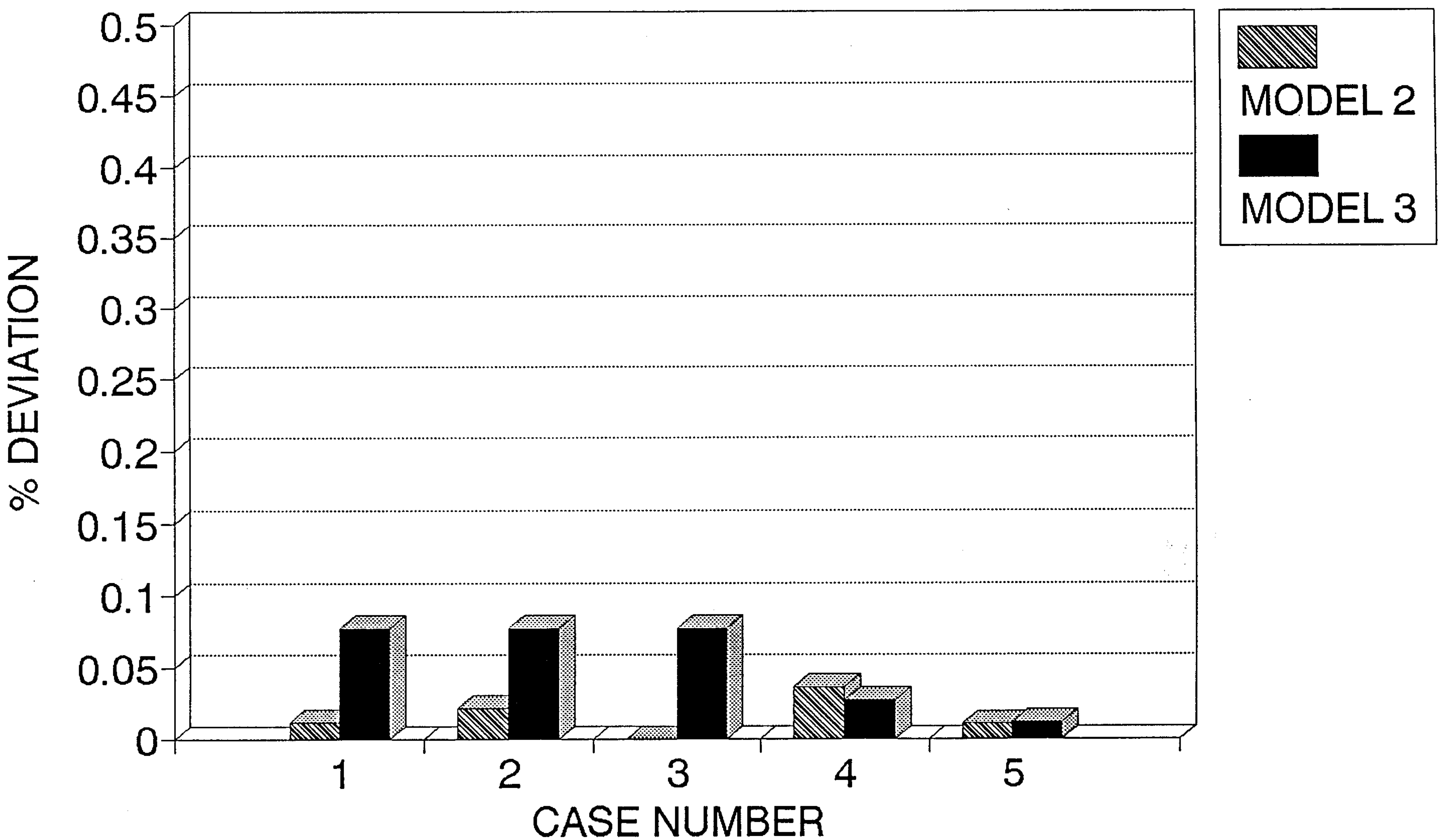


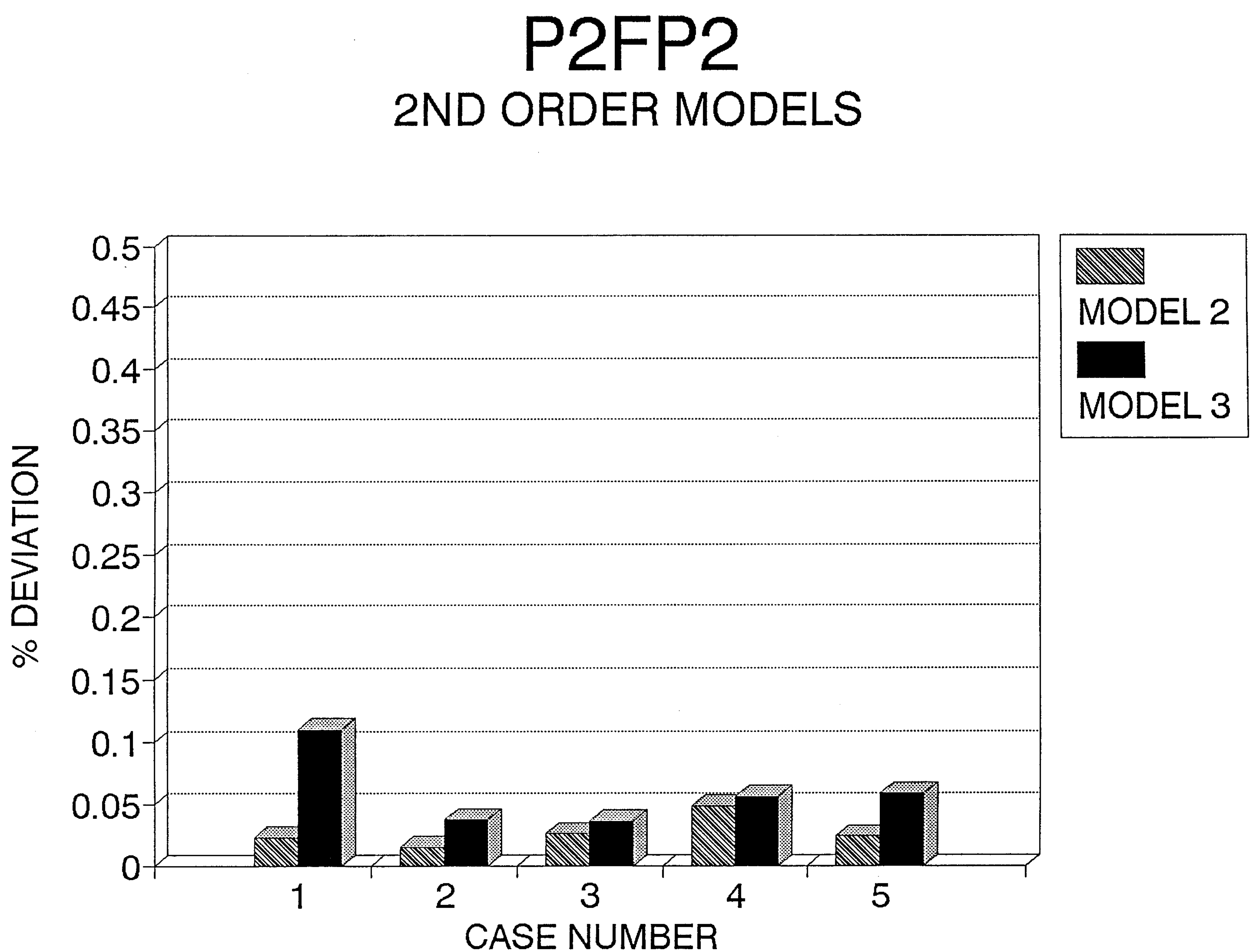

T2OT2
2ND ORDER MODELS
% DEVIATION
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
CASE NUMBER
MODEL 2
MODEL 3


T2FT2
2ND ORDER MODELS
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
% DEVIATION
1
2
3
4
5
CASE NUMBER
MODEL 2
MODEL 3

P2OP3

2ND ORDER MODELS

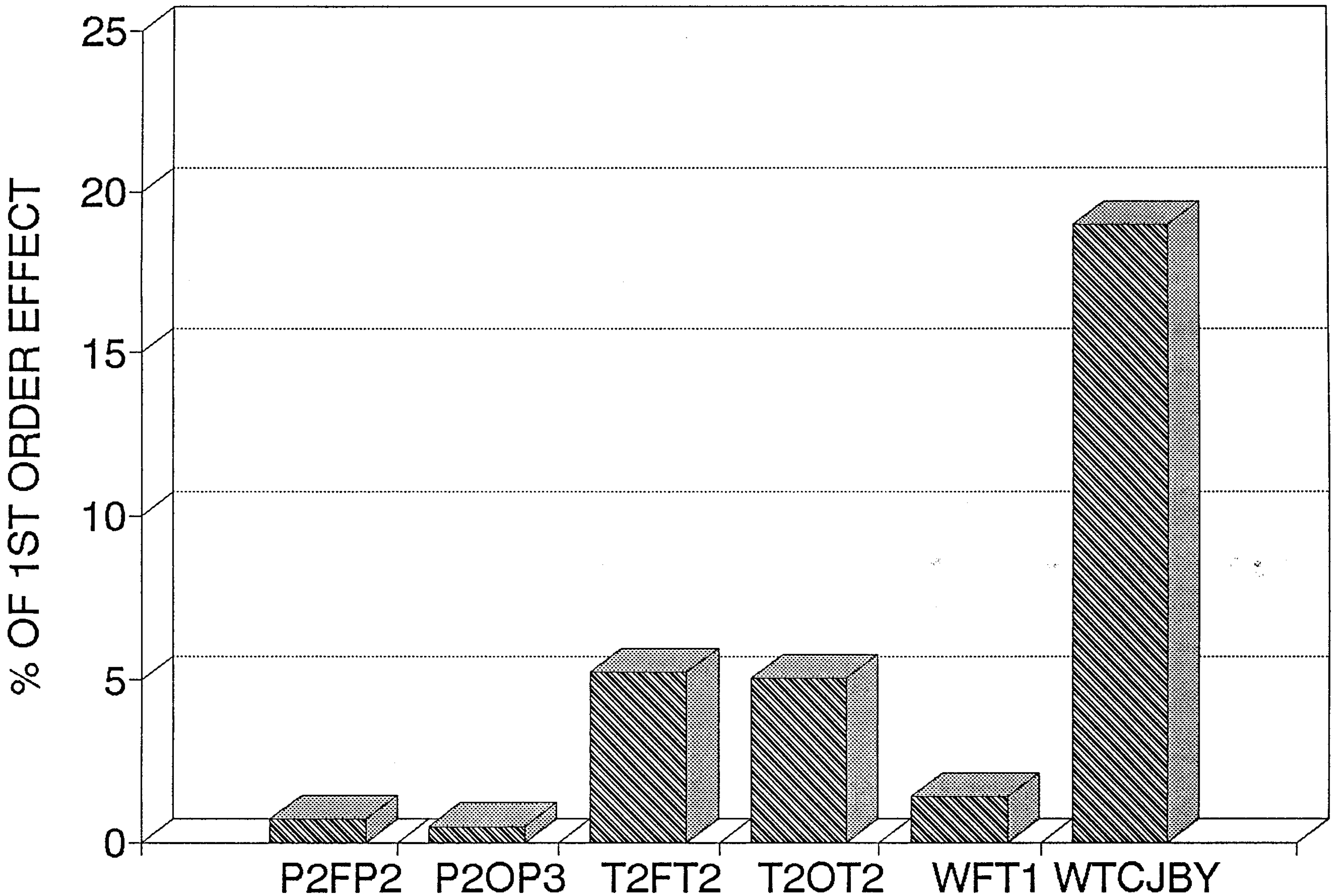



P2FP2
2ND ORDER MODELS
% DEVIATION
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5
CASE NUMBER
MODEL 2
MODEL 3

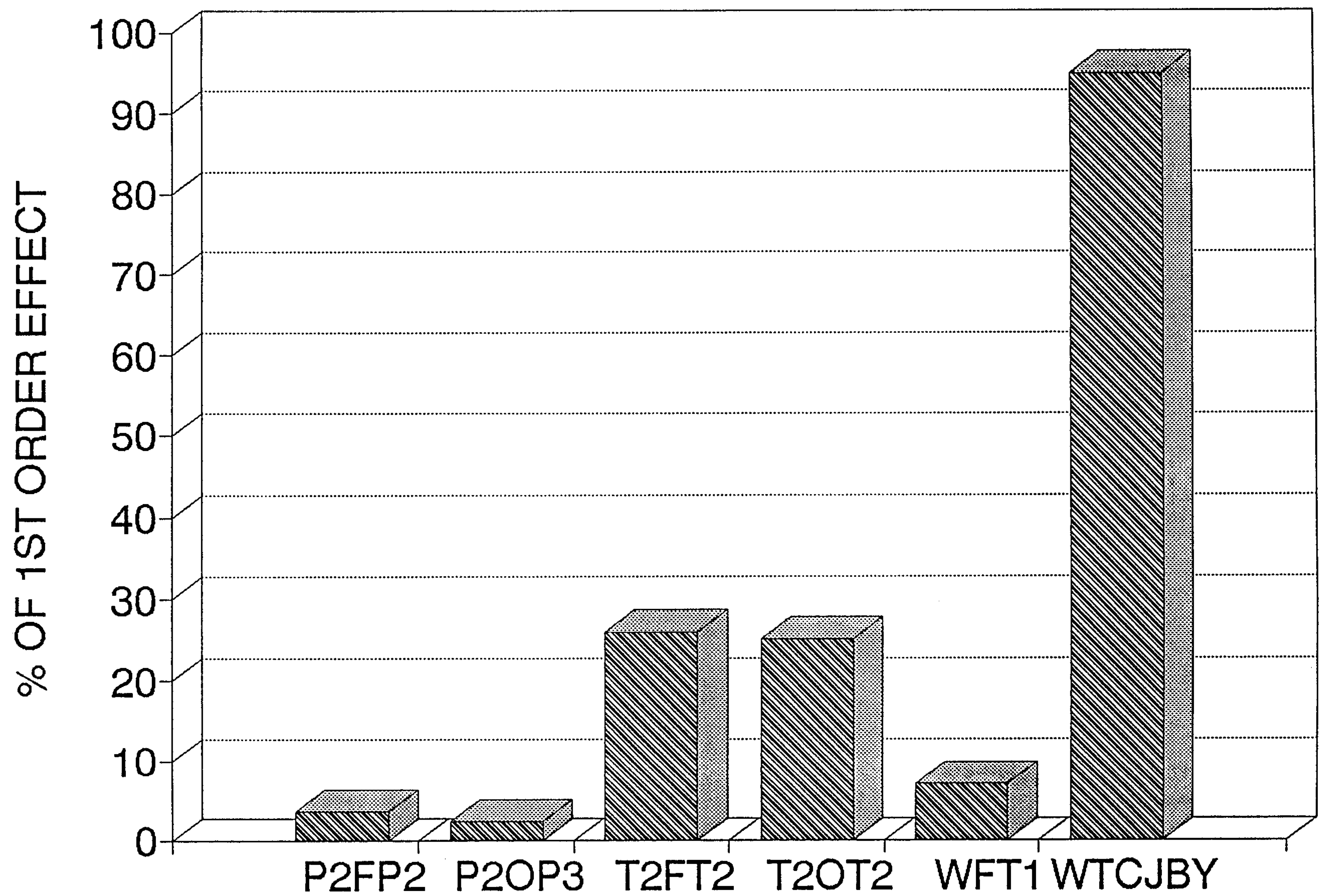
APPENDIX B5

SECOND ORDER CONTRIBUTIONS
WITH
UNIFORM VARIATIONS

1% UNIFORM VARIATIONS
2ND ORDER CONTRIBUTIONS



5% UNIFORM VARIATIONS
2ND ORDER CONTRIBUTIONS



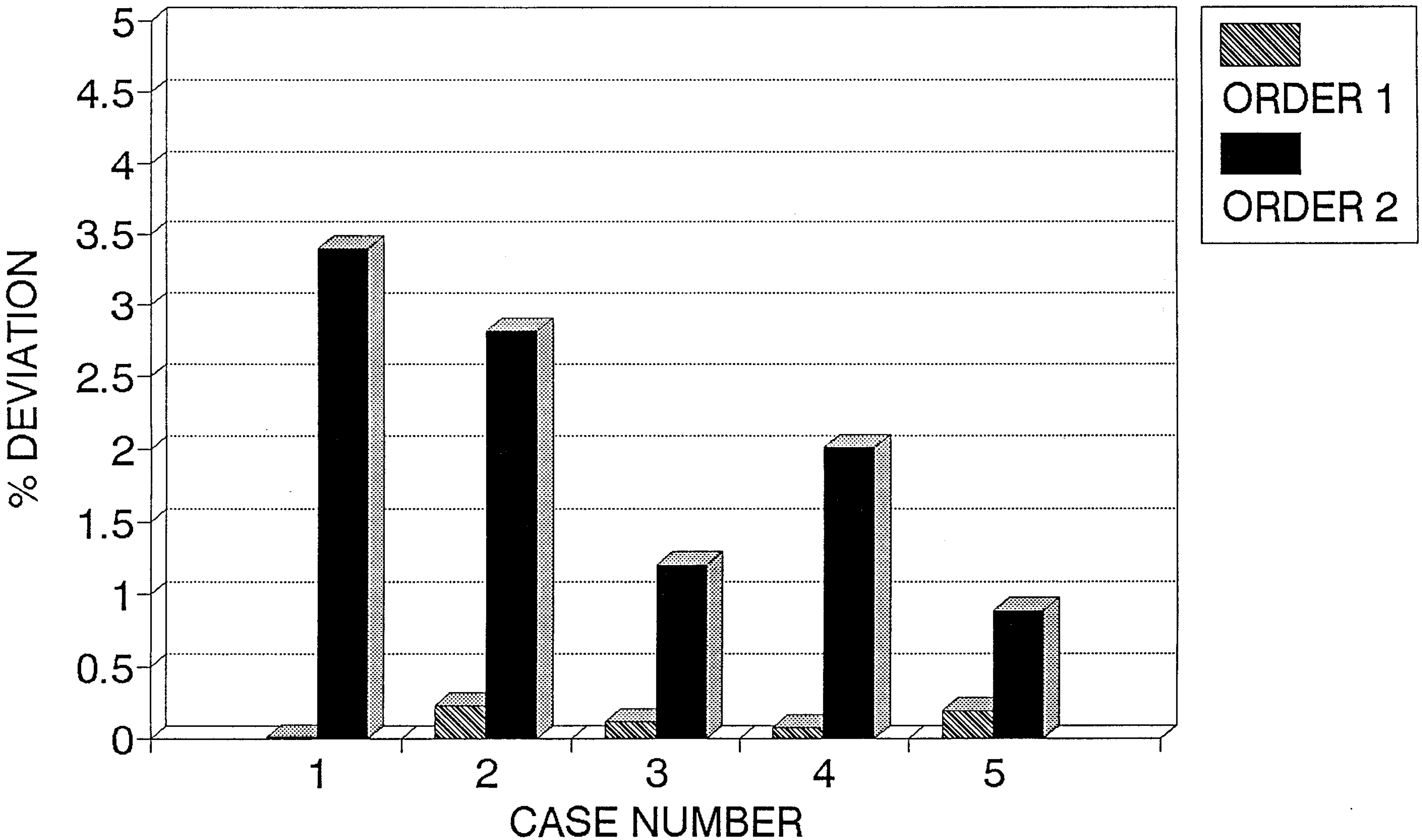
APPENDIX C1

ROUTINE EXTRACT

```
C  PROGRAM  EXTRACT
C
C*********************************************************************
C  INPUT
C
C    DATA FILE      VARIABLE          DEFINITION
C
C    EXTIO.DAT
C                   INPUT1     IDENTIFICATION OF INPUT FILE # 1
C                                   (CHARACTER*24 VARIABLE)
C                   INPUT2     IDENTIFICATION OF INPUT FILE # 2
C                                   (CHARACTER*24 VARIABLE)
C                   OUTPUT     IDENTIFICATION OF OUTPUT FILE
C                                   (CHARACTER*24 VARIABLE)
C
C    INPUT1
C                   IMODEL     IDENTIFICATION OF MODEL NUMBER
C                   NIVAR      NUMBER OF MODEL INDEPENDENT VARIABLES
C                   NDVAR      NUMBER OF MODEL DEPENDENT VARIABLES
C                   IIVAR(I)   A-ARRAY POSITION OF INDEPENDENT VARIABLE I
C                                   (I = 1-NIVAR)
C                   IDVAR(I)   A-ARRAY POSITION OF DEPENDENT VARIABLE I
C                                   (I = 1-NDVAR)
C
C    INPUT2
C                   A(I)       TEST DATA STORED IN POSITION I
C                                   (I = 1-1350, NOTE: A(I), I=1-1350
C                                    IS SEQUENTIALLY READ NRUNP1 TIMES
C                                    WHERE NRUNP1=1+NIVAR+((NIVAR+1)
C                                                 *NIVAR)/2,
C                                    FIRST A-ARRAY IS BASE CASE ARRAY,
C                                    SEBSEQUENT A-ARRAY'S ARE MODEL
C                                    DEFINITION CASES,
C                                    INPUT FORMAT FOR EACH A-ARRAY READ
C                                    IS (270(/8X,5E13.6)) )
C
C *********************************************************************
C  OUTPUT
C
C    DATA FILE      VARIABLE          DEFINITION
C
C    OUTPUT         IMODEL     SAME AS INPUT
C                   NIVAR      SAME AS INPUT
C                   NDVAR      SAME AS INPUT
C                   IIVAR(I)   SAME AS INPUT
C                   IDVAR(I)   SAME AS INPUT
C                   XI(I,J)    EXTRACTED VALUE OF INDEPENDENT VARIABLE
C                              J FOR RUN I
C                              (BASE CASE I=1, DEFINITION CASES I>1)
C                   XD(I,J)    EXTRACTED VALUE OF DEPENDENT VARIABLE J
C                              FOR RUN I
C                              (BASE CASE I=1, DEFINITION CASES I>1)
C
C *********************************************************************
```

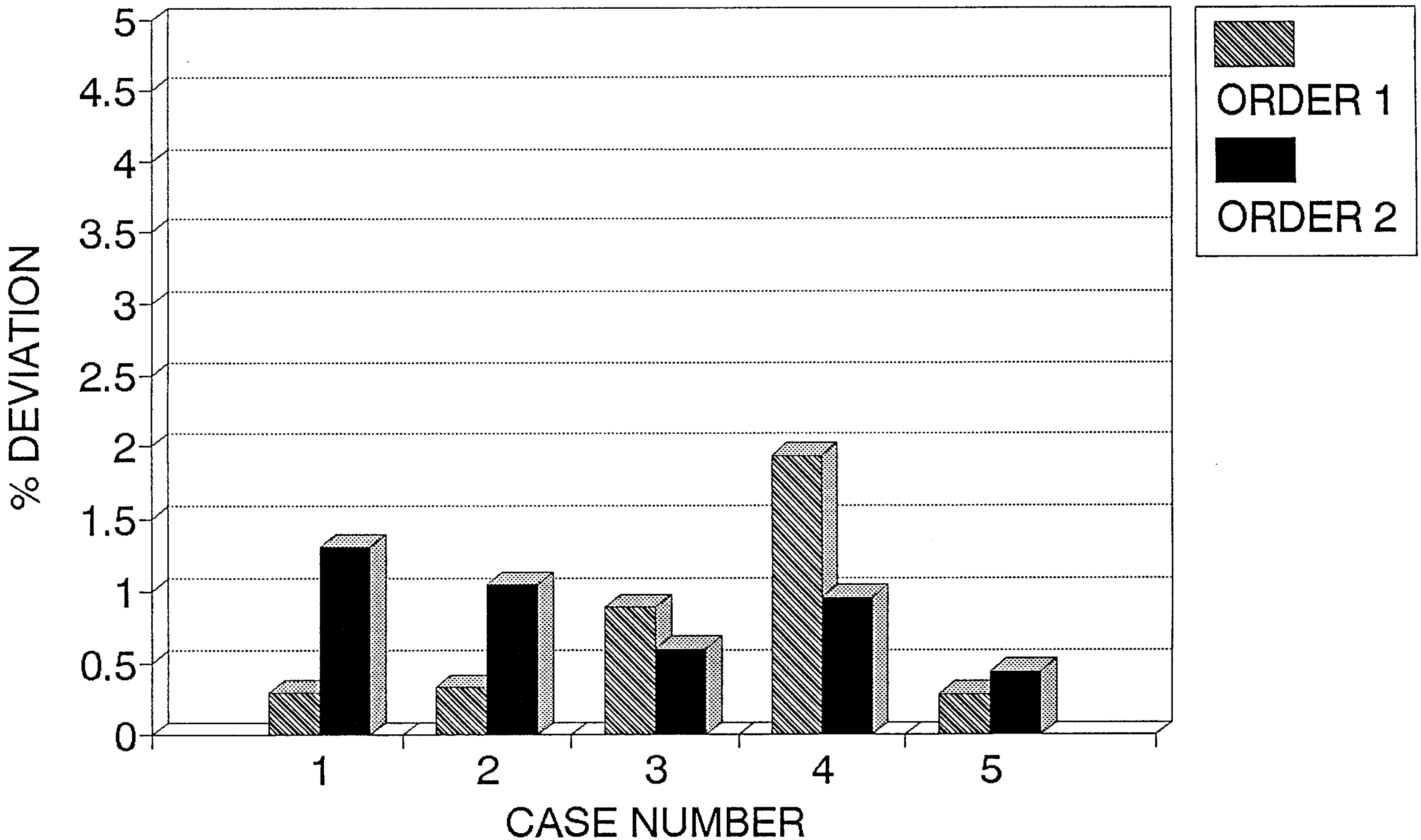
MODEL 1

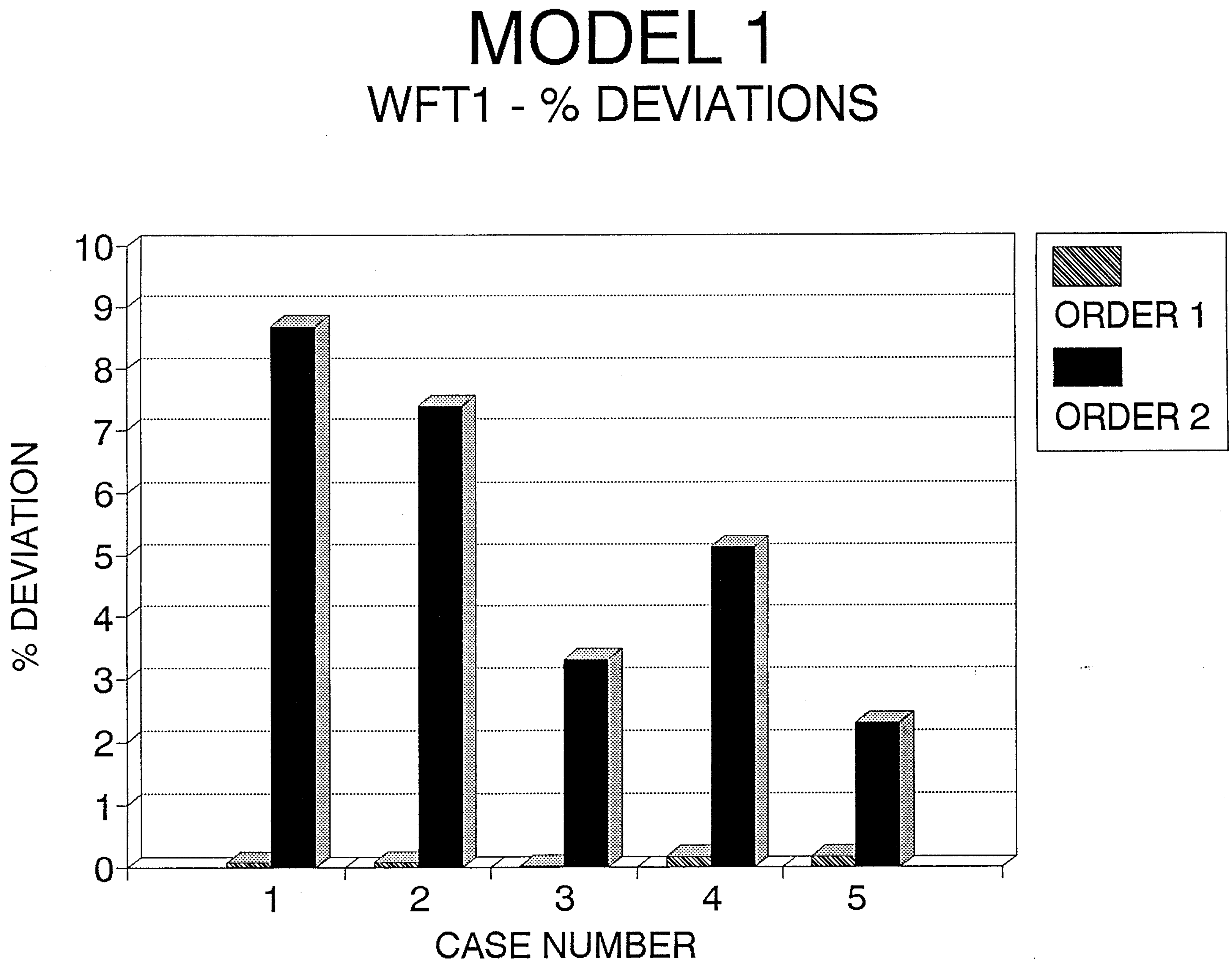
T2FT2 - % DEVIATIONS



MODEL 1

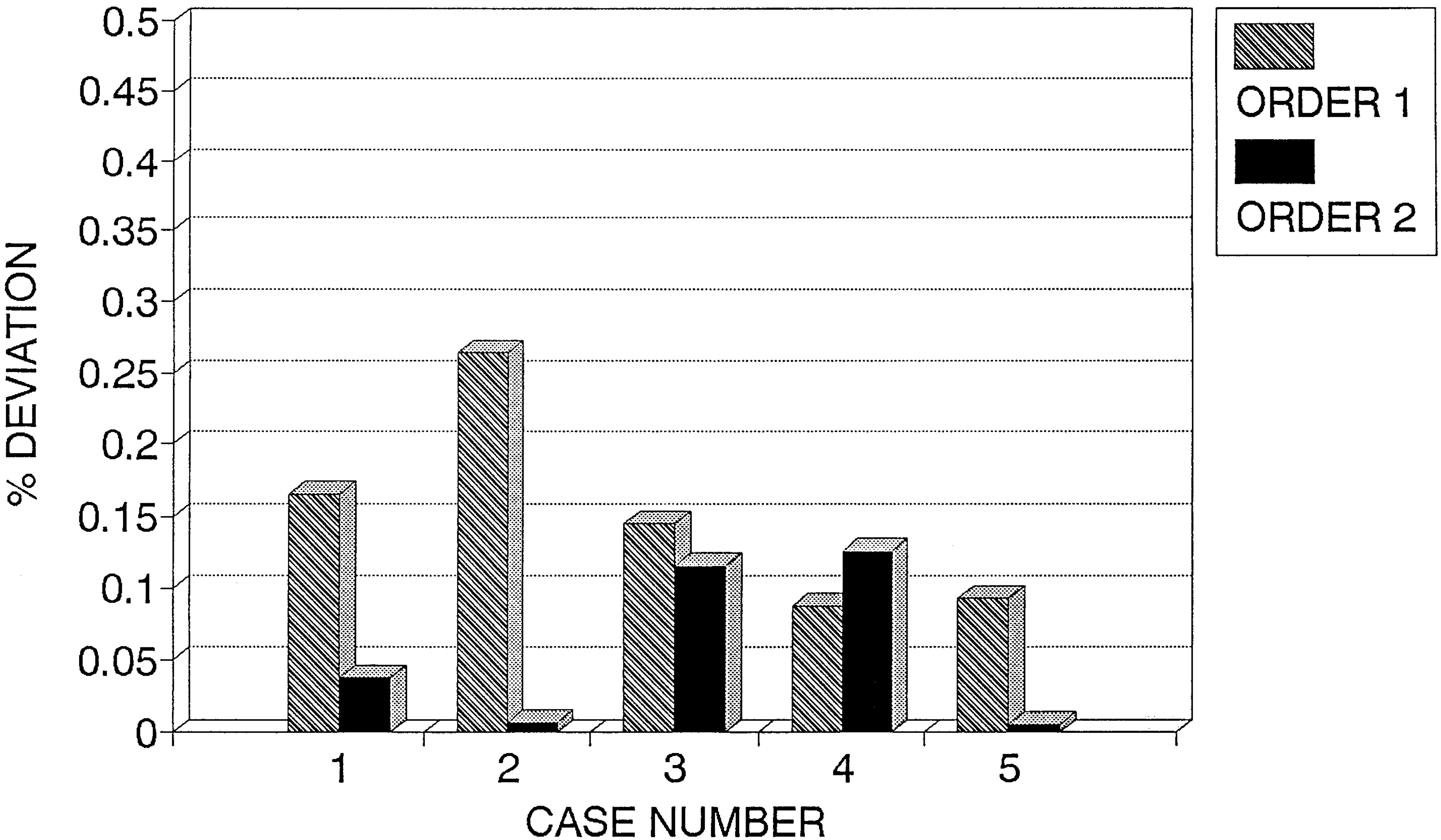
T2OT2 - % DEVIATIONS




MODEL 1
WFT1 - % DEVIATIONS
% DEVIATION
0
1
2
3
4
5
6
7
8
9
10
1
2
3
4
5
CASE NUMBER
ORDER 1
ORDER 2

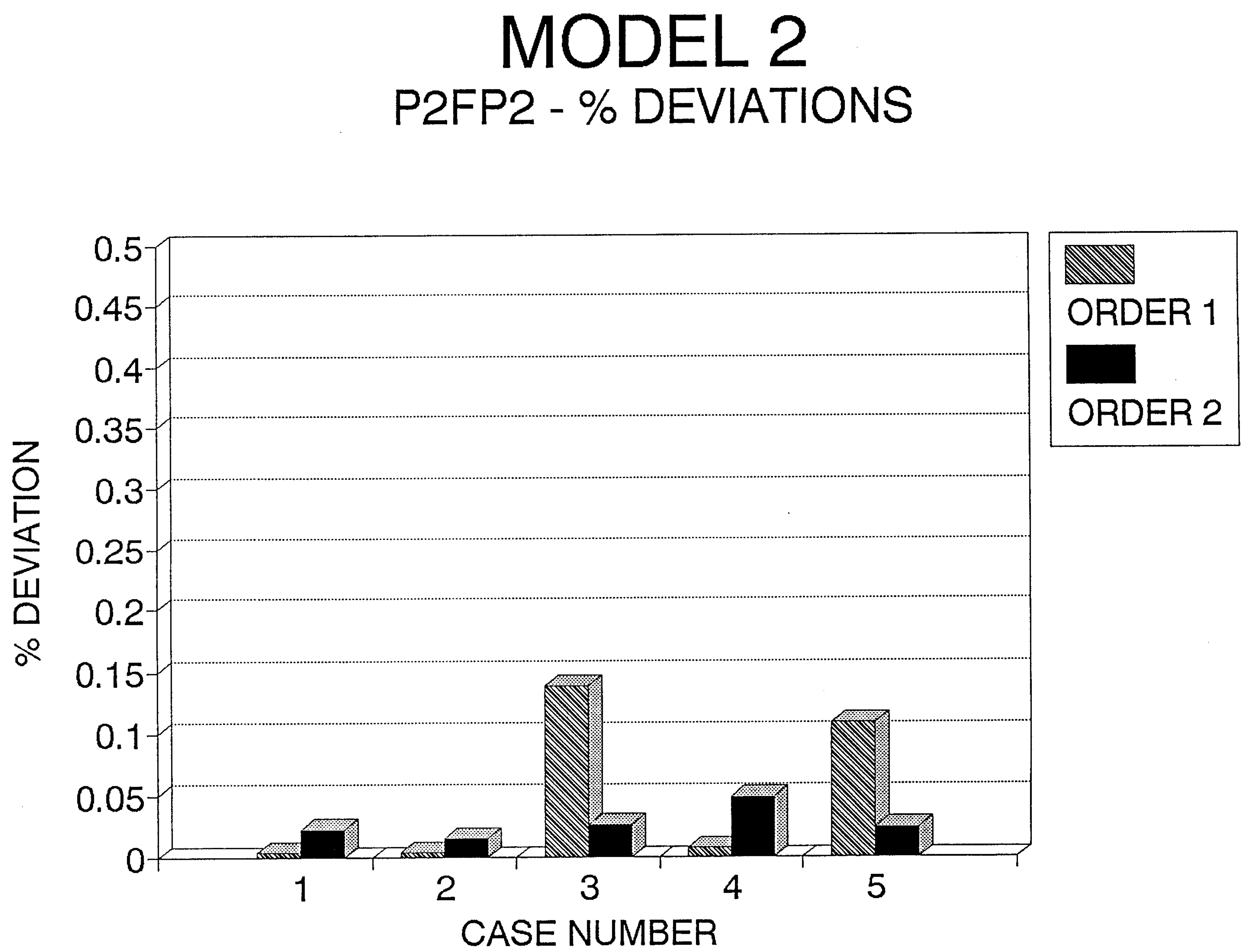
MODEL 1

WTCJBY - % DEVIATIONS



APPENDIX B2

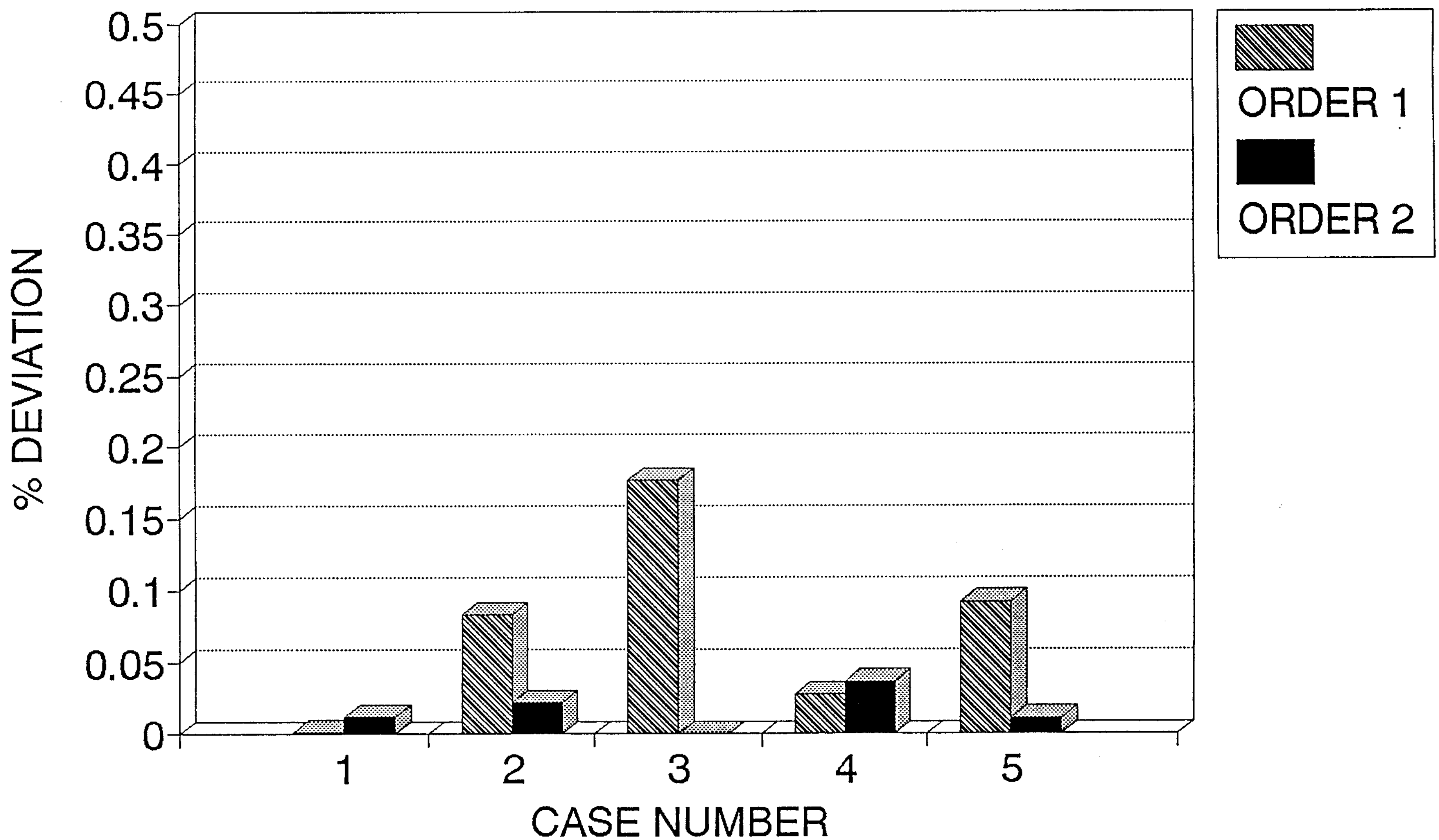
MODEL 2 DEVIATIONS

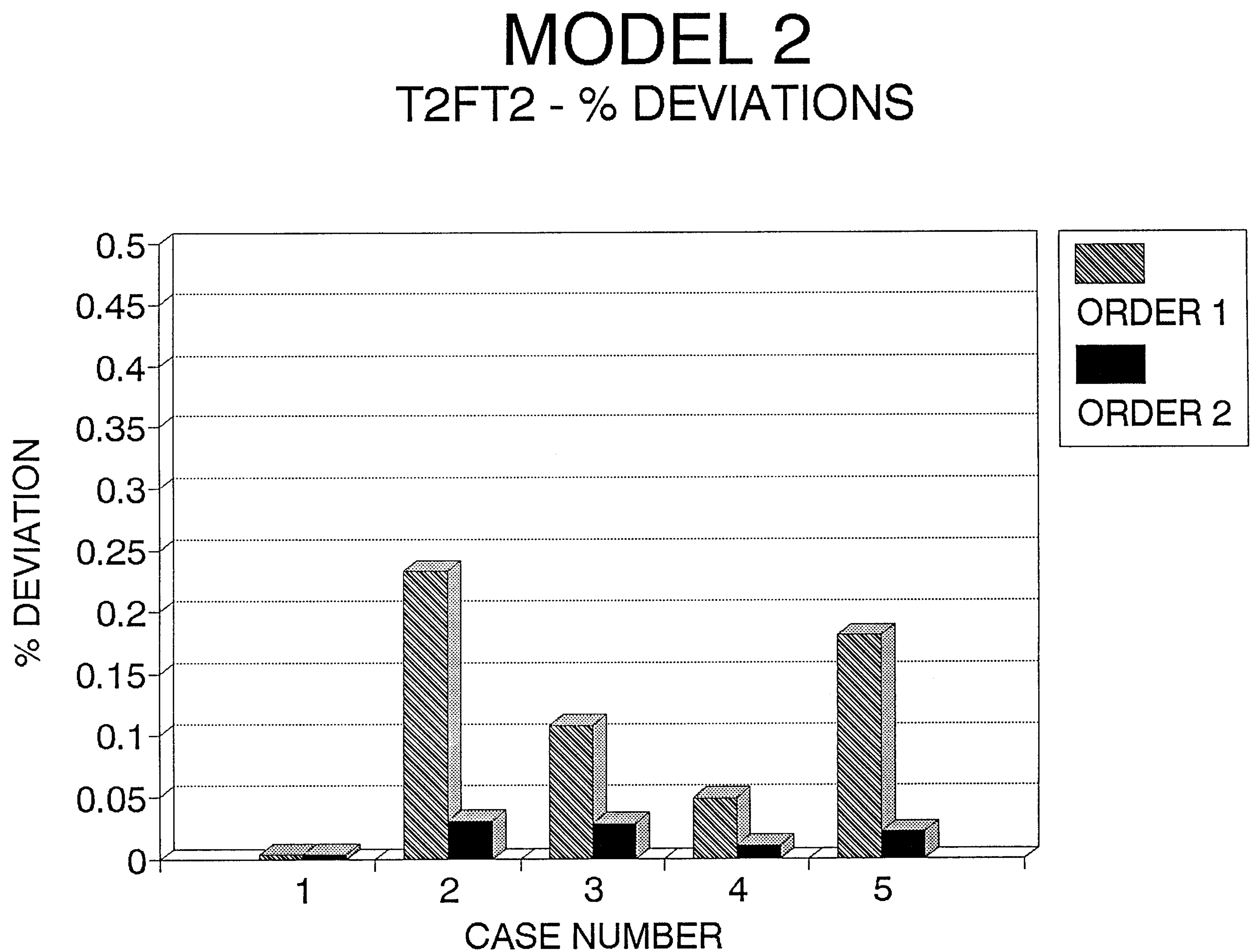


MODEL 2
P2FP2 - % DEVIATIONS
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
% DEVIATION
1
2
3
4
5
CASE NUMBER
ORDER 1
ORDER 2

MODEL 2

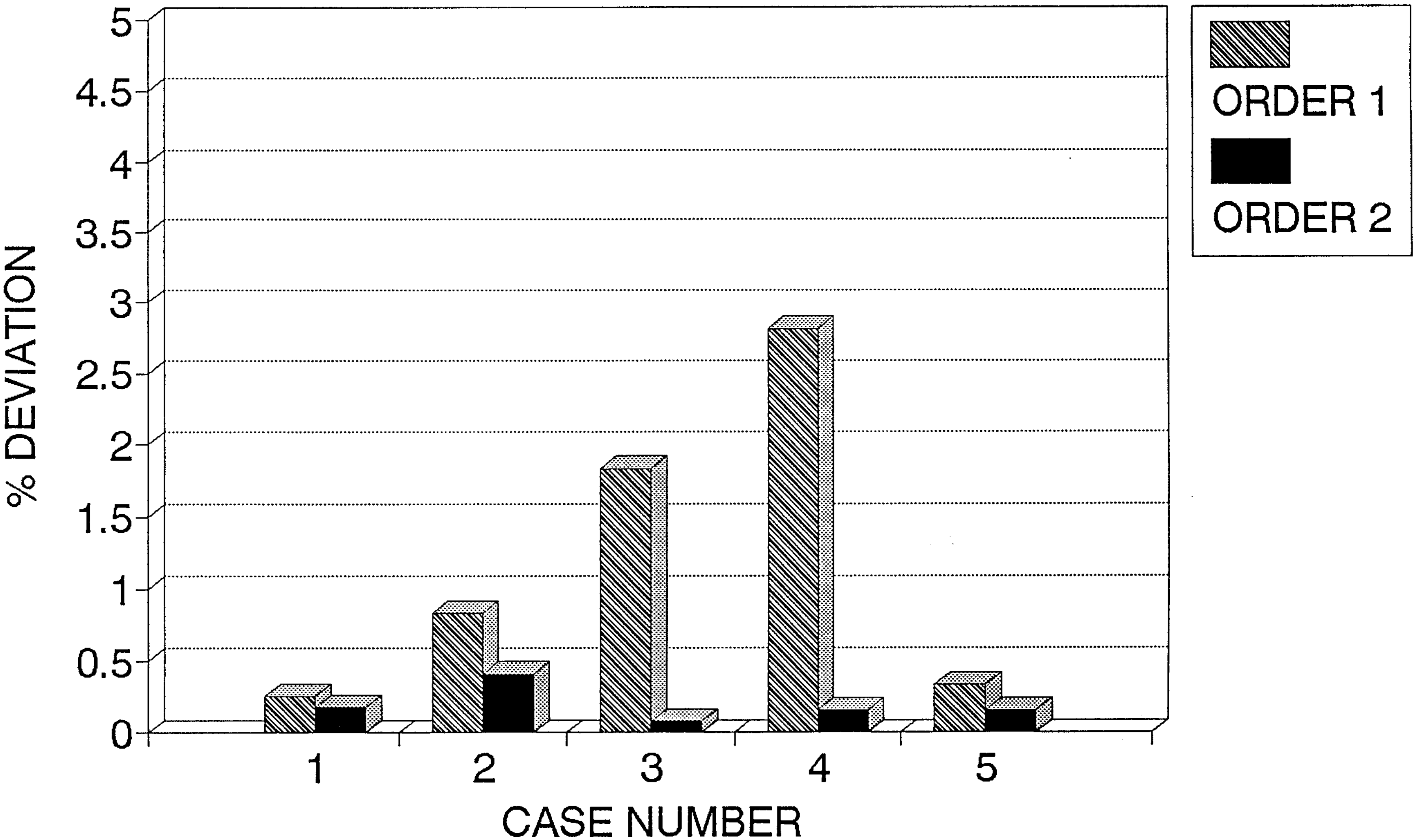
P2OP3 - % DEVIATIONS




MODEL 2
T2FT2 - % DEVIATIONS
% DEVIATION
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5
CASE NUMBER
ORDER 1
ORDER 2

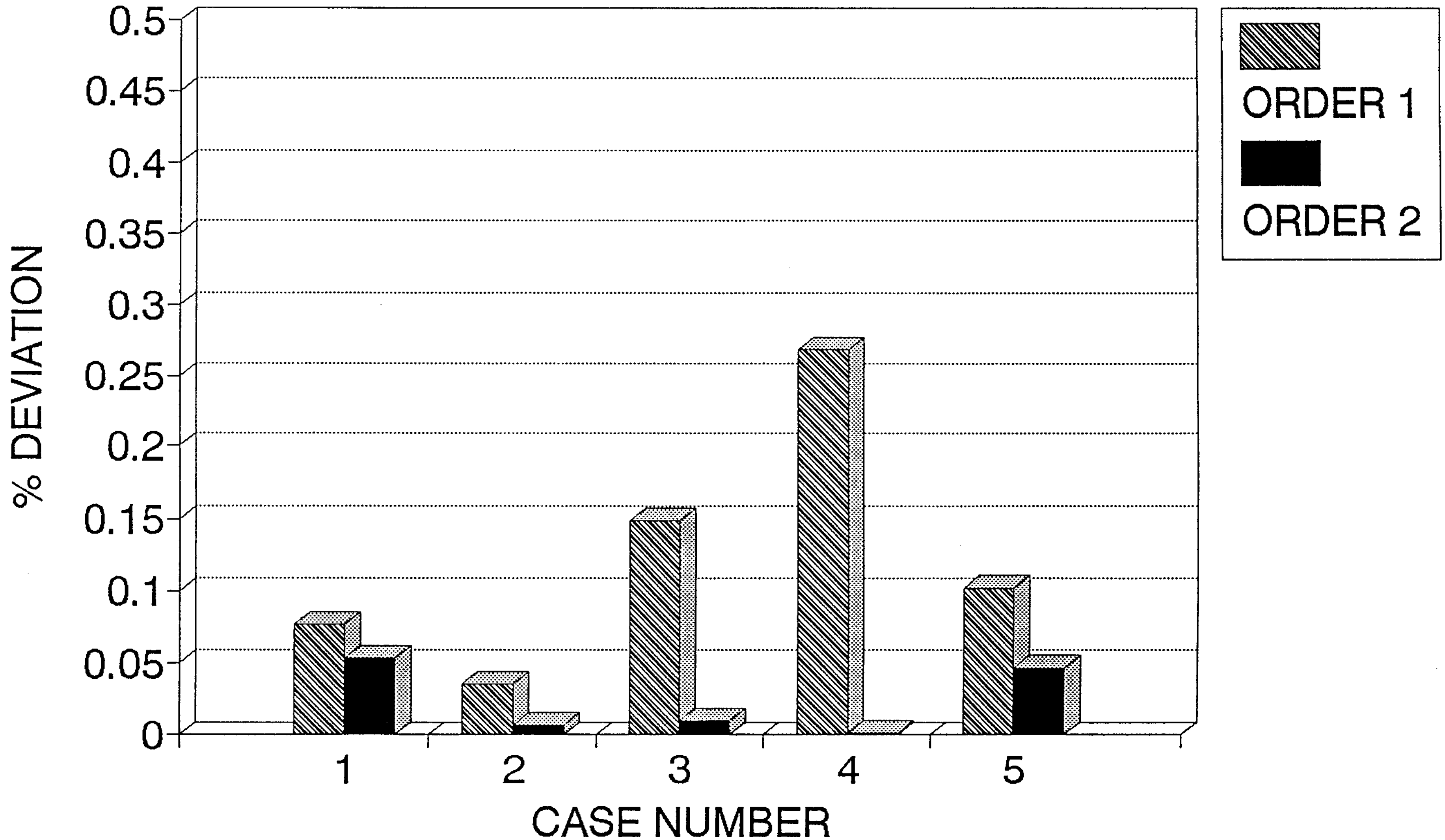
MODEL 2

T2OT2 - % DEVIATIONS

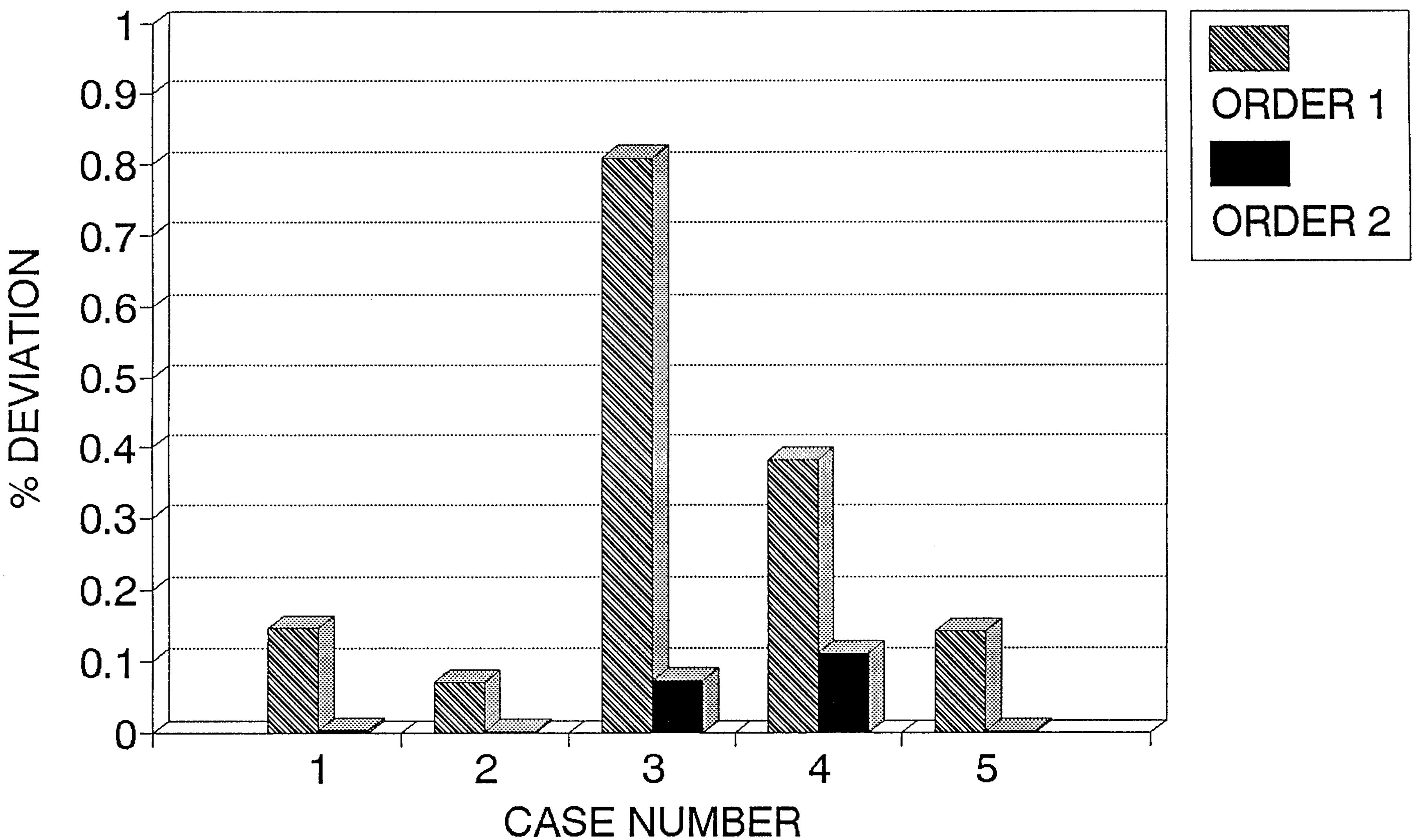


MODEL 2

WFT1 - % DEVIATIONS



MODEL 2
WTCJBY - % DEVIATIONS



APPENDIX B3

MODEL 3 DEVIATIONS

```
C
      CHARACTER*24 INPUT1,INPUT2,OUTPUT
      DIMENSION A(1500),XI(30,20),XD(30,20),IIVAR(20),IDVAR(20)
      OPEN ( 7,FILE='EXTIO.DAT',STATUS='OLD')
      READ ( 7,*) INPUT1,INPUT2,OUTPUT
      OPEN (10,FILE=INPUT1,STATUS='OLD')
      OPEN (11,FILE=INPUT2,STATUS='OLD')
      OPEN (21,FILE=OUTPUT)
      READ (10,*) IMODEL,NIVAR,NDVAR
      READ (10,*) (IIVAR(I),I=1,NIVAR),(IDVAR(I),I=1,NDVAR)
      NRUNP1=1+NIVAR+((NIVAR+1)*NIVAR)/2
      WRITE (21,*) IMODEL,NIVAR,NDVAR
      WRITE (21,*) (IIVAR(I),I=1,NIVAR),(IDVAR(I),I=1,NDVAR)
      DO 10  I = 1, NRUNP1
      READ (11,920)  (A(II),II=1,1350)
      DO 2  J=1,NIVAR
      XI(I,J)=A(IIVAR(J))
    2 CONTINUE
      DO 4  J=1,NDVAR
      XD(I,J)=A(IDVAR(J))
    4 CONTINUE
   10 CONTINUE
      DO 20  I = 1,NRUNP1
      WRITE (21,930) (XI(I,J),J=1,NIVAR)
      WRITE (21,930) (XD(I,J),J=1,NDVAR)
   20 CONTINUE
  920 FORMAT (270(/8X,5E13.6))
  930 FORMAT (6E12.5)
      END
```

APPENDIX C2

ROUTINE INFLUENCE

```
C                    NDVAR        SAME AS INPUT
C                    IIVAR(I)     SAME AS INPUT
C                    IDVAR(I)     SAME AS INPUT
C                    AI0(I)       SAME AS INPUT
C                    AD0(I)       SAME AS INPUT
C                    C1(I,J)      SAME AS CDP(I,J) IN OUT1 EXCEPT SINGLE
C                                 PRECISION
C
C    NOTE: FOR FIRST ORDER MODELS CDP(I,J) IS THE INFLUENCE
C          COEFFICIENT ASSOCIATED WITH CHANGES IN INDEPENDENT
C          VARIABLE I FOR DEPENDENT VARIABLE J.
C
C          FOR SECOND ORDER MODELS CDP(I,J) IS THE INFLUENCE
C          COEFFICIENT ASSOCIATED WITH CHANGES IN INDEPENDENT
C          VARIABLE I FOR DEPENDENT VARIABLE J IF I<=NIVAR.
C          IF I>NIVAR, CDP(I,J) IS THE INFLUENCE COEFFICIENT
C          ASSOCIATED WITH THE PRODUCT OF CHANGES IN INDEPENDENT
C          VARIABLES K AND L ACCORDING TO THE FOLLOWING RECURSION
C
C          I   =  NIVAR
C          DO 1  K = 1, NIVAR
C          DO 1  L = K, NIVAR
C          I   =  I + 1
C          CDP(I,J) = INFLUENCE COEFFICIENT ASSOCIATED WITH PRODUCT
C                     (CHANGE IN VARIABLE K) * (CHANGE IN VARIABLE L)
C       1  CONTINUE
C
C *******************************************************************
C
      CHARACTER*24 INPUT,OUT1,OUT2
      DOUBLE PRECISION   CDP
      DIMENSION  IIVAR(5), IDVAR(50),
     1           AD(50), AI(10), AD0(50), AI0(10),
     2           B(20,50), C(20,70), C1(20,70), CDP(20,70)
C
      OPEN ( 7,FILE='INFIO.DAT',STATUS='OLD')
      READ ( 7,*) INPUT,OUT1,OUT2
      OPEN (11,FILE=INPUT,STATUS='OLD')
      OPEN (20,FILE=OUT1)
      OPEN (21,FILE=OUT2)
C
      READ (11,*)  IMODEL, NIVAR, NDVAR
C
      READ (11,*)  ( IIVAR( I ),  I = 1, NIVAR ),
     1             ( IDVAR( I ),  I = 1, NDVAR )
      READ (11,*)  ( AI0( I ),     I = 1, NIVAR ),
     1             ( AD0( I ),  I = 1, NDVAR )
C
      NRUN   =  NIVAR + ( ( NIVAR + 1 ) * NIVAR ) / 2
C
      DO 40  IRUN = 1, NRUN
      READ (11,*)  ( AI( I ),  I = 1, NIVAR ),
     1             ( AD( I ),  I = 1, NDVAR )
C
```

```
C     PROGRAM INFLUENCE
C
C ***********************************************************************
C  INPUT
C
C    DATA FILE      VARIABLE           DEFINITION
C
C    INFIO.DAT      INPUT        IDENTIFICATION OF INPUT FILE NAME
C                                     (CHARACTER*24 FORMAT)
C                   OUT1         IDENTIFICATION OF OUTPUT FILE # 1
C                                     (CHARACTER*24 FORMAT)
C                   OUT2         IDENTIFICATION OF OUTPUT FILE # 2
C                                     (CHARACTER*24 FORMAT)
C
C    INPUT          IMODEL       IDENTIFICATION OF MODEL NUMBER
C                   NIVAR        NUMBER OF MODEL INDEPENDENT VARIABLES
C                   NDVAR        NUMBER OF MODEL DEPENDENT VARIABLES
C                   IIVAR(I)     A-ARRAY POSITION OF INDEPENDENT VARIABLE I
C                                     (I = 1-NIVAR)
C                   IDVAR(I)     A-ARRAY POSITION OF DEPENDENT VARIABLE I
C                                     (I = 1-NDVAR)
C                   AI0(I)       VALUE OF BASE CASE INDEPENDENT VARIABLE I
C                                     (I = 1-NIVAR)
C                   AD0(I)       VALUE OF BASE CASE DEPENDENT VARIABLE I
C                                     (I = 1-NDVAR)
C                   AI(I)        TEST CASE VALUE OF INDEPENDENT VARIABLE I
C                                     (I = 1-NIVAR)
C                   AD(I)        TEST CASE VALUE OF DEPENDENT VARIABLE I
C                                     (I = 1-NDVAR)
C                                (NOTE: AI(I), I=1-NIVAR, AND
C                                 AD(I), I=1-NDVAR ARE SEQUENTIALLY READ
C                                 NRUN TIMES, WHERE NRUN=NIVAR+((NIVAR+1)
C                                                         *NIVAR)/2
C                                 IS THE NUMBER OF MODEL DEFINITION CASES
C
C ***********************************************************************
C  OUTPUT
C
C    DATA FILE      VARIABLE           DEFINITION
C
C    OUT1           IMODEL       IDENTIFICATION OF MODEL NUMBER
C                   IIVAR(I)     SAME AS INPUT
C                   IDVAR(I)     SAME AS INPUT
C                   CDP(I,J)     DOUBLE PRECISION FORM OF INFLUENCE
C                                COEFFICIENT I ASSOCIATED WITH DEPENDENT
C                                VARIABLE J
C                                (FOR FIRST ORDER MODEL,
C                                 I=1-NIVAR, J=1-NDVAR,
C                                 FOR SECOND ORDER MODEL,
C                                 I=1-NRUN,  J=1-NDVAR)
C
C    OUT2           IMODEL       SAME AS INPUT
C                   IORDER       GAINS MODEL ORDER (1 OR 2)
C                   NIVAR        SAME AS INPUT
```

```
      DO 10  J = 1, NIVAR
      C(IRUN,J)  =  ( AI( J ) - AI0( J ) ) /
     1                AI0( J )
   10 CONTINUE
C
      INDEX  =  NIVAR
      DO 20  K = 1, NIVAR
      DO 20  L = K, NIVAR
      INDEX  =  INDEX + 1
      C(IRUN,INDEX)  =  C( IRUN, K ) * C( IRUN, L )
   20 CONTINUE

      DO 30  J = 1, NDVAR
      B(IRUN,J)  =  ( AD( J ) - AD0( J ) )
     1                             / AD0( J )
   30 CONTINUE
C
   40 CONTINUE
C
      DO 50  I = 1, NIVAR
      DO 50  J = 1, NIVAR
      C1(I,J)  =  C( I, J )
   50 CONTINUE
C
      DO 60  I = 1, NIVAR
      DO 60  J = 1, NDVAR
      C1(I,NIVAR+J)  =  B( I, J )
   60 CONTINUE
C
      IORDER  =  1
      CALL GSSJOR ( C1, CDP, NIVAR, NDVAR, 20, DET, NERR )
C
      IF ( NERR .GT. 0 ) GO TO 120
      WRITE (20,901)  IMODEL
      WRITE (20,902)  ( IDVAR( I ),  I = 1, NDVAR )
C
      DO 70  I = 1, NIVAR
      WRITE (20,903)  IIVAR( I ), ( CDP( I, J ),  J = 1, NDVAR )
   70 CONTINUE
C
      WRITE (21,*)  IMODEL, IORDER, NIVAR, NDVAR
      WRITE (21,*)  ( IIVAR( I ),  I = 1, NIVAR ),
     1              ( IDVAR( I ),  I = 1, NDVAR )
      WRITE (21,*)  ( AI0( I ),  I = 1, NIVAR ),
     1              ( AD0( I ),  I = 1, NDVAR )
C
      DO 75  J = 1, NDVAR
      WRITE (21,*)  ( C1( I, J ),  I = 1, NIVAR )
   75 CONTINUE
C
      DO 80  I = 1, NRUN
      DO 80  J = 1, NRUN
      C1(I,J)  =  C( I, J )
   80 CONTINUE
```

```
C
      DO 90  I = 1, NRUN
      DO 90  J = 1, NDVAR
      C1(I,NRUN+J)  =  B( I, J )
   90 CONTINUE
C
      IORDER  =  2
      CALL GSSJOR ( C1, CDP, NRUN, NDVAR, 20, DET, NERR )
C
      IF ( NERR .GT. 0 ) GO TO 120
      WRITE (20,904)  IMODEL
      WRITE (20,902)  ( IDVAR( I ),  I = 1, NDVAR )
C
      DO 100  I = 1, NIVAR
      WRITE (20,903)  IIVAR( I ), ( CDP( I, J ),  J = 1, NDVAR )
  100 CONTINUE
C
      INDEX  = NIVAR
      DO 110  K = 1, NIVAR
      DO 110  L = K, NIVAR
      INDEX  =  INDEX + 1
      WRITE (20,905)  IIVAR( K ), IIVAR( L ),
     1                ( CDP( INDEX, J ),  J = 1, NDVAR )
  110 CONTINUE
C
      WRITE (21,*)  IMODEL, IORDER, NIVAR, NDVAR
      WRITE (21,*)  ( IIVAR( I ),  I = 1, NIVAR ),
     1              ( IDVAR( I ),  I = 1, NDVAR )
      WRITE (21,*)  ( AI0( I ),  I = 1, NIVAR ),
     1              ( AD0( I ),  I = 1, NDVAR )
C
      DO 115  J = 1, NDVAR
      WRITE (21,*)  ( C1( I, J ),  I = 1, NRUN )
  115 CONTINUE
C
      GO TO 130
C
  120 WRITE (20,920)  IMODEL, NERR
C
  130 CONTINUE
C
  901 FORMAT (//2X,'MODEL',I3,13X,'1st ORDER INFLUENCE COEFFICIENTS')
  902 FORMAT (/27X,'DEPENDENT VARIABLE NUMBER',/12X,I5,5I10,/1X,
     1       'IND VAR #')
  903 FORMAT (2X,I4,4X,6E10.4)
  904 FORMAT (///2X,'MODEL',I3,13X,'2nd ORDER INFLUENCE COEFFICIENTS')
  905 FORMAT (1X,2I4,1X,6E10.4)
  920 FORMAT (//2X,'SOLUTION FAILURE',/2X,'MODEL NO = ',I5,
     1       /2X,'RANK     = ',I5)
C
      END
C
C
      SUBROUTINE GSSJOR(A,ADP,N,NRS,NR,DET,NERR)
```

```
      DOUBLE PRECISION ADP,SD,AH,Z,P,ZERODV
      DIMENSION A(NR,*),ADP(NR,*)
      DATA ZERODV / .5D-12 /
C        INITIALIZE
      NERR = 0
      JMAX = N+NRS
      SD = 1.0D0
      NP1 = N+1
      NM1 = N-1
C        MOVE INPUT A ARRAY TO DOUBLE PRECISION WORKING ARRAY
    5 DO 6  J=1,JMAX
      DO 6  I=1,N
    6 ADP(I,J) = A(I,J)
C        BEGIN PROCESS
      DO 39  K=1,N
      KP1 = K+1
C        REORDER EQUATIONS - CHOOSE LARGEST DIVISOR
   10 IH = K
      AH = DABS(ADP(K,K))
      DO 15  I=K,N
      IF(DABS(ADP(I,K)) .GT. AH) THEN
        IH = I
        AH = DABS(ADP(I,K))
      ENDIF
   15 CONTINUE
C        CHANGE ORDER
   20 IF(IH .NE. K) THEN
        SD =-SD
        DO 25  J=1,JMAX
        Z = ADP(K,J)
        ADP(K,J) = ADP(IH,J)
   25   ADP(IH,J) = Z
      ENDIF
C        ZERO OFF-DIAGONAL ELEMENTS
   30 DO 35  J=KP1,JMAX
      IF(DABS(ADP(K,K)) .GT. ZERODV)  GOTO 35
      NERR = K-1
      GOTO 999
   35 ADP(K,J) = ADP(K,J)/ADP(K,K)
      DO 39  I=1,N
      Z = ADP(I,K)
      DO 39  J=KP1,JMAX
      IF(I .EQ. K)  GOTO 39
      ADP(I,J) = ADP(I,J) - Z*ADP(K,J)
   39 CONTINUE
C        CALCULATE DETERMINANT
   40 P = ADP(1,1)
      DO 45  K=2,N
   45 P = P*ADP(K,K)
      IF(P .LT. 1.0D+38) THEN
        DET = SD*P
      ELSE
        DET = 1.0E+38
      ENDIF
```

```
      IF(NRS .EQ. 0)  GOTO 999
C        REPLACE COLUMNS 1 THRU NRS WITH SOLUTION VECTORS
   50 DO 55  J=NP1,JMAX
      DO 55  K=1,N
      ADP(K,J-N) = ADP(K,J)
   55 A(K,J-N) = ADP(K,J)
C
  999 RETURN
      END
```

APPENDIX C3

ROUTINE COMPARE

```
C  PROGRAM COMPARE
C
C ***********************************************************************
C  INPUT
C
C    DATA FILE      VARIABLE            DEFINITION
C
C    COMIO.DAT      INPUT1      IDENTIFICATION OF INPUT FILE # 1
C                                    (CHARACTER*24 VARIABLE)
C                   INPUT2      IDENTIFICATION OF INPUT FILE # 2
C                                    (CHARACTER*24 VARIABLE)
C                   OUTPUT      IDENTIFICATION OF OUTPUT FILE
C                                    (CHARACTER*24 VARIABLE)
C
C    INPUT1         NTEST       NUMBER OF GAINS MODEL TEST CASES
C                   AIT(I)      TEST CASE VALUE OF INDEPENDENT VARIABLE I
C                   ADT(I)      TEST CASE VALUE OF DEPENDENT VARIABLE I
C
C    INPUT2         IMODEL      IDENTIFICATION OF MODEL NUMBER
C                   IORDER      GAINS MODEL ORDER (1 OR 2)
C                   NIVAR       NUMBER OF MODEL INDEPENDENT VARIABLES
C                   NDVAR       NUMBER OF MODEL DEPENDENT VARIABLES
C                   IIVAR(I)    A-ARRAY ADDRESS OF INDEPENDENT VARIABLE I
C                                    (I = 1-NIVAR)
C                   IDVAR(I)    A-ARRAY ADDRESS OF DEPENDENT VARIABLE I
C                                    (I = 1-NDVAR)
C                   AI0(I)      BASE CASE VALUE OF INDEPENDENT VARIABLE I
C                                    (I = 1-NIVAR)
C                   AD0(I)      BASE CASE VALUE OF DEPENDENT VARIABLE I
C                                    (I = 1-NDVAR)
C                   CINF(I,J)   INFLUENCE COEFFICIENT I ASSOCIATED WITH
C                               DEPENDENT VARIABLE J
C                               (FIRST ORDER MODEL - I=1,NIVAR, J=1,NDVAR
C                                SECOND ORDER MODEL- I=1,NRUN , J=1,NDVAR
C                                WHERE NRUN=NIVAR+((NIVAR+1)*NIVAR)/2)
C ***********************************************************************
C  OUTPUT
C
C    DATA FILE      VARIABLE            DEFINITION
C
C    OUTPUT         ITEST       TEST CASE NUMBER
C                   IMODEL      SAME AS INPUT
C                   IORDER      SAME AS INPUT
C                   NIVAR       SAME AS INPUT
C                   IIVAR(I)    SAME AS INPUT
C                   IDVAR(I)    SAME AS INPUT
C                   ADG         GAINS MODEL PREDICTED VALUE OF DEPENDENT
C                               VARIABLE
C                   ADT(I)      SAME AS INPUT
C                   PCT         PERCENT DIFFERENCE BETWEEN GAINS MODEL
C                               PREDICTION AND TEST VALUE OF DEPENDENT
C                               VARIABLE
C
C ***********************************************************************
```

```
C
      CHARACTER*24 INPUT1,INPUT2,OUTPUT
      DIMENSION  IDVAR( 50 ), IIVAR( 10 ), AIO( 10 ),
     1           ADO( 50 ), AIT( 10 ), ADT( 50 ), CINF( 20, 50 )
C
      OPEN ( 7,FILE='COMIO.DAT',STATUS='OLD')
      READ ( 7,*) INPUT1,INPUT2,OUTPUT
      OPEN (11,FILE=INPUT1,STATUS='OLD')
      OPEN (21,FILE=INPUT2,STATUS='OLD')
      OPEN (30,FILE=OUTPUT)
C
      READ (11,*)  NTEST
C
      DO 200  ITEST = 1, NTEST
C
      READ (21,*)  IMODEL, IORDER, NIVAR, NDVAR
      READ (21,*)  ( IIVAR( I ),  I = 1, NIVAR ),
     1             ( IDVAR( I ),  I = 1, NDVAR )
      READ (21,*)  ( AIO( I ),      I = 1, NIVAR ),
     1             ( ADO( I ),  I = 1, NDVAR )
C
      DO 10  J = 1, NDVAR
      READ (21,*)  ( CINF( I, J ),  I = 1, NIVAR )
   10 CONTINUE
C
      READ (11,*)  ( AIT( I ),      I = 1, NIVAR ),
     1             ( ADT( I ),  I = 1, NDVAR )
C
      WRITE (30,901)  ITEST, IMODEL, IORDER, NIVAR,
     1               ( IIVAR( I ),  I = 1, NIVAR )
      WRITE (30,902)
C
      DO 30  J = 1, NDVAR
      DELTA  =  0.0
C
      DO 20  I = 1, NIVAR
      DELTA  =  DELTA + CINF( I, J ) * ( AIT( I )-AIO( I ) ) / AIO( I )
   20 CONTINUE
C
      ADG  =  DELTA * ADO( J ) + ADO( J )
      PCT  =  100. * ( ADG - ADT( J ) ) / ADT( J )
      WRITE (30,903)  IDVAR( J ), ADG, ADT( J ), PCT
   30 CONTINUE
C
      READ (21,*)  IMODEL, IORDER, NIVAR, NDVAR
      READ (21,*)  ( IIVAR( I ),  I = 1, NIVAR ),
     1            ( IDVAR( I ),  I = 1, NDVAR )
      READ (21,*)  ( AIO( I ),      I = 1, NIVAR ),
     1            ( ADO( I ), I = 1, NDVAR )
C
      NRUN  =  NIVAR + ( ( NIVAR + 1 ) * NIVAR ) / 2
C
      DO 40  J = 1, NDVAR
      READ (21,*)  ( CINF( I, J ),  I = 1, NRUN )
```

```
   40   CONTINUE
C
        WRITE (30,901)  ITEST, IMODEL, IORDER, NIVAR,
       1                ( IIVAR( I ),  I = 1, NIVAR )
        WRITE (30,902)
C
        DO 70  J = 1, NDVAR
        DELTA  =  0.0
C
        DO 50  I = 1, NIVAR
        DELTA  =  DELTA + CINF( I, J ) * ( AIT( I )-AIO( I ) ) / AIO( I )
   50   CONTINUE
C
        INDEX  = NIVAR
        DO 60  K = 1, NIVAR
        DO 60  L = K, NIVAR
        INDEX  =  INDEX + 1
        DELTA  =  DELTA + CINF( INDEX, J ) * ( ( AIT( K ) - AIO( K ) ) /
       1          AIO( K ) ) * ( ( AIT( L ) - AIO( L ) ) / AIO( L ) )
   60   CONTINUE
C
        ADG  =  DELTA * ADO( J ) + ADO( J )
        PCT  =  100. * ( ADG - ADT( J ) ) / ADT( J )
        WRITE (30,903)  IDVAR( J ), ADG, ADT( J ), PCT
   70   CONTINUE
C
        REWIND 21
C
  200   CONTINUE
C
  901   FORMAT (//2X,'TEST NUMBER  = ',I5,
       1        /2X,'MODEL NUMBER = ',I5,/2X, 'ORDER        = ',I5,
       2        /2X,'NUMBER IVARS = ',I5,/2X, 'IND VARIABLES= ',10I5)
  902   FORMAT (/2X,'DVAR#',8X,'GAINS',10X,'PBM',7X,'% DIFF')
  903   FORMAT (2X,I5,3F13.4)
C
        END
```

Aspects of Model-Based Rocket Engine Condition Monitoring and Control

A 6-Month Progress Report Submitted to

Dr. Gerald R. Karr, Director
NASA/ASEE 1991 Summer Faculty Fellowship Program
at the Marshall Space Flight Center

Mechanical Engineering Department
The University of Alabama at Huntsville
Huntsville, Alabama 35899

by

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May 1993

1 Review of Technical Objectives

Modern rocket engine systems often utilize closed-loop control in order to achieve requirements on system performance. In order to enhance reliability, extend system life, and reduce operating costs the development of condition monitoring systems, which would augment the control system, is currently under study. During my period at the Marshall space Flight Center as a 1991 Summer Faculty Fellow I began an investigation of the issues involved in the design of condition monitoring systems using a novel approach; one that viewed the condition monitoring problem and the control design problem from within an integrated system-theoretic framework. At the end of the Fellowship period a number of issues regarding the further development of this approach remained. Of these, I selected the following two for further study under the auspices of a Continuation Grant.

Validation of Off-Nominal Condition Modeling Schemes: The approach to integrated condition monitoring and control alluded to above is predicated on our ability to model off-nominal conditions arising in rocket engine systems in one of the two possible ways depicted in Figures 1 and 2. In each case the block p denotes a nominal engine component or subsystem. In the case of Figure 1 off-nominal conditions are represented by injecting an exogenous signal either at the component input or output. In the case of Figure 2 off-nominal conditions are represented by augmenting the component dynamics by some additional dynamics indicative of a certain type of failure/degradation.

Correspondingly, the first research task proposed here was to study the dynamics associated with rocket engine systems, the types of failures/degradations likely to occur, and the effects of these failures/degradations on rocket engine dynamics.

The purpose of this study was to determine if the assumption on our ability to model off-nominal engine conditions by one of the two schemes given in Figures 1 and 2 was valid.

Extensions of Current Framework: The second task proposed here involves the extension of the integrated approach to condition monitoring and control through the development of a more enhanced set of analysis and design tools. This task is made possible by virtue of the fact that the approach developed at MSFC last summer allows us to embed the integrated condition monitoring and controls problem in a general control systems architecture. Thus, this allows the extensive set of control system analysis and synthesis methods to be adapted for use on the integrated condition monitoring and control problem.

Correspondingly, this task would involve the modification and application of existing control design tools for use in condition monitoring system design.

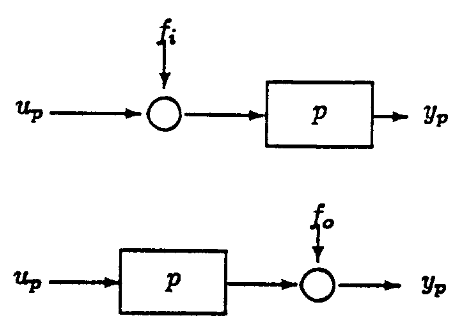

Figure 1: Signal Off-nominal Representations.

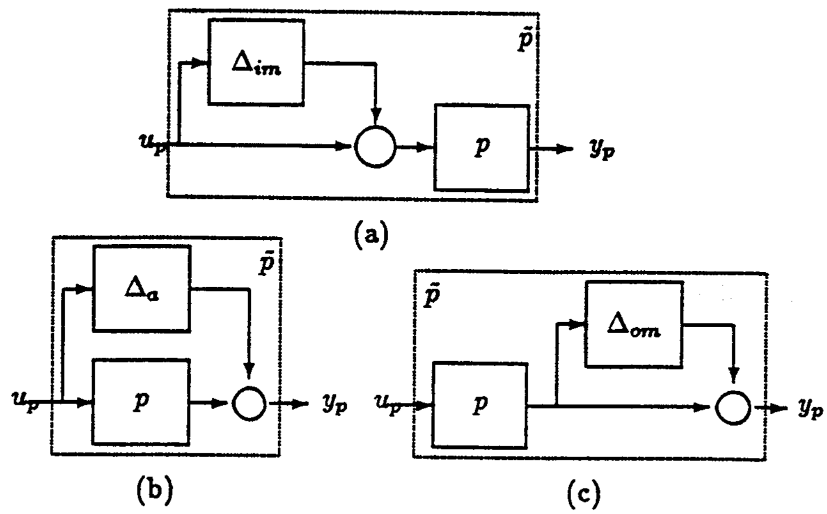

Figure 2: Dynamic System Off-nominal Representations.

2 Technical Progress

Research efforts to date have focused almost exclusively on the first task outlined above, i.e., Modeling Scheme Validation. These efforts have resulted in one published paper and one technical presentation. Both of these are provided in the appendix of this report. Specific technical developments which have been achieved can be summarized as follows:

- The dynamics for thermo-fluid systems such as rocket engines have been studied using a first principles approach based on equations obtained from conservation of mass, momentum, and energy.
- Using the understanding gained from the previous item, a modeling scheme has been developed for generating dynamic models of rocket engine systems. This scheme is modular in that it is applied to each engine component and then the resulting equations are assembled to form the overall engine model.
- Through the use of non-dimensionalization, scaling, and singular perturbations applied to the models obtained in the last item we have been able to characterize both nominal and off-nominal rocket engine systems with low order dynamical models which demonstrate good low-to-mid frequency fidelity. In addition, we have been able to distinguish when each of the two off-nominal modeling schemes given in Figures 1 and 2 represent satisfactory models of given off-nominal conditions.
- We have applied these results to develop dynamic computer simulations of the Space Transportation Main Engine (STME). These simulations are written in SIMULINK and have been demonstrated to compare favorable with NASA/MSFC STME MARSYAS simulations of much higher order.

Technical details can be found in the documents contained in the appendix. As a result of these developments the research is now at the stage where the first task outlined in Section 1 can effectively be considered to be complete. Discussions are now under way with my NASA colleague, Fred Kuo, to arrange a time when I can travel to MSFC to demonstrate the engine software models and demonstrate the application of these results.

3 Future Plans

The time remaining on this Continuation Grant will be spent focusing on the second technical objective discussed in Section 1 above, i.e., Framework Extension. Based on my discussions with Fred Kuo, I have identified one particular control system design concept which we feel has good potential for extension to the condition monitoring problem, that of an observer. For obvious reasons, rocket engine designers try to minimize the number of sensors mounted on flight configured engines. I would like to study the possibility of using such a reduced sensor suite together with the modeling results above to develop an observer-based filter which could aid in reconstructing internal engine variables not directly measured. In this way unmeasured engine data could be constructed either on-line or off-line for use in engine maintenance and monitoring decisions.

A Supporting Documentation

This section contains a copy of a paper published and presentation overheads which resulted directly from the research pursued during the first 6 months of this grant. Both appeared at the Fifth Annual Space System Health Management Technology Conference held in Cincinnati, Ohio, April 14th, 1993.

Modeling for Health Monitoring and Control with Application to the STME

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Fifth Annual Space System Health Management
Technology Conference
Cincinnati, OH
April 14, 1993

Summary

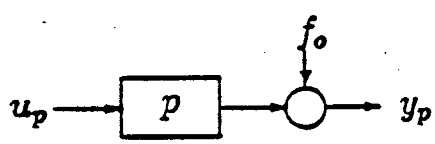
- An Architecture for Integrated Monitoring and Control (IMC)
- Modeling Issues: A Motivating Example
- Conservation Equations for Generic Component
- Analysis using Non-Dimensional Analysis and Scaling
- Comparison of Modeling Results
- Conclusions and Future Directions

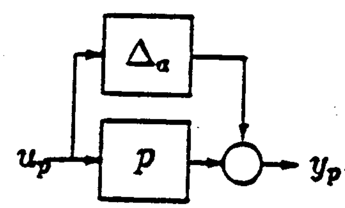
IMC Architecture

Basic Observation: Control and monitoring functions impact one another and hence an integrated approach is indicated.

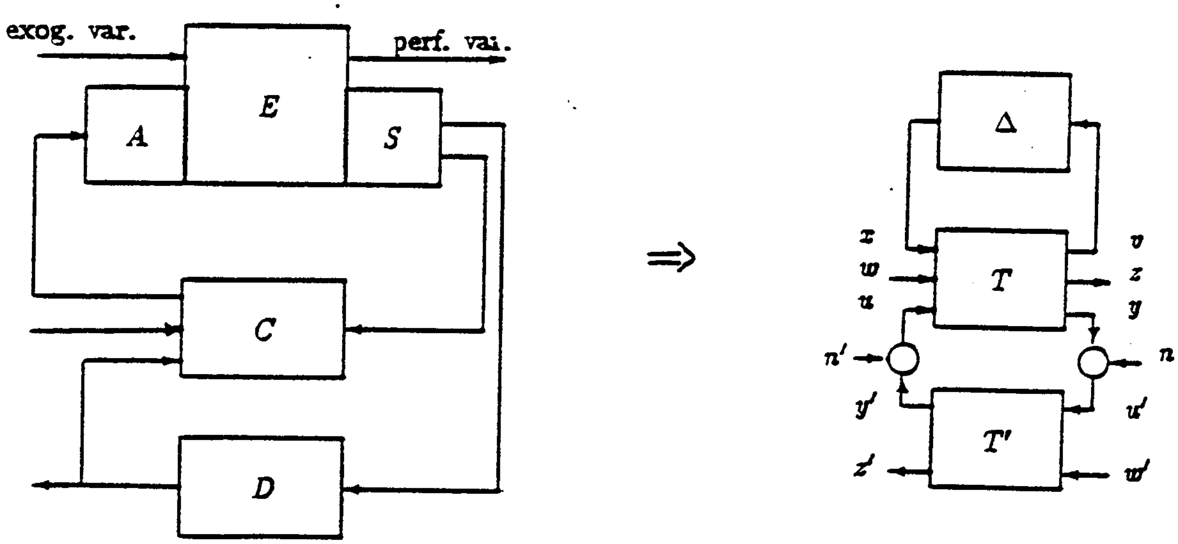
Basic Approach: Develop tools for Integrated Monitoring and Control (IMC) problem by select modeling scheme for off-nominal behavior which is compatible with existing systems methods, and then embed IMC problem within general systems structure.

Off-nominal modeling schemes:

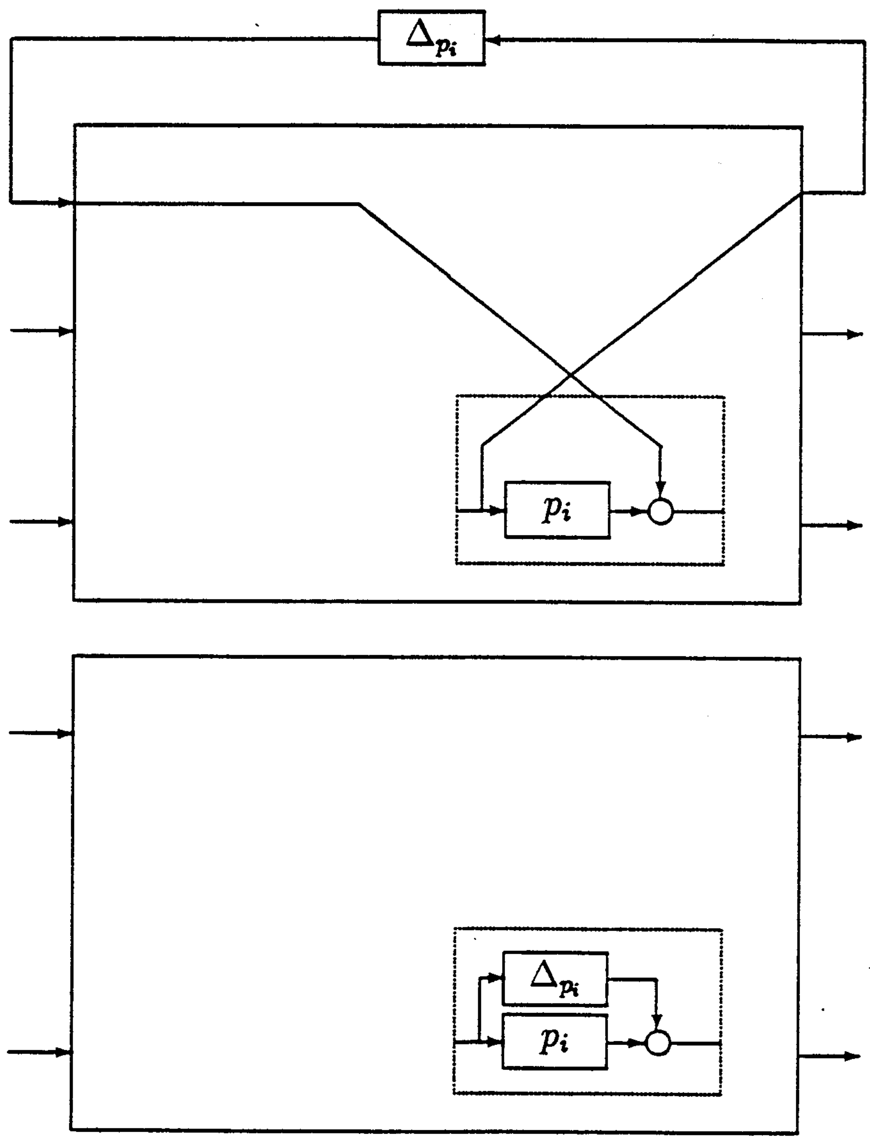




IMC embedding:



IMC Architecture



Motivating Example

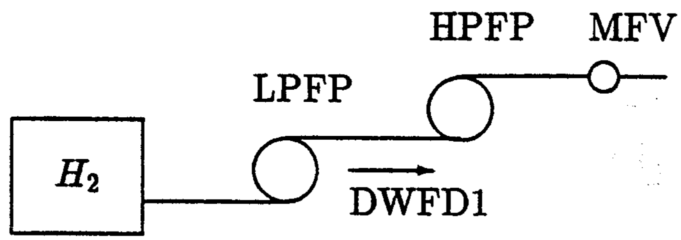


Figure 1: Isolated Nominal SSME Fuel Pump System.

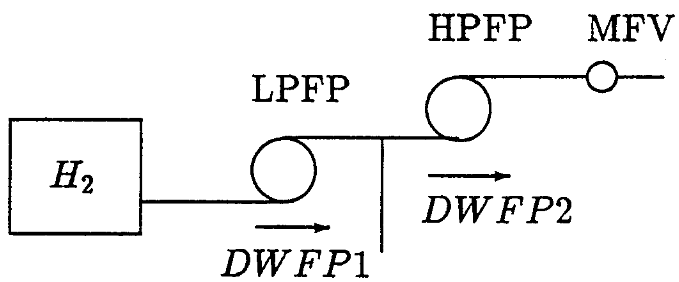


Figure 2: Possible Anomalous Condition Associated with Leak.

Nominal Case:

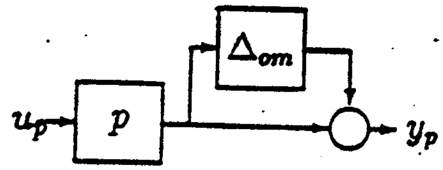
1 flow state

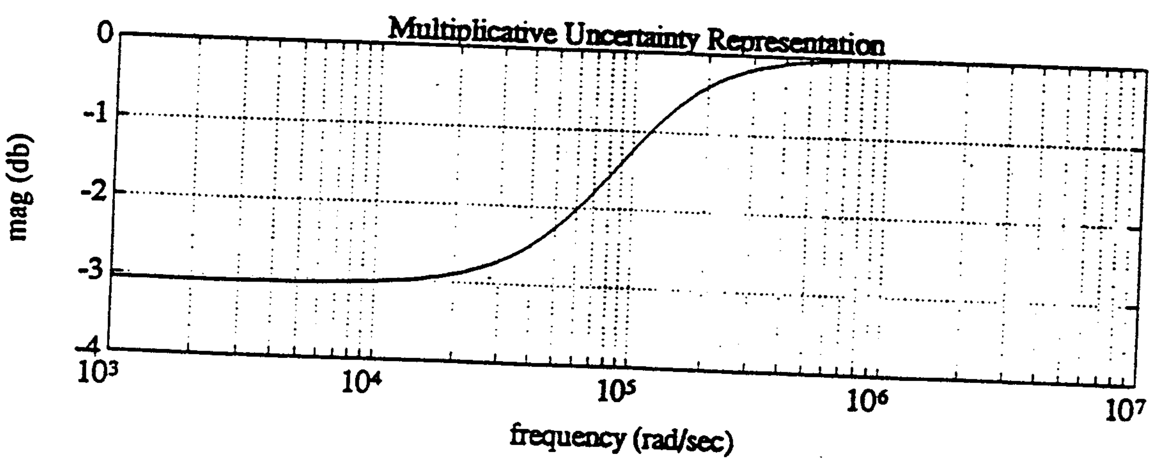
Leaky Case:

2 flow states

Indicates need for use of uncertainty-based modeling scheme!

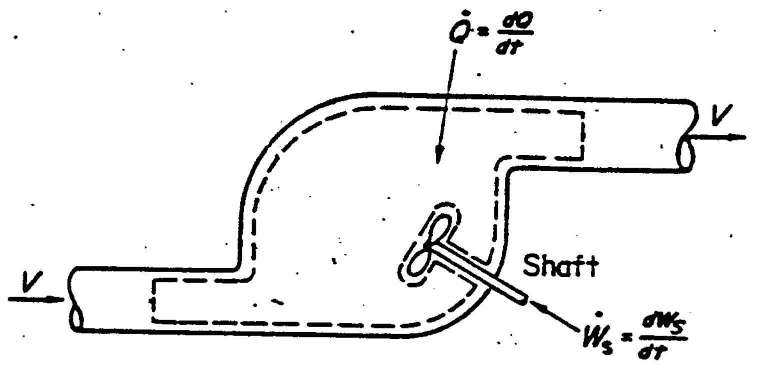
Motivating Example





Conservation Equations

Generic engine component:



Assume flow is quasi one-dimensional, inviscid liquid with negligible body forces, then the governing laws for fluid-thermo-mechanical system are given by:

1. State Equation

$$\tilde{\rho} = \frac{\tilde{\rho}_o \tilde{p}}{\beta}$$

2. Conservation of Mass

$$\frac{\partial(\tilde{\rho}\tilde{A})}{\partial\tilde{t}} = -\frac{\partial(\tilde{\rho}\tilde{V}\tilde{A})}{\partial\tilde{x}}$$

3. Conservation of Momentum

$$\frac{\partial(\tilde{\rho}\tilde{V}\tilde{A})}{\partial\tilde{t}} = -\frac{\partial}{\partial\tilde{x}}[(\tilde{\rho}\tilde{V}^2 + \tilde{p})\tilde{A}] + \tilde{p}\frac{\partial\tilde{A}}{\partial\tilde{x}} + \tilde{\rho}\tilde{A}\tilde{f}_s$$

Here:

- β denotes bulk modulus of elasticity,
- $\tilde{f}_s$ denotes shaft-on-fluid forces, and
- $\tilde{}$s denote that variables are dimensional.

Conservation Equations

By massaging these equations we can obtain:

1. State Equation

$$\tilde{\rho} = \frac{\tilde{\rho}_o \tilde{p}}{\beta}$$

2. Conservation of Mass

$$\frac{\partial}{\partial \tilde{t}}\tilde{\rho} = \frac{A_o}{\mathcal{V}}\left(\tilde{\rho}_1 \tilde{V}_1 \frac{\tilde{A}_1}{A_o} - \tilde{\rho}_2 \tilde{V}_2 \frac{\tilde{A}_2}{A_o}\right)$$

3. Conservation of Momentum

$$\frac{\partial}{\partial \tilde{t}}\tilde{w} = \frac{A_o}{L}\left(\tilde{\rho}_1 \tilde{V}_1^2 \frac{\tilde{A}_1}{A_o} - \tilde{\rho}_2 \tilde{V}_2^2 \frac{\tilde{A}_2}{A_o} + \tilde{p}_1 \frac{\tilde{A}_1}{A_o} - \tilde{p}_2 \frac{\tilde{A}_2}{A_o} - \hat{p}\frac{\tilde{A}_1 - \tilde{A}_2}{A_o}\right)$$

where $\tilde{w} = \tilde{\rho}\tilde{V}\tilde{A}$ denotes mass flow, A_o denotes mean control volume area, L denotes the control volume length, and $\mathcal{V}$ denotes control volume volume.

Thus, dynamics appear to depend on relative sizes of $A_o/\mathcal{V}$ and A_o/L.

The problem is that these quantities are dimensional.

Use of Non-dimensionalization and Scaling

Let

$$a = \sqrt{\beta/\rho_o} \quad , \quad p = \tilde{p}/(\rho_o a^2) \quad , \quad M = \tilde{V}/a$$
$$x = \tilde{x}/L \quad , \quad t = \tilde{t}/t_o \quad , \quad A = \tilde{A}/A_o$$
$$f_s = \tilde{f}_s/(\beta/\rho_o L) \ ,$$

then the governing laws for fluid-thermo-mechanical system are given by:

1. Conservation of Mass

$$\epsilon \frac{\partial(pA)}{\partial t} = -\frac{\partial(pMA)}{\partial x}$$

2. Conservation of Momentum

$$\epsilon \frac{\partial M}{\partial t} = -Mp\frac{\partial M}{\partial x} - \frac{\partial p}{\partial x} + pf_s$$

where

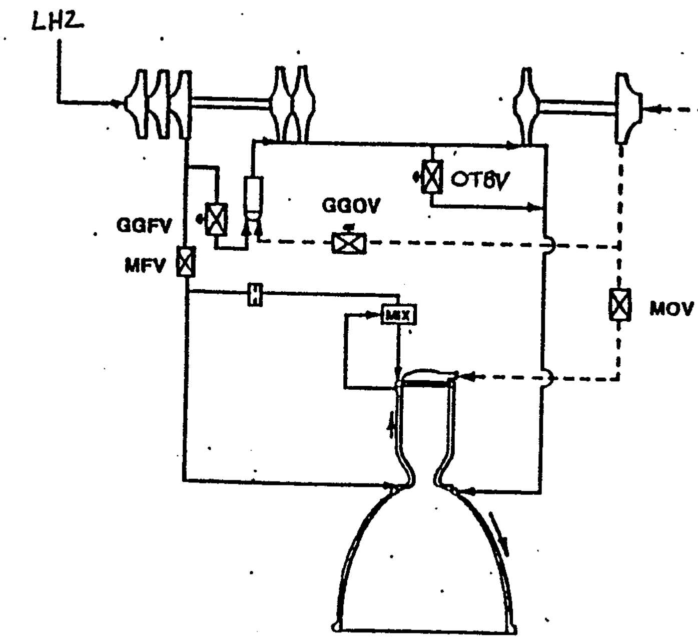
- $\epsilon = L/(t_o a)$ determines component dynamics, and
- each of the terms shown are non-dimensional.

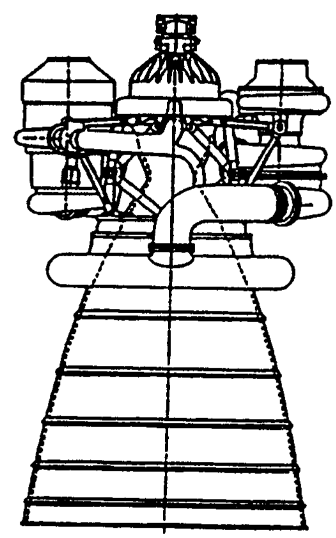
Use of Non-dimensionalization and Scaling

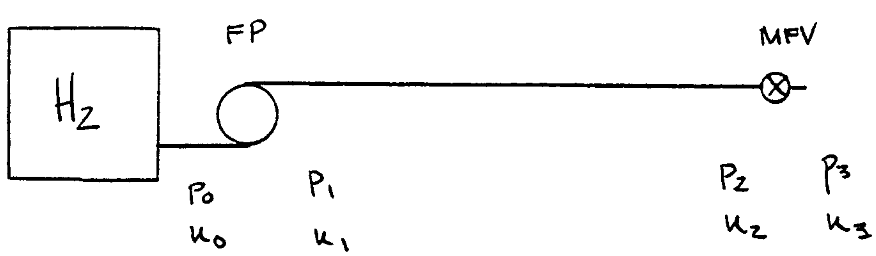
Approach Benefits:

1. Provides "rigorous" derivation of engine model as well handle for systematically adjusting model complexity.
2. Control-oriented: could aid in robustness analysis.
3. Monitoring-oriented: modeling for monitoring purposes?
4. Impacts development of "generic" models (e.g., ROCETS).
5. Numerically well conditioned for simulation purposes.

STME Fuel Side Model

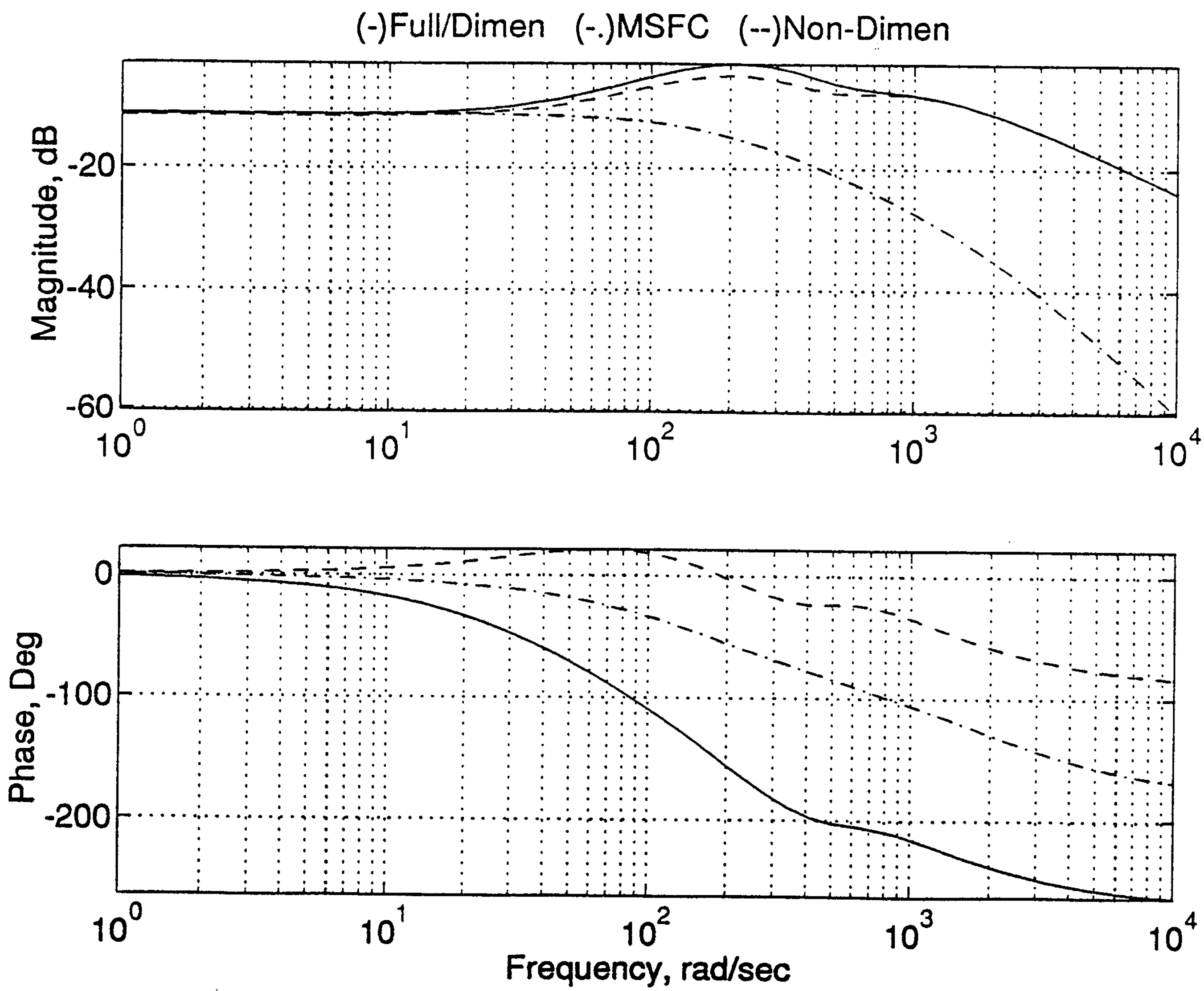






STME Fuel Side Model

	Pump	Duct	Valve	Totals
Full/Dimen.	C,M	C,M	C,M	6
MSFC	C,M	–	C	3
Non-Dimen.	C,M	C,M	–	4



Conclusions and Future Directions

- Further modeling studies.
  - Bond graph connections.
- Further sensitivity studies.
  - Detailed sensors suite studies for specified failure modes.
  - Overall monitoring specification study.
- Development of obsever-based filter for data reconstruction from reduced sensor set.

Modeling for Health Monitoring and Control with Application to the STME[1]

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Abstract

The Space Transportation Main Engine (STME) is a liquid hydrogen/oxygen, gas generator engine currently under development. A key feature of this engine is that it will be designed specifically for improved reliability and reduced cost of operation rather than increased performance and reduced weight. As a result, health monitoring and control functions will play a key role in the development of this engine system. A prerequisite for the analysis and design of health monitoring and control functions is the ability to adequately model the engine dynamics as well as the ability to understand how various off-nominal conditions of interest will effect engine performance. This paper will present the results of a simulation-based study of modeling issues for control and health monitoring of the STME. These results yield a set of modeling techniques which may be applied to general rocket engine systems in order to obtain dynamical models suitable for use in recently developed system-theoretic approaches to the design of integrated health monitoring and control systems.

[1]This work has been supported by the University of Cincinnati Space Engineering Research Center, the NASA Marshall Space Flight Center, and the NASA/ASEE Summer Faculty Fellowship Program.

1 Introduction

As NASA continues to develop various advanced propulsion technologies for space exploration two factors are becoming increasingly dominant in design specifications: increase operational reliability and decrease operational cost. No where is this trend more evident than in the development and design of the Space Transportation Main Engine (STME) [1, 4, 14]. One approach that has been proposed to meet these challenges is to incorporate into rocket propulsion systems, such as the STME, some sort of diagnostic/monitoring capability in the form of a Health Monitoring System (HMS).

HMS technology offers the promise of increased operational reliability through its ability to assess system performance, detect and isolate degradations and/or failures, and modify system operation so as to minimize effects on performance. Decreased operational costs are accrued by the use of the HMS technology to help automate inspection and checkout procedures and to help minimize maintenance activities by transitioning from a maintenance-on-routine basis to a maintenance-on-condition basis.

However, in order to fulfill this promise it is clear that HMS systems must be capable of performing these tasks when integrated as part of the overall rocket engine system. Recent research by the author [9, 10] has identified significant interaction effects which exist between control and health monitoring functions. The nature of these interaction effects is such that unless they are taken into account during the design phase, significant performance degradation may occur when the system is assembled. As a result, the authors have proposed an integrated approach to the analysis and design health monitoring and control systems (IHMCS).

This approach allows the designer to embed the IHMCS into a general system architecture wherein the wide array system analysis and design tools can be brought to bear. Within the this approach off-nominal conditions are modeled as indicated in Figure 1 and Figure 2. Here p denotes

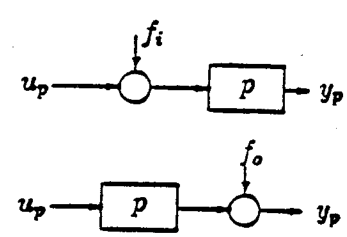

Figure 1: Signal type off-nominal conditions.

some nominal component or subsystem within the rocket engine.

In the first case (Figure 1), off-nominal conditions are represented by exogenous signals injected at either the component input or output. When off-nominal conditions of this sort are incorporated into the overall engine model, the analysis and design tasks can be reduced to problems in tracking and disturbance rejection [9, 10].

In the second case (Figure 2), off-nominal conditions are represented by exogenous component dynamics which may run in parallel or cascade with the nominal component. When off-nominal conditions of this sort are incorporated into the overall engine model, the analysis and design tasks can be reduced to problems in uncertainty accommodation and robustness [9, 10].

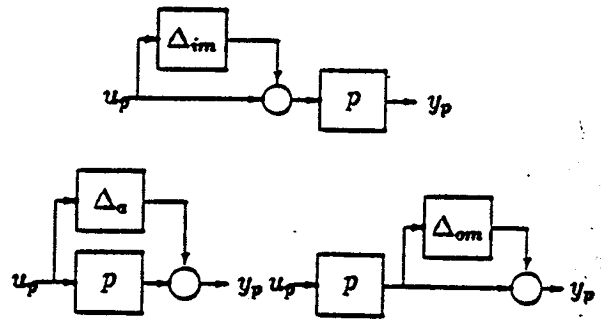

Figure 2: Uncertainty type off-nominal conditions.

In either case, it is clear that the approach discussed above is model-based. Thus, in order to apply these results it necessary to have access dynamical models for both the nominal rocket engine system and all off-nominal conditions of interest. Alternatively, and perhaps more favorable, would be to have access to a methodology for obtaining such models in a systematic way. It is the task of this paper to present just such a methodology.

This methodology is based on the application and manipulation of the fundamental laws of conservation in order to derive dynamical models for thermo-fluid systems such as those contained in chemical propulsion systems. It has a number of significant features which make it well suited for use addressing the problems encountered in IHMCS design: First, it allows the assembly of dynamical nominal and off-nominal engine models of low order by indicating the significant dynamics. Second, it allows us to distinguish between those off-nominal conditions which are likely to be modeled as signal type (Figure 1), and those likely to be modeled as uncertainty type (Figure 2). Third, it allows for the easy incorporation of various sensor and actuator types. Finally, it provides models in a format suitable for direct computer simulation.

The rest of this paper is organized as follows: In Section 2 the basic modeling principles discussed above are developed. In Section 3 a brief overview of the STME is given. In Section 4 the modeling principles developed in Section 2 are applied to the STME in order to obtain an engine model. Conclusions and discussion of future work is given in Section 5. Finally, an appendix provides a listing the symbols and notation used throughout the paper.

2 Development of Thermo-fluid Modeling Principles

Chemical propulsion systems such as the STME are basically thermo-fluid systems whose dynamics are governed by the laws of conservation of mass, momentum, and energy. Discussion of these basic laws of physics can readily be found in the literature (see, for example, [5, 6, 12]). In this section we apply these laws in order to develop a set of principles for model development by studying the dynamical behavior of the fluid in a generic engine component represented by the variable area control volume given in Figure 3. Here fluid is allowed to cross the boundary only at the inlet ($x = x_1$) and outlet ($x = x_2$). The results given here are based on the application of the conservation laws and concepts from perturbation theory for differential equations. These results are motivated by treatments given in [12, 8, 11]. The development below proceeds as follows: First, the basic laws of conservation are introduced. Next, the generic engine component dynamical equations are developed for the case where the fluid flow in the component is a liquid. Finally, the generic engine component dynamical equations are developed for the case where the fluid flow in

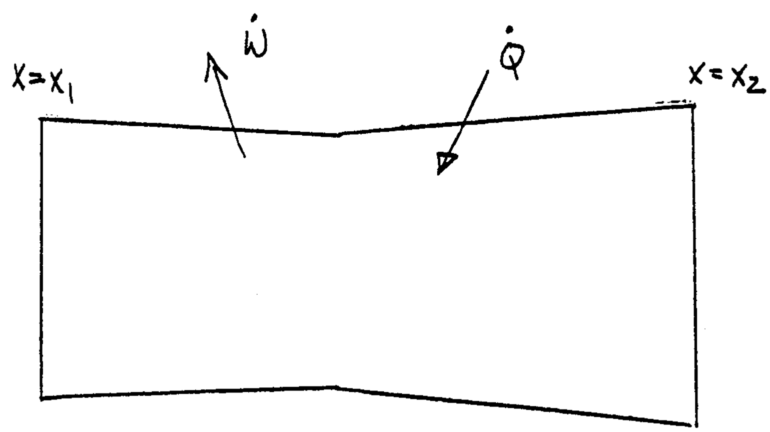

Figure 3: Generic engine component.

the component is a gas.

2.1 Conservation Laws

Considering the control volume depicted in Figure 3, it is well established that one can associate with the flow in this volume 4 state variables:

- $\mathbf{u}(\mathbf{z},t)$ - flow velocity
- $p(\mathbf{z},t)$ - pressure
- $T(\mathbf{z},t)$ - temperature
- $\rho(\mathbf{z},t)$ - density

These 4 variables are constrained to obey the fundamental conservation laws of physics, namely—conservation of mass, conservation of momentum, and conservation of energy. These laws yield 3 equations in 4 unknowns. A fourth equation, the equation of state, can be given which relates three of these four variables.[3] Thus under these conditions we have 4 equation in 4 unknowns which can be used to uniquely characterize the 4 state variables above.

The conservation laws discussed above are well known in the literature and are given below, with only brief explanation, for completeness. The various notation used here is summarized in Appendix A in Table 3. In addition, for the purposes of this section we let V and S denote, respectively, the volume and surface area of the control volume depicted in Figure 3.

The principle of conservation of mass is expressed can be expressed in integral form by the equation

$$\frac{\partial}{\partial t}\iiint_V \rho\, dV = -\iint_S \rho\mathbf{u}\cdot d\mathbf{S}\ .$$

Each of the terms in this expression can be given the following physical interpretations:

$$\begin{aligned}\frac{\partial}{\partial t}\iiint_V \rho\, dV &= \text{rate of increase of mass in } V\\ \iint_S \rho\mathbf{u}\cdot d\mathbf{S} &= \text{net mass flow across } S\end{aligned}$$

[3]The state equation can be mathematically derived for the case where ideal gas flows are considered. However, in the case of liquids, it is necessary to rely on experimental observations in obtaining an equation of state.

The principle of conservation of linear momentum can be expressed in integral form by the equation

$$\frac{\partial}{\partial t}\iiint_V \rho\mathbf{u}\, dV = -\iint_S (\rho\mathbf{u}\cdot d\mathbf{S})\mathbf{u} + \iiint_V \rho\mathbf{f}\, dV - \iint_S p\, d\mathbf{S} + \mathbf{F}_{\text{visc}}\,.$$

Each of the terms in this expression can be given the following physical interpretations:

$$\begin{aligned}
\frac{\partial}{\partial t}\iiint_V \rho\mathbf{u}\, dV &= \text{rate of increase of momentum in } V\\
\iint_S (\rho\mathbf{u}\cdot d\mathbf{S})\mathbf{u} &= \text{net momentum flow across } S\\
\iiint_V \rho\mathbf{f}\, dV &= \text{total body force in } V\\
\iint_S p\, d\mathbf{S} &= \text{total pressure force on } S\\
\mathbf{F}_{\text{visc}} &= \text{total friction force on } S
\end{aligned}$$

The principle of conservation of energy can be expressed in integral form by the equation

$$\frac{\partial}{\partial t}\iiint_V \rho\left(e+\frac{u^2}{2}\right) dV = -\iint_S \rho\left(e+\frac{u^2}{2}\right)\mathbf{u}\cdot d\mathbf{S} +$$
$$\dot{Q} + \dot{W}_{\text{shaft}} + \dot{W}_{\text{visc}} - \iint_S p\mathbf{u}\cdot d\mathbf{S} + \iint_V \rho(\mathbf{f}\cdot\mathbf{u})\, dV\,.$$

Each of the terms in this expression can be given the following physical interpretations:

$$\begin{aligned}
\dot{Q} &= \text{rate of heat added across } S\\
\dot{W}_{\text{shaft}} &= \text{rate of shaft work done in } V\\
\dot{W}_{\text{visc}} &= \text{rate of friction work done on } S\\
\iint_S p\mathbf{u}\cdot d\mathbf{S} &= \text{rate of work done on } S \text{ by pressure forces}\\
\iint_V \rho(\mathbf{f}\cdot\mathbf{u})\, dV &= \text{rate of work done in } V \text{ by body forces}\\
\frac{\partial}{\partial t}\iiint_V \rho\left(e+\frac{u^2}{2}\right) dV &= \text{rate of change of energy in } V\\
\iint_S \rho\left(e+\frac{u^2}{2}\right)\mathbf{u}\cdot d\mathbf{S} &= \text{rate of flow of energy across } S
\end{aligned}$$

2.2 Some Assumptions

The basic assumptions which are made for the purposes of the development below are given as follows:

1. We assume that the fluid flows in the engine are quasi-one-dimensional, i.e., flow properties are axisymmetric and uniform in any plane normal to the direction of flow,
2. the flow is inviscid, i.e., friction effects are assumed to be negligible,
3. body forces are negligible,

4. where the working fluid is in a gaseous state, it behaves as an ideal gas. Specifically, $p = \rho RT$, $e = c_v T$, and $h = c_p T$.

5. where the working fluid is in a liquid state, it behaves as an incompressible fluid, i.e., ρ =const.

Given these assumptions we now proceed to develop dynamical expressions characterizing the behavior of the fluid in the generic engine component of Figure 3.

2.3 Incompressible Flow

We begin with the case where the fluid flow is considered incompressible. It is well known in the literature that under this assumption, provided there is no heat transfer, only the mass and momentum equations need be considered [12, 13]. Hence the energy equation will not be needed here.

The assumption of incompressibility is especially reasonable in the front stages of a rocket engine system where the propellants are still in liquid phase.

Application of the principle of conservation of mass to the situation depicted in Figure 3 under the assumption of incompressibility proceeds as follows: Since ρ is a constant we have

$$\frac{\partial}{\partial t}\iiint_V \rho dV = \frac{\partial}{\partial t}(\rho V) = 0 ,$$

and since the inlet and outlet planes (1 and 2 resp.) of the control volume are normal to the flow direction (x-direction), and no flow is permitted to cross section the boundary of the control volume elsewhere, we have

$$\begin{aligned} -\iint_S \rho \mathbf{u} \cdot d\mathbf{S} &= -\left(\iint_{A(x_2)} \rho u(x,t) dS - \iint_{A(x_1)} \rho u(x,t) dS\right) \\ &= \rho\left(A(x_1)u(x_1,t) - A(x_2)u(x_2,t)\right) . \end{aligned}$$

Substituting these expressions into the mass equation we obtain

$$A(x_1)u(x_1,t) - A(x_2)u(x_2,t) = 0 . \tag{1}$$

Thus we see that for the case of incompressible flow the mass equation contributes only algebraically, and not dynamically, to the component model.

Application of the principle of conservation of momentum to the situation depicted in Figure 3 under the assumption of incompressibility is slightly more complicated then the development above. We begin by considering the left hand term from the momentum equation. Using the assumption on quasi-one-dimensional flow we obtain

$$\frac{\partial}{\partial t}\iiint \rho \mathbf{u} dV = \frac{\partial}{\partial t}\int_{x_1}^{x_2} A(x)\rho u(x,t)dx .$$

Applying the Integral Mean Value Theorem (IMVT) [12] we can further simplify this expression to obtain

$$\frac{\partial}{\partial t}\iiint \rho \mathbf{u} dV = (x_2 - x_1)\frac{\partial}{\partial t}\left(A(x^*)\rho u(x^*,t)\right)$$

for some point $x_1 \le x^* \le x_2$. Using standard arguments the terms on the right hand side of the momentum equation can be simplified to obtain

$$\begin{aligned} &-\iint_S (\rho \mathbf{u} \cdot d\mathbf{S})\mathbf{u} + \iiint_V \rho \mathbf{f}\, dV - \iint_S p\, d\mathbf{S} + \mathbf{F}_{\text{visc}} = \\ &\quad \rho u(x_1,t)^2 A(x_1) - \rho u(x_2,t)^2 A(x_2) + p(x_1,t)A(x_1) - p(x_2,t)A(x_2) - p_3(A(x_1) - A(x_2)) . \end{aligned}$$

Substituting into the momentum conservation equation one obtains the following differential equation

$$\begin{aligned}\frac{\partial}{\partial t}w(x^*,t) &= \frac{A}{x_2-x_1}\left[\rho u(x_1,t)^2\frac{A(x_1)}{A}-\rho u(x_2,t)^2\frac{A(x_2)}{A}+\right.\\ &\left. p(x_1,t)\frac{A(x_1)}{A}-p(x_2,t)\frac{A(x_2)}{A}-p_3\frac{A(x_1)-A(x_2)}{A}\right]\,. \end{aligned} \tag{2}$$

where $w = \rho u A$ denotes mass flow, p_3 pressure distribution denotes the surface of the control volume excluding the inlet and outlet, and A denotes the mean control volume cross sectional area.

2.4 Compressible Flow

By employing the same techniques as above we can develop a set of dynamical equations which characterize the behavior of the control volume depicted in Figure 3 when the assumption on incompressibility is not valid. Here we will in general need all three conservation laws: mass, momentum, and energy.

The assumption of compressibility is especially reasonable in the later stages of a rocket engine system where the propellants are still in gas phase.

The resulting mass, momentum and energy, respectively, equations are obtained

$$\frac{\partial\rho(x^*,t)}{\partial t} = \frac{A}{V}\left(\rho(x_1,t)u(x_1,t)\frac{A(x_1)}{A}-\rho(x_2,t)u(x_2,t)\frac{A(x_2)}{A}\right) \tag{3}$$

$$\begin{aligned}\frac{\partial w(x^*,t)}{\partial t} &= \frac{A}{x_2-x_1}\left(\rho(x_1,t)u(x_1,t)^2\frac{A(x_1)}{A}-\rho(x_2,t)u(x_2,t)^2\frac{A(x_2)}{A}+\right.\\ &\left. p(x_1,t)\frac{A(x_1)}{A}-p(x_2,t)\frac{A(x_2)}{A}-p_3\frac{A(x_1)-A(x_2)}{A}\right)\end{aligned} \tag{4}$$

$$\frac{\partial}{\partial t}\left[\rho\left(e(x^*,t)+\frac{u(x^*,t)^2}{2}\right)\right] = \frac{1}{V}(w(x_1,t)c_pT(x_1,t)-w(x_2,t)c_pT(x_2,t)+\dot{Q})\,. \tag{5}$$

2.5 Component Modeling

Equations 1–5 above characterize the behavior of the fluid flow in the generic engine component depicted in Figure 3. Direct application of these expression in modeling a rocket engine system would lead to a model order dynamic order $3N+2M$ where N denotes the number of engine components with liquid flow and M denotes the number of engine components with gas flow. However, dynamic models of lower order can be obtained by observing that depending on the physical attributes of the various engine components certain of these equations will tend to equilibrium much faster than others. These equations can be approximated by their steady state conditions without appreciably altering the dynamic response of the overall model, especially in the low frequency range.

To make this point more clear consider Equation 3 describing the dynamics associated with the mass equation for a compressible flow. It is clear from this expression that if the ratio of the mean area-to-volume for a given component is large, i.e., $A/V >> 1$, then the dynamics corresponding to this equation tend to equilibrium quickly. In such cases, the mass equation for the given component can be replaced by the algebraic equation

$$\rho(x_1,t)u(x_1,t)A(x_1) = \rho(x_2)u(x_2,t)A(x_2)\,, \tag{6}$$

and the mass equation does not contribute to the system's dynamic order.

Equation	*Dynamic Contribution*	*Algebraic Contribution*
Mass (Eqn 3)	$A/V << 1$	$A/V >> 1$
Momentum (Eqn 2,4)	$A/L << 1$	$A/L >> 1$
Energy (Eqn 5)	$1/V << 1$	$1/V >> 1$

Table 1: Engine component modeling conditions.

Similar arguments apply for the other dynamic equations developed above. The results are summarized below in Table 1 where $L = x_2 - x_1$.

The algebraic mass equation, Equation 6, which replaces the dynamic mass equation, Equation 3, under the conditions given in Table 1 was given above. The algebraic counterparts of the other equations are obtained similarly and are stated below for completeness. The algebraic counter parts of the momentum equations (2, 4) are given, respectively, by

$$\rho u(x_1,t)^2 A(x_1) - \rho u(x_2,t)^2 A(x_2) + p(x_1,t)A(x_1) - p(x_2,t)A(x_2) - p_3(A(x_1) - A(x_2)) = 0 \ . \quad (7)$$

and

$$\begin{aligned} &\rho(x_1,t)u(x_1,t)^2 A(x_1) - \rho(x_2,t)u(x_2,t)^2 A(x_2) + \\ &\qquad p(x_1,t)A(x_1) - p(x_2,t)A(x_2) - p_3(A(x_1) - A(x_2)) = 0 \ . \end{aligned} \quad (8)$$

The algebraic counter part of the energy equation (5) are given by the same expression

$$w(x_1,t)c_p T(x_1,t) - w(x_2,t)c_p T(x_2,t) + \dot{Q} = 0 \ . \quad (9)$$

Thus, based on the analysis given here, the relative sizes for A/V, A/L, and $1/V$ for a given engine component can be used to decide which dynamics are required to model that component, and which dynamics can be replaced by algebraic relationships. Moreover, as we shall see below these expressions can be applied to both nominal and off-nominal components to obtain the models required for IHMCS for the specific case of the STME.

3 Engine Overview

In this section we give some background information on the STME which will be useful in the next section when we apply the modeling principles developed above.

The STME is a liquid hydrogen/oxygen, gas generator engine currently under development to power the National Launch System (NLS). The NLS is comprised of a family of earth-to-orbit launch vehicles designed to meet the needs of future commercial, civil, and military payloads. This engine/vehicle system will provide the following unique capabilities [1, 4, 14]:

- It will support a wide range of payloads.
- It will provide more operability (as measured by reliability, launch on schedule, payload exchange, launch call-up and surge, launch configurability, etc.) than existing systems.
- It will provide a significant reduction in per-flight costs.
- It will be evolutionary (initially unman-rated and expendable, ultimately man-rated and reusable).

As noted above a key feature of this launch system is that it will be designed specifically for improved reliability and reduced cost of operation rather than increased performance and reduced weight. As a result, it is envisioned that health monitoring and control functions will play a key role in the development.

3.1 Engine System Description

Figure 4 provides a preliminary indication of the physical layout of the STME to scale [1, 4].

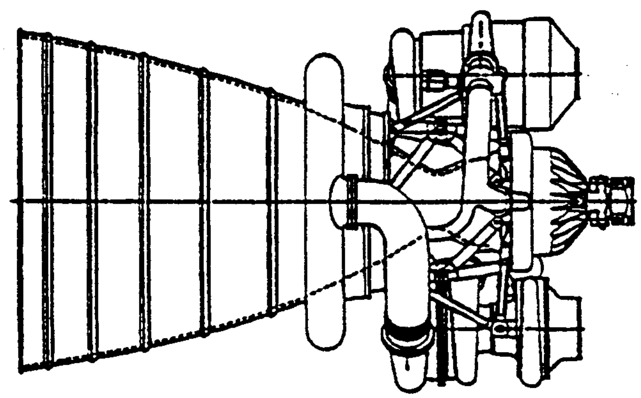

Figure 4: STME physical layout.

Figure 5 provides a schematic diagram of the engine as well as expected pressure, temperature, and flow levels for selected sites at 100% RPL [1, 4].

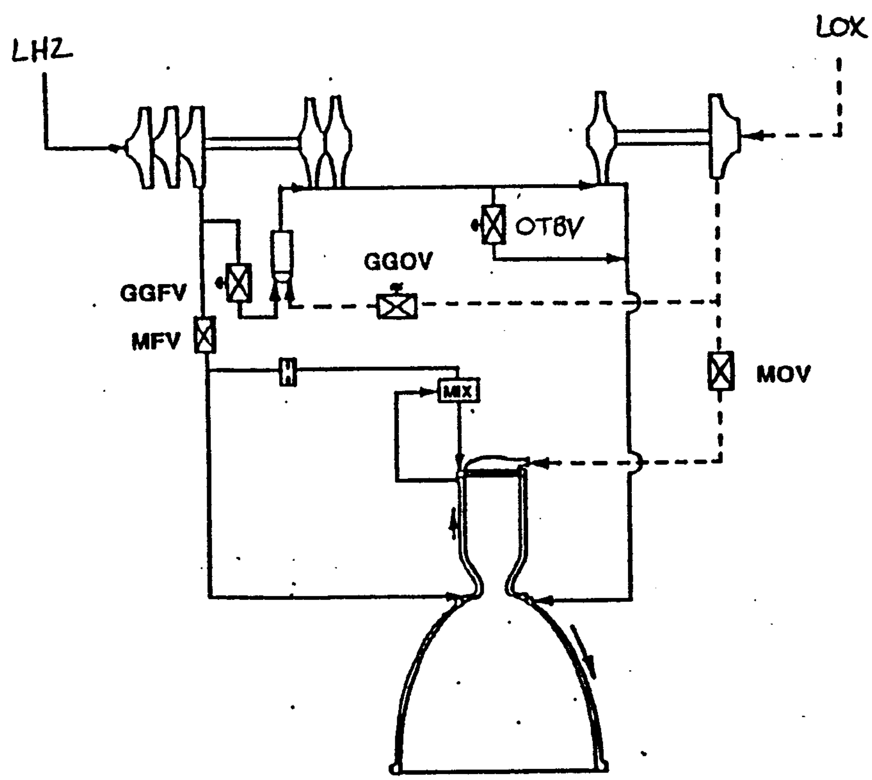

Figure 5: STME flow schematic.

A detailed list of operating specifications/characteristics are provided in Table 2 below [1, 4].

Thrust	553Klb±2%
Throttleability	70%, 100%
Mixture Ratio	6.0±1.5%
Specific Impulse	364.6sec±0.7%

Table 2: STME operating specifications.

4 Development of STME Model

Using the modeling concepts described in Section 2, a dynamic model of the STME has been constructed. Based on the schematic diagram given in Figure 5 we divide the STME up into four major assemblies (Main Combustion Chamber, Fuel Turbo-pump, Oxidizer Turbo-pump, and Gas Generator Chamber) each consisting of one or more components. The discussion of modeling issues for each of these components is handled separately below.

4.1 Fuel Turbo-Pump Assembly

We begin with the Fuel Turbo-pump Assembly (FTPA). For the purposes of modeling we have split this assembly into three primary components: the pump (FP) , the turbine (FT), the turbo-pump shaft, and the main valve (MFV) and adjacent duct work. Each of these is discussed individually below:

The turbo-pump shaft model clearly does not involve fluid dynamics. Its dynamics are governed by a torque balance relationship defined by the torque produced by the turbine and the torque required by the pump.

The pump is modeled assuming the flow is in liquid state. Hence Equations 1, 2 and their algebraic counterparts are employed. Based on the relative sizing indicated in Figure 4, both dynamic mass and momentum equations are employed.

The main valve is also modeled assuming liquid flow. Again using the relative sizing indicated in Figure 4, both dynamic mass and momentum equations are employed.

The turbine is modeled assuming the flow is in gas state. Hence Equations 3, 4, 5 and their algebraic counterparts are employed. Again using the relative sizing indicated in Figure 4, both dynamic mass and energy equations are employed. Momentum dynamics are taken to be fast relative to these and so are modeled algebraically.

4.2 Oxidizer Turbo-Pump Assembly

Next we consider the Oxidizer Turbo-pump Assembly (OTPA). As with the FTPA we have split this assembly into three primary components: the pump (OP), the turbine (OT), the turbo-pump shaft, and the main valve (MOV) and adjacent ducting. Additionally, we include the bypass valve (OBPV) and its adjacent ducting.

As above turbo-pump shaft model does not involve fluid dynamics and governed by a torque balance relationship.

The pump is modeled assuming the flow is in liquid state, and both dynamic mass and momentum equations are employed.

The main valve is also modeled assuming liquid flow. However, since its associated duct work is relatively minor and so it is modeled using algebraic mass and momentum equations.

The turbine is modeled assuming the flow is in gas state. As with the fuel turbine both dynamic mass and energy equations are employed in modeling this component. Momentum dynamics are taken to be fast relative to these and so are modeled algebraically.

Due to the small relative size of the bypass valve structure, no dynamics are associated with it and algebraic equations are used to model its effects.

4.3 Gas Generator Chamber Assembly

Next we consider the Gas Generator Chamber Assembly (GGCA). For the purposes of modeling we have split this assembly into three primary components: the fuel valve/injector and ducting, the oxidizer valve/injector and ducting, and the combustion chamber. Each of these is discussed individually below:

The fuel valve/injector and ducting is modeled using the liquid flow equations. The mass equation is modeled dynamically and the momentum equation is modeled algebraically.

Likewise, the oxidizer valve/injector and ducting is modeled using the liquid flow equations. As above, the mass equation is modeled dynamically and the momentum equation is modeled algebraically.

The combustion chamber is modeled using the gas flow equations. Dynamic mass and energy and algebraic momentum equations are employed.

4.4 Main Combustion Chamber Assembly

Next we consider the Main Combustion Chamber Assembly (MCCA). For the purposes of modeling we have split this assembly into three primary components: the fuel injector and ducting, the oxidizer injector and ducting, and the combustion chamber. Each of these is discussed individually below:

The fuel injector and ducting are modeled using the liquid flow equations. The mass equation is modeled dynamically and the momentum equation is modeled algebraically.

Likewise, the oxidizer valve/injector and ducting is modeled using the liquid flow equations. As above, the mass equation is modeled dynamically and the momentum equation is modeled algebraically.

The combustion chamber is modeled using the gas flow equations. Dynamic mass and energy and algebraic momentum equations are employed.

4.5 Modeling Results

Based on the discussions above a 18 state dynamical model of the STME was constructed and implemented in both MARSYAS [2] and SIMULINK [3]. MARSYAS is a simulation software package in use at the NASA Marshall Space Flight Center and SIMULINK is a menu driven simulation package which has been almost seemlessly integrated with MATLAB and all of its toolboxes.

Though space does not permit the inclusion of the full model here. software copies may be obtained directly from the authors. This model represents the open-loop engine and, as such, does not include any control algorithms. We note, however, that some actuator and sensor dynamics are available to be included in the model.

This simulation was run against the smallest order dynamic STME simulation available at MSFC, a 23 state MARSYAS simulation, and was found to have good fidelity at low frequencies and to track with zero error all steady state values.

The plots in Figures 6 and 7 given an indication of the kind of analysis that for which this model has been used. These plot show the frequency response for the transfer function from fuel and oxidizer, respectively, pump inlet pressure to engine thrust for the STME simulation linearized about 100% RPL values. These plots indicate that the thrust will have a significant response to variations in the pump inlet pressures. Since such variations are likely to occur as a result of acceleration during ascent, this indicates a the need for some type of control authority in order to meet the stringent specifications outlined in Section 3.

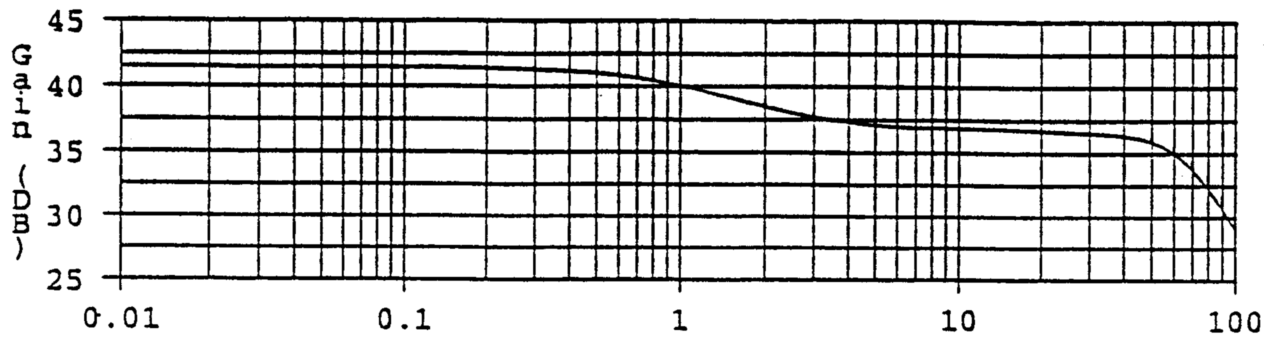

Figure 6: Fuel pump inlet pressure to thrust frequency response.

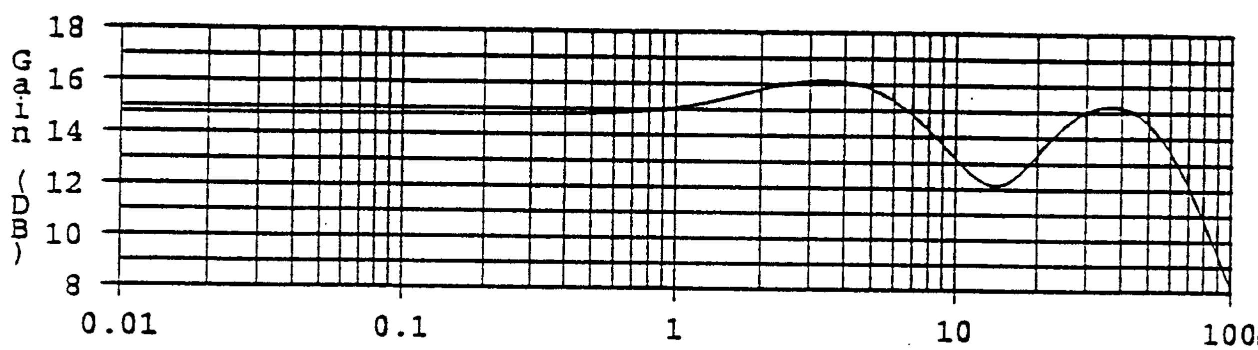

Figure 7: Oxidizer pump inlet pressure to thrust frequency response.

5 Conclusions and Future Work

In this paper we have described a methodology for deriving dynamic models for rocket engine systems. This methodology is based on the basic conservation laws from physics. In addition, by incorporating concepts from the area of singular perturbation theory, this methodology allows for the assembly of low order, yet dynamically accurate model of rocket engine systems.

This methodology has a number of significant features which make it well suited for use addressing the problems encountered in IHMCS design: First, it allows the assembly of dynamical nominal and off-nominal engine models of low order by indicating the significant dynamics. Second, it allows us to distinguish between those off-nominal conditions which are likely to be modeled as signal type (Figure 1), and those likely to be modeled as uncertainty type (Figure 2). Third, it allows for the easy incorporation of various sensor and actuator types. Finally, it provides models in a format suitable for direct computer simulation.

Future work in this area is currently directed in two areas. The first area is to make use of the models obtained using process outlined herein for analysis and design purposes. Issues of interest include: design studies for rocket engine control and monitoring, analysis of effects on engine dynamics of off-nominal conditions, the development of observer-based filters for engine data reconstruction using a reduced sensor set, sensor suite selection studies, etc. The second area is to improve on the modeling methods described herein. One improvement to which has recently been proposed in the literature merge the approach described here with the methods of non-dimensionalization and scaling [12, 7]. Two primary advantages of such an approach would be: (i) By using non-dimensional equations key parameters could be isolated which determine the dynamic nature of the physical phenomena thereby ensuring a good dynamic model. For example, the size of Mach numbers and Reynolds numbers have long been known to be correlated to the dominance of certain dynamic phenomena in fluid systems. Moreover, the use of non-dimensional equations would make decisions about which dynamics to retain independent of the unit system chosen. (ii) Also, the proper use of scaling techniques would result in dynamic models with good numerical properties relative to simulation.

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A Notation

We define here some of the standard notation which is used throughout this paper.

Symbol	*Variable*
p	pressure
e	internal energy
h	enthalpy
ρ	density
T	temperature
u	flow velocity
w	mass flow rate
A, S	area
V	volume
Q	heat

Table 3: General variable definitions.