

# Acceleration-Augmented LQG Control of an Active Magnetic Bearing

Joseph J. Feeley

Department of Electrical Engineering and Microelectronics Research Center  
University of Idaho  
Moscow ID 83843

*Abstract* - A linear-quadratic-gaussian (LQG) regulator controller design for an acceleration-augmented active magnetic bearing (AMB) is outlined. Acceleration augmentation is a key feature in providing improved dynamic performance of the controller. The optimal control formulation provides a convenient method of trading-off fast transient response and force attenuation as control objectives.

## 1 Introduction

Active magnetic bearings (AMB) are being considered for an increasing number of applications. In particular, their simplicity and reliability offer many advantages for space applications. For example, magnetic bearings and actuators are used exclusively in a space-borne cryogenic cooler developed by Philips Laboratories for NASA [1].

In many applications it is desirable that forces originating on the moving shaft be attenuated as much as possible before being transmitted to the bearing. In space applications this is especially important since these forces may cause undesirable motion of the host satellite. Careful mechanical design and various counterbalancing schemes have been used to achieve this objective. In addition, the flexibility afforded by AMB control can also play an important role in this regard. While the primary objective of the AMB control system is to keep the shaft centered in the bearing, active control can also be used to dynamically adjust bearing stiffness to attenuate force transmittal from the shaft to the bearing. In this context the AMB control system design becomes an optimization problem, trading-off transient response in shaft centering with force attenuation. The techniques of modern automatic control theory are well-suited to solving this design problem.

## 2 Acceleration-Augmented Design

At first glance, the most direct method of dealing with shaft forces appears to be to measure the acceleration of the shaft and use that signal for explicit force cancellation in the AMB controller. Unfortunately, it is seldom practical or economical to measure shaft acceleration directly; shaft position is usually the only measurement available for AMB control. It is also generally impractical to compute shaft acceleration by twice differentiating position because of the noise associated with the position measurement. Neither is it generally possible to use observer theory directly to compute acceleration because acceleration is not a state variable in traditional shaft dynamic models. However, if the shaft disturbance force is modeled as a random process, the resulting state variable model permits estimation of the shaft

acceleration in the context of a Kalman filter. The estimated acceleration may then be used in a stochastic optimal control scheme based on the separation principle. This procedure is outlined below.

A complete set of model equations for a magnetic bearing with eddy currents in the actuator has been previously developed [2]. Only a simplified form of the equations of shaft dynamics are required to illustrate the acceleration-augmented control scheme suggested here. The simplified equations of motion of the shaft, in state variable form, are

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{m}(u_c + x_3)\end{aligned}$$

where

$$\begin{aligned}x_1 &= \text{position} \\ x_2 &= \text{velocity} \\ x_3 &= \text{disturbance force} \\ u_c &= \text{control force} \\ m &= \text{mass of shaft}\end{aligned}$$

To a first order approximation, it is quite reasonable to model the disturbance force as a random walk

$$\dot{x}_3 = w$$

where

$$w = \text{white noise process with power spectral density } Q$$

Assuming that the position measurement is corrupted by additive white noise, the measurement equation becomes

$$z = x_1 + v$$

where

$$v = \text{white noise process with power spectral density } R$$

The foregoing equations can be written in standard state variable form as

$$\dot{\hat{x}} = Fx + G_c u_c + G_d w$$

$$z = Hx + v$$

The usual LQG techniques employing the separation principle can now be used to find the steady state gains for the optimal controller and estimator.

The optimal control is found to be

$$u_c = -K_c \hat{x}$$

where

$$\begin{aligned}\hat{x} &= \text{state estimate} \\ K_c &= \text{controller gain}\end{aligned}$$

The controller gain is given by

$$K_c = B^{-1} G_c^T S$$

where  $S$  is the steady-state solution to the matrix Riccati equation

$$\dot{S} = -SF - F^T S + SG_c B^{-1} G_c^T S - A$$

and

$A =$  performance index weighting on states

$B =$  performance index weighting on control input

The controller gain and the bearing stiffness can be adjusted by changing the performance index weighting on the states and the control effort.

The optimal state estimates are the solutions to

$$\dot{\hat{x}} = F\hat{x} + G_c u + K_e(z - H\hat{x})$$

where the optimal estimator gain is

$$K_e = PH^T R^{-1}$$

and  $P$  is the steady state solution of the matrix Riccati equation

$$\dot{P} = FP + PF^T + G_d Q G_d^T - PH^T R^{-1} HP$$

To the extent that the controller model is accurate this control scheme provides optimal AMB control. The relative importance the controller places on centering control versus force attenuation is controlled by the relative magnitudes of the  $A$  and  $B$  matrices, respectively.

### 3 Results

The results of applying this acceleration-augmented design approach to the simplified magnetic bearing model are described below. It is instructive to use the simplified model because it yields closed form solutions [3] which provide considerable insight into the problem. Assuming unit mass and direct measurement of position only, the relevant matrices to characterize the system are

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_c = [0 \ 1 \ 0]^T$$

$$G_d = [0 \ 0 \ 1]^T$$

$$H = [1 \ 0 \ 0]$$

$$Q = q$$

$$R = r$$

Under these conditions the Kalman gain becomes [3]

$$K_e = [2(\sigma)^{1/6} 2(\sigma)^{1/3} (\sigma)^{1/2}]$$

8.3.4

where  $\sigma = \frac{q}{r}$  is a measure of the system signal to noise ratio. The estimator equations then become

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2 + 2(\sigma)^{1/6}(z - \hat{x}_1) \\ \dot{\hat{x}}_2 &= \hat{x}_3 + u + 2(\sigma)^{1/3}(z - \hat{x}_1) \\ \dot{\hat{x}}_3 &= (\sigma)^{1/2}(z - \hat{x}_1)\end{aligned}$$

Since the state is uncontrollable, the optimal control problem is solved by considering only states and in the performance index. With unit weighting on the position, zero weighting on the velocity and weighting on the control input, the weighting matrices become

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \frac{1}{c}$$

The optimal gain is found to be [3]

$$K_c = [c (2c)^{1/2} \ 1]$$

A block diagram of the closed-loop system is shown in Figure 1.

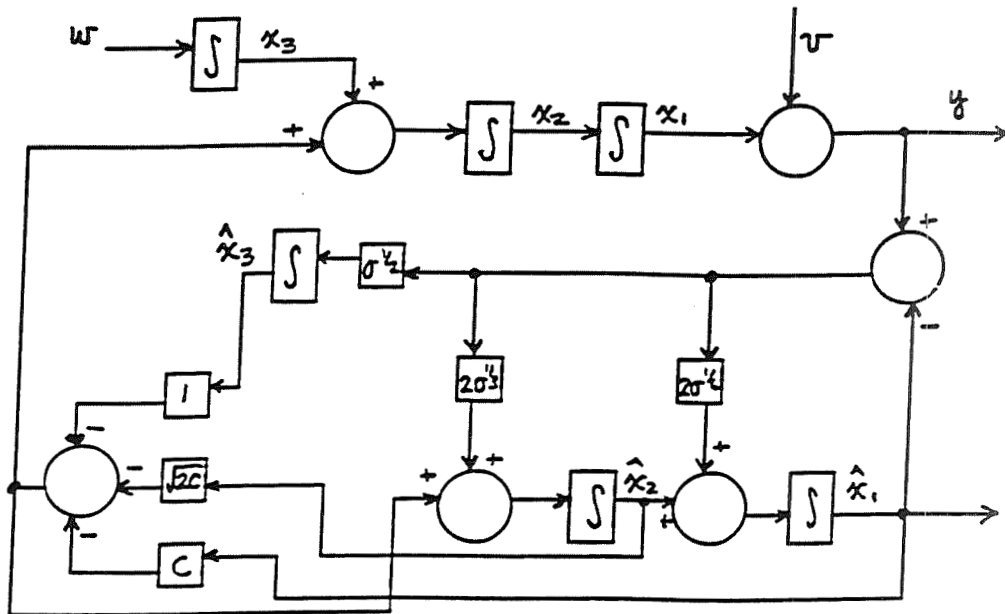


Figure 1: Block diagram of acceleration-augmented AMB

The closed-loop system has eigenvalues at

$$s_1 = 0$$

$$s_{2,3} = -\frac{(2c)^{1/2}}{2}(1 \pm j)$$

$$s_4 = -(\sigma)^{1/6}$$

$$s_6 = -(\sigma)^{1/6} \left( \frac{1}{2} \pm \frac{(3)}{2} \right)$$

It is clear from the above that the control effort weight  $c$  and the signal to noise ratio  $\sigma$  can be used as design parameters to adjust system performance.

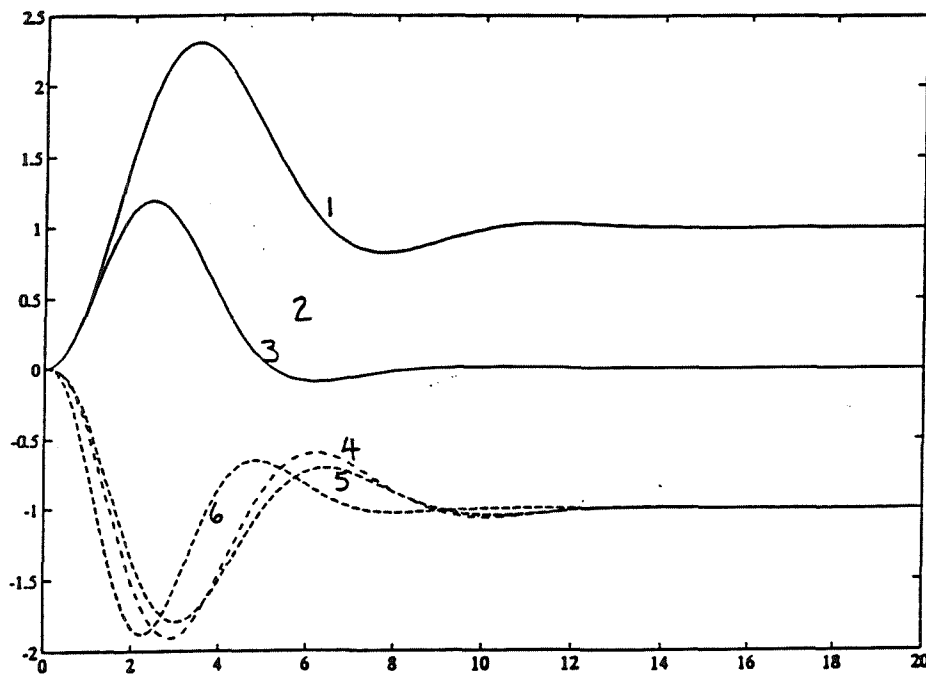


Figure 2: (1) Position response, no augmentation. (2) Position response with augmentation. (3) Position response with augmentation and large  $\sigma$ . (4) Control effort, no augmentation. (5) Control effort with augmentation. (6) Control effort with augmentation and large  $\sigma$ .

The effects of acceleration-augmented control and the use of the design parameters is illustrated in Figure 2. Traces 1, 2, and 3 show the response of shaft position to an impulsive input disturbance (unit step increase in disturbance force). Trace 1 is for the system without acceleration-augmentation, while traces two and three show the response of the system with acceleration-augmentation included. Not surprisingly, acceleration-augmentation reduces the error in desired shaft position to zero since (somewhat surprisingly) for this simple case acceleration augmentation has the effect of introducing an integral control mode. Traces 2 and 3 show the effect of increasing the signal to noise ratio. Increasing  $\sigma$  to 10 moves the

estimator poles away from the origin and reduces the response time of the system. The control effort associated with each of these responses is shown in traces 4,5, and 6, respectively. Notice that not only does the system respond faster with acceleration-augmentation and large  $\sigma$ , but it also uses less control effort as shown by trace 6.

## 4 Conclusions and Future Work

This paper shows that previously used "force control" schemes [4] are readily explained in terms of the stochastic control of a properly augmented linear dynamic model of a magnetic bearing system. Performance is readily modified to meet the desired trade-off between transient centering control and force attenuation by adjustment of control system design parameters. Nonlinear simulation studies are underway to quantify the performance improvement afforded by acceleration-augmented LQG AMB control. Efficient digital implementation of the controller is also under investigation.

## References

- [1] F. Stolfi et. al., "Design and Fabrication of a Long-Life Stirling Cycle Cooler for Space Application," Philips Laboratories Report, March, 1983.
- [2] J.J. Feeley and D.J. Ahlstrom, "A New Eddy Current Model for Magnetic Bearing Control System Design," Fourth NASA Symposium on VLSI Design, October, 1992.
- [3] B. Friedland, *Control Systems Design*, McGraw-Hill Book Company, pp. 451-455, 1986.
- [4] H.M. Chen and M.S. Darlow, "Magnetic Bearing With Rotating Force Control," Transactions of the ASME, Journal of Tribology, Vol. 10, pp. 100-105, January, 1988.