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Evaluation of a Nonlinear Method for the Enhancement of Tonal Signal Detection

Donald P. Garber

Lockheed Engineering & Sciences Company
Hampton, Virginia

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National Aeronautics and
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Langley Research Center
Hampton, Virginia 23681-0001

Abstract

A method is presented for biasing spectral estimates to enhance detection of tonal signals against a background of broadband noise. In this method, a nonlinear average of an ensemble of individual spectral estimates is made where broadband noise energy is biased downward, pure tone energy is unbiased, and a mixture of the two is biased by an amount that depends on the ratio of tonal energy to broadband energy. The method is analyzed to provide estimates of the extent of tonal signal detection enhancement.

Symbol List

C	contrast of adjacent spectral bands
C_q	apparent contrast of adjacent spectral bands exhibited by W_q
E_0	expectation operator for density function of noise
E_1	expectation operator for density function of noise plus tonal signal
E_2	expectation operator for density function of tonal signal
f_0	density function of single spectral estimate of noise
f_1	density function of single spectral estimate of noise plus tonal signal
f_2	density function of single spectral estimate of tonal signal
F_0	cumulative function of single spectral estimate of noise

F_1	cumulative function of single spectral estimate of noise plus tonal signal
$\bar{f}_0$	density function of biased spectral estimate of noise
$\bar{f}_1$	density function of biased spectral estimate of noise plus tonal signal
$\bar{f}_2$	density function of biased spectral estimate of tonal signal
$\hat{f}_0$	density function of ensemble averaged spectral estimate of noise
$\hat{f}_1$	density function of ensemble averaged spectral estimate of noise plus tonal signal
$\hat{f}_2$	density function of ensemble averaged spectral estimate of tonal signal
${}_1F_1$	degenerate hypergeometric function
G	arbitrary function
I_ν	modified Bessel function
m	number of spectral estimates in an ensemble
$P(D)$	probability of detection
$P(FA)$	probability of false alarm
q	power of a power function

R	signal–to–noise ratio
R_q	apparent signal–to–noise ratio exhibited by W_q
$\bar{R}$	apparent detection signal–to–noise ratio from biased spectral estimate processing gain
$\hat{R}$	apparent detection signal–to–noise ratio from ensemble average processing gain
T	detection threshold
$\bar{T}$	detection threshold of biased spectral estimate
$\hat{T}$	detection threshold of ensemble averaged spectral estimate
W	statistic composed of an inverse function of the average of an arbitrary function of an ensemble of single spectral estimates
W_q	statistic composed of the q^{th} root of the average of the power function, of power q, of an ensemble of single spectral estimates
W_1	statistic composed of the arithmetic mean of an ensemble of single spectral estimates

W_0	statistic composed of the geometric mean of an ensemble of single spectral estimates
W_{-1}	statistic composed of the harmonic mean of an ensemble of single spectral estimates
$W_{-\infty}$	the limit of W_q as $q \to -\infty$, equivalent to the first order statistic of an ensemble of single spectral estimates
x	spectral estimate consisting of the power spectrum, or squared magnitude of the Fourier transform, at a single frequency
x_i	i^{th} spectral estimate of an ensemble
$x_{(1)}$	first order statistic
$\mathbf{x}_m$	ensemble of m independent single spectral estimates in a frequency band
$\mathbf{x}_{(m)}$	ensemble of m ordered single spectral estimates in a frequency band
γ	Euler's constant
Γ	gamma function
δ	Dirac delta function
ζ	dummy variable of integration
η	dummy variable of integration

ξ	root of false alarm equation
σ_n^2	noise power
σ_s^2	tonal signal power

Introduction

Spectral estimates for helicopter acoustic signatures are generally made by calculating a set (or ensemble) of presumably independent power spectra using a finite Fourier transform and then linearly averaging those spectra. By averaging, the uncertainty of the spectral estimate is reduced, and a peak (tone) in the spectrum is made more distinguishable from the random background noise. Although the process of averaging individual spectra is linear (in terms of squared pressure), the individual spectra and the average spectrum are biased in the region of a tone. Despite the bias associated with a finite Fourier transform estimate of a spectrum, the process of linearly averaging a set of spectra introduces no further bias. Also, by reducing uncertainty, the average provides a better estimate of spectral levels than an individual spectrum.

By assuming that a random process is essentially broadband in nature (and avoiding the bias associated with a finite Fourier transform by letting the integration time approach infinity), it is possible to derive the probability density function of a single spectral estimate

$$f_0(x) = \left(\frac{1}{\sigma_n^2}\right) e^{-x/\sigma_n^2}$$

where x is the spectral estimate (in terms of squared pressure) in a frequency band, and σ_n^2 is the portion of broadband (noise) power in that band. It is also possible to derive the probability density function of a single spectral estimate of a composite process composed of both sinusoidal (tonal) and broadband (noise) terms

$$f_1(x) = \left(\frac{1}{\sigma_n^2}\right) e^{-[x+\sigma_s^2]/\sigma_n^2}\ \mathrm{I}_0\left(\frac{2\sigma_s\sqrt{x}}{\sigma_n^2}\right)$$

where $\sigma_s = P^2/2$ is the power of a sinusoidal signal of pressure amplitude P, and $\mathrm{I}_0()$ is the zero order modified Bessel function *[1]*. When the signal-to-noise ratio becomes very large, the probability density function approaches that of a sinusoidal signal without noise

$$f_2(x) = \delta\left(x - \sigma_s^2\right)$$

where $\delta()$ is the Dirac delta function.

The different functional forms of the noise probability density and the signal-plus-noise probability density suggest that it may be possible to enhance the difference in their statistical behavior to provide a better apparent signal-to-noise ratio at the expense of generating biased spectral estimates. Specifically, an estimation method that tends to emphasize smaller values of an ensemble of individual spectra, rather than equally weighting all spectra in a spectral average, should bias a pure noise spectrum more than one composed of both tonal signal and noise. Ideally, the estimate should be completely unbiased for a sinusoidal signal with no noise.

A Nonlinear Estimate Method

A direct method to form a biased spectral estimate is to evaluate the inverse function of an averaged function of the individual spectra

$$W = G^{-1}\left(\frac{1}{m}\sum_{i=1}^{m} G(x_i)\right)$$

where x_i is the i^{th} member of an ensemble of m independent spectral estimates in a particular frequency band

$$\mathbf{x}_m = \{x_1, x_2, \ldots, x_m\}$$

A power function

$$G(x) = x^q$$

where $q \leq 1$, forms a relatively simple statistic

$$W_q = W_q(\mathbf{x}_m) = \left(\frac{1}{m}\sum_{i=1}^{m} x_i^q\right)^{\frac{1}{q}}$$

with a very useful characteristic, namely, that $W_{q_1} < W_{q_2}$ if $q_1 < q_2$ and the x_i are not all equal. For $q = 1$, W_1 is the arithmetic (unbiased) mean, as $q \to 0$, W_0 approaches the geometric mean, and for $q = -1$, W_{-1} is the harmonic mean. Each frequency band in an ensemble of spectra is processed independently of the others so that the collection of frequency bands so averaged forms an ensemble averaged spectral estimate.

Analysis of the Method

Unbiased Estimate

The expected value of a linear ensemble average (unbiased) spectral estimate for broadband noise alone is

$$E_0[W_1] = E_0[x] = \int_0^\infty x f_0(x)dx = \sigma_n^2$$

For signal plus noise, the expected value is

$$E_1[W_1] = E_1[x] = \int_0^\infty x f_1(x) dx = \sigma_n^2 + \sigma_s^2$$

and for signal alone, the expected value is

$$E_2[W_1] = E_2[x] = \int_0^\infty x f_2(x) dx = \sigma_s^2$$

Indeed, for signal alone, the joint probability density function of m independent spectra is given by

$$f_2(\mathbf{x}_m) = \prod_{i=1}^{m} \delta\left(x_i - \sigma_s^2\right)$$

so that the expected value for signal alone of any estimate W_q is given by

$$E_2[W_q] = \int_0^\infty W_q(\mathbf{x}_m) f_2(\mathbf{x}_m) d\mathbf{x}_m = \sigma_s^2$$

which indicates that W_q is, as desired, an unbiased estimator for any value of q when a signal is present without noise.

Geometric Mean

To determine the effect of the biased estimator, W_q, on the signal–to–noise ratio when $q \neq 1$, the expected values of the estimator for noise and signal–plus–noise must be compared with the expected values of an unbiased estimator, W_1. The simplest analytical case is $q = 0$. Because W_0 is the geometric mean, it can be expressed as

$$W_0(\mathbf{x}_m) = \sqrt[m]{x_1 x_2 \ldots x_m}$$

so that the expected value for noise only is

$$E_0[W_0] = \{E_0\left[\sqrt[m]{x}\right]\}^m = \sigma_n^2\left[\Gamma\left(1+\frac{1}{m}\right)\right]^m$$

and the expected value for signal plus noise is

$$\begin{aligned} E_1[W_0] &= \{E_1\left[\sqrt[m]{x}\right]\}^m \\ &= \sigma_n^2\left[\Gamma\left(1+\frac{1}{m}\right)\right]^m\left[{}_1F_1\left(-\frac{1}{m},1,-R\right)\right]^m \end{aligned}$$

where the signal-to-noise ratio is $R = \sigma_s^2/\sigma_n^2$, and the degenerate hypergeometric function is

$${}_1F_1(\alpha,\beta,z) = 1 + \left(\frac{\alpha}{\beta}\right)\frac{z}{1!} + \left(\frac{\alpha}{\beta}\right)\left(\frac{\alpha+1}{\beta+1}\right)\frac{z^2}{2!} + \ldots$$

which must be evaluated numerically.

The signal-to-noise ratio of an unbiased estimate can be expressed as

$$R = \frac{E_1[W_1]}{E_0[W_1]} - 1 = \frac{\sigma_s^2}{\sigma_n^2}$$

so the apparent signal-to-noise ratio of a biased estimate should be

$$R_q = \frac{E_1[W_q]}{E_0[W_q]} - 1$$

For $q = 0$, the apparent signal-to-noise ratio is then

$$R_0 = \frac{E_1[W_0]}{E_0[W_0]} - 1 = \left[{}_1F_1\left(-\frac{1}{m},1,-R\right)\right]^m - 1$$

which depends only on the unbiased signal-to-noise ratio, R, and the number of independent spectra, m, included in the estimate.

Harmonic Mean

A more involved analysis is required for the case of $q = -1$. For noise alone, the joint probability density function of m independent spectra is given by

$$f_0(\mathbf{x}_m) = \sigma_n^{-2m} e^{-(\sum_{i=1}^{m} x_i)/\sigma_n^2}$$

The expected value for noise alone of an estimate W_{-1} is given by

$$E_0[W_{-1}] = \int_0^\infty W_{-1}(\mathbf{x}_m) f_0(\mathbf{x}_m) d\mathbf{x}_m$$

Substituting the appropriate expressions in the equation above gives

$$E_0[W_{-1}] = m\sigma_n^2 \int_0^\infty \left(\sum_{i=1}^{m} z_i^{-1} \right)^{-1} e^{-(\sum_{i=1}^{m} z_i)} \; dz_1 dz_2 \ldots dz_m$$

Only for the simplest non-trivial case, $m = 2$, can a closed form solution be derived

$$E_0[W_{-1}] = 2\sigma_n^2 \left(\frac{1}{3} \right)$$

No attempt was made to analyze signal-plus-noise.

First Order Statistic

If an ensemble of independent spectral estimates in a specified frequency band is given by

$$\mathbf{x}_m = \{x_1, x_2, \ldots, x_m\}$$

then, when these spectral estimates are arranged in ascending order, the (dependent) ordered samples are given by

$$\mathbf{x}_{(m)} = \left\{x_{(1)}, x_{(2)}, \ldots, x_{(m)}\right\}$$

where

$$x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(m)}$$

and $x_{(i)}$ is referred to as the i^{th} order statistic. It can be shown that

$$W_{-\infty} = \lim_{q \to -\infty} W_q = x_{(1)}$$

which means that the smallest possible value of the biased estimate is the first order statistic or the smallest of the spectral levels in a frequency band.

Because the first order statistic, $x_{(1)}$, provides a lower bound for the suggested biased estimator, an analysis of the apparent signal–to–noise ratio for this estimator will provide an indication of the maximum signal–to–noise ratio enhancement. The probability density function of the first order statistic of m independent samples is given by

$$\tilde{f}(x_{(1)}) = m\left[1 - F(x = x_{(1)})\right]^{m-1} f(x = x_{(1)})$$

where $F(x)$ is the cumulative distribution function

$$F(x) = \int_0^x f(\zeta)d\zeta$$

of a single spectral estimate *[2]*. For noise alone this gives

$$F_0(x) = 1 - e^{-x/\sigma_n^2}$$

which, in turn, gives

$$\bar{f}_0(x_{(1)}) = \left(\frac{m}{\sigma_n^2}\right) e^{-m x_{(1)}/\sigma_n^2}$$

which has an expected value of

$$E_0[x_{(1)}] = \int_0^\infty \eta \bar{f}_0(\eta) d\eta = \sigma_n^2/m$$

For signal–plus–noise, the cumulative distribution function is

$$F_1(x) = \int_0^x f_1(\zeta) d\zeta$$

which, in turn, gives

$$\bar{f}_1(x_{(1)}) = m\left[1 - \int_0^{x_{(1)}} f_1(\zeta) d\zeta\right]^{m-1} f_1(x = x_{(1)})$$

which can be integrated numerically to determine the expected value

$$E_1[x_{(1)}] = m \int_0^\infty \eta \left[1 - \int_0^\eta f_1(\zeta) d\zeta\right]^{m-1} f_1(\eta) d\eta$$

The apparent signal–to–noise ratio of the biased estimate provided by the first order statistic can then be expressed as

$$\begin{aligned} R_{-\infty} &= \frac{E_1[W_{-\infty}]}{E_0[W_{-\infty}]} - 1 = \frac{E_1[x_{(1)}]}{E_0[x_{(1)}]} - 1 \\ &= \frac{m^2}{\sigma_n^2} \int_0^\infty \eta \left[1 - \int_0^\eta f_1(\zeta) d\zeta\right]^{m-1} f_1(\eta) d\eta - 1 \end{aligned}$$

which gives the best signal–to–noise ratio enhancement achievable by these methods. When the unbiased signal–to–noise ratio is very large, the apparent signal–to–noise ratio is bounded by

$$R_{-\infty} < \frac{E_2[W_{-\infty}]}{E_0[W_{-\infty}]} - 1 = \frac{E_2[x_{(1)}]}{E_0[x_{(1)}]} - 1 = mR - 1$$

On a logarithmic scale, the best signal–to–noise ratio improvement that can be expected is given by

$$\Delta SNR = 10\log_{10}(R_{-\infty}/R) \approx 10\log_{10}(m)\text{dB}$$

Not surprisingly, the method of biased spectral estimates for enhancing signal–to–noise ratio works best when the unbiased signal–to–noise ratio is very high.

Signal–to–Noise Ratio and Contrast

A useful concept for examining the effect of signal–to–noise ratio enhancement is contrast. If two adjacent frequency bands contain the same level of broadband noise while only one contains a tonal signal, then the signal–to–noise ratio in classical analysis is given by

$$R = \frac{E_1[x]}{E_0[x]} - 1 = \frac{\sigma_s^2}{\sigma_n^2}$$

The contrast can be expressed as the ratio of the two spectral levels (or the difference between the two spectral levels on a dB scale)

$$C = \frac{E_1[x]}{E_0[x]} = 1 + \frac{\sigma_s^2}{\sigma_n^2} = 1 + R$$

For the case of a biased estimate, the same basic definitions hold true. The signal–to–noise ratio is given by

$$R_{-\infty} = \frac{E_1[x_{(1)}]}{E_0[x_{(1)}]} - 1$$

while the contrast is given by the ratio of the two spectral levels

$$C_{-\infty} = \frac{E_1[x_{(1)}]}{E_0[x_{(1)}]} = 1 + R_{-\infty}$$

For example, when the unbiased signal-to-noise ratio is 0 dB, $R = 1$, then the unbiased contrast is about 3 dB, $C = 2$.

The utility of biased spectral estimates can be shown by comparing the apparent signal-to-noise ratio of a biased estimate with that of a classical one, R_q/R, or by comparing the apparent contrast of a biased estimate with that of a classical one, C_q/C. The only two parameters analyzed were $q = 0$ and $q = -\infty$. Figure 1 shows the signal-to-noise ratio enhancement in dB that can be expected from using the harmonic mean, W_0, instead of the arithmetic mean, W_1, for several different signal-to-noise ratios. It is clear that the greater the signal-to-noise ratio, the greater the enhancement. However, regardless of the signal-to-noise ratio, the limit to the extent of enhancement is given by

$$\lim_{R\to\infty} \frac{R_0}{R} = \left[\Gamma\left(1+\frac{1}{m}\right)\right]^{-m}$$

It is also clear that the greater the number of spectra included in the technique, the greater the enhancement. However, regardless of the number of spectra, the limit to the extent of enhancement is given by

$$\lim_{m\to\infty} \frac{R_0}{R} = e^{\gamma} \approx 2.51\text{dB}$$

where γ is the Euler constant. Figure 2 shows the corresponding contrast enhancement.

Figure 3 shows the signal-to-noise ratio enhancement in dB that can be expected from using the first order statistic, $x_{(1)} = W_{-\infty}$, instead of the arithmetic mean, W_1, for several different signal-to-noise ratios. The greater the signal-to-noise ratio, the greater the enhancement, but the limit to the extent of enhancement regardless of the signal-to-noise ratio

$$\lim_{R \to \infty} \frac{R_{-\infty}}{R} = m$$

Figure 4 shows the corresponding contrast enhancement.

Signal Detection and the Threshold Effect

A simple analysis of the first order statistic method shows that there is a signal–to–noise ratio enhancement based on the mean value of the distribution of the statistic. However, signal detection depends on exceeding a threshold which is determined from an acceptable probability of false alarm, $P(FA)$. Because the detection threshold depends both on the mean and the variance of the statistic, calculating a signal–to–noise ratio enhancement for detection analysis that depends only on the mean can be misleading. For a single spectral estimate, the noise density function is given by

$$f_0(x) = \left(\frac{1}{\sigma_n^2}\right) e^{-x/\sigma_n^2}$$

from which the threshold, T, can be determined by solving

$$P(FA) = \int_T^\infty f_0(x)dx$$

so that

$$T = -\sigma_n^2 \ln[P(FA)]$$

The probability of detection when a sinusoidal signal is present can then be written

$$P(D) = \int_T^\infty f_1(x)dx$$

where the density function of signal plus noise is

$$f_1(x) = \left(\frac{1}{\sigma_n^2}\right) e^{-[x+\sigma_s^2]/\sigma_n^2} \ \mathrm{I}_0\left(\frac{2\sigma_s\sqrt{x}}{\sigma_n^2}\right)$$

The noise density function for the first order statistic, $x_{(1)}$, from an ensemble of m spectra is given by

$$\tilde{f}_0(x_{(1)}) = \left(\frac{m}{\sigma_n^2}\right) e^{-mx_{(1)}/\sigma_n^2}$$

The threshold, $\tilde{T}$, can be determined by solving

$$P(FA) = \int_{\tilde{T}}^{\infty} \tilde{f}_0(x_{(1)})dx_{(1)}$$

which gives

$$\tilde{T} = -\sigma_n^2 \ln\left[P(FA)\right]/m$$

The probability of detection when a sinusoidal signal is present can then be written

$$P(D) = \int_{\tilde{T}}^{\infty} \tilde{f}_1(x_{(1)})dx_{(1)}$$

where the density function of signal plus noise is

$$\tilde{f}_1(x_{(1)}) = m\left[1 - \int_0^{x_{(1)}} f_1(\zeta)d\zeta\right]^{m-1} f_1(x = x_{(1)})$$

For the statistic W_1, which is a linear average of an ensemble of m spectra, the noise density function is given by

$$\hat{f}_0(W_1) = \frac{m}{\sigma_n^2\Gamma(m)}\left(\frac{mW_1}{\sigma_n^2}\right)^{m-1} e^{-mW_1/\sigma_n^2}$$

Similarly, the threshold, $\hat{T}$, can be determined by solving

$$P(FA) = \int_{\hat{T}}^{\infty} \hat{f}_0(W_1)dW_1$$

so that

$$\hat{T} = \xi\sigma_n^2/m$$

where ξ is found by solving

$$P(FA) = e^{-\xi} \sum_{k=0}^{m-1} \frac{\xi^k}{k!}$$

The probability of detection when a sinusoidal signal is present can then be written

$$P(D) = \int_{\hat{T}}^{\infty} \hat{f}_1(W_1)dW_1$$

where the density function of signal plus noise is

$$\hat{f}_1(W_1) = \frac{m}{\sigma_n^2}\left(\frac{W_1}{\sigma_s^2}\right)^{\frac{m-1}{2}} e^{-m[W_1+\sigma_s^2]/\sigma_n^2}\ \mathrm{I}_{m-1}\left(\frac{2m\sigma_s\sqrt{W_1}}{\sigma_n^2}\right)$$

The simplest way to examine the effect on detection of using $x_{(1)}$ or $W_{-\infty}$, rather than W_1, is to assume that the estimates are approximately unbiased for a sinusoidal signal plus noise, as when the signal-to-noise ratio is very high, and examine the ratio of the respective thresholds

$$\tilde{T}/\hat{T} = -\ln[P(FA)]/\xi$$

where ξ is as above. Figure 5 shows the ratio of the classical threshold to the first order statistic threshold, in dB, for various probabilities of false alarm. Because these curves are only appropriate for high sign-to-noise ratios and detection is desired for marginal signal-to-noise ratios, this approach can be misleading.

A less extreme simplification is to assume that the $W_{-\infty}$ estimate is biased when a sinusoidal signal is present and that detection occurs when the expected value of signal plus noise is equal to the threshold. For W_1, detection would then occur when

$$\hat{T} = \sigma_n^2 + \sigma_s^2$$

from which the classical signal-to-noise ratio can be derived

$$\hat{R} = \xi/m - 1$$

where ξ is, again, as above. For $W_{-\infty}$, detection would occur when

$$\tilde{T} = m \int_0^\infty x \left[1 - \int_0^x f_1(\zeta) d\zeta\right]^{m-1} f_1(x) dx$$

and the amount the biased technique would enhance detection is then given by the ratio of the two signal-to-noise ratios. Determination of the signal-to-noise ratio, $\tilde{R}$, requires finding the root of the equation

$$\begin{aligned} 0 = &\ln[P(FA)] \\ &+ m^2 \int_0^\infty \eta \left[1 - \int_0^\eta g(\tilde{R}, \zeta) d\zeta\right]^{m-1} g(\tilde{R}, \eta) d\eta \end{aligned}$$

where

$$g(\tilde{R}, \eta) = e^{-(\eta + \tilde{R})} \; \mathrm{I}_0\left(2\sqrt{\tilde{R}\eta}\right)$$

The signal-to-noise ratio enhancement is then $\hat{R}/\tilde{R}$. Figure 6 shows curves of the signal-to-noise enhancement for detection purposes. Because the first order statistic shows greater variability than the classical average despite the fact that is has a much lower mean, the enhancement shown is not as great as indicated by simple signal-to-noise ratio or contrast methods.

Although a probability of detection was not specified, using the expected value of the density function to define detection is not entirely unfounded. For a symmetric density function, the expected value would yield a probability of detection of 50%. The classical, W_1, density function is asymmetric but can be shown to approach a symmetric Gaussian form when either the signal-to-noise ratio is high or when the number of spectra in the average is great. There should, however, be minor differences between the detection signal-to-noise ratio enhancement curves shown here and those curves which might be derived for a fixed detection probability.

Conclusions

A method was presented for calculating biased spectral estimates that enhance tonal signals against a background of broadband noise. The method was shown to differentially bias different mixtures of broadband noise and pure tones as a function of the ratio of tonal energy to broadband energy. The method was analyzed and shown to provide the best enhancements for large ratios of tonal energy to broadband energy. The method provides some enhancement for any tone, but is unable to significantly improve detection of tones that are very much lower than broadband noise levels.

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2. Reiss, R.-D.: *Approximate Distributions of Order Statistics.* Springer–Verlag New York, Inc., 1989.

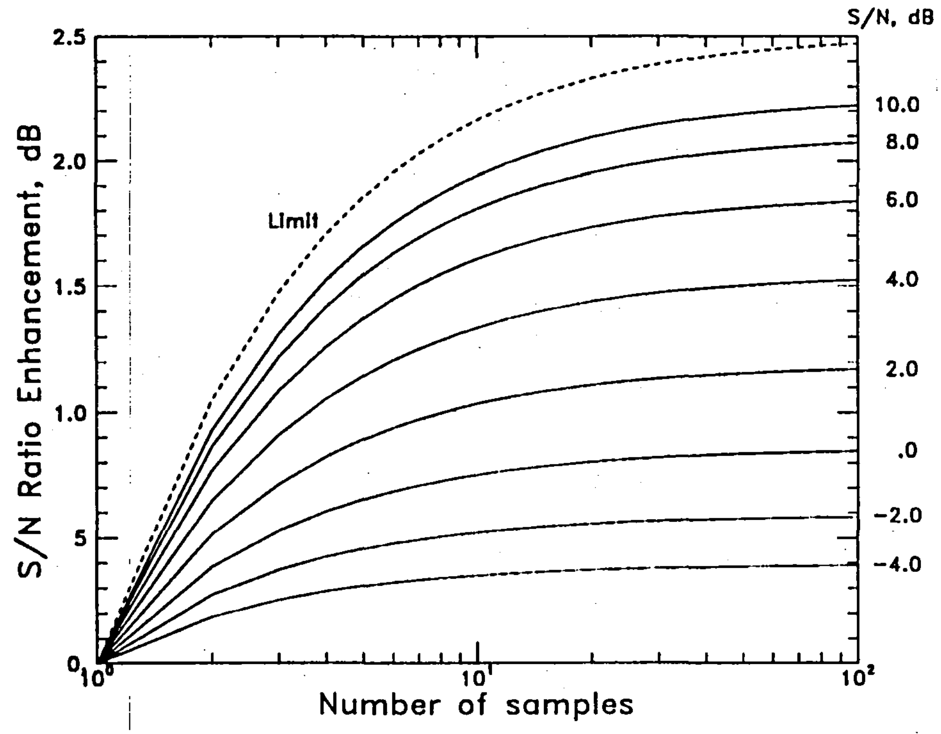

Figure 1. Signal-to-noise ratio enhancement from the harmonic mean.

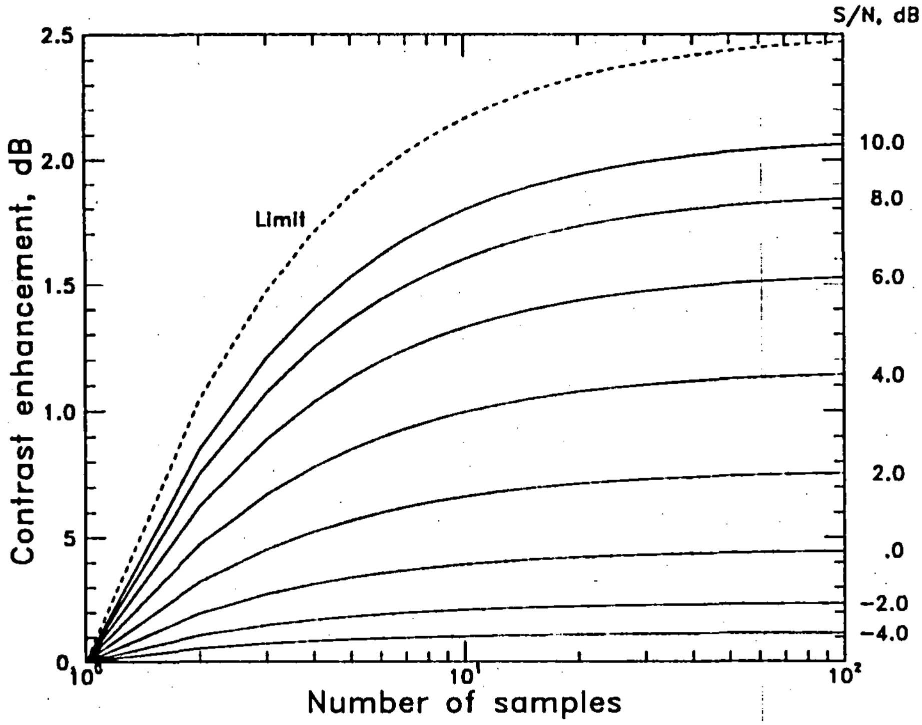

Figure 2. Contrast enhancement from the harmonic mean.

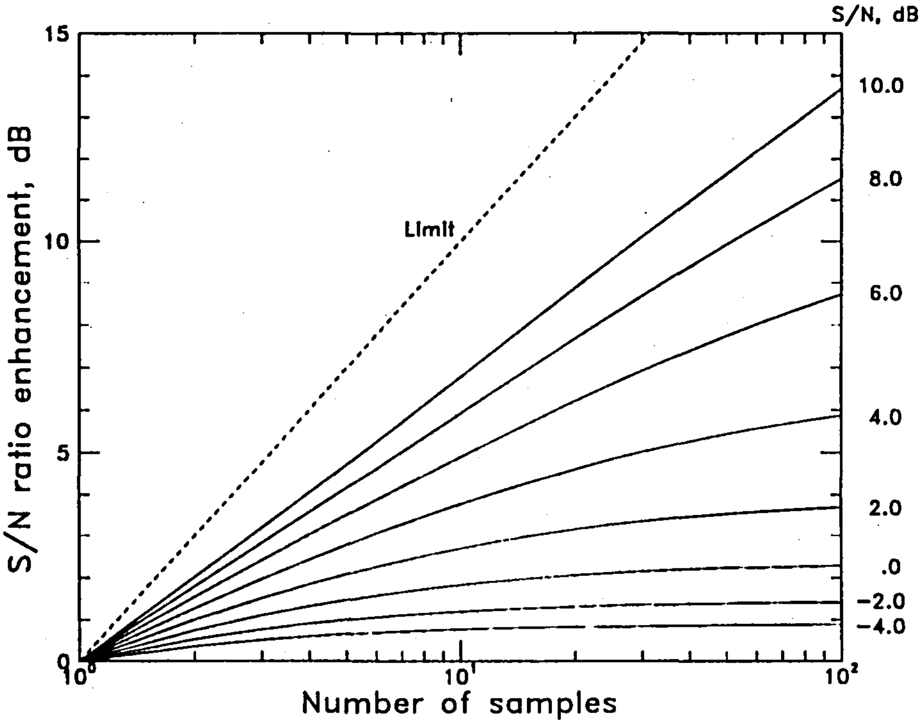

Figure 3. Signal-to-noise ratio enhancement from the first order statistic.

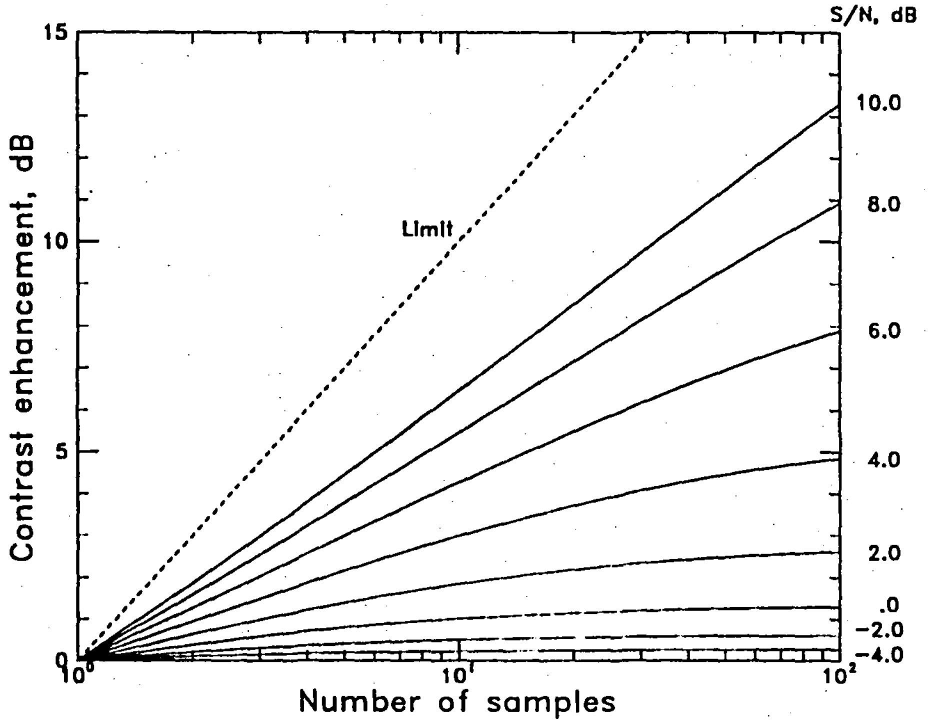

Figure 4. Contrast enhancement from the first order statistic.

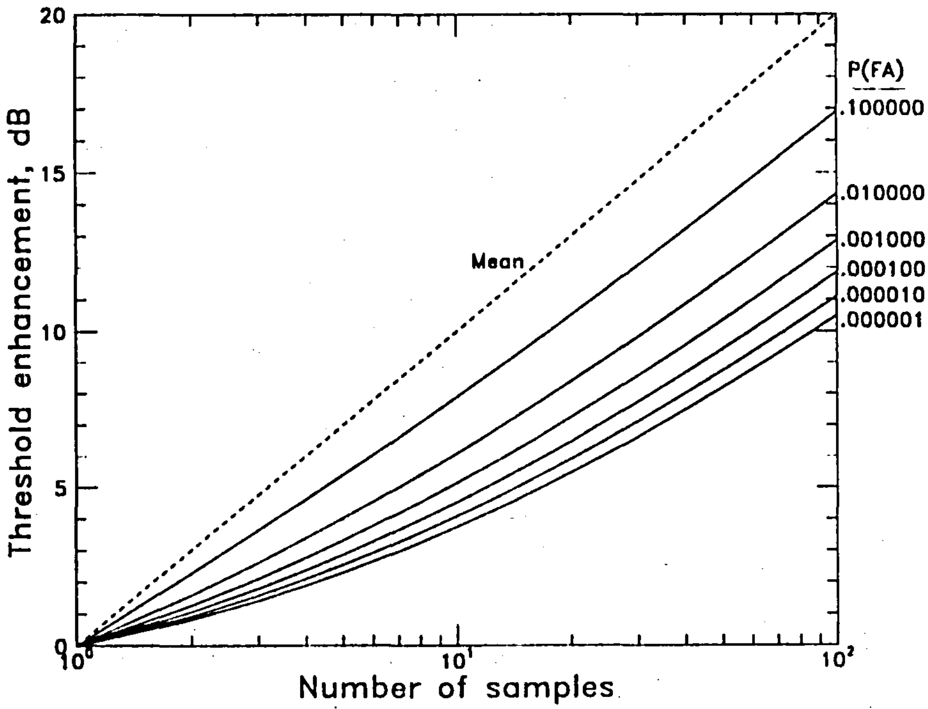

Figure 5. Detection threshold enhancement from the first order statistic.

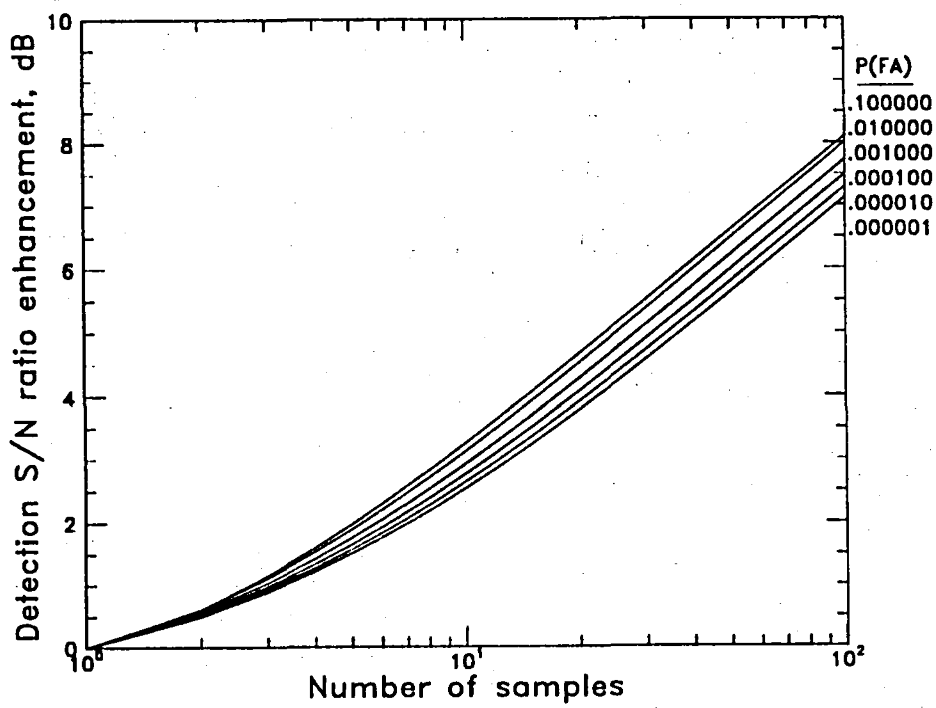

Figure 6. Detection signal–to–noise ratio enhancement from the first order statistic.

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