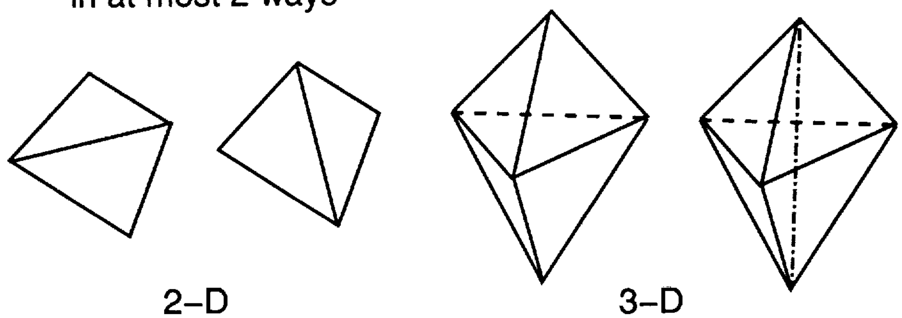
N94-22356

3-D UNSTRUCTURED MESH GENERATION USING LOCAL TRANSFORMATIONS

TIMOTHY J. BARTH
NASA AMES RESEARCH CENTER

3–D Combinatorial Edge Swapping

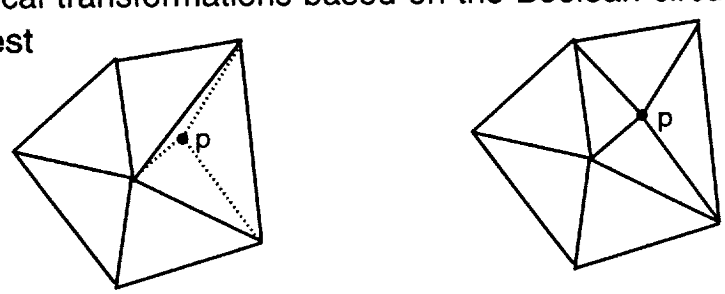
- Convex sets of n+2 sites in $\mathbf{R}^n$ can be configured in at most 2 ways



- This **local transformation** based on a Boolean decision serves as mechanism for local optimization

3–D Incremental Triangulation via Local Transformations

- Joe (1989) and Rajan (1991) showed that 3–D Delaunay triangulations can be constructed using local transformations based on the Boolean circumsphere test



2–D Example of Incremental Insertion and Optimization

- We have constructed triangulation algorithms in 3–D which locally optimize other mesh qualities: max–min dihedral angles, min–max dihedral angles, etc.

Motivations

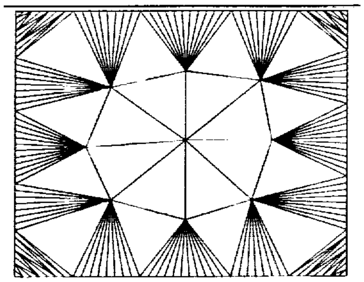
- Develop a mesh generation capability suitable for generating highly stretched meshes required for viscous flow computations at high Reynolds numbers
  - Experience has shown that existing triangulation methods such as Delaunay triangulation are not suitable for the generation of highly stretched meshes
- Investigate triangulation algorithms which accommodate mesh generation and adaptation while maintaining high robustness

Randomized △ Algorithms Based on Local Transformations

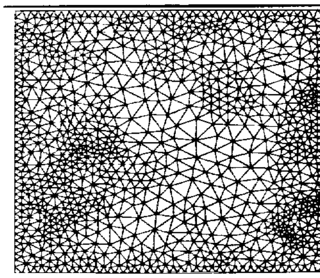
- Worst case optimal complexity can be achieved by *randomizing* the order in which sites are introduced into the triangulation (Guibas, Knuth, Sharir, 1992)
  - n log (n) expected performance in 2–D
  - n^2 expected worst case performance in 3–D
- Suggests a new "continuous" data structure which encodes a family of triangulations (coarsest to finest)
  - 2–D randomized theory predicts O(n) size of this structure
  - We have exploited this construction to produce a novel multigrid scheme and theory for solving differential eqns

A New Approach to Multigrid for Unstructured Meshes

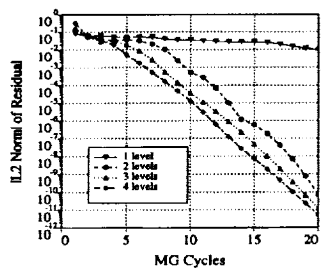
- Solution of Burgers' equation using continuous data structure



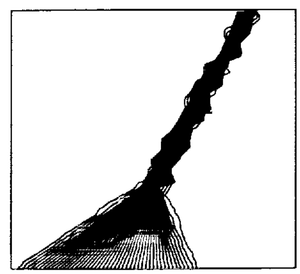
Coarsest Mesh



Finest Mesh



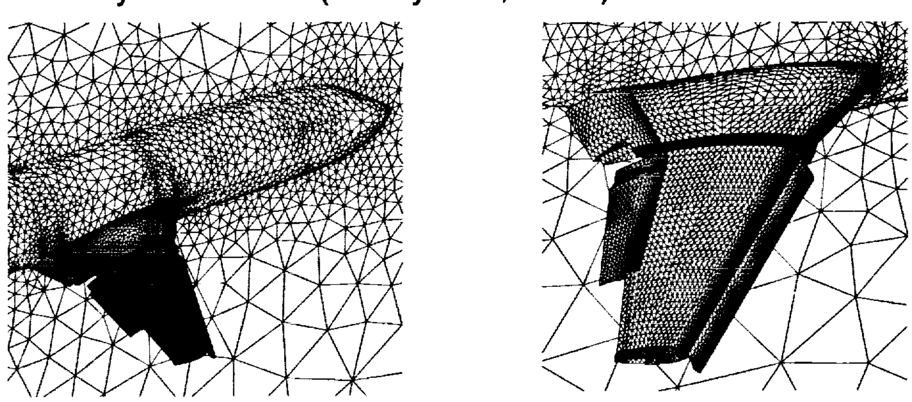
Convergence History



Solution Contours

Surface Mesh Generation Using Local Transforms

- Exploring new techniques capable of generation isotropic or stretched elements on tensor product spline patches
- Method supports adaptation based on geometrical or soln error
- Extension to manifold B–rep objects is being carried out by Code RFG (Maksymiuk, Chou)



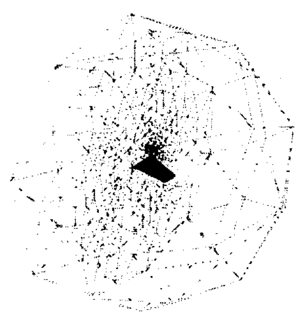
Mesh with isotropic and stretched elements

Volume Triangulations

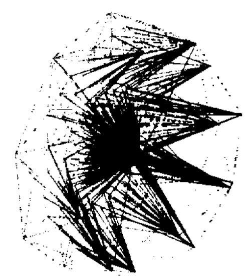
(1) Initial Triangulation of Surface Data

(2) Constrained/Conforming Triangulation to Preserve Body Integrity

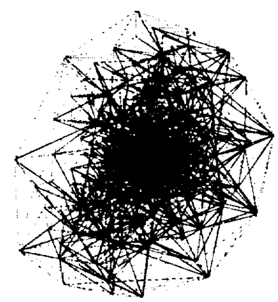
(3) Incremental Insertion and Optimization of Specified Sites



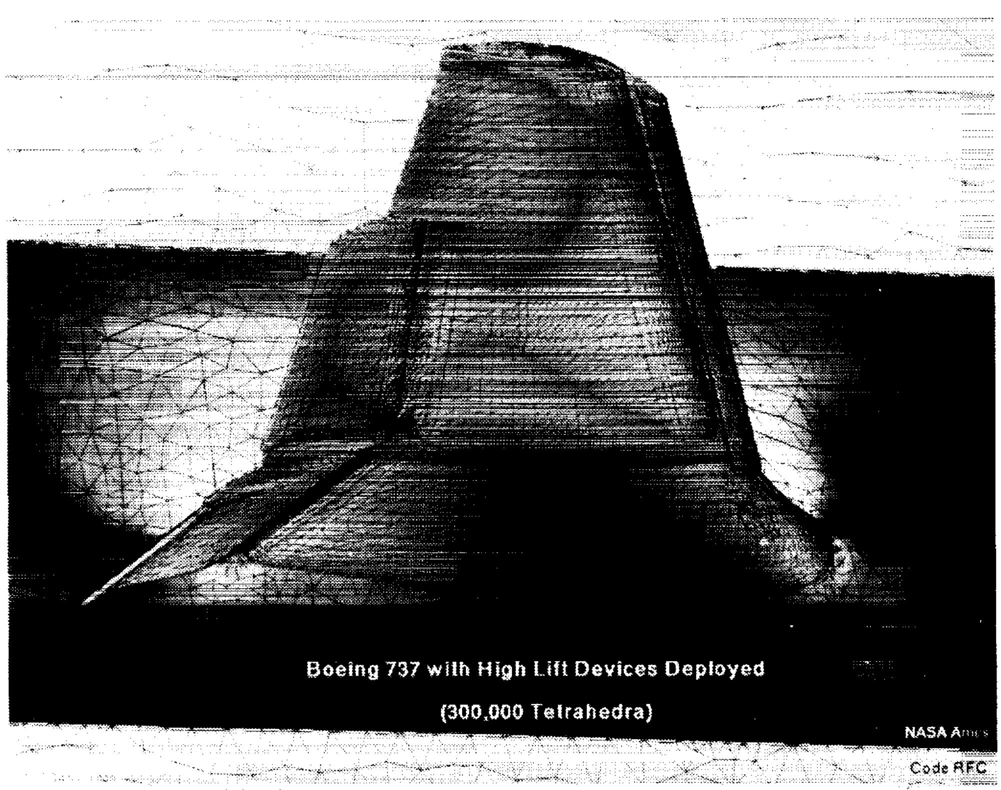
Surface Triangulation



Constrained/Conforming Triangulation of Boundary

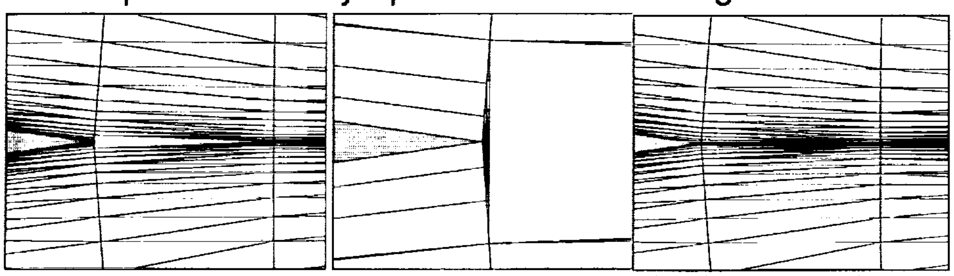


Final Volume Triangulation



Why Some Standard Triangulation Methods Fail

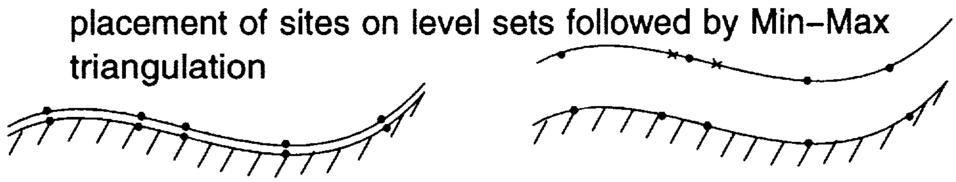
- Delaunay triangulation has a well known characterization that it maximizes the minimum angle for triangle pairs
- Theoretical and practical considerations indicate that it may be more beneficial to minimize the maximum angle for triangle pairs
- Incremental insertion and local optimization can be used to produce locally optimal Min–Max triangulations



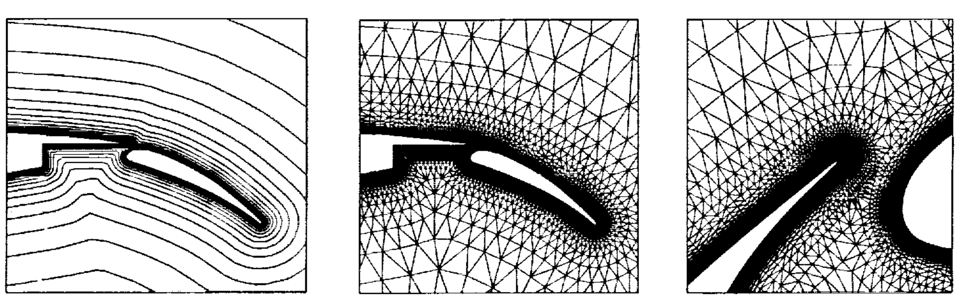
Delaunay triangulation near an airfoil trailing edge | Extreme closeup of DT in trailing edge region | Min–Max triangulation obtained by local optimization

Viscous Mesh Generation

- Automatic generation of viscous meshes by adaptive placement of sites on level sets followed by Min–Max triangulation



Point Selection (AR >>1) | Point Selection and Adaptation (AR ≅ 1)



Distance Function | Min–Max Triangulation | Closeup in Flap Region

Future Directions

- Continue investigating optimization criteria for tetrahedral meshes
- Develop new strategies for site placement
  - Level set strategies
  - Steiner point strategies
- Solution adaptation based on *a priori* error estimates