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UNSTRUCTURED LOW-MACH NUMBER VISCOUS FLOW SOLVER

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Outline

- Governing Equations
- Grid Generation
- Numerical Approach
  - Discretization Technique
  - Preconditioning
  - Artificial Dissipation
  - Boundary Conditions
  - Sparse Matrix Solvers
- Results
- Conclusions

Unstructured Low-Mach Number Viscous Flow Solver

- Navier-Stokes equations(2-D)
  - Conservation law form in terms of primative variables substitute $\rho = \dfrac{P}{RT}$
  - Cell centered finite volume discretization
  - Implicit delta formulation written as: $A\vec{x} = \vec{b}$

Navier-Stokes equations nondimensionalized

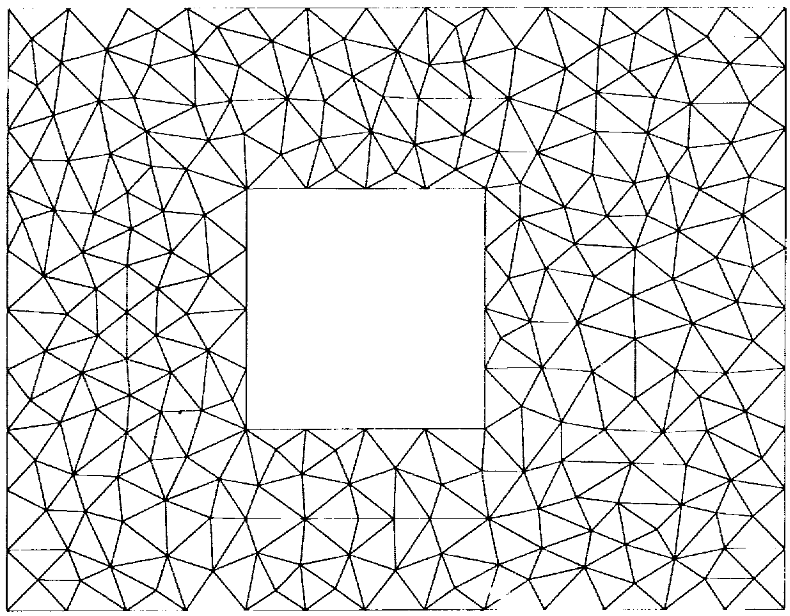
$$\frac{\partial Q(\tilde{w})}{\partial \tilde{t}} + \frac{\partial G(\tilde{w})}{\partial \tilde{x}} + \frac{\partial H(\tilde{w})}{\partial \tilde{y}} = 0$$

$$\tilde{w} = \begin{bmatrix} \tilde{P} \\ \tilde{u} \\ \tilde{v} \\ \tilde{T} \end{bmatrix}, \quad Q = \begin{bmatrix} \frac{\tilde{P}}{\tilde{T}} \\ \frac{\tilde{P}\tilde{u}}{\tilde{T}} \\ \frac{\tilde{P}\tilde{v}}{\tilde{T}} \\ \frac{\tilde{P}}{\tilde{T}}\left[\left(\tilde{C}_p - \tilde{R}\right)\tilde{T} + \frac{\tilde{u}^2}{2} + \frac{\tilde{v}^2}{2}\right] \end{bmatrix}$$

Unstructured Grid Generation

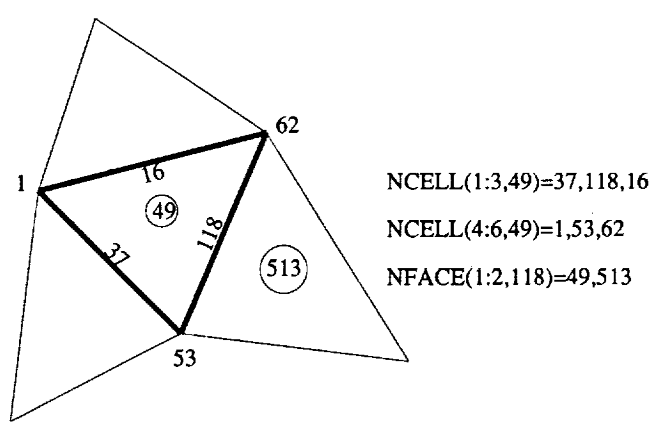
- Delaunay Triangulation
    - Bowyer's Algorithm
      Grid refinement based on aspect ratio, area, circumcircle radius

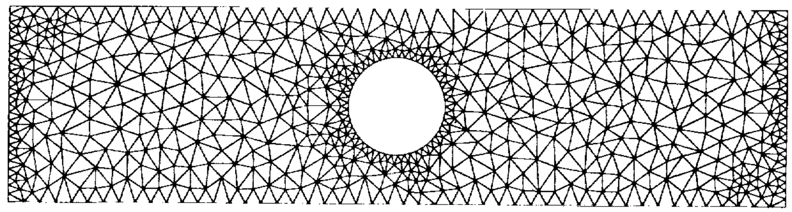
- Connectivity



Grid Code Output: Geometry, Connectivity

- Node point x, y coordinates
- Cell nodes, cell faces, face cells





Preconditioning(N-S eq. 1-d)

$$\frac{\partial Q(w)}{\partial t} + \frac{\partial G(w)}{\partial x} - \frac{\partial G_v(w)}{\partial x} = 0$$

where

$$w = \begin{bmatrix} P \\ u \\ T \end{bmatrix}, \quad Q = \begin{bmatrix} \dfrac{P}{RT} \\ \dfrac{Pu}{RT} \\ \dfrac{P}{\gamma R} + \dfrac{M^2(\gamma - 1)}{2}\dfrac{Pu^2}{RT} \end{bmatrix}$$

$$\mathbf{A}_t \frac{\partial w}{\partial t} + \mathbf{A}_x \frac{\partial w}{\partial x} = \frac{\partial G_v(w)}{\partial x}; \quad R = (\gamma M^2)^{-1}$$

where

$$\mathbf{A}_t = \begin{bmatrix} \frac{\gamma M^2}{T} & 0 & -\frac{P}{RT^2} \\ \frac{\gamma M^2 u}{T} & \frac{P}{RT} & -\frac{Pu}{RT^2} \\ M^2 + \frac{\gamma M^4(\gamma-1)u^2}{2T} & \frac{\gamma M^4(\gamma-1)Pu}{T} & -\frac{\gamma M^4(\gamma-1)Pu^2}{2T^2} \end{bmatrix},$$

$$\mathbf{A}_x = \begin{bmatrix} \frac{\gamma M^2 u}{T} & \frac{P}{RT} & -\frac{Pu}{RT^2} \\ \frac{\gamma M^2 u^2}{T} + 1 & \frac{2Pu}{RT} & -\frac{Pu^2}{RT^2} \\ \gamma M^2 u + \frac{\gamma M^4(\gamma-1)u^3}{2T} & \frac{P}{R} + \frac{3M^4(\gamma-1)Pu^2}{2T} & -\frac{\gamma M^4(\gamma-1)Pu^3}{2T^2} \end{bmatrix}$$

$$\frac{\partial w}{\partial t} + \mathbf{A}_t^{-1}\mathbf{A}_x \frac{\partial w}{\partial x} = \mathbf{A}_t^{-1} \frac{\partial G_v(w)}{\partial x}$$

instead

$$\mathbf{A}_p \frac{\partial w}{\partial \tau} + \mathbf{A}_t \frac{\partial w}{\partial t} + \mathbf{A}_x \frac{\partial w}{\partial x} = \frac{\partial G_v(w)}{\partial x}$$

$$\mathbf{A}_p = \begin{bmatrix} \frac{1}{T} & 0 & -\frac{P}{RT^2} \\ \frac{u}{T} & \frac{P}{RT} & -\frac{Pu}{RT^2} \\ \frac{1}{\gamma} + \frac{M^2(\gamma-1)u^2}{2\gamma T} & \frac{\gamma M^4(\gamma-1)Pu}{T} & -\frac{\gamma M^4(\gamma-1)Pu^2}{2T^2} \end{bmatrix}$$

Boundary Conditions

- Implicit treatment
- Solid wall specified as viscous no-slip or inviscid tangency.
- Symmetry and periodic boundaries are treated through connectivity

Characteristics of Preconditioned System(N-S eq. 1-d)

Find characteristics of $\mathbf{A}_p^{-1}\mathbf{A}_x$ instead of $\mathbf{A}_x$.

$$A_p^{-1} = \begin{bmatrix} \frac{\gamma M^2(\gamma-1)u^2}{2} & -\gamma M^2(\gamma-1)u & \gamma \\ -\frac{uRT}{P} & \frac{RT}{P} & 0 \\ -\frac{((\gamma-1)u^2-2RT)T}{2P} & -\frac{(\gamma-1)Tu}{P} & \frac{\gamma RT}{P} \end{bmatrix}$$

$$\lambda_1 = U\,, \text{ and } \lambda_{2,3} = \frac{U \pm \sqrt{U^2+4\gamma T}}{2}$$

when letting $M \to 0$. Preconditioning gives a finite instead of infinite value of λ.

Sparse Matrix Solvers

- Point Gauss-Seidel scheme
- Point block Gauss-Seidel scheme
- Conjugate gradient like method(SITRSOL)

used to solve

$$A\vec{x} = \vec{b}$$

Point Gauss-Seidel Scheme

- Every element of matrix except diagonal of block moved to RHS
- Prone to divergence with poor initial conditions
- Very sensitive to lack of diagonal dominance

Point Block Gauss-Seidel Scheme

- All blocks except diagonal block moved to RHS
- Uses LU decomposition to the remaining matrix equation
- More robust than the point G-S scheme

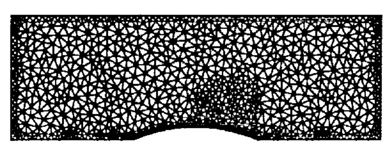
A grid coloring scheme was used to vectorize the Gauss-Seidel method since it suffers from recurrence. The four color theorem was used to remove the recurrence from the convective terms. Recurrence remains in the viscous terms but doesn't seem to affect the convergence rate. The coloring scheme was done by sweeping the computational cells twice.

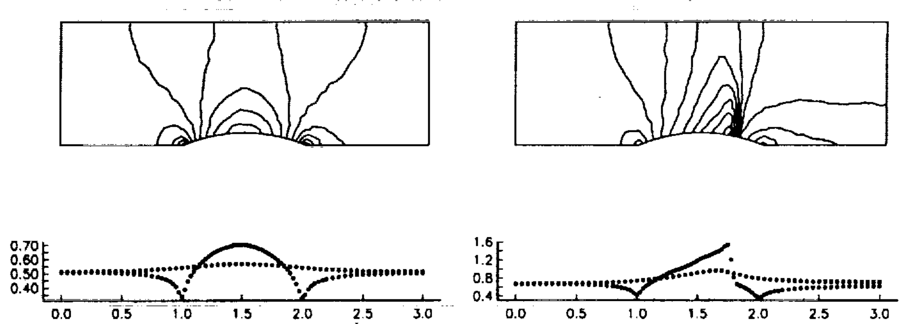
Conjugate gradient like solver(SITRSOL)

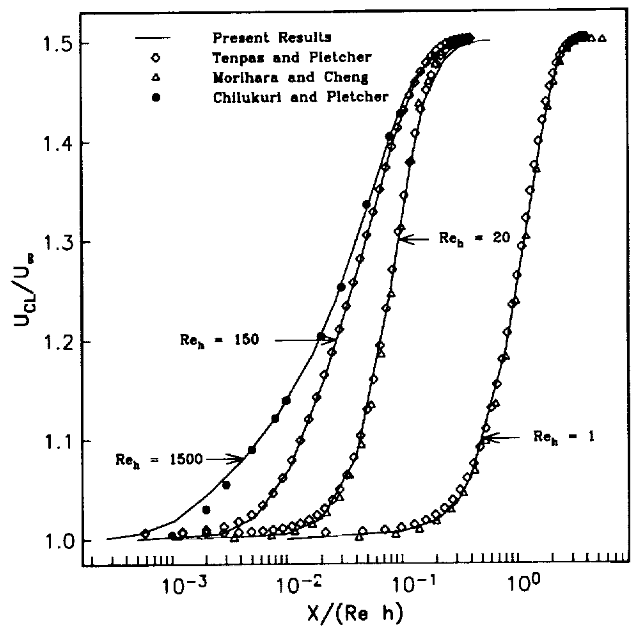
- Iterative solver based loosely on the conjugate gradient method
- Several iterative methods are available for solving non-symmetric positive indefinite sparse linear systems
    - Bi-conjugate gradient method
    - Generalized minimal residual method
    - Generalized conjugate residual method
- The incomplete LU preconditioner was used

Results

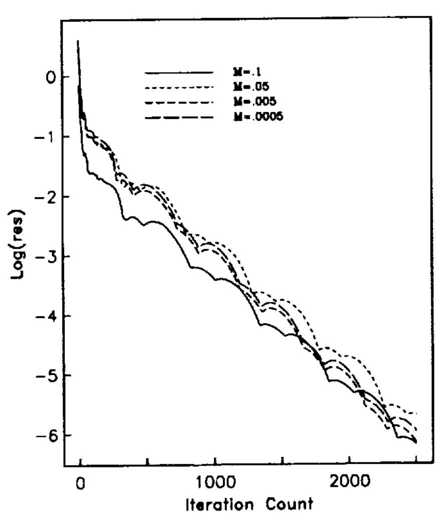
- Bump on Wall
- Developing Channel Flow
- Sudden Expansion
- Periodic Tandem Circular Cylinders in Cross Flow
- Four Port Valve



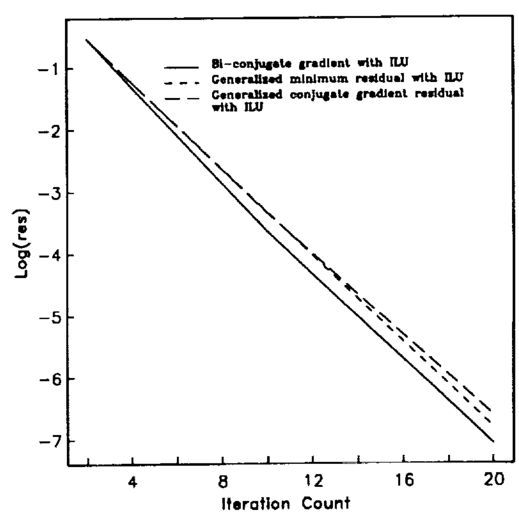




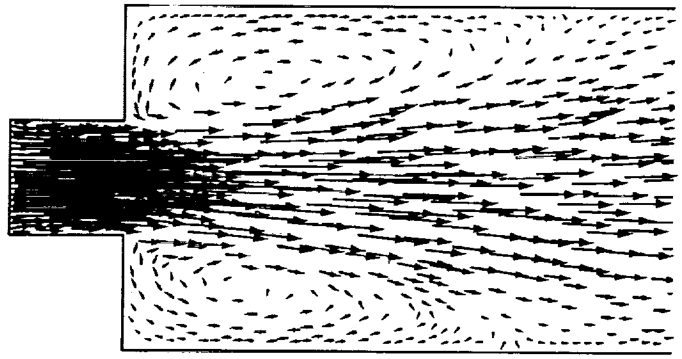
Present Results
Tenpas and Pletcher
Morihara and Cheng
Chilukuri and Pletcher
Re_h = 20
Re_h = 150
Re_h = 1500
Re_h = 1
U_{CL}/U_{∞}
X/(Re h)

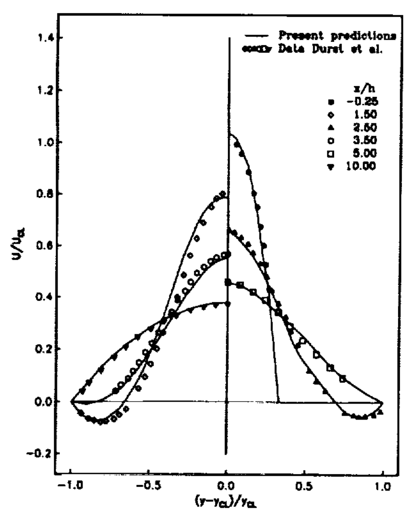


M=.1
M=.05
M=.005
M=.0005
Log(res)
Iteration Count

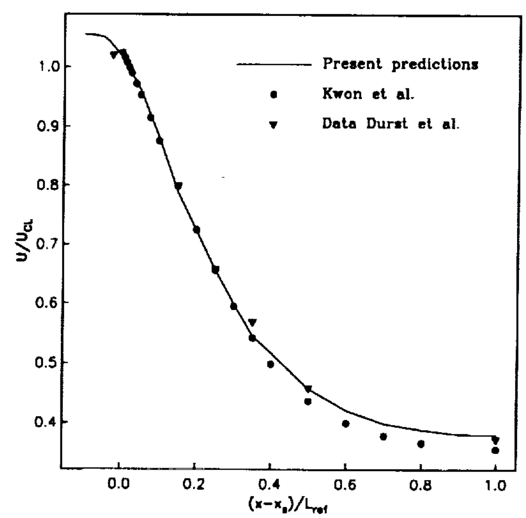


Bi-conjugate gradient with ILU
Generalized minimum residual with ILU
Generalized conjugate gradient residual with ILU
Log(res)
Iteration Count

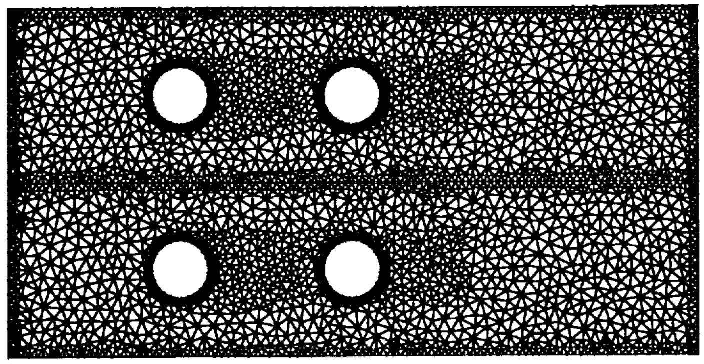


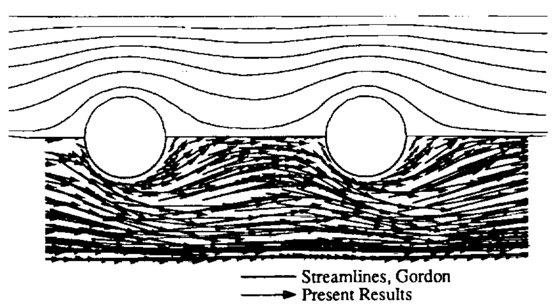


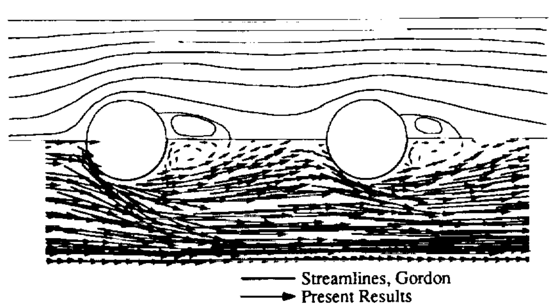
Present predictions
Data Durst et al.
x/h
-0.25
1.50
2.50
3.50
5.00
10.00
U/U_CL
(y-y_CL)/y_CL

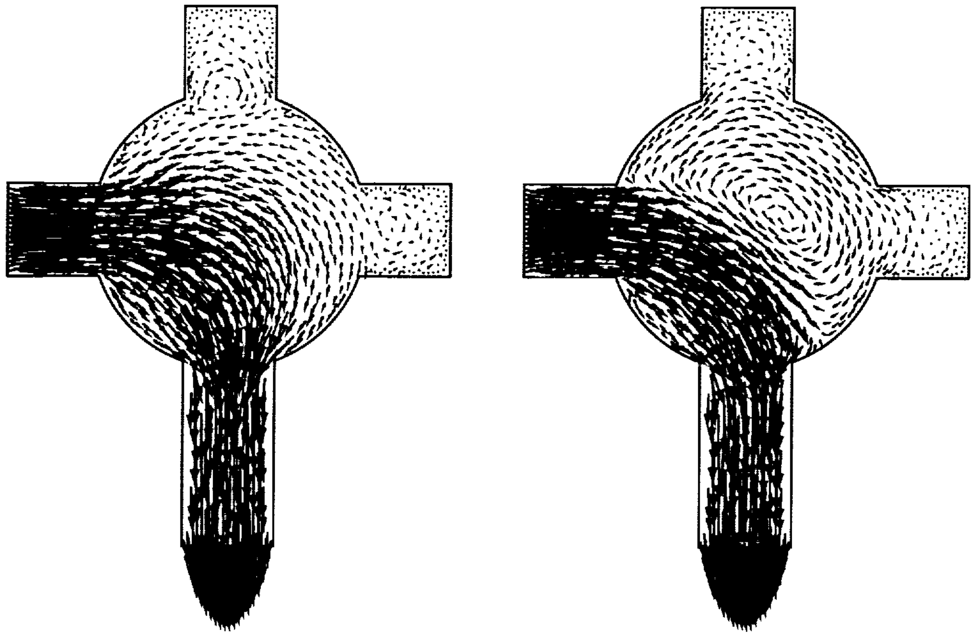


Present predictions
Kwon et al.
Data Durst et al.
U/U_CL
(x-x_s)/L_ref




Streamlines, Gordon
Present Results


Streamlines, Gordon
Present Results



Conclusions

- Grids can be generated about complex geometries
- Diagonal block Gauss-Seidel solver more robust than point diagonal Gauss-Seidel version of solver
- Coloring scheme allowed the vectorization of the implicit Gauss-Seidel solver
- Sparse iterative solver(SITRSOL) allowed a much larger time step than Gauss-Seidel(ran 2 to 2.5 times faster)
- Temporal preconditioning allowed the compressible code to run at very low Mach numbers