N94-22368

ACCURACY ASSESSMENT FOR GRID ADAPTATION

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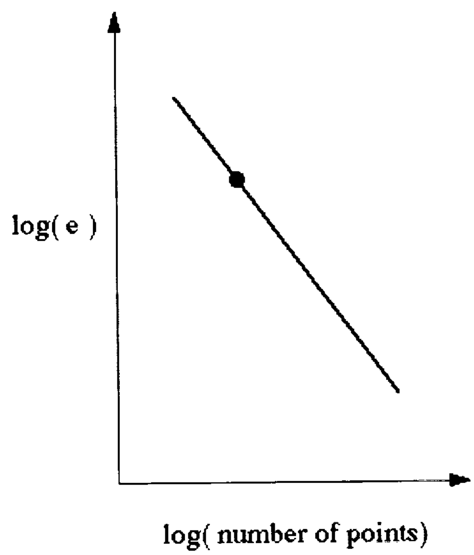
Outline

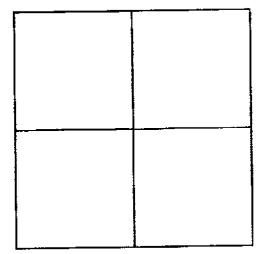
- Introduction
- Grid Convergence Study + Adaptive Methods
- Ongoing O.D.E. Work
- Discussion

Introduction

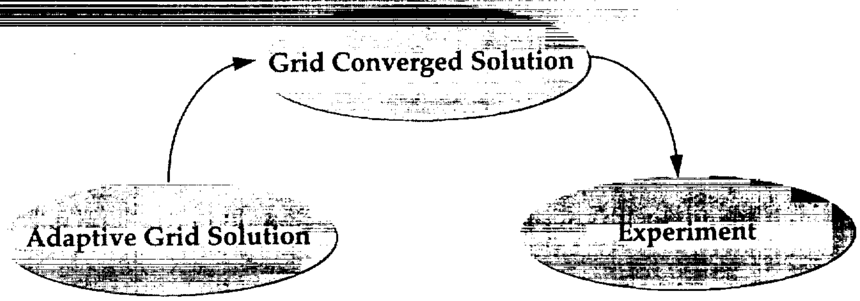
- Adaptive methods will be necessary for large problems
- Adaptive point movement methods
  - redistribute grid points to obtain optimal topology
- Adaptive point addition methods
  - Add grid points to obtain optimal topology
  - continued point addition will result in grid convergence (hopefully with fewer grid points)
- The first part of this talk examines grid convergence using several refinement criteria
  - Two adaptive point addition Euler solvers
  - One block-structured Euler solver (for grid convergence study)

Uniform Refinement

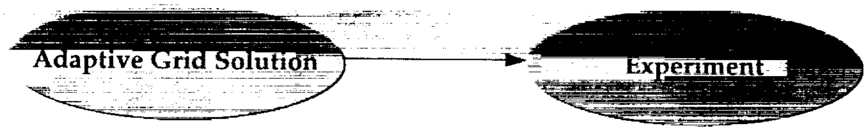




What we should be doing

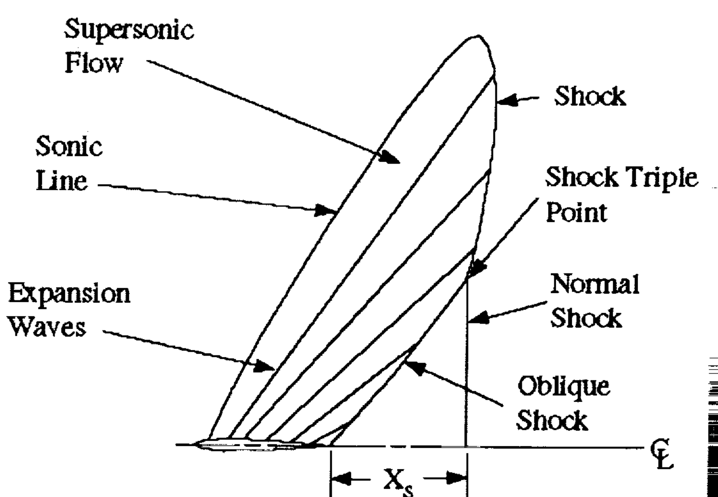


What we should not be doing (Until we have the bugs out)



AGARD 03 Test Case

NACA 0012
Mach = 0.95
α = 0 degrees



Grid Convergence Study

- Conditions for grid convergence

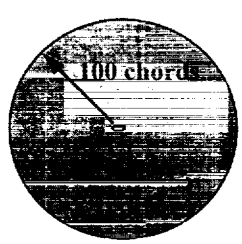
$$\lim_{n\to\infty} \| e \|_{\infty} = 0$$

This occurs if method is consistent and

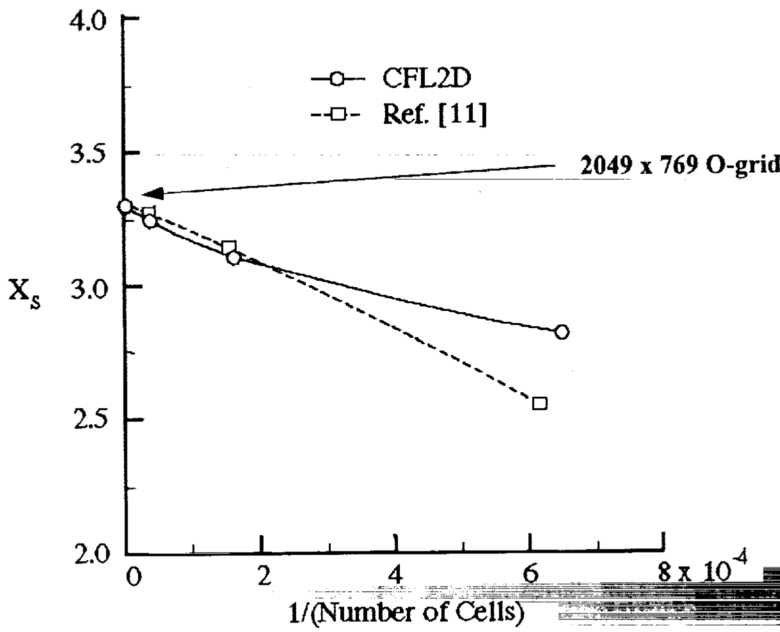
$$\lim_{n\to\infty} h_i = 0$$

Grid Convergence Study

- O-grid with fixed outer boundary of 100 chords
- Use sequence of finer grids
  - 65 x 25
  - 129 x 49
  - 257 x 97
  - 2049 x 769
- Extrapolate shock location to "infinitely refined grid"



Grid Convergence Study



Common Adaptive Methods

Divided differences

$$\tilde{e} = \frac{\partial p}{\partial x} \approx \frac{\Delta p}{\Delta x}$$

Undivided differences

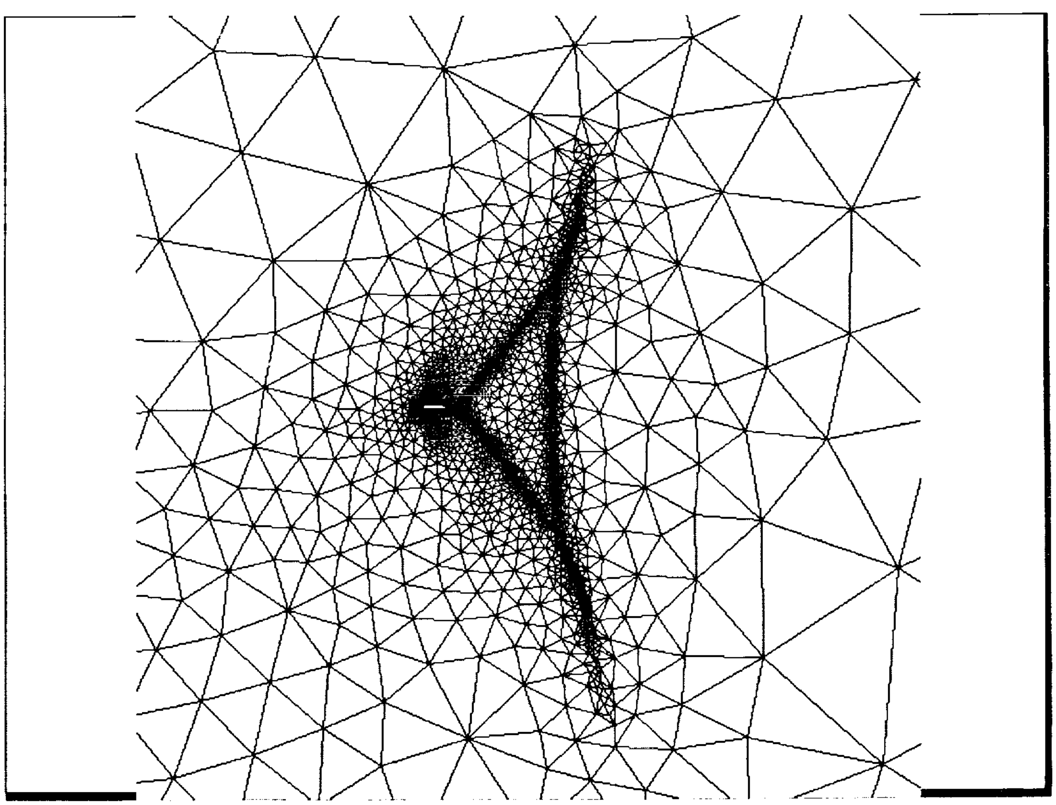
$$\tilde{e} = \tilde{\Delta} p$$

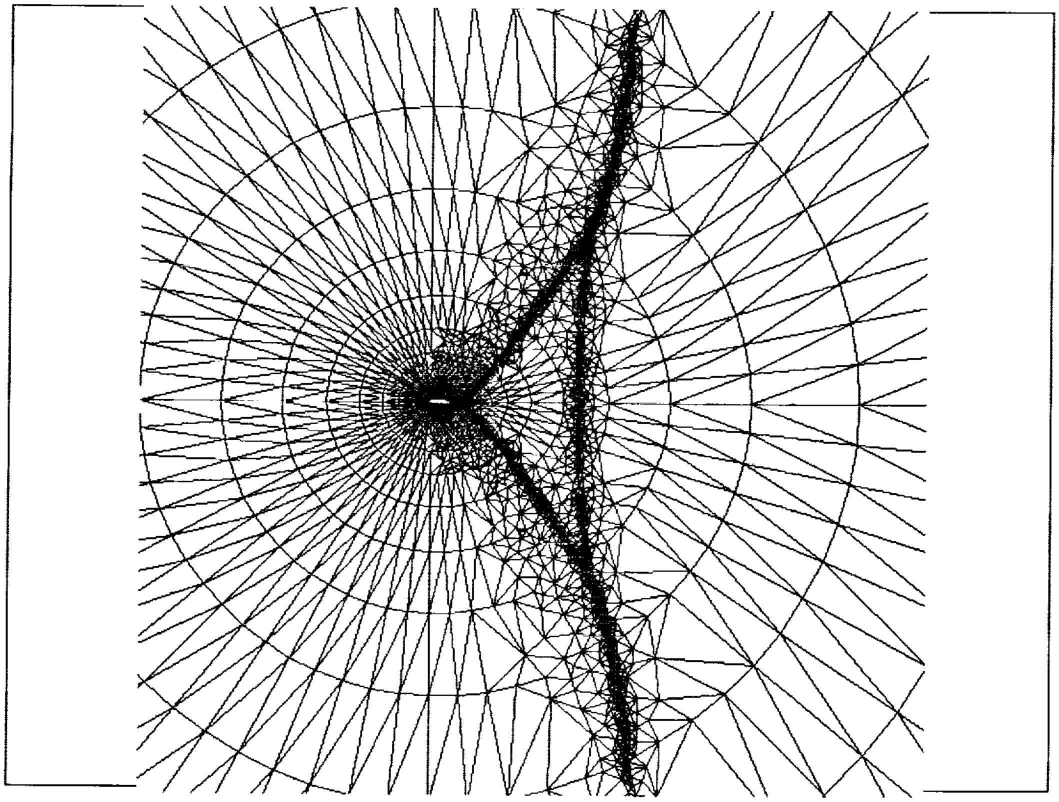
$$\tilde{e} = h^2 \frac{\partial^2 p}{\partial x^2} \approx \tilde{\Delta}^2 p$$

Truncation error estimates

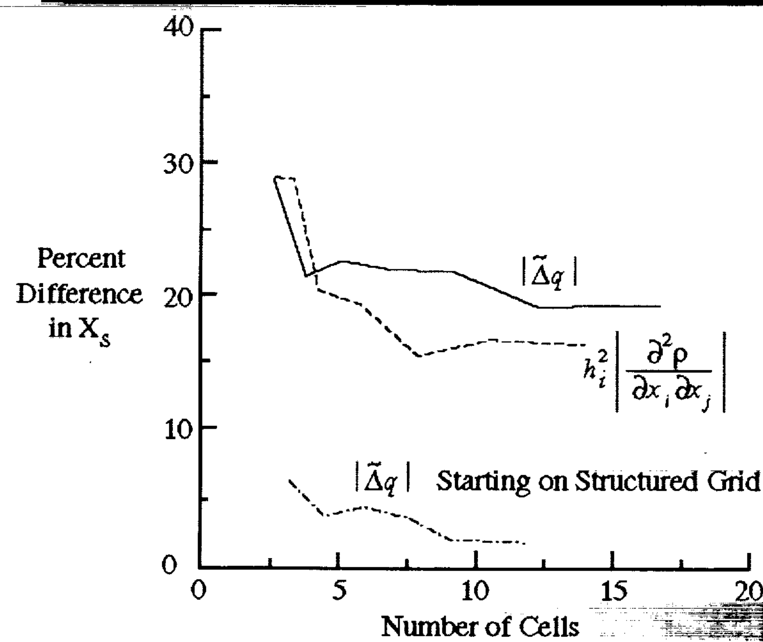
Threshold Determination

- Based on experience (?)
- Statistical approach
  - ➤ Threshold = average + standard deviation

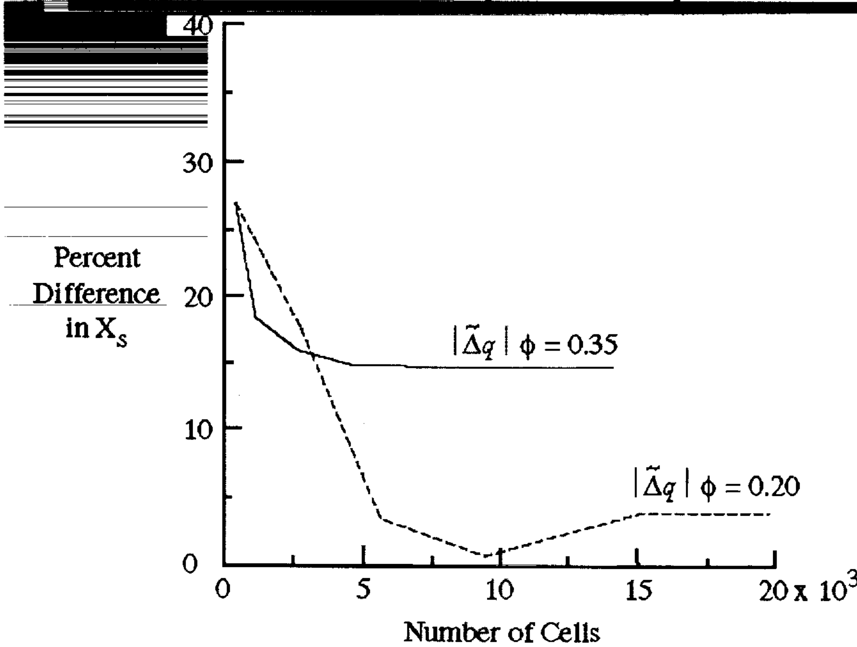




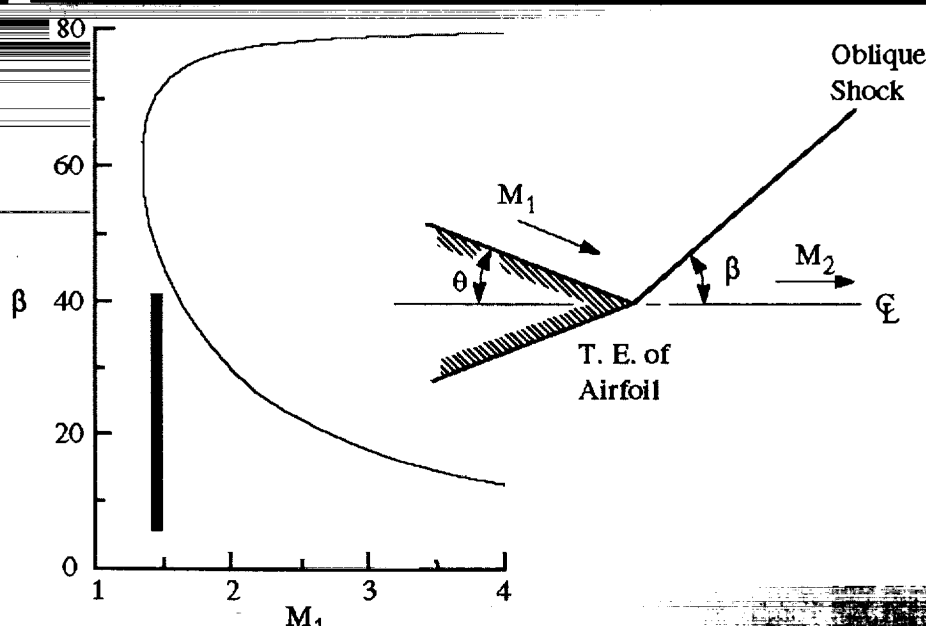
Adaptive Results (FUN2D)



Adaptive Results (SUN2D)



Shock Polar for θ = 7.99 degrees



C.4

Corrected Adaptation Method

Problem occurs when

$$\lim_{n \to \infty} \| e \|_\infty \neq 0$$

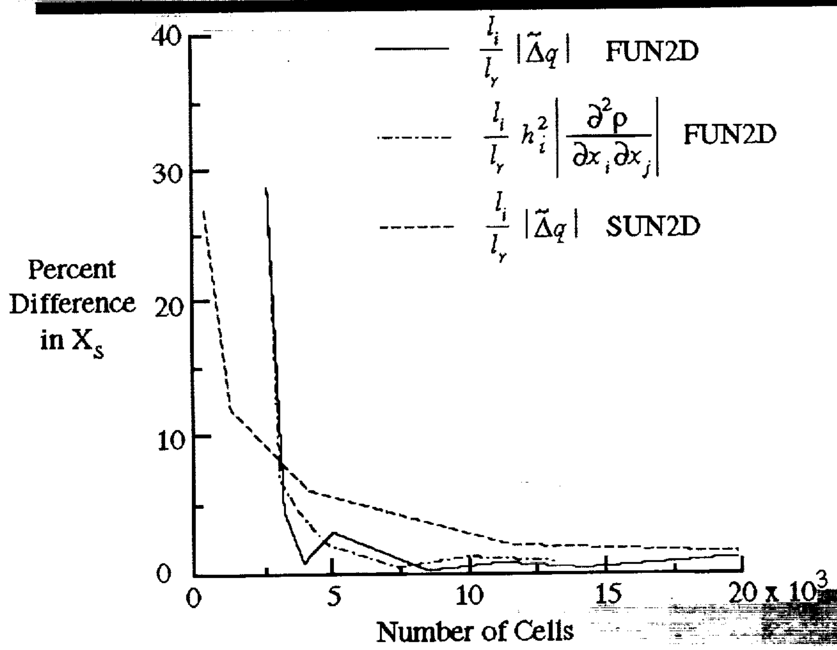
which causes

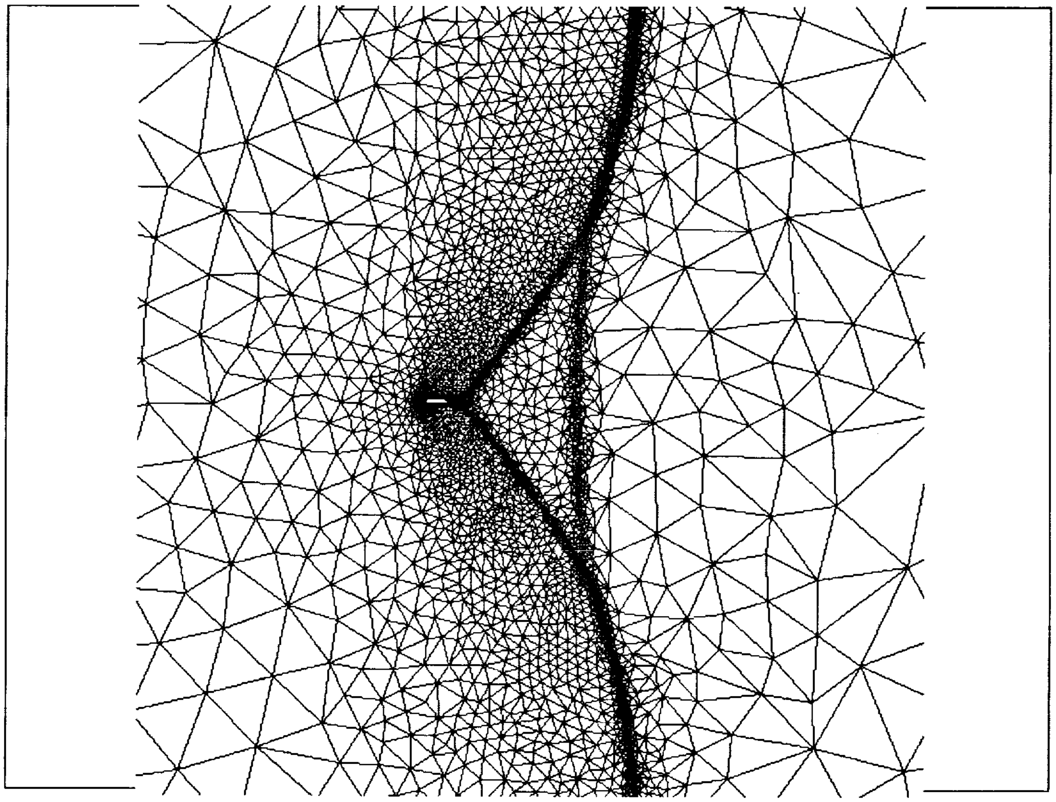
$$\lim_{n \to \infty} h_i \neq 0$$

Desirable limit properties can be enforced by multiplying by local length scale

$$\tilde{e} = \frac{l_i}{l_r} \tilde{\Delta} q \qquad \text{... etc}$$

Results From Corrected Adaptive Criteri





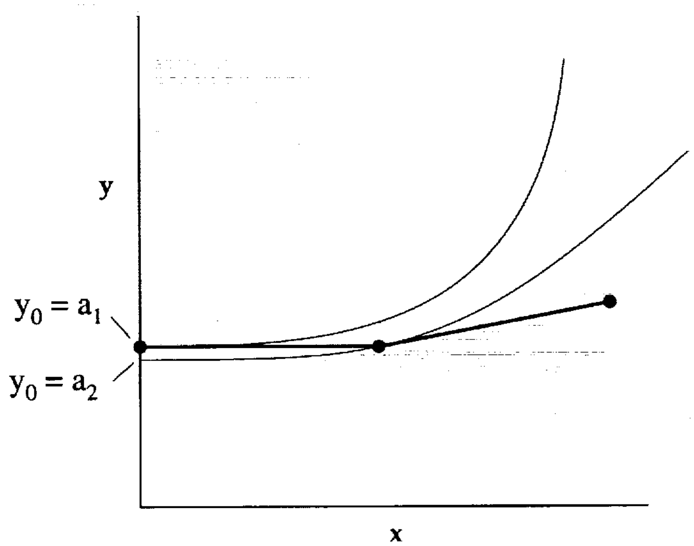
Lesson Learned

- Beware of adaptive criteria that refine "gradients" only and do not approach zero for all cells.

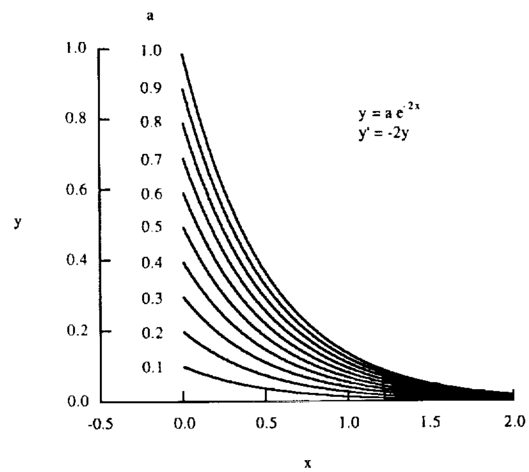
One-Dimensional O.D.E's

- Two-Point Boundary Value Problems
  - Babuska - Optimal grid spacing occurs when error is evenly distributed
  - Models elliptic and parabolic p.d.e. behavior
- Initial Value Problem
  - Models hyperbolic p.d.e. behavior
  - Must account for error propagation and accumulation

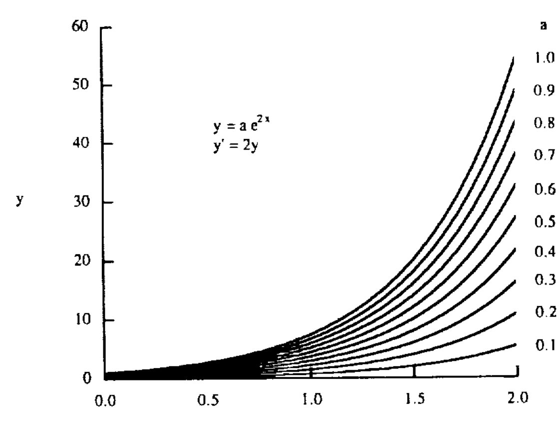
Error and I.V.P.'s



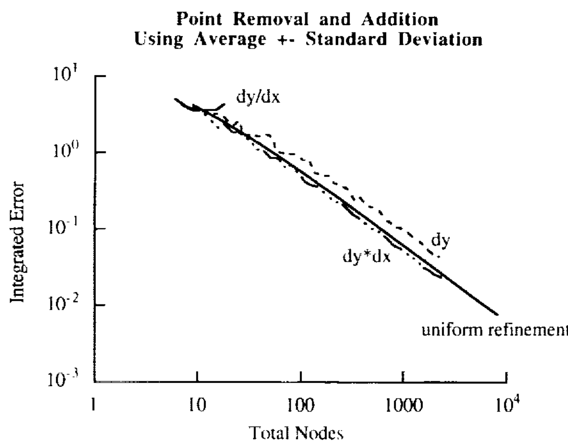
Model O.D.E.



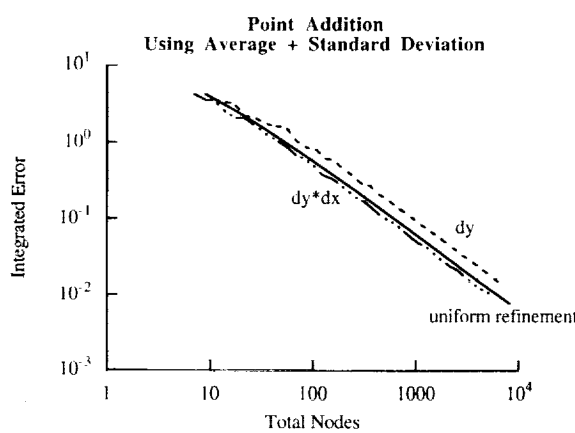
Model O.D.E.



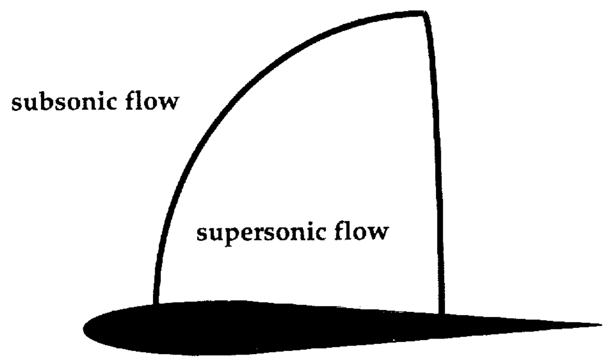
O.D.E. Adaptation



Model O.D.E.



How do we adapt to transonic flow??



Discussion

- Adaptation criteria must approach zero in all cells as they are refined (like local error) to guarantee grid convergence
- Adapting to marching problems is not the same as for two-point boundary value problems
- Marching problems must take into account spatial stability + zone of influence