Recent Advances in Computational Structural Reliability Analysis Methods

Ben H. Thacker, Y.-T. (Justin) Wu, Harry R. Millwater,
Tony Y. Torng, and David S. Riha
Southwest Research Institute
San Antonio, Texas

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INTRODUCTION

The goal of structural reliability analysis is to determine the probability that the structure will adequately perform its intended function when operating under the given environmental conditions. Thus, the notion of reliability admits the possibility of failure. Given the fact that many different modes of failure are usually possible, achievement of this goal is a formidable task, especially for large, complex structural systems. The traditional (deterministic) design methodology attempts to assure reliability by the application of safety factors and conservative assumptions. However, the safety factor approach lacks a quantitative basis in that the level of reliability is never known and usually results in overly conservative designs because of compounding conservatisms. Furthermore, problem parameters that control the reliability are not identified, nor their importance evaluated.

This paper presents a summary of recent advances in computational structural reliability assessment. A significant level of activity in the research and development community has been seen recently, much of which has been directed towards the prediction of failure probabilities for single mode failures. The focus of this paper is to present some early results and demonstrations of advanced reliability methods applied to structural system problems. This includes structures that can fail as a result of multiple component failures (e.g., a redundant truss), or structural components that may fail due to multiple interacting failure modes (e.g., excessive deflection, resonate vibration, or creep rupture). From these results, some observations and recommendations are made with regard to future research needs.

PROBABILISTIC STRUCTURAL ANALYSIS METHODS (PSAM) PROGRAM

The methodologies presented in this paper have been developed by the Southwest Research Institute (SwRI) under sponsorship from the NASA Probabilistic Structural Analysis Methods (PSAM) Program [1,2]. The objective of the NASA/PSAM program is to develop probabilistic structural analysis methods for critical space shuttle main engine (SSME) components such as turbine blades, transfer ducts, piping systems, and liquid oxygen posts. These components are considered critical because of the severe consequence of failure. A major accomplishment of the PSAM program is the development of the NESSUS computer program, which integrates advanced reliability methods with general structural analysis capabilities. Rocketdyne (Rockwell Corporation) has and is currently applying NESSUS to critical SSME components [3].

The methodologies developed by PSAM are applicable to a wide range of applications. Under several other projects, SwRI is applying PSAM technology to geomechanics, nuclear waste, rotordynamics, industrial design and optimization, biomechanics, and numerous other structural and mechanical reliability problems [4, 5, 6].

- Ten-Year Research and Development Program
  - Phase 1: Probabilistic Structural Response Analyses of the Space Shuttle Main Engine Components (1984-1989)
  - Phase 2: Structural System Risk Assessment, Qualification, Certification, and Health Monitoring (1990-)

- Simulate Uncertainty/Variability in Loads, Material Properties, Geometries

- Computer Code NESSUS Integrates Reliability Methods with Structural Analysis Methods (FEM, BEM)

- PSAM Methodology and Code are General Structural Risk Assessment and Reliability Design Tools

Prime Contractor: Southwest Research Institute
Project Sponsor: NASA Lewis Research Center

SUMMARY OF NESSUS 6.0 CAPABILITIES

The probabilistic structural analysis methods used in this paper are implemented in the NESSUS^{tm} probabilistic computer program [7]. NESSUS couples numerous advanced probabilistic algorithms with general-purpose structural analysis capabilities to provide a very efficient means of computing probabilistic results for complex applications. Thus, the key feature of NESSUS is its ability to establish the cumulative distribution function (CDF) for complex structures with a minimum number of response re-solutions. Figure 1 summarizes the capabilities in NESSUS Version 6.0.

Random Variables
Loads
- Forces
- Pressures
- Temperatures
- Vibrations (PSD)

Material properties
- Moduli
- Poisson's ratio
- Yield stress
- Hardening parameters
- Material orientation

Geometry
User-defined

Probabilistic Methods
Fast Probability Analysis
- Advanced Mean-Value
- First and Second-Order
- Fast Convolution

Sampling
- Standard Monte Carlo
- Latin Hypercube
- Adaptive Importance

Probabilistic Fault Tree

Probability of Creep Rupture

1.0

0

360 3D Solid Elements
810 Nodes, 2100 DOF

Service Life

Probabilistic Results
- Full probability distribution
- Component reliability
- System/multi-failure-modes rel.
- Probabilistic sensitivities
- Probability-based risk/cost

Performance Functions
- Structural responses:
  stress, strain, disp., freq., etc.
- Fatigue and fracture life
- Creep rupture life
- User-defined subroutines
- External analysis programs
  (requires custom-made interface)

Analysis Types
Static
Transient dynamics
Buckling
Vibrations
Nonlinearities
- Plasticity
- Large displacement

Element Library
Beam
Plate
Plane strain
Plane stress
Axisymmetric
3D solid
Enhanced solids

Operating Systems
Mainframes
- CRAY
- VAX
Workstations
- HP
- SUN
- APOLLO

Figure 1. Summary of NESSUS Version 6.0 Capabilities.

One of the challenges in computational structural reliability is the development of efficient and accurate probabilistic analysis algorithms for analyzing structures where the computations of the performance and its sensitivities are very time-consuming. Recently, probabilistic methods based on the limit state approach have been developed and successfully integrated with finite element and boundary element methods. In these methods, the primary computational effort is typically spent on locating the most probable point (MPP) for a limit state function, $Z = z_i$, as illustrated in Fig. 2. Once the MPP is identified, the probability of failure can be estimated. Several approaches are available to search for the MPP. One efficient method, developed and implemented in NESSUS, is the advanced mean value method (AMV), which has been shown to be extremely efficient [8]. One of the limitations of the limit state approach is that the probability estimate is based on a low-order polynomial approximation. The adaptive importance sampling (AIS) method provides a quick way to check the AMV solution by sampling in the most likely failure region [9].

- Input Variables Defined Using Probability Distributions

- Fast Probability Integration Estimates MPP on Response Surface

- Advanced Mean Value (AMV+) Iteration Computes Converged MPP and Estimates Probability of Failure (p_f)

- Adaptive Importance Sampling (AIS) Can be Used to Check p_f Calculation
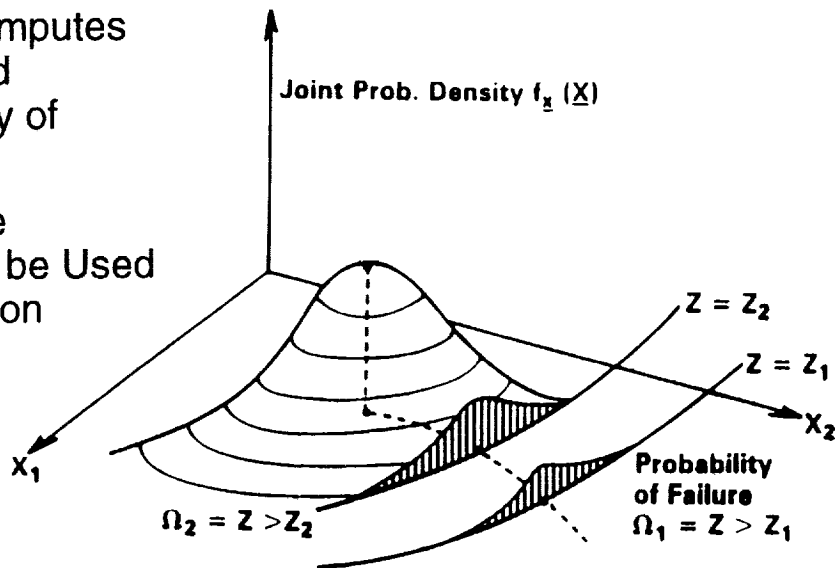
Figure 2. Joint Density Function and Most Probable Point (MPP) for Two Random Variables.

RELIABILITY ANALYSIS USING COMPONENT STRESS AND MATERIAL RESISTANCE CURVES

The example shown in Fig. 3 considers a circular disk subjected to two equal and opposite point loads. The disk is assumed to fail when the equivalent stress at the center of the disk exceeds the material yield stress, which is a function of temperature. Random variables considered include the loading, thickness of the disk, and the temperature. A simple relationship is used to describe the degradation of yield stress as a function of temperature.

Using NESSUS, the distribution functions for stress and strength were computed. The reliability analysis, also performed with NESSUS, then computed the probability of strength being less than stress, which was considered failure. The probabilistic analysis was verified using both AMV and Monte Carlo with the circular disk modeled in closed-form and with finite elements.

Component Stress

Maximum von
Mises Stress in
Disk Under Load

Thin Disk
Modeled Using
Finite Elements

Material Resistance

Yield Stress Modeled as
Function of Thermal Cycles

$$\sigma_y = \sigma_{y0}\left(\frac{T_F - T}{T_F - T_0}\right)^n$$

Probability of Failure Computed Using NESSUS

Strength Variable
Yield Stress
Mean=35.4 ksi
Std=1.8 ksi

Stress Variable
Equivalent Stress
Mean=23.0 ksi
Std=3.23 ksi

$p_f = 0.00291$
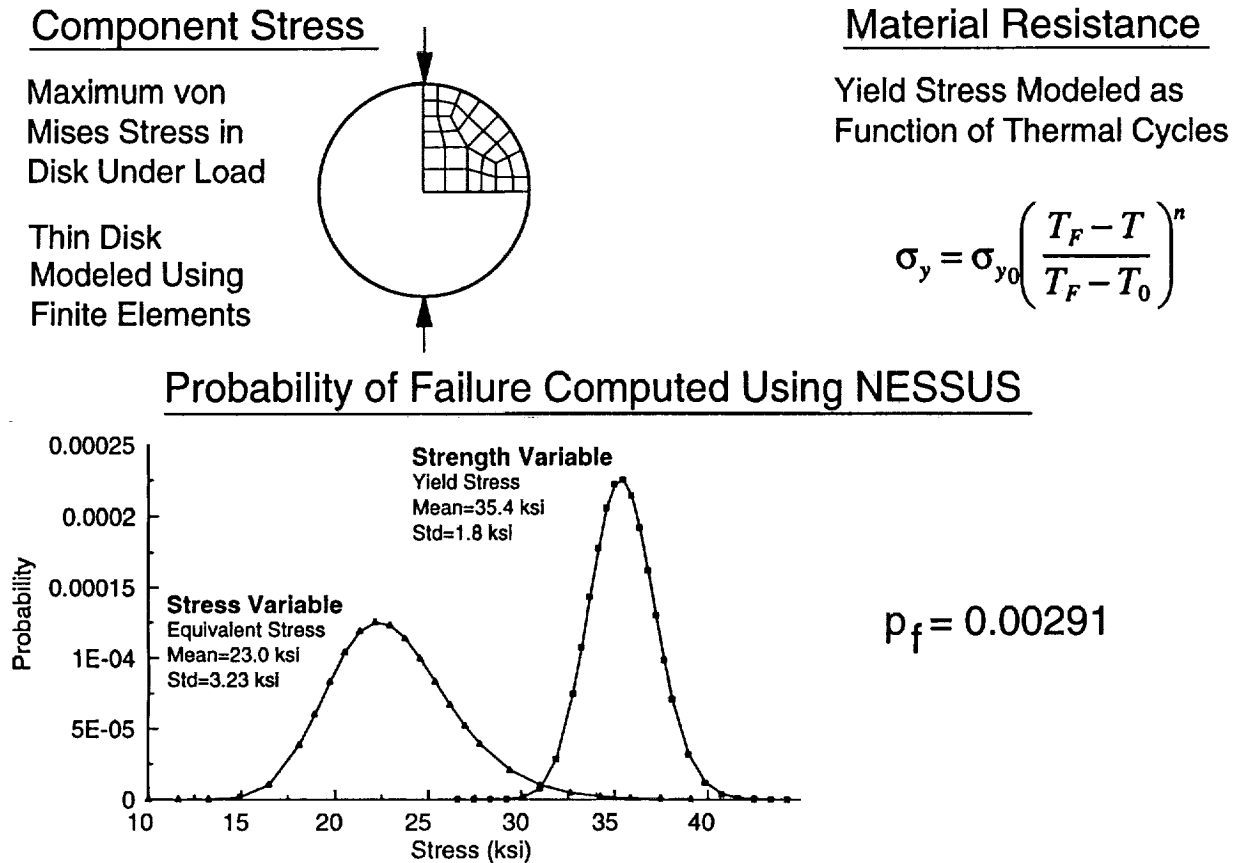
Figure 3. Component Reliability Analysis Demonstrated for Simple Example.

ROCKET ENGINE TURBINES SUBJECT TO SEVERE CONDITIONS

Representative of a more real world component reliability problem, a model of an SSME component was considered. The high-pressure fuel turbopump blade, shown in Fig. 4, represents a critical component in the SSME engine in that failure of a blade can result in loss of the complete engine. Stringent limitations on size and weight coupled with the hot hydrogen enriched steam turbine fluid and the cold hydrogen cooling fluid results in a very severe thermal environment. Probabilistic methods are ideally suited for the SSME turbine blade analysis where the lack of available local measurements results in significant uncertainty in loads such as thermal response. The lack of data is attributed to the difficulty in making measurements in the extreme environments and operating conditions within the engine.

- Severe Design Requirements
  - Comparatively Short but Severe Service Life
  - Strict Limitations on Size and Weight
  - High Energy Content of Fluids
  - High Specific Work Output
  - Rapid Start and Short Run Duration
  - Severe Thermal Shock Conditions
  - High Stage Loading and Stress
- Blades Prone to High Cycle Fatigue Cracking
- Operating Stresses and Deflections Must be Closely Controlled
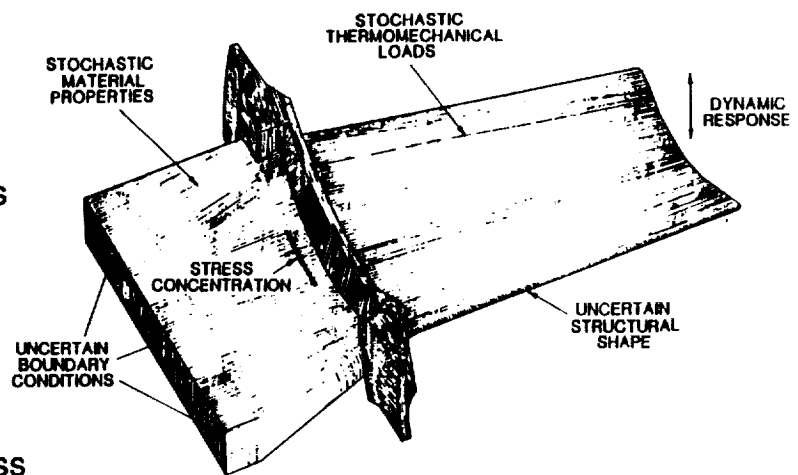
Figure 4. Space Shuttle Main Engine (SSME) Turbine Blade.

TURBINE BLADE RANDOM VARIABLE INPUT

Several methods exist for modeling uncertainty, such as probability distributions, fuzzy sets, convex models, bounding, etc. The probability distribution approach is used here to model each engineering variable in terms of its statistical parameters, namely its mean, standard deviation and distribution type. The standard deviation characterizes the magnitude of the scatter in the data and the distribution describes how the scatter is distributed about the mean. The variables listed in Table 1 were used for the demonstration analyses presented in this paper.

Table 1. Random Variable Definitions Used for the Turbine Blade Analysis.

Variable	Symbol	Mean Value	Standard Deviation	Distribution
Crystal Orientation about z	θ_z	+3°	3.9°	Normal
Crystal Orientation about y	θ_y	-2°	3.9°	Normal
Crystal Orientation about x	θ_x	+5°	3.9°	Normal
Young's Modulus	E	18.38E6 psi	0.46E6 psi	Normal
Poisson's Ratio	v	0.386	.00965	Normal
Shear Modulus	G	18.63E6 psi	.47E6 psi	Normal
Material Parameter	B_0	86.0	0.086	Normal
Density	ρ	0.805E - 3 lbm/in^3	0.493E - 5 lbm/in^3	Normal

TURBINE BLADE CREEP RUPTURE RELIABILITY ANALYSIS

A component reliability analysis of the turbine blade considering failur by creep rupture was performed. The turbine blade is modeled using the finite element model shown in Fig. 5. The blade is composed of a single crystal material described by three elastic constants, and three orientation angles. In the analysis, the blade is rotating at a constant speed and operating at a constant temperature. The failure mode being studied is creep rupture due to operation of the blade under elevated temperature. The temperature, stress and rupture life are related using a Larson-Miller relation, given by $P = T(C + \log t)$ where P is the Larson-Miller parameter; T is the absolute temperature; and t is the rupture life. More detail of this analysis is given in Ref. [10].

Possibly one of the most valuable products of the probabilistic analysis are the probabilistic sensitivity factors (PSF), shown by the bar chart in Fig. 5. The PSF's provide an important ranking of the problem variables with respect to the total *uncertainty* in the response. Therefore, since the statistics of the input variables are contained in these sensitivity data, the PSF data provide a more realistic and useful ranking of the variables.

Loads:
Pressure
Thermal
Centrifugal

● Turbine Blade Operating at High Speed in High Temperature Environment

● Failure by Creep Rupture Possible if Operated Past Critical Time

● AMV+ Procedure Used to Assess Reliability
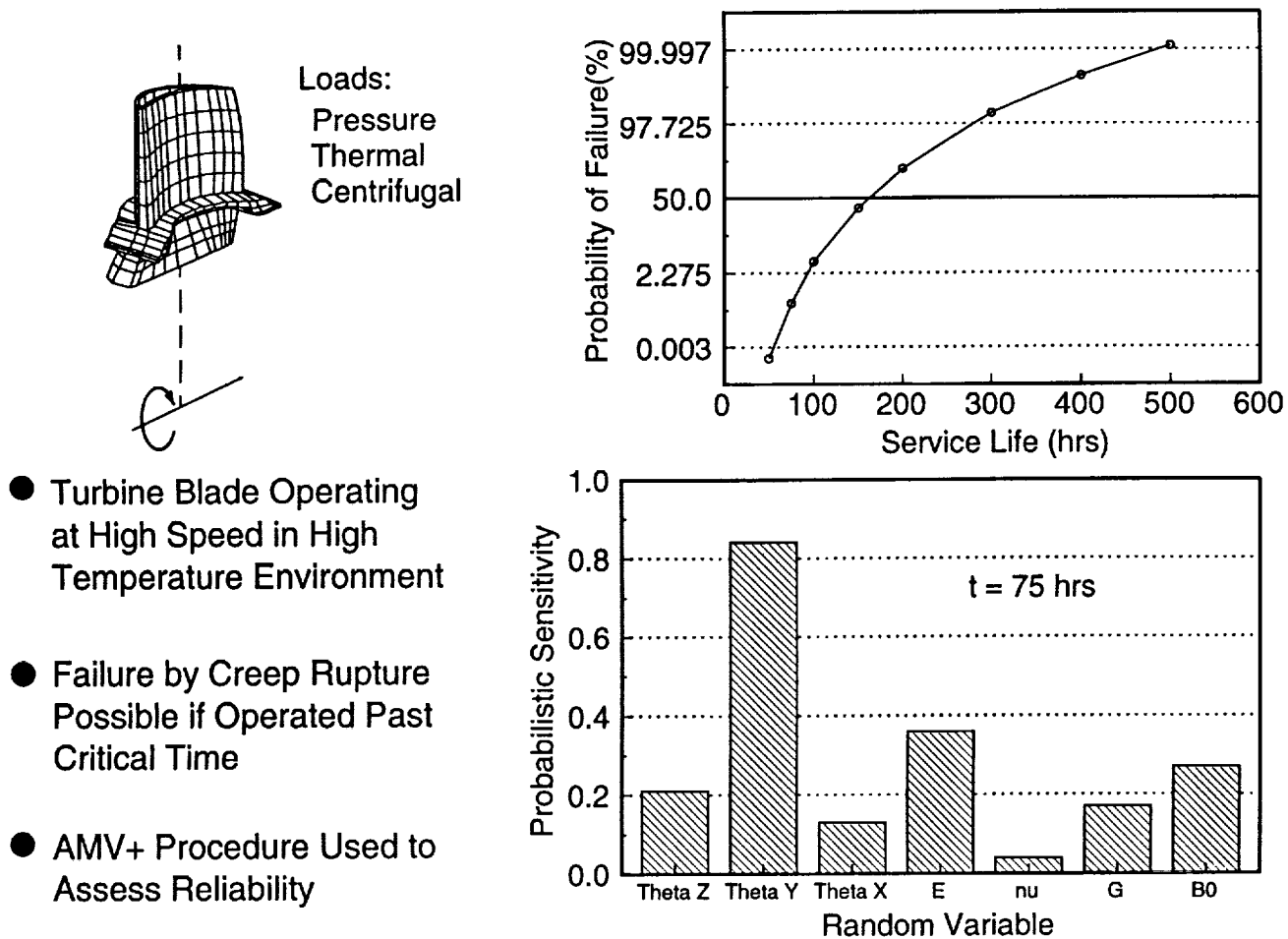
Figure 5. Creep Rupture Reliability Analysis of the Turbine Blade.

PROBABILITY OF EXCEEDING 60 KSI EFFECTIVE STRESS

Another component reliability analysis of the turbine blade considers stress exceedance. Figure 6 shows the probability of the von Mises stress exceeding 60 ksi. The plot was obtained by computing the probability of exceeding 60 ksi stress at each node using the NESSUS program and plotting the results using a general-purpose finite element graphics program.

The probabilistic information was obtained from a mean-based sensitivity analysis and must therefore be interpreted accordingly. The contours indicate where the high failure probability regions are located and where more refined analyses should be directed. It should be noted that the high probability regions may not be the same as the high stress regions from a deterministic analysis. This is because the standard deviation in stress varies from location to location in the mesh. For example, although the mean stress at some location (A) may be lower than the mean stress at some other location (B), the standard deviation may be higher at A than at B (i.e., more variation in stress). Thus, the *probability* of exceeding a certain stress level could be higher at A than at B even though the mean stress is lower at A than at B.
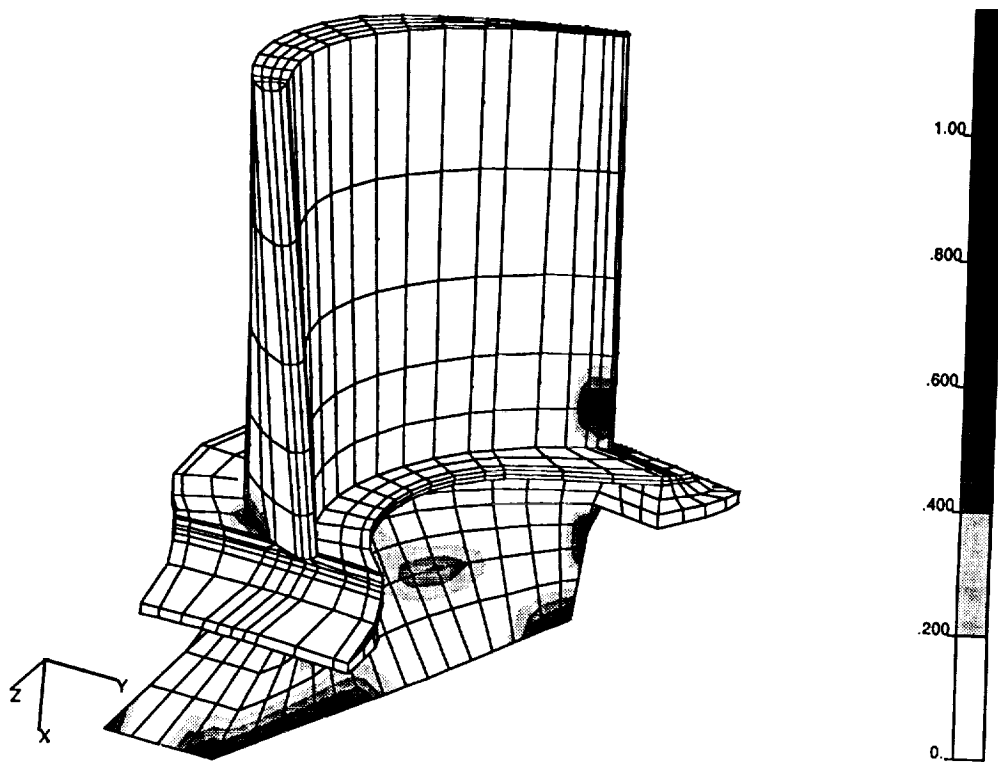
Figure 6. Probability of Exceeding 60 ksi Effective Stress.

NESSUS PROBABILISTIC FAULT TREE ANALYSIS METHODOLOGY

What dictates failure in real structures, as previously described, will usually be a sequence or interaction of individual component failure modes. A popular method for modeling system failure as a function of component failures is with a fault tree. A fault tree provides a systematic way to deal with multiple, possibly complicated, failure paths composed of multiple components or multiple failure modes (bottom events). In traditional fault tree analysis, probabilities are assigned to the bottom events, and propagated through gates (AND, OR, etc.). For typical structural reliability analysis problems, however, the failure events will often times be correlated due to common problem variables. To account for this dependency, it is necessary that the limit state functions, rather than simply the probabilities, be used to define the bottom events. In addition, conditional limit state functions must be established that represent updated structural system configurations as a result of sequential failures (for modeling redundancy or progressive fracture for example). Figure 7 shows how failure modes and sequential failure can be modeled using a fault tree. Sequential failures can be modeled using the PRIORITY AND gate. A sequence of limit state functions corresponding to a sequence of updated structural configurations with load redistribution, must be generated during the analysis. This fault tree methodology has been developed and is implemented in NESSUS [9].
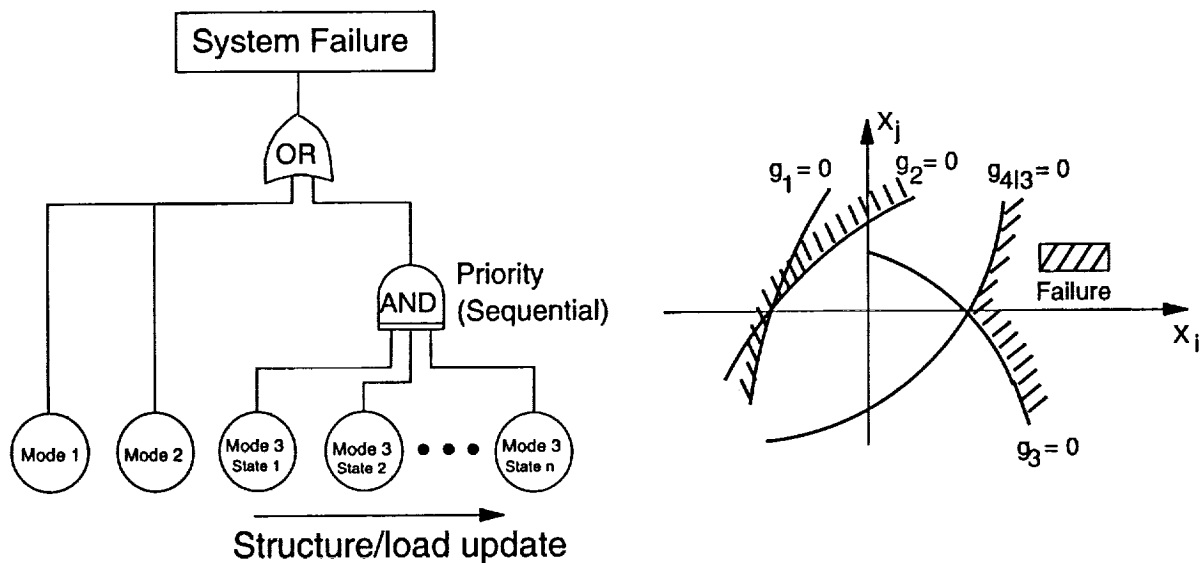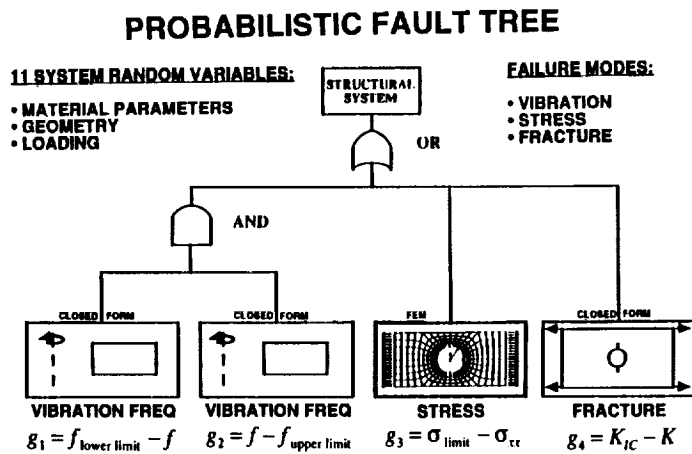

- Fault tree used for modeling multiple failure modes and paths
- Bottom events modeled using FEM model, approximate response surface, analytical equation
- Dependencies between bottom events accounted for
- Reliability calculated by adaptive importance sampling

Figure 7. NESSUS Probabilistic Fault Tree Analysis Methodology.

NESSUS SYSTEM RELIABILITY ANALYSIS EXAMPLE

Figure 8 shows a simple fault tree for a simple structural component. The hypothetical structure has three failure modes: vibration, stress and fracture. The structure is considered failed if any of the three failure conditions are satisfied. This is represented with the "OR" gate. Failure is defined as the first natural frequency being within a certain range, the stress exceeding a stress limit, and the mode I stress intensity factor exceeding the fracture toughness. For vibration, a two-sided limit is used to define failure and is represented in terms of an "AND" gate. As indicated, three of the bottom events are modeled analytically and one is given by finite element model.

Probability of system failure was obtained using several sampling methods; conventional Monte Carlo and adaptive importance sampling (AIS) using the exact limit state functions, and AIS on an approximate limit state. The approximate limit state consists of a closed-form approximation to the finite element model computed using the AMV+ procedure described earlier. The results indicate that the AIS method achieved results comparable to those obtained using Monte Carlo at 1/78 the computational cost. As also indicated, good agreement is obtained using the approximate limit state also, with further reductions in calculation cost. This example indicates the practical application of structural system reliability using a fault tree approach with advanced probabilistic methods.

PROBABILISTIC FAULT TREE

11 SYSTEM RANDOM VARIABLES:
- MATERIAL PARAMETERS
- GEOMETRY
- LOADING

STRUCTURAL SYSTEM

OR

AND

FAILURE MODES:
- VIBRATION
- STRESS
- FRACTURE

CLOSED FORM — CLOSED FORM — FEM — CLOSED FORM

VIBRATION FREQ | VIBRATION FREQ | STRESS | FRACTURE

$g_1 = f_{lower\ limit} - f$ $g_2 = f - f_{upper\ limit}$ $g_3 = \sigma_{limit} - \sigma_{tt}$ $g_4 = K_{IC} - K$

SYSTEM RELIABILITY RESULTS

(< 20% ERROR, 95% CONFIDENCE)

$P_f = Prob.[\{(g_1 < 0) \cap (g_2 < 0)\} \cup (g_3 < 0) \cup (g_4 < 0)]$

Method	P_f	System Samples*	CPU Time (sec) HP 700 Series
Monte Carlo	0.01170	10,000 (117 Failures)	54,800
AIS Exact**	0.01176	97 (41 Failures)	706
AIS Approx.***	0.01183	-	19

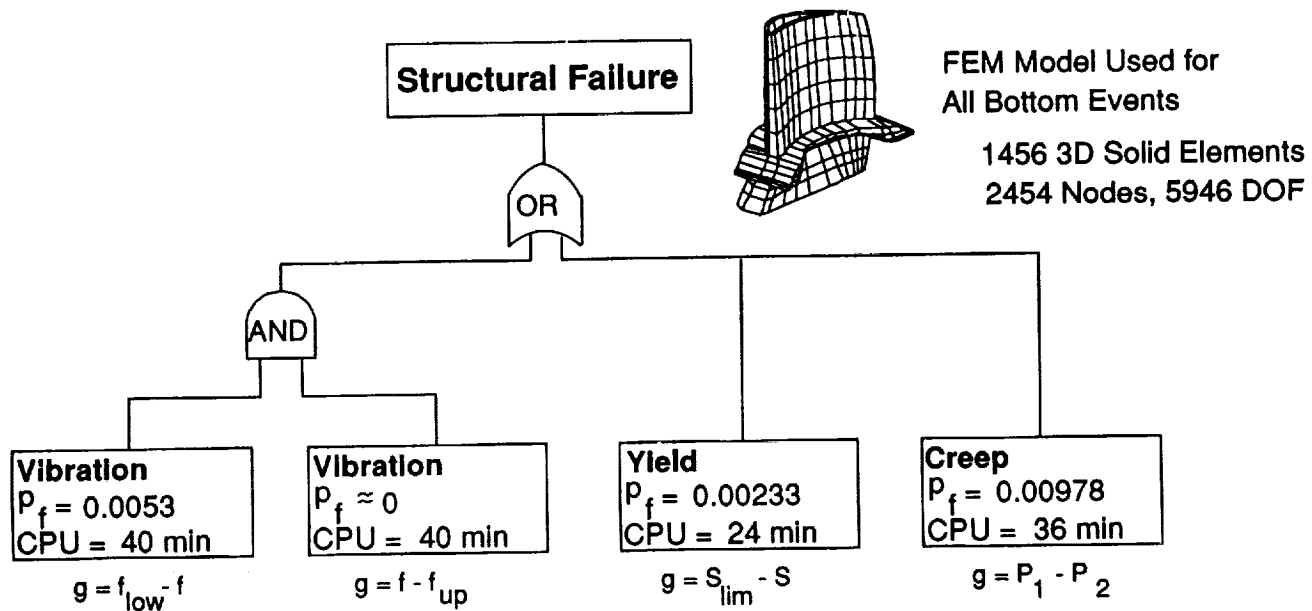* Four bottom events per system sample not including initial AMV+ function evaluations

** Adaptive Importance Sampling of exact functions

*** Adaptive Importance Sampling of approximate functions

Figure 8. NESSUS System Reliability Analysis Example.

SYSTEM RELIABILITY ANALYSIS OF THE TURBINE BLADE

To demonstrate the computational system reliability methodology on a realistic-sized problem, the turbine blade was analyzed probabilistically considering three modes of failure: frequency, yield, and rupture. The procedure employing approximate limit states for the component modes is recommended for this level of deterministic modeling. Each mode of failure is first analyzed using the AMV+ procedure to establish the approximate limit state. The computational effort required for this step is given in each bottom event box in Fig. 9. Next, the system failure is computed using AIS. This procedure is automated in NESSUS. This example demonstrates that system reliability assessment is now possible for complex structural systems.

Structural Failure

OR

AND

Vibration	Vibration	Yield	Creep
$P_f = 0.0053$	$P_f \approx 0$	$P_f = 0.00233$	$P_f = 0.00978$
CPU = 40 min	CPU = 40 min	CPU = 24 min	CPU = 36 min
$g = f_{low} - f$	$g = f - f_{up}$	$g = S_{lim} - S$	$g = P_1 - P_2$

FEM Model Used for
All Bottom Events

1456 3D Solid Elements
2454 Nodes, 5946 DOF

Probability of Structural Failure: 0.01711

Total CPU Time Required: 2.5 hours

CPU Times for HP Series 750 Workstation (64Mb RAM)

Figure 9. System Reliability Analysis of the Turbine Blade.

SYSTEM RELIABILITY ANALYSIS IDENTIFIES
CRITICAL FAILURE MODES AND RANDOM VARIABLES

As in the component reliability analysis procedure described earlier, the system reliability analysis also provides a probabilistic ranking of the inputs. The inputs for the system analysis are the component failure modes, whereas the inputs for the component analysis are the problem variables. Thus, not only are the problem variables ranked by importance, but the dominate failure modes are also identified. As shown in Fig. 10, for example, the creep rupture mode is seen to contribute the most to the overall system probability. The bar chart on the right shows the problem variable ranking for each failure mode. This type of information is required to establish a reliability based design procedure.
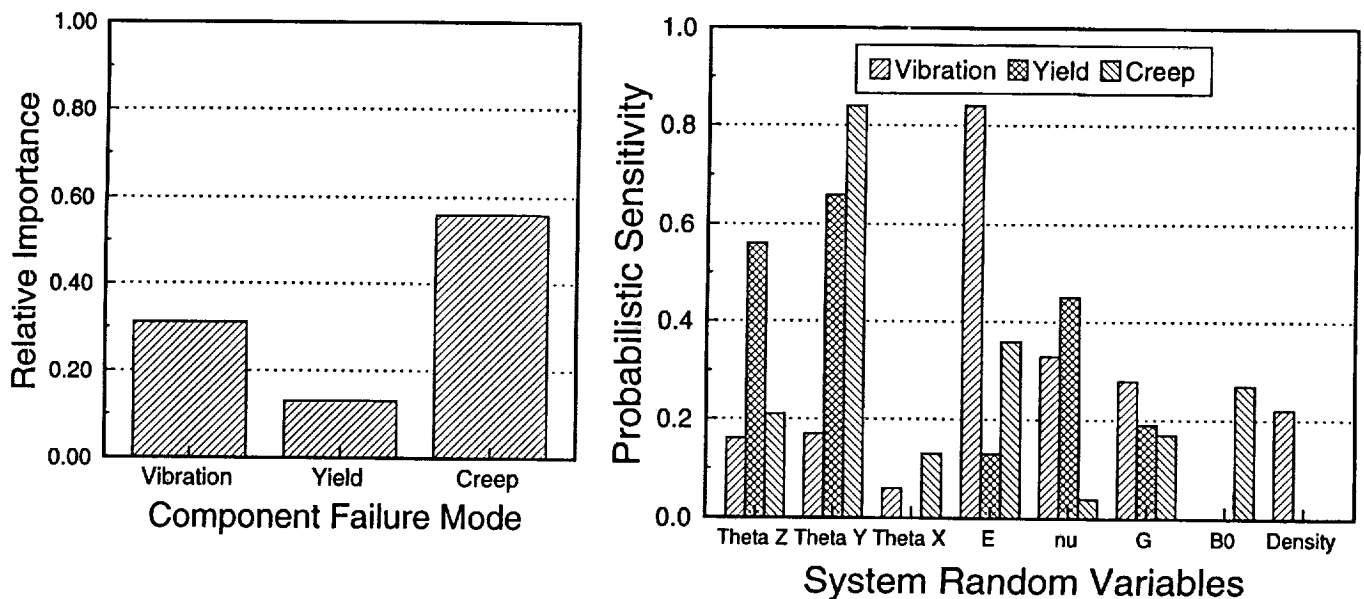
Figure 10. Dominate Failure Modes and Problem Variables are Identified in the System Reliability Analysis.

RELIABILITY ANALYSIS AND DESIGN PROCEDURE

How the probabilistic methodologies presented here would fit into a typical design procedure is shown in Fig. 11. The shadowed boxes indicate the position of the component and system reliability analyses in the overall design cycle. The first step in the process is to identify all of the physical variables and sources of uncertainty. How these physical variables affect the problem variables comprises the next step. For example, in the SSME, the fuel mixture ratio (a physical variable) effects the pressure, thermal, and centrifugal loading (problem variables) on the turbine blade. In the next steps, the individual failure modes are identified and analyzed both deterministically and probabilistically. Next, the overall system failure and dominate failure modes and problem variables are identified, which are used in subsequent steps to alter the design subject to the design requirements (e.g., cost, weight, reliability).
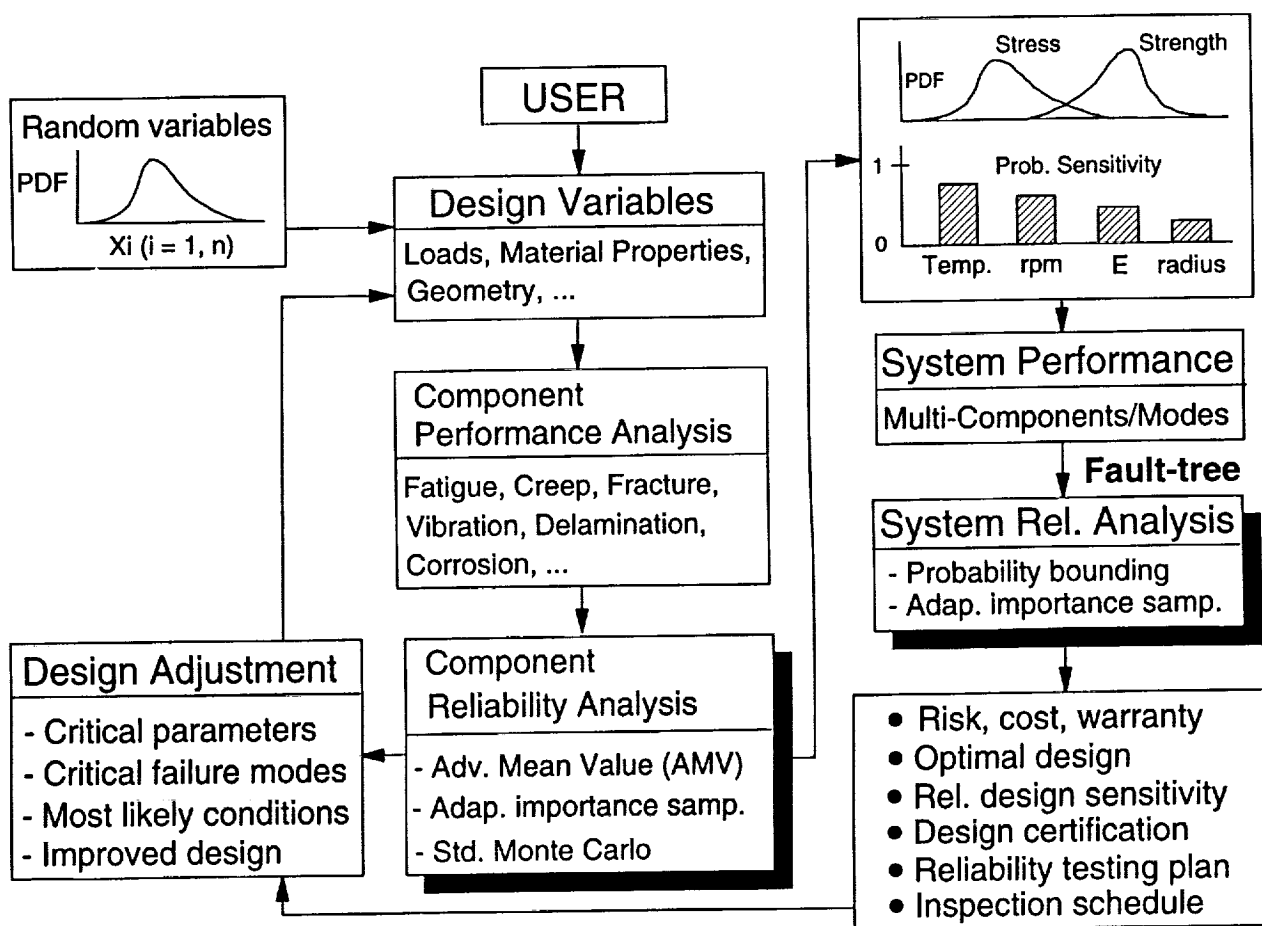

Figure 11. Component and System Reliability Analysis Integrated Into The Design Procedure.

200

STATE-OF-THE-ART TECHNOLOGY IN COMPUTATIONAL STRUCTURAL RELIABILITY ANALYSIS

It must be recognized that probabilistic structural analysis methods are not as well developed as deterministic design methodologies. Computational tools, such as those described in this paper, are just recently becoming available and have only been applied to limited numbers of problems. Moreover, training and experience are required to conduct a probabilistic analysis, neither of which are easily acquired at the present time. Consequently, before probabilistic methods can be successfully integrated into the current design cycle, several challenges, such as those listed in Fig. 12, must be overcome.

● What is Possible Now:
- Moderately Detailed Component Reliability Analyses (10,000 DOF)
- Simplified System Reliability Analyses (< 5 Modes)
- Simplified Integrated Risk/Reliability Analyses

● Current Challenges:
- Awareness, Comprehension and Acceptance
- Identification of Uncertainties or Randomness
- Probabilistic Data Bases
- Robustness and Validation of Recently Developed Computational Tools

Figure 12. State-of-The-Art In Computational Structural Reliability Analysis.

APPLICATIONS WILL SUPPORT (DRIVE) FUTURE
RESEARCH AND DEVELOPMENT

The requirement for a probabilistic approach is dictated by need, and current application needs will certainly impact the future research directions. A few of these application areas are listed in Fig. 13. One of the more promising areas is probabilistic fracture mechanics. This is because fracture usually results in sudden catastrophic failure of the system. Both progressive fracture and multi-site damage are of current concern and will require probabilistic methods.

- Probabilistic Progressive Fracture
- Multi-Site Damage (Linkup)
- Parallel Processing
- Certification
- Health Monitoring
- Multi-Disciplinary Reliability Assessment
- Optimum Inspection Scheduling and Retirement
- Human/Modeling Error
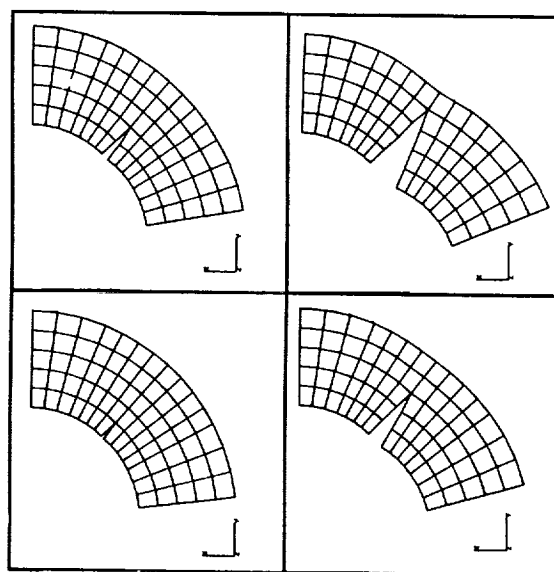
Figure 13. Some Current Application Areas That Will Require Probabilistic Methods.

REFERENCES

1. Chamis, C. C., "Probabilistic Structural Analysis Methods for Space Propulsion System Components," *Probabilistic Engineering Mechanics*, Vol. 2, No. 2, 1987.

2. Cruse, T. A., Burnside, O. H., Wu, Y.-T., Polch, E. Z., Find, P. K., Dias, J. B. and Rajagopal, K. R., "Probabilistic Structural Analysis Methods for Select Space Propulsion System Structural Components (PSAM)," *Computers and Structures*, Vol. 29, No. 5, 1988, pp. 891-901.

3. Cruse, T. A., Chamis, C. C. and Rajagopal, K. R., "Probabilistic Structural Analysis, Reliability and Risk of Critical SSME Components," NASA-MSFC Conference on Advanced Earth-to-Orbit Propulsion Technology, May 1990.

4. Thacker, B. H. and Senseny, P. E., "Probabilistic Structural Analysis of Deep Tunnels," *Probabilistic Methods in Geomechanics*, A. F. Fossum (ed.), AMD Vol. 134, American Society of Mechanical Engineers, NY, 1992, pp. 1-13.

5. Wu, Y.-T., Journel, A. G., Abramson, L. R. and Nair, P. K., Uncertainty Evaluation Methods for Waste Package Performance Assessment, NUREG/CR-5639, U.S. Nuclear Regulatory Commission, Jan. 1991.

6. Wu, Y. T., Torng, T. Y., Millwater, H. R., Fossum, A. F. and Rheinfurth, M. H., "Probabilistic Methods for Rotordynamics Analyses," SAE-911110, April 1992.

7. Millwater, H. R., Wu, Y.-T., Torng, T. Y., Thacker, B. H., Riha, D. S. and Leung, C.-P., "Recent Developments of the NESSUS Probabilistic Structural Analysis Computer Program," AIAA-92-2411, April 1992.

8. Wu, Y.-T., Millwater, H. R. and Cruse, T. A., "An Advanced Probabilistic Structural Analysis Method for Implicit Performance Functions," *AIAA Journal*, Vol. 28, No. 9, Sept. 1990, pp. 1840-1845.

9. Wu, Y.-T., "An Adaptive Importance Sampling Method for Structural System Reliability Analysis," ASME Winter Annual Meeting, Nov. 1992.

10. Thacker, B. H. and Wu, Y.-T., "A New Response Surface Approach for Structural Reliability Analysis," AIAA-92-2408, April 1992.

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