

# Pseudo-Coherent Demodulation for Mobile Satellite Systems\*

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## Abstract

This paper proposes three so-called pseudo-coherent demodulation schemes for use in land mobile satellite channels. The schemes are derived based on maximum-likelihood (ML) estimation and detection of an  $N$ -symbol observation of the received signal. Simulation results for all three demodulators are presented to allow comparison with the performance of differential PSK (DPSK), and ideal coherent demodulation for various system parameter sets of practical interest.

## Introduction

A land mobile satellite channel can be characterized by AWGN, phase noise, frequency offsets due to local oscillator instabilities, Doppler and Doppler rate, Rician fading, and shadowing due to vegetation, terrain, and man-made buildings. The severity of these impairments varies for different applications depending on the satellite transponder, data rate, carrier frequency, type of antenna, and so on. For example, if a high gain antenna is used for the mobile, then Rician fading can be ignored. The modem designed to operate in such a channel must be robust in the face of frequent but rapid signal outages and must be able to reacquire the signal quickly. In the absence of a mid-band pilot, coherent demodulation is not an appropriate choice for this channel due to the long signal reacquisition time caused by the above mentioned impairments. Therefore, differentially coherent PSK (DPSK) can be selected as an alternate modulation scheme because of its simplicity and ability to recover quickly from the fade events. The bit error rate performance of uncoded DPSK is worse than that for coherent demodulation by about 1 dB in  $E_b/N_0$ . Convolutionally coded DPSK, however, requires about 3 dB higher  $E_b/N_0$  than coherently demodulated PSK at a bit error rate of  $10^{-3}$  in an AWGN channel assuming 3-bit soft quantization. To reduce this penalty, we consider pseudo-coherent demodulation which represents a compromise between the two extremes of coherent and differentially coherent demodulation.

This paper proposes three so-called pseudo-coherent demodulation schemes. The schemes are

derived based on maximum-likelihood (ML) estimation and detection of an  $N$ -symbol observation of the received signal. Typical values of  $N$  range from 5 to 15 symbols. The first scheme is based on ML estimation of carrier phase assuming that the unknown frequency offset is perfectly compensated by a frequency estimator *prior* to the phase estimation process. The second scheme is based on direct ML estimation of a time varying phase caused by the presence of the frequency offset. This scheme does not require a separate frequency estimator as in the first scheme. Last of all, a scheme is proposed which is a hybrid of a simple open loop frequency estimator and the ML estimate of carrier phase *conditioned* on perfect knowledge of the frequency offset. Due to the time-varying phase, both the second and third demodulators are equipped with a 180 degree phase jump detector to resolve the periodic 180 degree phase ambiguities that occur. Simulation results for all three demodulators are presented to allow comparison with the performance of DPSK for various system parameter sets of practical interest.

## Derivation of a pseudo coherent demodulation scheme in the absence of frequency offset

Consider the transmission of a BPSK modulated signal over AWGN channel with unknown carrier phase. For simplicity we use the complex envelope signal representation. The transmitted signal in the interval  $kT \leq t < (k+1)T$  is

$$s(t) = \sqrt{2P}e^{j\phi_k} \quad (1)$$

where  $P$  denotes the constant signal power,  $T$  denotes the PSK symbol duration and  $\phi_k$  the transmitted phase which takes on one of two values 0 and  $\pi$ . The corresponding received signal in this same interval is

$$r(t) = \sqrt{2P}e^{j\phi_k}e^{j\theta} + n(t) \quad (2)$$

where  $n(t)$  is a zero mean complex Gaussian noise with two-sided power spectral density  $2N_0$  and  $\theta$  is an arbitrary carrier phase introduced by the channel. In (2) we have assumed that the received signal is down-converted to baseband by a frequency reference signal

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$\exp(j2\pi\hat{f}_c t)$ , where  $\hat{f}_c$  is the estimate of the received carrier frequency  $f_c$  and is provided by a Doppler frequency estimator. If we assume perfect frequency estimation i.e.  $\hat{f}_c = f_c$ , then at the output of the integrate-and-dump (I&D) filter we obtain

$$r_k = \sqrt{2P}e^{j(\phi_k + \theta)} + n_k \quad kT \leq t < (k+1)T \quad (3)$$

where  $n_k$  is a sample of zero mean complex Gaussian noise with variance  $\sigma_n^2 = N_0 / T$  per dimension.

Consider now a received sequence  $\mathbf{r} = (r_{k-1}, \dots, r_{k-N})$  of length  $N$  and assume that the carrier phase  $\theta$  is constant over the length of this sequence. Then the likelihood function is

$$p(\mathbf{r}|\phi, \theta) = \left(\frac{1}{2\pi\sigma_n^2}\right)^N e^{-\frac{1}{2\sigma_n^2} \sum_{i=1}^N |r_{k-i} - \sqrt{2P}e^{j(\phi_{k-i} + \theta)}|^2} \quad (4)$$

where vector  $\phi = (\phi_{k-1}, \dots, \phi_{k-N})$  is the transmitted phase sequence. Equation (5) can be written as

$$p(\mathbf{r}|\phi, \theta) = F e^{\alpha \sum_{i=1}^N \text{Re}\{r_{k-i} e^{-j(\phi_{k-i} + \theta)}\}} \quad (5)$$

where

$$F = \left(\frac{1}{2\pi\sigma_n^2}\right)^N e^{-\frac{1}{2\sigma_n^2} \sum_{i=1}^N |r_{k-i}|^2 - \frac{NP}{\sigma_n^2}} \quad (6)$$

which is independent of the data, and

$$\alpha = \frac{\sqrt{2P}}{\sigma_n^2} \quad (7)$$

We would like to obtain the maximum-likelihood estimation of the carrier phase  $\theta$  given the observation  $\mathbf{r} = (r_{k-1}, \dots, r_{k-N})$ . Thus,  $\hat{\theta}_{ML}$  should satisfy

$$\ln p(\mathbf{r}|\hat{\theta}_{ML}) = \max_{\theta} \ln p(\mathbf{r}|\theta) \quad (8)$$

$$p(\mathbf{r}|\theta) = E\{p(\mathbf{r}|\phi, \theta)\} \\ \text{But} \quad = F \prod_{i=1}^N \left( \frac{1}{2} \sum_{\phi_{k-i}} e^{\alpha \text{Re}\{r_{k-i} e^{-j(\phi_{k-i} + \theta)}\}} \right) \quad (9) \\ = F \prod_{i=1}^N \cosh(\alpha \text{Re}\{r_{k-i} e^{-j\theta}\})$$

or

$$\ln p(\mathbf{r}|\theta) = \ln F + \sum_{i=1}^N \ln \cosh(\alpha \text{Re}\{r_{k-i} e^{-j\theta}\}) \quad (10)$$

Then, the solution to

$$\left. \frac{\partial \ln p(\mathbf{r}|\theta)}{\partial \theta} \right|_{\theta = \hat{\theta}_{ML}} = 0 \quad (11)$$

results in  $\hat{\theta}_{ML}$ . Using (11) in (12) we obtain

$$\frac{\partial \ln p(\mathbf{r}|\theta)}{\partial \theta} = -\alpha \sum_{i=1}^N \tanh(\alpha \text{Re}\{r_{k-i} e^{-j\theta}\}) \cdot (\text{Re}\{jr_{k-i} e^{-j\theta}\}) = 0 \quad (12)$$

For small signal to noise ratio we have the approximation  $\tanh x \approx x$  (13)

Therefore,  $\hat{\theta}_{ML}$  should satisfy

$$\sum_{i=1}^N \text{Re}\{r_{k-i} e^{-j\hat{\theta}_{ML}}\} \text{Re}\{jr_{k-i} e^{-j\hat{\theta}_{ML}}\} = 0 \quad (14)$$

which results in the maximum-likelihood estimate of phase

$$e^{j\hat{\theta}_{ML}} = \left( \frac{\sum_{i=1}^N r_{k-i}^2}{\sum_{i=1}^N |r_{k-i}|^2} \right)^{\frac{1}{2}} \quad (15)$$

The structure of the demodulator corresponding to (15) is shown in Fig. 1.

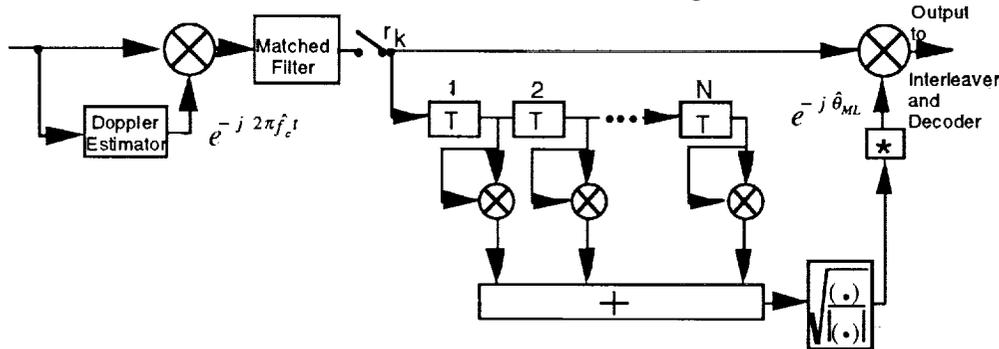


Figure 1. Pseudo-Coherent Demodulator (Scheme 1)

This scheme can be shown to be equivalent to a similar scheme proposed in [1] if the nonlinearity used in [1] is a squaring device. However, the authors of [1] did not specifically show that their structure is based on maximum-likelihood estimation.

We now present the relative bit error performances of a communication system employing the proposed demodulation scheme with respect to comparable systems using DPSK and ideal coherent demodulation. To do this we consider the three systems illustrated in Figs. 2, 3, and 4 respectively. In all three cases, the transmitter uses a constraint length  $K=7$ , rate  $r=1/2$  convolutional encoder with an interleaving size of  $4 \times 32$  symbols. A Viterbi decoder with infinite bit quantization input and decoder buffer size of 32 bits has been used at the receiver.

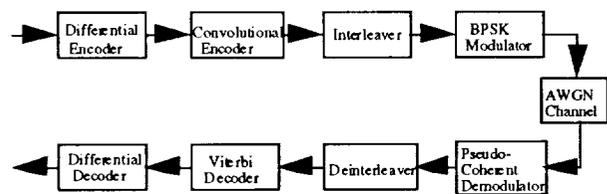


Figure 2. System 1, with Pseudo Coherent Demodulation.

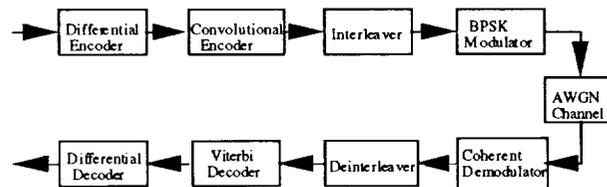


Figure 3. System 2, with Coherent Demodulation.

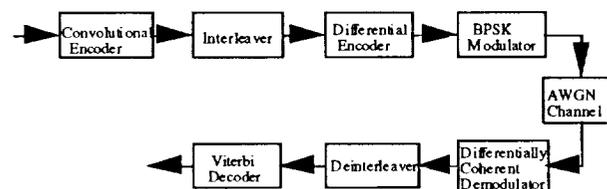


Figure 4. System 3, with Differentially Coherent Demodulation (DPSK).

Simulation results for the bit error performance of the three systems are shown in Figure 5. As can be seen from this figure, the proposed pseudo-coherent demodulator requires 2.40 dB less  $E_b/N_0$  than the DPSK demodulator, and 0.85 dB more  $E_b/N_0$  than an ideal coherent demodulator at a bit error rate of  $10^{-3}$ . In obtaining the simulation results, perfect Doppler frequency tracking was assumed.

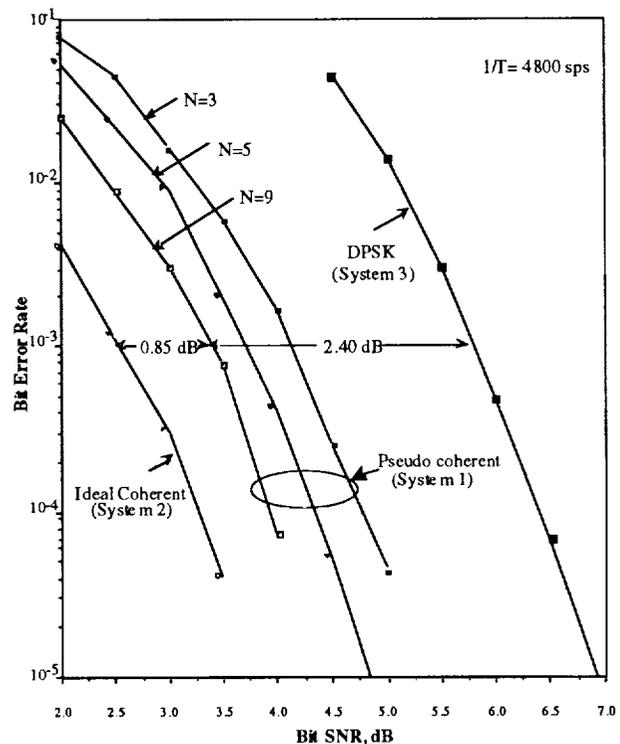


Figure 5. Simulation Results

#### Derivation of a pseudo coherent demodulation scheme in the presence of frequency offset

Let  $\theta_k$  be the unknown time-varying carrier phase (due to frequency offset) at time  $t = kT$ . Then,  $\theta_k$  can be written as

$$\theta_k = \theta_0 + 2\pi f kT \quad (16)$$

where  $\theta_0$  is the initial carrier phase and  $f$  is the frequency offset. Similarly, the phase at time  $t = (k-i)T$  is related to the phase at time  $t = kT$  by

$$\theta_{k-i} = \theta_k - 2\pi f iT \quad (17)$$

We are interested in finding the maximum-likelihood estimate of  $\theta_k$  by observing  $N$  past received samples  $r_{k-1}, r_{k-2}, \dots, r_{k-N}$  denoted by the vector  $\mathbf{r}$ , i.e.

$$p(\mathbf{r} | \hat{\theta}_{k,ML}) = \max_{\theta_k} p(\mathbf{r} | \theta_k) \quad (18)$$

To obtain  $p(\mathbf{r} | \theta_k)$ , we first find

$$p(\mathbf{r} | \phi, \theta_k, f) = F e^{\alpha \sum_{i=1}^N \text{Re} \{ r_{k-i} e^{-j(\phi_{k-i} + \theta_k - 2\pi f iT)} \}} \quad (19)$$

where  $F$  and  $\alpha$  are given by (6) and (7) and  $\phi = (\phi_{k-1}, \dots, \phi_{k-N})$  is a vector of BPSK phases, each

taking values 0 or  $\pi$ . Next, we compute  $p(\mathbf{r}|\theta_k, f)$  by averaging (20) over the binary-valued equiprobable data phases producing

$$p(\mathbf{r}|\theta_k, f) = E\{p(\mathbf{r}|\phi, \theta_k, f)\} \\ = F \prod_{i=1}^N \cosh(\alpha \operatorname{Re}\{R_{k-i} e^{-j\theta_k}\}) \quad (20)$$

where  $R_{k-i} \stackrel{\Delta}{=} r_{k-i} e^{j2\pi f iT}$ . Finally, we compute  $p(\mathbf{r}|\theta_k)$  by averaging (20) over the frequency shift  $f$  assuming as its density function a uniform distribution between  $-f_{\max}$  and  $f_{\max}$  where  $f_{\max}$  corresponds to the maximum expected (Doppler) frequency shift. The result of this averaging gives

$$p(\mathbf{r}|\theta_k) = E\{p(\mathbf{r}|\theta_k, f)\} \\ = \frac{F}{2f_{\max} - (-f_{\max})} \int_{-f_{\max}}^{f_{\max}} \prod_{i=1}^N \cosh(\alpha \operatorname{Re}\{R_{k-i} e^{-j\theta_k}\}) df \quad (21)$$

To obtain  $\hat{\theta}_{k,ML}$  we need to determine the solution to

$$\left. \frac{\partial p(\mathbf{r}|\theta_k)}{\partial \theta_k} \right|_{\theta_k = \hat{\theta}_{k,ML}} = 0 \quad (22)$$

Using small signal-to-noise ratio (SNR) approximations for  $\sinh(\cdot)$  and  $\cosh(\cdot)$ , we obtain the final result as

$$e^{j\hat{\theta}_{k,ML}} = \left( \frac{\sum_{i=1}^N r_{k-i}^2 \operatorname{sinc}(4f_{\max} iT)}{\sum_{i=1}^N r_{k-i}^2 \operatorname{sinc}(4f_{\max} iT)} \right)^{\frac{1}{2}} \quad (23)$$

where  $\operatorname{sinc}(x) = \sin(\pi x)/\pi x$ .

The structure of the demodulator corresponding to (23) is shown in Fig. 6. Due to time-varying phase and the structure of the ML estimator, the demodulator is equipped with a 180 degree phase jump detector to resolve the periodic 180 degree phase ambiguities. Without a 180 degree phase jump detector, the differential decoder fails at the end of each 180 degree phase jump period. The phase jump detector compares the present and the previous phase estimates for detection. The bottom portion of Fig. 6 shows the phase jump detector.

Simulation results for the bit error rate of the pseudo-coherent demodulator using scheme 1 and scheme 2 are compared and the results are shown in Fig. 7. Scheme 1 is the best in the absence of frequency error, but it is very sensitive even to very small frequency offsets. Scheme 1 will fail for frequency offsets even as small as 1 Hz. Therefore, in practice, it is preferable to use scheme 2 even though there is a small penalty due to using a phase jump detector.

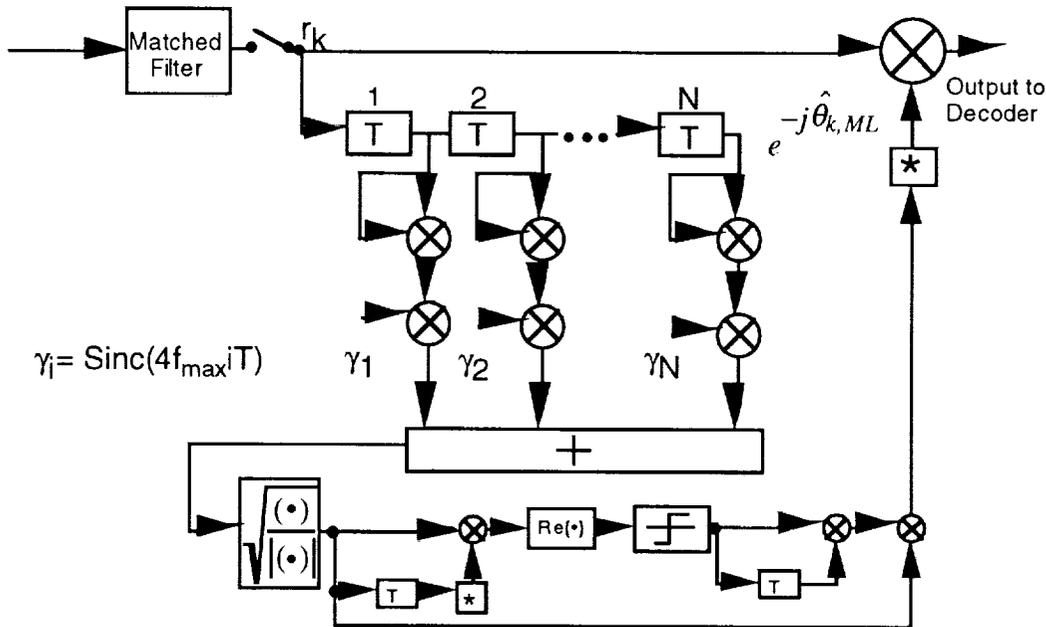


Figure 6. Pseudo-Coherent Demodulator (Scheme 2)

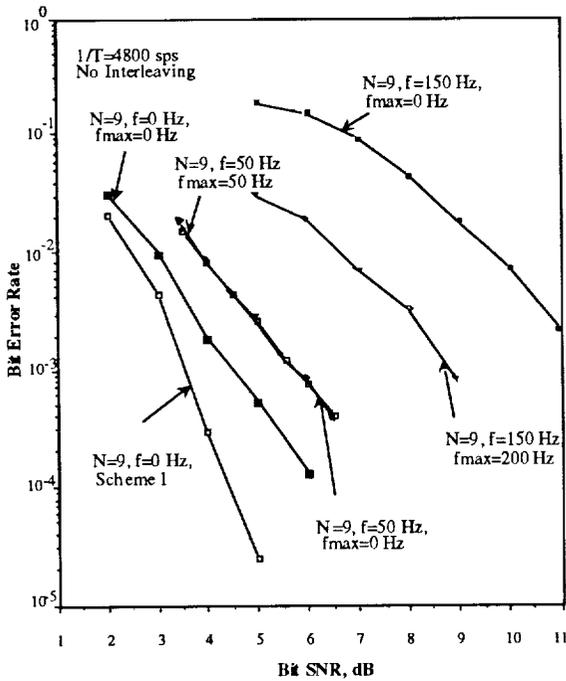


Figure 7. Simulation results.

### A pseudo-coherent demodulation scheme with an open loop frequency estimator

The third proposed pseudo-coherent demodulation scheme uses a simple open loop frequency estimator together with the ML estimate of the carrier phase conditioned on perfect knowledge of the frequency offset. Due to time-varying phase and the structure of the ML estimator, the demodulator is again equipped with a 180 degree phase jump detector to resolve the periodic 180 degree phase ambiguities. The structure of the demodulator is shown in Fig. 8. A typical sample function of the phase estimate in the presence and absence of noise is shown in Fig. 9. Simulation results have been obtained to compare the bit error probability of this scheme with DPSK for various cases of practical interest. These results are illustrated in Fig. 10.

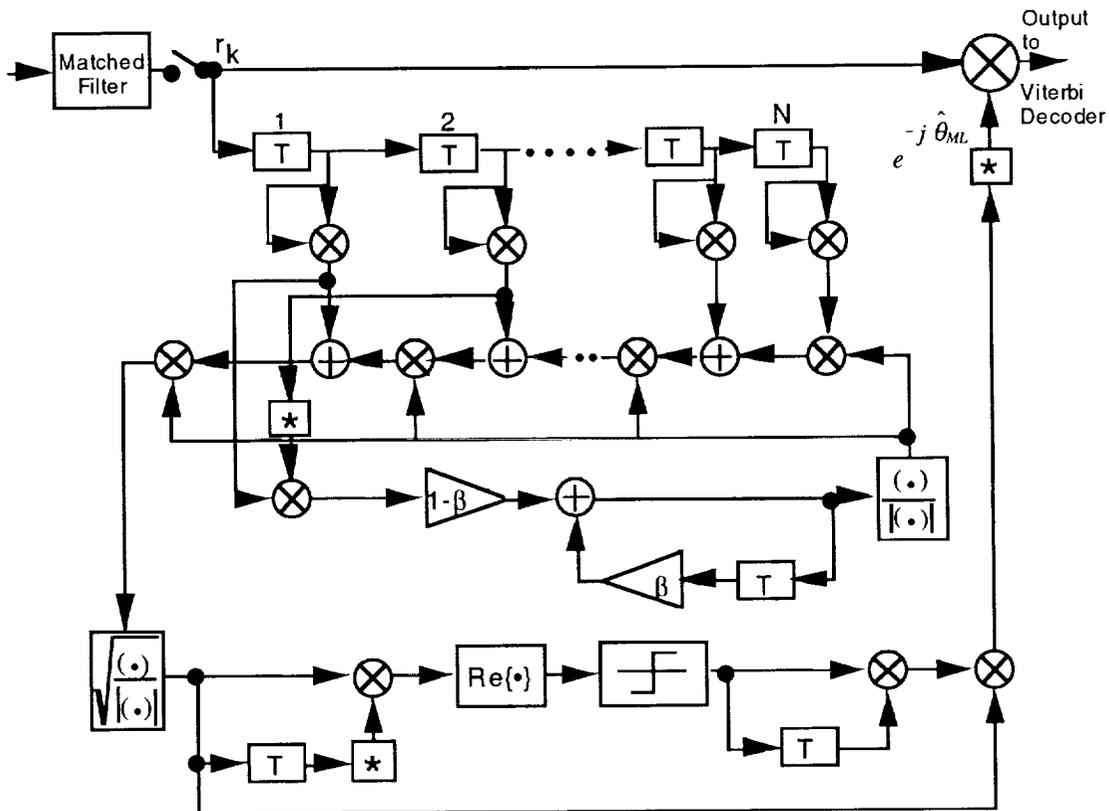


Figure 8. Pseudo coherent Demodulator (Scheme 3)

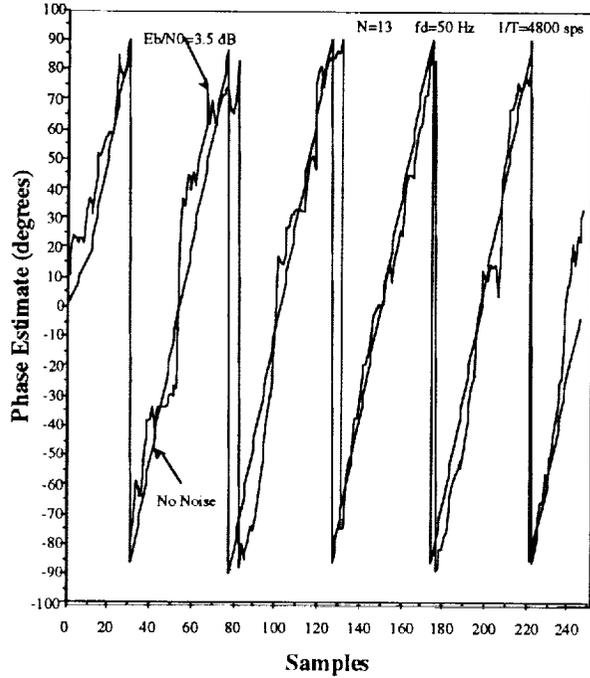


Figure 9. Sample function of phase estimate

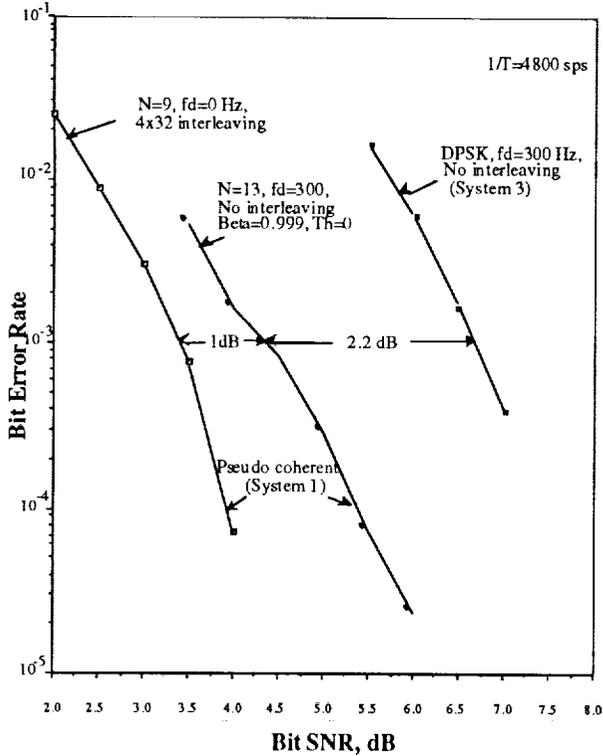


Figure 10. Simulation results

## Conclusion

In this paper, three pseudo-coherent demodulation schemes were proposed. The schemes were derived based on maximum-likelihood (ML) estimation and detection of an  $N$ -symbol observation of the received signal. Simulation results for all three demodulators were presented to allow comparison with the performance of differential PSK (DPSK) and ideal coherent demodulation for various system parameter sets of practical interest.

Scheme 1 can be used if there is an external frequency estimator with very small frequency error variance. Scheme 2 can be used if the frequency offset is small, or the receiver is equipped with an external frequency estimator. Scheme 3 can be used if the frequency offset is large and no external frequency estimator is used.

## Acknowledgment

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## References

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## Session 11

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