RELIABILITY ENHANCEMENT OF NAVIER-STOKES CODES THROUGH CONVERGENCE ENHANCEMENT

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SUMMARY

Reduction of total computing time required by an iterative algorithm for solving Navier-Stokes equations is an important aspect of making the existing and future analysis codes more cost effective. Several attempts have been made to accelerate the convergence of an explicit Runge-Kutta time-stepping algorithm. These acceleration methods are based on local time stepping[1], implicit residual smoothing[1], enthalpy damping[1] and multigrid techniques[2]. Also, an extrapolation procedure based on the power method and the Minimal Residual Method (MRM) were applied[2] to the Jameson's multigrid algorithm. The MRM uses same values of optimal weights for the corrections to every equation in a system and has not been shown to accelerate the scheme without multigriding. Our Distributed Minimal Residual (DMR) method[3-4] based on our General Nonlinear Minimal Residual (GNLMR) method[5] allows each component of the solution vector in a system of equations to have its own convergence speed. The DMR method was found capable of reducing the computation time by 10-75% depending on the test case and grid used. Recently, we have developed and tested a new method termed Sensitivity Based DMR or SBMR method that is easier to implement in different codes and is even more robust and computationally efficient than our DMR method.

TECHNICAL DISCUSSION

This method predicts an optimum amount of correction to the solution vector by combining the information from several previous iteration levels. Each of the corrections obtained from the past iterations is multiplied by a different weighting factor and these weighting factors are determined so that they minimize the overall future residual. Although it is based on general Krylov subspace methods, the DMR method differs from them by the fact that weighting factors are different from one variable to another in the system. Recently, we divided the computational domain into several zones and the DMR method was applied separately in each of these zones. This approach did not show noticeable improvement over the original DMR method. Also it was difficult to decide how to divide the domain systematically into several zones. The DMR concept was also applied to minimize the future residual at each grid point rather than to minimize the residual integrated over the whole domain. This approach was not successful, because the weighting factors obtained for each grid point differ too much from one point to another thus making convergence history erratic and often diverging.

SENSITIVITY BASED MINIMAL RESIDUAL (SBMR) METHOD

The residual at a grid point depends on the solution vector Q at the neighboring points including the point itself. The sensitivity of residual R_m (m=1,2,3: number of equations) with respect to Q_k (k=typical neighboring points) is $\dfrac{\partial R_m}{\partial Q_k}$. Notice that the sensitivity can be determined from the finite difference equation used in the scheme.

For a two-dimensional incompressible flow solved using Chorin's artificial compressibility method the solution vector is $Q = [p \ u \ v]^T$. Suppose we have calculated the solution vector Q at iteration levels up to t+n where n is the number of regular iteration steps between two iteration levels. Then the change in the solutions between the iteration levels can be written as follows.

$$\Delta p_k = (p_k)^{t+n} - (p_k)^t \qquad \Delta u_k = (u_k)^{t+n} - (u_k)^t, \qquad \Delta v_k = (v_k)^{t+n} - (v_k)^t, \qquad (1)$$

Using the first two terms in a Taylor series expansion in artificial time direction, each residual for a two-dimensional system after n iterations will be

$$R_m^{t+n} = R_m^t + \left[\sum_k \frac{\partial R_m^t}{\partial p_k}\Delta p_k\right] + \left[\sum_k \frac{\partial R_m^t}{\partial u_k}\Delta u_k\right] + \left[\sum_k \frac{\partial R_m^t}{\partial v_k}\Delta v_k\right] \; ; m=1,2,3 \qquad (2)$$

Similarly, future residual at t = (t+n)+1 can be approximated by

$$R_m^{(t+n)+1} = R_m^t + \left[\sum_k \frac{\partial R_m^t}{\partial p_k}\Delta p_k\right]\alpha_p + \left[\sum_k \frac{\partial R_m^t}{\partial u_k}\Delta u_k\right]\alpha_u + \left[\sum_k \frac{\partial R_m^t}{\partial v_k}\Delta v_k\right]\alpha_v \qquad (3)$$

Here, α's are the factors that multiply Δ's to estimate the future solution vector so that it satisfy a desired objective. For now, each α is assumed to have the same value over the whole domain. The future solutions are estimated as

$$(p_k)^{(t+n)+1} = (p_k)^t + \alpha_p \Delta p_k \qquad (4)$$

with similar expressions for u and v. The α's are determined such that the L-2 norm of the overall future residual will be minimized, that is,

$$\sum_D \left[\frac{\partial (R^{(t+n)+1})^2}{\partial \alpha_q}\right] = 2 \sum_D \left[R^{(t+n)+1}\frac{\partial R^{(t+n)+1}}{\partial \alpha_q}\right] = 0 \qquad (5)$$

where the subscript q stands for each flow variable p, u and v. For simplicity, let us denote the bracketed terms in equation (3) as a_{mp}, a_{mu} and a_{mv} so that

$$a_{mp} = \sum_k \frac{\partial R_m}{\partial p_k}\Delta p_k \qquad (6)$$

with similar expressions for a_{mu} and a_{mv}. Therefore,

$$R_m^{(t+n)+1} = R_m^t + a_{mp}\alpha_p + a_{mu}\alpha_u + a_{mv}\alpha_v \qquad (7)$$

Substituting (7) into (5) gives the following three equations for optimal global α's.

$$\sum_{D} \left[\sum_{m=1}^{3} \left(R_m^t + a_{mp}\alpha_p + a_{mu}\alpha_u + a_{mv}\alpha_v \right) a_{mp} \right] = 0 \qquad (8)$$

with similar expressions involving a_{mu} and a_{mv}. In equation (8), R's and a's are known from the past iteration results. Since each α is assumed to have the same value over the computational domain, equation (8) gives a system of simultaneous equations for α_p, α_u and α_v.

$$\left[\sum_{D} \left(\sum_{m=1}^{3} a_{mp}a_{mp} \right) \right]\alpha_p + \left[\sum_{D} \left(\sum_{m=1}^{3} a_{mu}a_{mp} \right) \right]\alpha_u + \left[\sum_{D} \left(\sum_{m=1}^{3} a_{mv}a_{mp} \right) \right]\alpha_v = - \sum_{D} \left(\sum_{m=1}^{3} R_m^t a_{mp} \right) \qquad (9)$$

with similar expressions arising from u and v components. As the grids are clustered (higher cell aspect ratio), local time steps become smaller in those regions in order to meet the stability criterion. Numerical results (Figures 1-6) obtained at low and high Reynolds numbers on non-clustered and moderately clustered grids for straight and U-shaped two-dimensional channel flow demonstrate that SBMR method can: a) be used in conjunction with any basic iterative algorithm, b) be used with only minor modifications in the existing codes, c) significantly accelerates iteration procedure, d) perform more effectively at lower Reynolds numbers, e) perform well on moderately clustered grids.

The SBMR method applied so far calculates the same α's for the entire computational domain, which cannot represent optimum α's for both coarse grid regions and fine grid regions. Therefore, a natural conclusion is to allow α's to have different values in the clustered regions. The future work on SBMR will concentrate on applying this method by lines such that each grid line normal to the clustering direction has its own α's. We expect that with this approach, SBMR concept will calculate the optimal local corrections to solution vectors on highly clustered grids.

REFERENCES

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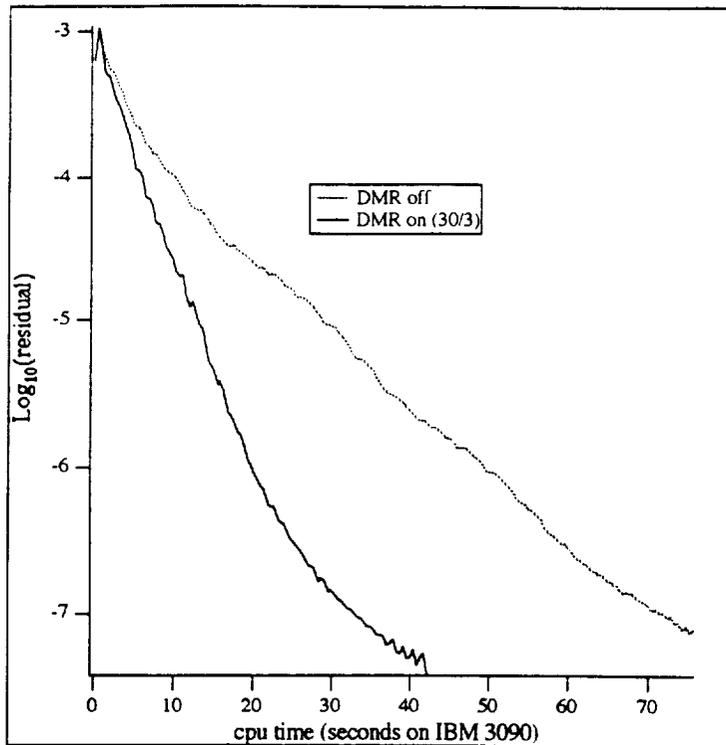
Figure 1. Convergence histories for a cascade flow: Re=500, non-clustered 40x30 grid, CFL=2.8, β=3.0. No artificial dissipation was used. Half of NACA 0012 airfoil on top and bottom walls.
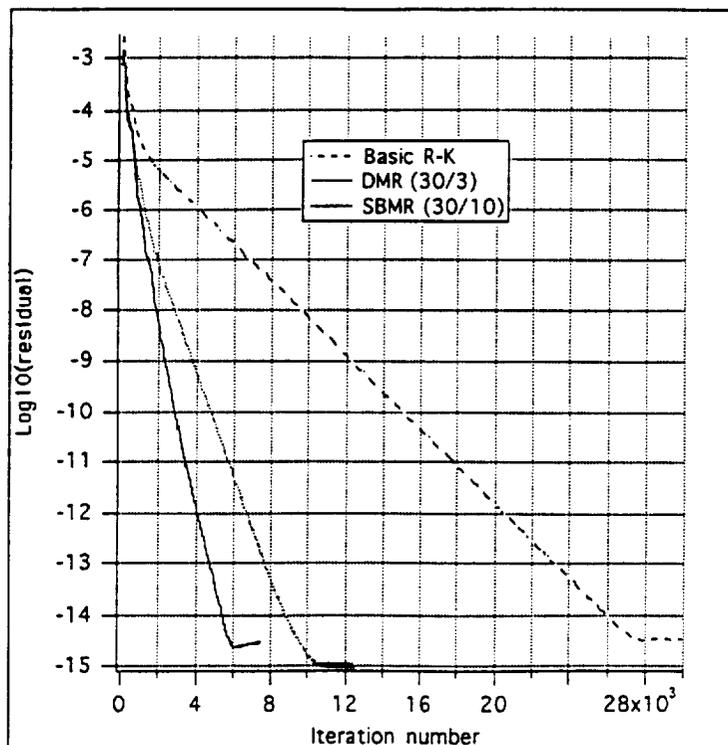

Figure 2. Convergence history for a straight channel flow: Re=1600, non-clustered 60x60 grid, x-length=5, y-length=1, β=5, von=0.4, CFL=2.5, omega=0.001. Initial guess: u=1.0E-5, v=0, p=0. Inlet velocity profile: u=parabolic, v=0.
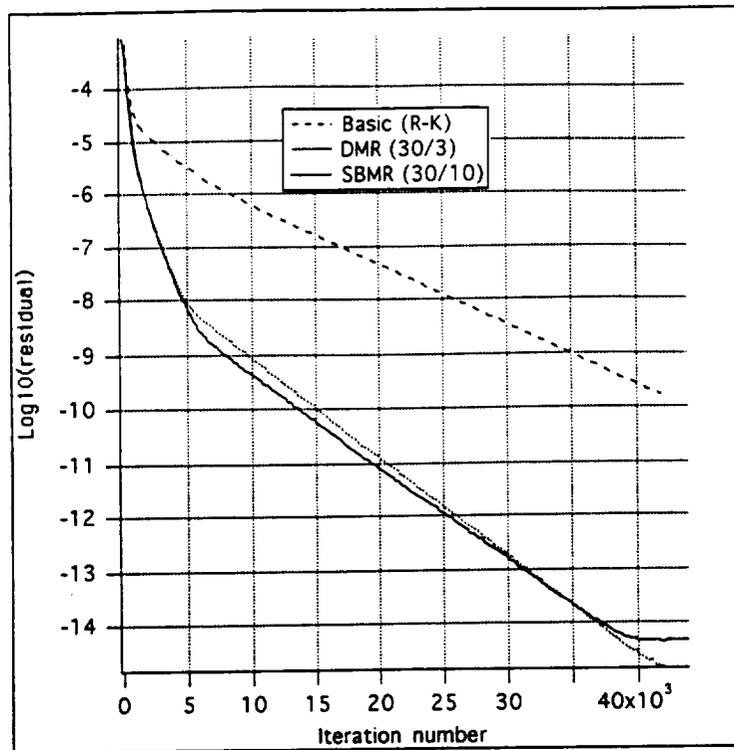
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Figure 3. Convergence history for a straight channel flow: Re=1.6 million, non-clustered 60x60 grid, x-length=5, y-length=1, β=5, CFL=2.8, von=0.4^{-4}, omega=0.005. Initial guess: u=1.0E-5, v=0, p=0. Inlet velocity profile: u=parabolic, v=0.
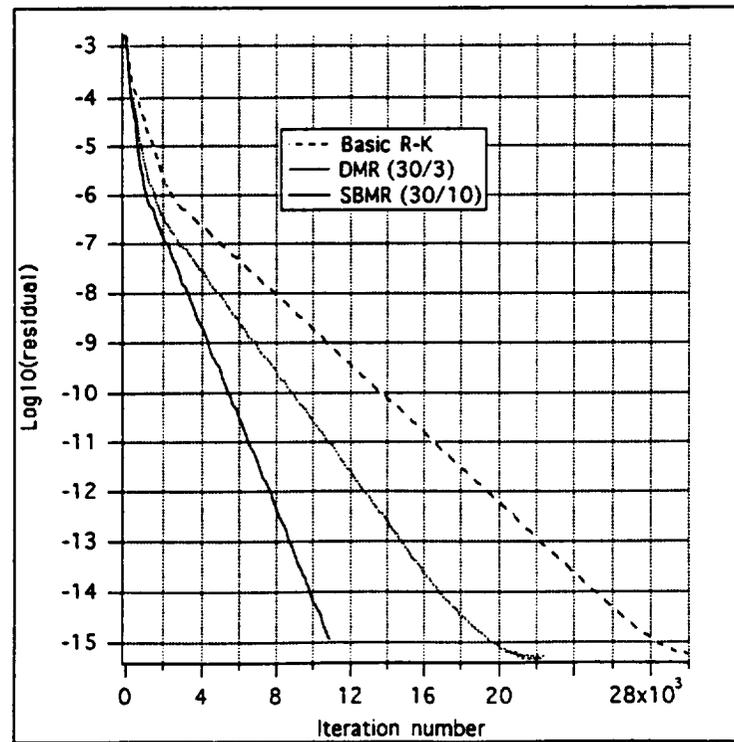

Figure 4. Straight channel flow: Re=1600, max AR=100 on 60x60 grid, x-length=5, y-length=1, β=5, CFL=2.5, von=0.4, omega=0.0. Initial guess: u=1.0E-5, v=0, p=0 everywhere. Inlet velocity profile: u=parabolic, v=0.
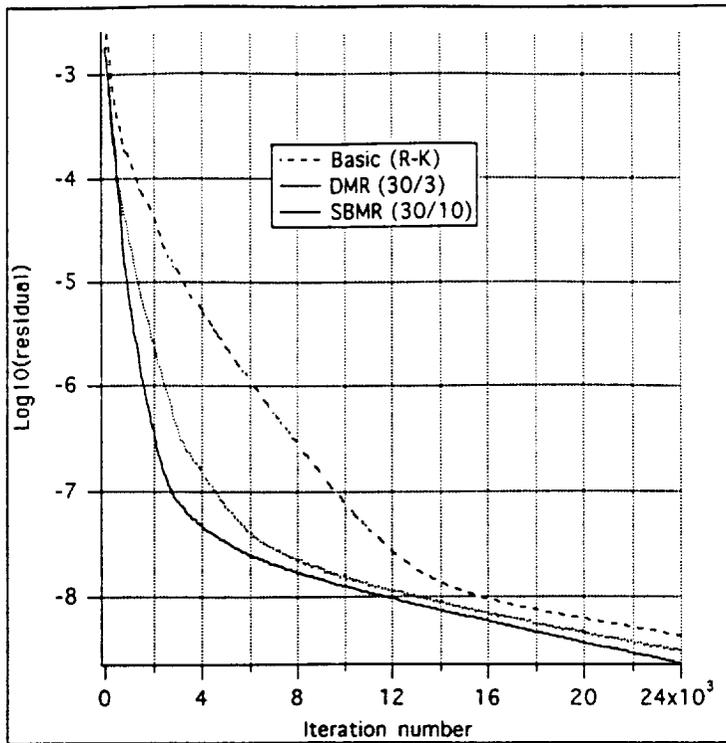
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Figure 5. Straight channel flow: Re=1.6 million, max AR=100 on 60x60 grid, x-length=5, y-length=1, β=5, CFL=2.5, von=0.4x10^{-4}, omega=0.005. Initial guess: u=1.0E-5, v=0, p=0. Inlet velocity profile: u=parabolic, v=0.
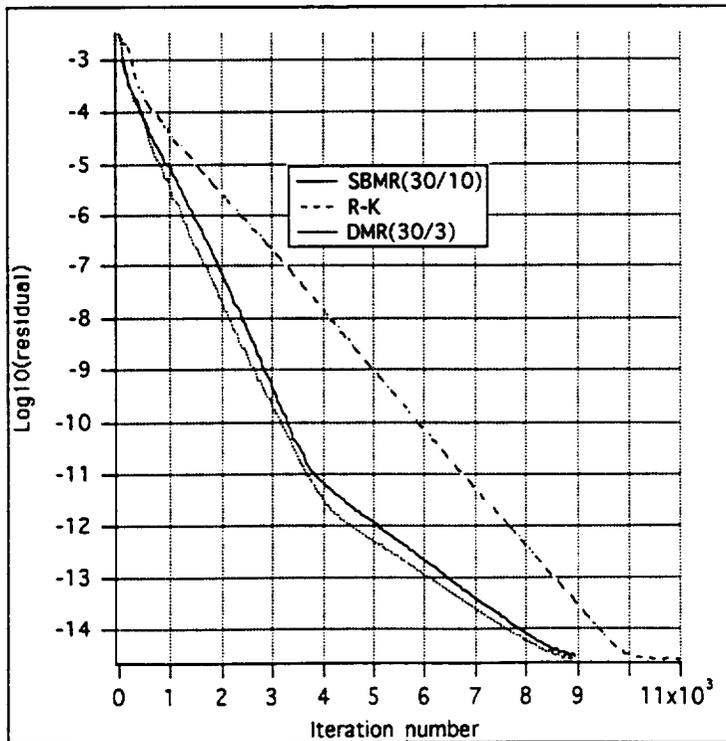

Figure 6. Convergence history for a U-shaped channel flow (Re=100, 129x30 grid cells); β=5, CFL=2.8, von=0.4, omega=0.0.