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INTERPRETATION OF DISCREPANCIES IN MASS SPECTROSCOPY DATA OBTAINED FROM DIFFERENT EXPERIMENTAL CONFIGURATIONS

PREPARED BY:	Dr. John M. Russell
ACADEMIC RANK:	Associate Professor
UNIVERSITY AND DEPARTMENT:	Florida Institute of Technology Department of Mechanical and Aerospace Engineering
NASA/KSC	
DIVISION:	Electronic Systems
BRANCH:	Ground Support Equipment
NASA COLLEAGUE:	Frederick W. Adams Carolyn A. McCrary
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This report owes much to a document titled "Report of KSC-STS GHe Leak Check Independent Assessment" by DENNIS PETERSON, dated May 24, 1991. The PETERSON report identifies a 1968 Boeing study as the source of the so-called *Apollo factor* (for estimating the ratio of actual leak rate to the leak rate indicated by a leak detector equipped with a hand held sniffer probe). The PETERSON report points out that probe-type sampling systems with internally branched conduits have now replaced earlier ones in which the conduits were unbranched. These changes in instrumentation have induced changes in the conventions for the interpretation of meter readings during leakage inspections, which may, in turn, result in overestimation or underestimation of the rate of leakage from a test object. The PETERSON report argues that the possibility of underestimation is real.

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ABSTRACT

Many helium-mass-spectrometer leak detectors at KSC employ sampling system that feature hand held sniffer probes. Authors of general leakage-testing literature recommend sniffer probes for leak location but not for quantitative leakage measurement. Their use in the latter application at KSC involves assumptions that may be subtle. The purpose of the research effort reported herein was to establish the significance of indicated leak rates displayed by sniffer-probe-equipped leak detectors and to determine whether the use of alternative hardware or testing procedures may reduce the uncertainty of leakage measurements made with them. The report classifies probe-type sampling systems for helium leak detectors according to their internal plumbing (direct or branched), presents a basic analysis of the fluid dynamics in the sampling system in the branched-conduit case, describes the usual test method for measuring the *internal* supply-to-sample flowrate ratio (*a.k.a* permeation ratio), and describes a concept for a sponge-tipped probe whose *external* supply-to-sample flowrate ratio promises to be lower than that of a simple-ended probe. One conclusion is that the main source of uncertainty in the use of probe-type sampling systems for leakage measurement is uncertainty in the *external* supply-to-sample flowrate ratio (In contrast, the present method for measuring the *internal* supply-to-sample flowrate ratio is quantitative and satisfactory). The implication is that probes of lower external supply-to-sample flowrate ratio must be developed before this uncertainty may be reduced significantly.

SUMMARY

As was stated in the Abstract above, the purpose of the research effort reported herein was to determine the significance of numbers displayed by helium-mass-spectrometer leak detectors equipped with hand held sniffer probes (which are normally of value in leak location but not in the measurement of leak rates) and to determine whether modifications of hardware or procedures may enable one to employ such devices for quantitative leakage measurement.

The report classifies probe-type sampling systems for helium leak detectors according to their internal plumbing, which may be direct or branched. In a direct conduit sampling system, gas flows from the probe tip directly to the analysis cell of the mass spectrometer and the user interface for the spectrometer displays infomration from which one may deduce the rate of transport of helium through that cell (or may, indeed, display that information directly). In a branched conduit sampling system, gas flows from the probe tip to a place (a mass separator cell) where the flow branches, a portion entering the analysis cell of the mass spectrometer via helium-permeable window and the rest entering the low pressure port of a vacuum pump. In a branched-conduit sampling system, the rate of transport of helium through the probe tip is typically a large multiple of the rate of transport of helium through the analysis cell. This large multiple (the *internal supply to sample flowrate ratio* or *permeation ratio*) is special to each particular branched conduit sampling system. The pressure in the mass separator cell is small relative to pressure in the atmosphere but large relative to the pressure in the analysis cell of the spectrometer. Under such circumstances, the fluid in the conduit is a continuum and obeys the ordinary equations of fluid dynamics.

The report then presents a basic analysis of the fluid dynamics in the sampling system in the branched-conduit case. The equations of motion of a gas admit a simple analytical solution that holds when $R\epsilon \ll 1$, where $R = \rho\bar{w}d/\mu$ is the REYNOLDS number based on mass density, ρ, and viscosity, μ, of the sampled gas, tube diameter, d, and cross sectionally averaged fluid speed, $\bar{w}$, and $\epsilon = d/\ell$ is a slenderness factor based on the tube diameter, d, and the tube length, ℓ.

The report then describes a standard test method for measuring the internal supply-to-sample flowrate ratio of a branched conduit sampling system. The probe is first exposed to atmospheric air and the rate of transport of gas into the probe tip is measured with a flowmeter. The probe is then fed gas of known helium concentration from a bottle containing a calibrated gas mixture. A valve between the calibrated gas bottle and the probe is adjusted until the flow rate of calibrated gas matches the flow rate of atmospheric air determined earlier. The rate of transport of helium into the probe tip is then the rate of transport of the ambient gas mixture times the helium concentration. Knowing the helium transport into the probe tip and the helium transport through the analysis cell (which may be deduced from information displayed by the user interface), the ratio of the former to the latter must equal the internal supply to sample flowrate ratio. The report illustrates the application of this test method in the case of an Alcatel leak detector, where the internal supply to sample flowrate ratio was found to be approximately 1100.

The report then describes a concept for a sponge-tipped probe to be used in conjunction with a branched conduit sampling system. The concept is meant to apply in the case when the rate of transport of sampled gas through the branched conduit is large in comparison with the rate of leakage of helium one is trying to measure. The aim is to lower the *external* supply to sample flowrate ratio to a value well below that of a simple open-ended probe. One places the sponge-tipped probe in contact with a suspect leak (much the way one would place the brush head of a vacuum cleaner over something one wants to vacuum up). The assumptions are meant to ensure that the rate of outward transport of helium by diffusion is small in comparison with the rate of inward transport by convection.

The main conclusion is that leak detectors equipped with sniffer probes are capable of determining the rate of transport of helium into the probe (even if the sampling system is internally branched), but that quantitative measurement of the leakage from a suspect leak site will become feasible only after probes of higher external sypply-to-sample flowrate ratio are developed. The topics discussed in the report have implications in regard to the acceptability criteria of leakage from different parts of the shuttle orbiter and the report also discusses some of these implications.

TABLE OF CONTENTS

LIST OF TABLES

SECTION ONE

INTRODUCTION

1.1 INSPECTION FOR LEAKS. DOCUMENTS. MOTIVATION FOR THE PRESENT PROJECT

Uncontrolled leaks pose hazards to personnel and equipment and compromise the ability of spacecraft and ground support hardware to carry out their missions. Accordingly, shuttle operations personnel perform a variety of leak tests on the shuttle orbiter between flights.

Technicians involved in space shuttle oprations at KSC perform routine tasks according to checklists. Such checklists belong to a generic class of documents called *Operations and Maintenance Instruction*, abbreviated OMI. Ref. 1, titled "MPS leak and functional test (LPS). (LH2 feed system),"is an example of an OMI governing leak checking of the Main Propulsion System, specifically the system for the transport of liquid hydrogen. Every OMI is subject to rules laid down in one or another *Operations and Maintenance Requirements and Specification Document*, (OMRSD). Ref. 2, titled "Orbiter OMRSD main propulsion system," (Rockwell International) is an example of an OMRSD, in this case the OMRSD that governs the procedures described in Ref. 1. A third category of governing document is a *Specification*. Ref. 3, titled "Proof pressure and leak detection–Aerospace plumbing systems and assemblies," (Rockwell International) is an example of a Specification. The Rockwell Specification (Ref. 3) expresses acceptance criteria for leak rates at various suspect leak sites on the orbiter and states what kinds of instrumentation are permissable to verify that the acceptance criteria are satisfied. This Specification allows for leakage measurement by means of hand-held sniffer probes. The OMI that pertains to leak testing of liquid hydrogen systems (Ref. 1) specifically names the mass spectrometer leak detector as the instrument to be used to perform various inspections.

A report titled "Report of KSC-STS GHe Leak Check Independent Assessment" by DENNIS PETERSON, dated May 24, 1991 (Ref. 4) points out that probe-type sampling systems with internally branched conduits have now replaced earlier ones in which the conduits were unbranched. These changes in instrumentation have induced changes in the conventions for the interpretation of meter readings during leakage inspections, which may, in turn, result in overestimation or underestimation of the rate of leakage from a test object. The PETERSON report argues that the possibility of underestimation is real.

The research reported herein was motivated by the need to address some of the questions raised in the PETERSON report (Ref. 4) and to consider possible changes in the design of probes and sampling systems that may eliminate some of the uncertainties of existing sysems.

1.2 ON THE EXPRESSION OF LEAK RATES: DIMENSIONS

Before one can attribute differences in leak rate indications to differences in experimental configuration one must discuss what exactly one means by the term *leak rate*. A value of the leak rate is a quantity with certain dimensions and some discussion of these dimensions is in order. This subsecton supplies preliminary information of this nature.

In fluid dynamics, a *streamline* is a mathematical curve in space that is tangent at all of its points to the local velocity direction. Given a velocity field and a geometric point in that field, one can always draw a streamline that passes through that point. If, alternatively, one is given a geometric *figure* in the form of a closed loop, all of whose points lie in a velocity field, then one can draw a streamline through every point on the perimeter of that loop. The family of streamlines so generated describes a tube-shaped surface and is called a *streamtube*. Since a streamtube is a surface generated by streamlines and streamlines are tangent to the velocity direction at all of their points, one concludes that a streamtube is also tangent to the local velocity direction at all of *its* points. It follows that *a streamtube is impermeable to fluid motion*—fluid never passes across it. Now every leak belongs to a particular streamtube. One may therefore apply all of the ideas that fluid dynamicists have developed for general streamtubes to the particular case when the streamtube represents a leak.

Let S denote a geometric figure in the form of a surface that cuts across a streamtube at one particular station along it; let $\mathbf{u}$ be the local fluid velocity vector; and let $d\mathbf{A}$ be the differential directed area element normal to S and directed away from the downstream side of it (the magnitude $|d\mathbf{A}|$ represents the geometric area of the element described by the vector $d\mathbf{A}$). Then the *volumetric rate of transport* of fluid across the infinitesimal directed area element $d\mathbf{A}$ per unit time is $\mathbf{u}{\cdot}d\mathbf{A}$. Let $\dot{\Delta}$ be the *resultant* volumetric rate of transport of fluid across the *whole* surface S. One may compute $\dot{\Delta}$ by summing the separate contributions from all of the infinitesimal subsurfaces of S and express the result in the form of the following double integral

$$\dot{\Delta} \equiv \iint_S (\mathbf{u}{\cdot}d\mathbf{A}) . \tag{1.1}$$

The parameter $\dot{\Delta}$ has the dimensions of volume per unit time. One may, for example, express a value of $\dot{\Delta}$ in $(\mathrm{cm})^3/\mathrm{sec}$.

A *gram-mole* of a substance whose molecular mass is M is an amount of that substance whose mass in grams is numerically equal to M. Thus, a gram-mole of helium, whose molecular mass is 4 is an amount of helium whose mass equals 4 grams. Alternatively, one may say that M for helium has the value 4 grams/(gram-mole). Let $\dot{n}$ be the *molar rate of transport* of fluid through a streamtube across a cross sectional surface S (one may, for example, express a value of $\dot{\Delta}$ in $\mathrm{gm}-\mathrm{moles/sec}$). Let ρ^* be the molar density of a gas (one may, for example, express a value of ρ^* in $\mathrm{gm}/(\mathrm{cm})^3$). The number of moles of gas per unit time that passes across the infinitesimal subsurface $d\mathbf{A}$ equals the product of the number of moles per unit volume, ρ^*, times the volume transport rate, $\mathbf{u}{\cdot}d\mathbf{A}$ across that subsurface, and therefore has the value $\rho^*(\mathbf{u}{\cdot}d\mathbf{A})$. One may compute $\dot{n}$ by summing the separate contributions from all of the infinitesimal subsurfaces of S and express the result in the form of the following double integral

$$\dot{n} = \iint_S \rho^*(\mathbf{u}{\cdot}d\mathbf{A}) . \tag{1.2}$$

The parameter $\dot{n}$ has the dimensions of amount of substance per unit time. If the flow is steady, the equation of conservation of mass implies that the value of $\dot{n}$ is the same for all stations along the streamtube. In this respect, the value of $\dot{n}$ is an intrinsic property of the streamtube as a whole. This streamwise uniformity of $\dot{n}$ is to be contrasted with the possible streamwise nonuniformity of the volumetric rate of transport $\dot{\Delta}$. Variations of $\dot{\Delta}$ may occur as a result of compressibility of the gas.

According to the ideal gas law $p = \rho^*\mathcal{R}T$, in which p is the pressure; T is the absolute temperature; and $\mathcal{R}$ is the *universal gas constant*, whose value is

$$\mathcal{R} = 82.08\ \frac{\mathrm{atm}{\cdot}(\mathrm{cm})^3}{\mathrm{gm\text{-}mole}{\cdot}^\circ\mathrm{K}} . \tag{1.3}$$

The word "universal" in the phrase "universal gas constant" is meant to imply that $\mathcal{R}$ is the same for all gases (at least those that can be modeled as ideal gases). If one multiplies (1.2) by $\mathcal{R}T$ and restricts attention to the case when T is uniform over S, one obtains

$$\dot{n}\mathcal{R}T = \mathcal{R}T \iint_S \rho^*(\mathbf{u}{\cdot}d\mathbf{A}) = \iint_S \rho^*\mathcal{R}T(\mathbf{u}{\cdot}d\mathbf{A}) = \iint_S p(\mathbf{u}{\cdot}d\mathbf{A}) , \tag{1.4}$$

whose rightmost member has the dimensions of pressure times volume flow rate.

If the pressure is uniform over the cross section S, one may move p across the integral sign in (1.4) and obtain

$$\dot{n}\mathcal{R}T = p \iint_S (\mathbf{u}{\cdot}d\mathbf{A}) = p\dot{\Delta} , \tag{1.5}$$

in which the last equality follows from (1.1).

If the absolute temperature T does not differ greatly from laboratory conditions (say $T = 295°\text{K} = 71.33°\text{F}$), then one may carry out approximate calculations by assuming that $\mathcal{R}T$ has the room-temperature value

$$\mathcal{R}T = 24,214 \,\frac{\text{atm}\cdot(\text{cm})^3}{\text{gm-mole}} \qquad (\text{for } T = 295°\text{K} = 71.33°\text{F}). \tag{1.6}$$

If one accepts the value of $\mathcal{R}T$ given in (1.6) then equation (1.5) furnishes a formula from which one may calculate $\dot{n}$ from the value of the quantity $p\dot{\Delta}$ or $p\dot{\Delta}$ from the value of the quantity $\dot{n}$. In this respect, the two parameters $\dot{n}$ and $p\dot{\Delta}$, though they have different dimensions, both measure the flux of gas through a streamtube and are both intrinsic properties of that streamtube.

The relation (1.5) between the molar rate of transport $\dot{n}$, the parameter $p\dot{\Delta}$, and the value of the conversion factor $\mathcal{R}T$ given by (1.6) is independent of the choice of gas. If, for example, the absolute temperature has the value $T = 295°\text{K}$ and the parameter $p\dot{\Delta}$ has the value $p\dot{\Delta} = 1 \times 10^{-6}$ atm cc/sec, then these data furnish enough information to calculate the molar rate of transport $\dot{n}$ from (1.5) (in the present example, $\dot{n}$ would have the value 4.13×10^{-11} gm-moles/sec). This value of $\dot{n}$ is the same whether the gas is helium, hydrogen, nitrogen, or any other gas that can be modeled as an ideal gas. The observation that the parameter $p\dot{\Delta}$ has meaning as a measure of the rate of leakage and that this meaning is not specific to the choice of gas (or, for that matter, on the barometric pressure of the environment into which the gas discharges) helps account for why some leak detectors display leakage rate in units such as millibar·liters/sec or atm·(cm)^3/sec (both of which are suitable units for the parameter $p\dot{\Delta}$).

The parameter $p\dot{\Delta}$ also has a interpretation in terms of energy concepts. Thus, when gas leaks from the inside to the outside of a test object via an orifice, some of the gas in the exterior region that had initially abutted the leak site is *displaced* by the leaked gas. Thus, as fluid leaks out of a test object, it performs work on the surrounding gas to displace it. The parameter $p\dot{\Delta}$ thus represents the rate at which the leaking fluid performs such displacement work per unit time. In mechanics, the rate at which work is performed per unit time is *power*, and the standard unit for power is the Watt (W). One concludes that $p\dot{\Delta}$ may be expressed in Watts. Now one atmosphere (atm) is a unit of pressure equal to 1.01325×10^5 Newtons per square meter (N/(m^2)), one Watt (W) is defined to be 1 N·m/sec, and one centimeter (cm) equals 10^{-2} meters (m). It follows that

$$1\,\frac{\text{atm}\cdot(\text{cm})^3}{\text{sec}} = \frac{[1.01325 \times 10^5\ \text{N}/(\text{m}^2)](1 \times 10^{-2}\ \text{m})^3}{\text{sec}} = 0.101325\,\frac{\text{N}\cdot\text{m}}{\text{sec}} = 0.101325\ \text{W}\,. \tag{1.7}$$

Motivated by the foregoing considerations, I will refer to the parameter $p\dot{\Delta}$ as the *leakage rate*, *flowrate*, or *throughput* in this report. In this context, one may regard the familiar unit sccs (for *standard cubic centimeter per second*) as a shorthand for the more precise notation atm·(cm)^3/sec or atm·cc/sec. Equation (1.7) shows that 1 sccs is equivalent to 101.325 milliwatts (exactly) regardless of the choice of gas.

1.3 THE ANALYSIS CELL OF A MASS-SPECTROMETER LEAK DETECTOR

A 1968 NASA Contractor Report titled *Leakage testing handbook*, prepared by J. W. Marr on behalf of General Electric Company for Jet Propulsion Laboratory, Padadena, CA (Ref. 5), furnishes a general reference on the subject of leak testing. Marr's description of the action of the analysis cell of a mass spectrometer (*op. cit.*, p 150) suffices for the purposes of the present report:

> In the sector field mass spectrometer..., gas molecules entering the evacuated body of the instrument are ionized by a bombarding beam of electrons. The resultant positive ions are accelerated by the influence of a high voltage into a magnetic field whose direction is perpendicular to the plane of the mean path of the ions. Under the influence of this magnetic field, ions of different masses travel on arcs of different radii. Ions of a particular mass pass through a slit and fall on

a collecting electrode; from it they leak to ground through a high-value resistor. The ion current through the resistor is amplified and read on a meter. By varying the magnetic field or accelerating electric field, ions of any chosen mass may be collected.

One may surmise from the foregoing description of the principle of operation of the mass spectrometer that

$$i = e\eta\dot{n}\,, \tag{1.8}$$

in which: i is the electric current that passes through the receiver electrode belonging to a particular charge-to-mass ratio of molecule; e is the charge of one electron; $\dot{n}$ is the total flux of molecules of a given charge-to-mass ratio through the instrument; η is the ionization efficiency, *i.e.* the ratio of the flux of current-bearing molecules to $\dot{n}$. The following calculation illustrates a self-consistency check of the units of measurement of the various factors in (1.8):

$$\frac{\text{Coulombs}}{\text{time}} = \frac{\text{Coulombs}}{\text{ion}} \cdot \frac{\text{ions}}{\text{molecule}} \cdot \frac{\text{molecules}}{\text{time}} \tag{1.9}$$

The basic presumption in the calibration of a mass-spectrometer leak detector is that $e\eta$ is constant, so $i \propto \dot{n}$. One also assumes that $\dot{n}$ is proportional to the partial pressure of the helium on the upstream side of the inlet leak to the instrument.

To standardize a mass spectrometer leak detector, one must determine the instrument response to a known leak rate. A common method for producing a known leak rate is with the aid of a *calibrated leak*. According to MARR (*op. cit.*, p 112)

> Calibrated leaks, leaks which deliver gas at a known rate, are sold by a number of vendors. . .Calibrated leaks may be divided into two distinct categories—the reservoir leaks, those which contain their own gas supply, and the non-reservoir leaks to which tracer gas is added during test

Among the kinds of reservoir leaks are *permeation leaks*. According to MARR (*op. cit.*, p 114), such leaks

> employ the principle [of] gas diffusion through a thin wall.... Tracer gas [is] permeated from the area of high concentration to air or vacuum on the other side at a rate governed by the permeability of the thin membrane.

Tracer gas is the gas whose flux is measured by the analysis cell. Here, and elsewhere, I will assume that the tracer gas is helium. A reservoir-type leak based on the permeation principle may resemble a small bottle with a hand operated valve. The valve attaches to standard size vacuum couplings. In use, the operator attaches the standard leak to an input port on the leak detector and opens the valve. In some leak detectors, the meter reading on the electronic cabinet is expressed in the same units as the leak rate parameter $p\dot{\Delta}$ (say atm$\cdot$(cm)^3/sec). The label on the reservoir-type calibrated leak indicates its nominal leakage. By comparing the instrument reading with the indication on the label, one adjusts the gain in the electronic interface circuitry so as to force agreement between the two numbers. On other leak detectors the meter reading may be in the same units as the electrical current in the ion beam, say Amperes. For such a device, the ratio of the instrument reading (in Amperes) to the known leak rate for the calibrated leak is interpreted as an *instrument sensitivity factor*. Regardless of how the instrument is standardized, the meter reading of a helium mass spectrometer leak detector is proportional to $(p\dot{\Delta})_{\mathrm{He,Analysis\ cell}}$, namely *the rate of leakage of helium into the analysis cell of the mass spectrometer.*

1.4 TESTING METHODS EMPLOYING ENCLOSURES AND SNIFFER PROBES

1.4.1 ENCLOSURE TESTING FOR LEAK MEASUREMENT. According to one scheme for the quantitative measurement of leak rate, a fluid line or other test object is filled with gaseous helium under pressure. One measures the rate of discharge of helium from a leak by first enclosing the test object in some kind of enclosure (*e.g.* a hood, boot, or bag), then flushing the enclosure with a suitable purge gas (the rate of transport of purge gas being known), and finally measuring the helium concentration of the effluent.

Let $(p\dot{\Delta})_{\text{purge}}$ be the rate of transport of purge gas into the enclosure. Let $(p\dot{\Delta})_{\text{He, leak}}$ be the rate of leakate of helium into the enclosure and let $(p\dot{\Delta})_{\text{efft}}$ be the rate of discharge of gas mixture in the effluent. If the absolute temperature of the gas is effectively uniform throughout the enclosure and the flow is stationary in time, the net inflow must equal the net outflow and one deduces that

$$(p\dot{\Delta})_{\text{purge}} + (p\dot{\Delta})_{\text{He, leak}} = (p\dot{\Delta})_{\text{efft}} \,. \tag{1.10}$$

Let c denote the concentration of helium in the effluent. Then

$$c = \frac{(p\dot{\Delta})_{\text{He, leak}}}{(p\dot{\Delta})_{\text{efft}}} \,. \tag{1.11}$$

If one elimates $(p\dot{\Delta})_{\text{efft}}$ from (1.11) by means of (1.10), one obtains

$$c = \frac{(p\dot{\Delta})_{\text{He, leak}}}{(p\dot{\Delta})_{\text{purge}} + (p\dot{\Delta})_{\text{He, leak}}} \,. \tag{1.12}$$

One may manipulate this equation algebraically to obtain a formula for $(p\dot{\Delta})_{\text{He, leak}}$, namely

$$(p\dot{\Delta})_{\text{He, leak}} = \left(\frac{c}{1-c}\right)(p\dot{\Delta})_{\text{purge}} \,. \tag{1.13}$$

If one incorporates a flow metering device into the purge gas delivery system then $(p\dot{\Delta})_{\text{purge}}$ is known. If one then measures c by sampling the gas in the effluent and determining its helium concentration by mass spectrometry, then formula (1.13) furnishes a formula for the determination of $(p\dot{\Delta})_{\text{He, leak}}$. If c is very small compared to one, then one may approximate the expression (1.13) by $(p\dot{\Delta})_{\text{He, leak}} \approx c(p\dot{\Delta})_{\text{purge}}$. A 1987 AIAA Paper titled "Development of the Helium Signature Test for orbiter main propulsion system revalidation between flights" by VINCENT J. BILARDO, FRANCISCO IZQUIERDO, & R. SMITH (Ref. 6) describes an enclosure-type method now in use for the measurement of the aggregate helium leakage resulting from all individual leaks in the aft section of the shuttle orbiter. The Rockwell Specification on Proof pressure and leak detection (Ref. 3) specifies that the term *actual leak rate* refers to the leak rate measured by one or another of such enclosure-type tests (*op. cit.*, §6.2 p 13)

1.4.2 SNIFFER-PROBE TESTING FOR LEAK LOCATION. An enclosure-type test of a large or complicated test object may establish that a leak exists somewhere but may give no indication of how many leaks there are or where they are situated. A convenient method for locating such leaks involves the sampling of the air in the vicinity of a suspect leak site with a hand held *sniffer probe*. The sampled gas is then transported to a helium-mass-spectrometer leak detector where the rate of transport of helium into the leak detector is measured and displayed by the instrumentation. In such applications of sniffer probes, only a fraction of the helium that escapes from the leak in the original test object is collected by the sniffer probe, (the uncollected helium escapes to the surrounding atmosphere where it is dispersed). Here, and elsewhere, I will use the term *external supply-to-sample flowrate ratio* for the fraction

$$\frac{(p\dot{\Delta})_{\text{He, test object}}}{(p\dot{\Delta})_{\text{He, probe}}} \,, \tag{1.14}$$

i.e. the ratio of the rate of release of helium from a leak in a test object to the rate of transport of helium into the sniffer probe. Methods for standardizing helium-mass-spectrometer leak detectors enable one to measure the rate of transport of helium into the sniffer probe quantitatively, but the measurement of the external supply-to-sample flowrate ratio is far more problematic. Notwithstanding the uncertainties in the external supply-to-sample flowrate ratio, the meter deflection that results when a sniffer probe is passed over a leak provides useful information, namely information on where the leak is located.

1.4.3 SNIFFER-PROBE TESTING FOR LEAK MEASUREMENT. A 1968 report titled "Measurement and correlation of helium and fluid leak rates" prepared by A. E. SENEAR & A. W. BLAIR on behalf of the Boeing Company for what was then the Manned Spacecraft Center, Houston (Ref. 7) reports results of extensive tests on artificially produced leaks. The aim of of these tests was to separate the dependence of leakage rates on leak configuration (*e.g.* boundary shape and endpoint conditions for the pressure) from the dependence of leakage rates on the specific choice of leaked fluid (*e.g.* helium gas *versus* hypergolic liquids). In the course of this study, the Boeing investigators performed some tests that amount to a measurement of the external supply-to-sample flowrate ratio. Thus, SENEAR & BLAIR write (*op. cit.*, page 42)

> The helium sniffer probe attached to a CEC helium leak detector with 5 feet of Tygon tubing and fitted with a 1/8" ID Tygon tip (per Grumman Specification LSP-1A-50121A) was used to test an AN scribed fitting leaking at a rate of 1×10^{-5} cc He/sec at 380 psig applied helium pressure. The absolute sensitivity of the leak detector at the end of the probe had previously been determined to be 3.4×10^{-10} atm cc/(sec division). When tested in the open laboratory and when holding the probe perpendicular to the axis of the leaking fitting a vary large variation was obtained dependent upon precise positioning, with over an order of magnitude difference recorded 60° either side of the top of the fitting. Variations of about 50% at any one position were also noted. The maximum indicated leak rate detected during this experiment (265 divisions) was 9×10^{-8} cc/sec, by an operator with prior knowledge of the location of the leak.
>
> Experiments were also attempted with discouraging results, using the fingers to hood the leak in order to improve sensitivity. These readings proved quite unstable and non-reproducible although an increase in sensitivity level was produced. Checking in the oper air, if relatively still, seems to be the best, most reproducible method.
>
> Table 8 [whose content is reproduced here as Table 1-1 on the next page] gives the results obtained by four different operators on four different leaks utilizing both the hooded-by-fingers and open still air probing techniques. These operators had no prior knowledge of the location of the leaks. Examination of the data will reveal that the sniffer probe technique is not a quantitative measurement. It is, to be sure, quite useful as a screening technique for locating relatively large leaks ($> 1 \times 10^{-6}$ cc/sec of helium).

SENEAR & BLAIR state the conclusion they draw from the foregoing results as follows (*op. cit.*, p 70)

> The sniffer probe is of qualitative value only. The observed leak rates are approximately three orders of magnitude lower than the actual leak rate, and variations of 1300% are found between different operators, and different probe attitudes.

The ratio of the number in the leftmost column of Table 1-1 to the number in the rightmost column of the same row represents the ratio of actual to apparent leak rate. This ratio is typically on the order of one thousand. In these experiments, the conduit that transferred helium from the sniffer probe to the analyzer cell of the mass spectrometer was direct, *i.e.* all of the helium that entered the probe tip was counted in the computation of the apparent leak rate. One concludes that *in the Boeing data of Ref. 7 the ratio of actual to apparent leak rate (whose value was approximately one thousand) was equal to the external supply-to-sample flowrate ratio of the sampling system* that the investigators employed. Table 1-2 (on the second page after this one) lists the values of the external supply-to-sample flowrate ratio corresponding to the data in Table 1-1 (specifically, the data corresponding to the label "w/o hood").

The Rockwell Specification titled "Proof pressure and leak detection–Aerospace plumbing systems and assemblies" (Ref. 3) states what methods of measurement are acceptable for the determination of leakage rates. Method A of the Rockwell Specification, titled "Probe Technique" (*op. cit.* §3.5.1), indicates that sniffer probes may be used for leak measurement. An appendix to the Specification titled "Leak Point Matrix" defines the acceptable leakages from different parts of the shuttle orbiter. Table 1-3 (on the third page after this one) reproduces the content of the Rockwell Leak Point Matrix

The Leak Point Matrix has two columns of numbers headed "Actual He Leakage" and "Indicated He Leakage". For any one part of the orbiter (represented by a particular row in the Leak Point Matrix), the

Table 1-1. Sniffer-probe evaluation results after Senear & Blair 1968 (Ref. 4, p 44)

Known Leak Rate (cc/sec)	Operator	Leak Indication (divisons)		L.D. Sensitivity [(cc/sec)/div]	Apparent Leak Rate (cc/sec)	
		w/hood	w/o hood		w/hood	w/o hood
1×10^{-5}	A	212	18	3.62×10^{-10}	7.67×10^{-8}	6.51×10^{-9}
	B	695	14	3.73×10^{-10}	2.59×10^{-7}	5.22×10^{-9}
	C	219	17	3.73×10^{-10}	8.16×10^{-8}	6.35×10^{-9}
	D	265	16	3.40×10^{-10}	9.01×10^{-8}	5.45×10^{-9}
5×10^{-6}	A	84	17	3.62×10^{-10}	3.04×10^{-8}	6.18×10^{-9}
	B	339	8	3.73×10^{-10}	1.26×10^{-7}	2.98×10^{-9}
	C	69	6	3.73×10^{-10}	2.67×10^{-8}	2.24×10^{-9}
	D	180	14	3.73×10^{-10}	6.70×10^{-8}	5.22×10^{-9}
1×10^{-6}	A	16	2	3.62×10^{-10}	5.8×10^{-9}	7.5×10^{-10}
	B	62	2	3.73×10^{-10}	2.31×10^{-8}	7.45×10^{-10}
	C	10	1	3.73×10^{-10}	3.73×10^{-9}	3.73×10^{-10}
	D	13	3	3.73×10^{-10}	4.85×10^{-9}	1.12×10^{-9}
5×10^{-7}	A	10	2	3.62×10^{-10}	3.62×10^{-9}	7.5×10^{-10}
	B	30	_†	3.73×10^{-10}	1.12×10^{-8}	_____†
	C	3	_†	3.73×10^{-10}	1.12×10^{-9}	_____†
	D	14‡	3‡	3.73×10^{-10}	5.22×10^{-9}‡	1.1×10^{-9} ‡

† Could not possibly identify the number of divisions (if any)
‡ Questionable

numbers in these two columns differ by a factor of exactly one thousand. The report titled "Report of KSC-STS GHe leak check Independent Assessment" by DENNIS PETERSON of KSC (Ref. 4) suggests that the Leak Point Matrix in the Rockwell Specification was developed at a time when sniffer-probe equipped leak detectors used by shuttle inspection personnel at KSC featured direct-conduit sampling systems similar to the ones used to generate the data in Tables 1-1 and 1-2. According to PETERSON, the factor one-thousand between the Actual and Indicated leak rates in the Rockwell Leak Point Matrix was developed during the Apollo era. Some personnel with whom I have spoken refer to this three-order-of-magnitude adjustment as *the Apollo factor*.

Now the sniffer-probe equipped leak detectors that feature direct conduit sampling systems of the sort used to generate the data in Table 1-1 have some disadvantages. Thus, the gas in conduit is at nearly the same pressure as that of the analyzer cell of the mass spectrometer. At such a low pressure, the mean free path of the gas molecules is large compared to the tube diameter. In order to transport a measurable amount of gas, the sniffer-probe hose must be thick and unwieldy. The response time is also slow when the hose is longer than a few meters.

Such practical considerations have motivated personnel in charge of leak testing the shuttle orbiter to consider alternative configurations of test equipment. In the late eighties, they abandoned sniffer-probe equipped leak detectors featuring direct-conduit sampling systems in favor of sniffer probe equipped leak detectors featuring systems of *branched conduit* type. In a branched conduit sampling system, the sniffed gas is is allowed to flow at a significant fraction of atmospheric pressure through much of its passage from the probe end to the leak detector. In particular, the mean free path of the sampled gas is small compared to the inside diameter of the conduit (even when the latter is on the order of a millimeter or so) and the flow moves in accordance with the laws of viscous fluid motion. For such conduits, the response time may be no longer than eight seconds even when the conduit is one hundred feet long. To satisfy the condition of very

Table 1-2. External supply-to-sample flowrate ratio deduced from data in Senear & Blair 1968 (Ref. 4, p 44)

Known Leak Rate (cc/sec)	Operator	$\frac{(p\dot{\Delta})_{\text{He, test object}}}{(p\dot{\Delta})_{\text{He, probe}}}$
1×10^{-5}	A	1536
	B	1916
	C	1575
	D	1835
5×10^{-6}	A	809
	B	1678
	C	2232
	D	958
1×10^{-6}	A	1333
	B	1342
	C	2681
	D	893
5×10^{-7}	A	668
	B	____†
	C	____†
	D	455‡

† Could not possibly identify the number of divisions (if any)
‡ Questionable

low pressure in the analysis cell of the mass spectrometer, the fluid at the downstream end of the conduit branches. One branch is separated from the analysis cell by a helium permeable window (which is capable of reacting the difference in gas pressure between the analysis cell and the downstream end of the conduit) and the other branch is connected to the low pressure port of a vacuum pump (whose exhaust is discarded at atmospheric pressure). In the mean time, the only flow rate that the mass spectrometer measures is the rate of flow of helium into the analysis cell. Thus the branch of the the gas stream that passes through the the probe tip but does not pass through the analysis cell is *not* measured by the mass spectrometer (and so does not contribute to the leak rate value displayed by the instrumentation on the electronic cabinet of the leak detector).

Let $(p\dot{\Delta})_{\text{He, probe}}$ be the flux of helium into the probe and let $(p\dot{\Delta})_{\text{He, Analysis cell}}$ be the flux of helium into the analysis cell. Here, and elsewhere, I will refer to the ratio

$$\frac{(p\dot{\Delta})_{\text{He, probe}}}{(p\dot{\Delta})_{\text{He, Analysis cell}}} \tag{1.15}$$

as *the internal supply-to-sample flowrate ratio*. There is an Operation and Maintenance Instruction (OMI) that governs the procedures for validating the Leybold Hereaus Ultratest F mass spectrometer leak detector equipped with the optional Quick Test attachment (a sampling system of the internally branched type). The procedure described in this OMI (Ref. 8) involves the determination of a parameter (*the permeation ratio*), whose meaning is equivalent to the internal supply-to-sample flowrate ratio defined by (1.15) (see "cylinder samples check", sequence 02-010, p 26, *op. cit.*). Messers. KELVIN R. POLK and J. PERRY GODFREY of the Mass Spectrometer Laboratory of the Orbiter Processing Facility Shops (LSO-417) perform the procedures

Table 1-3 Leak-Point Matrix from Rockwell Specification MF0001-003 (Ref. 3, Appendix I, pp 13–15)

Test Point	Actual He Leakage (scc/sec)	MSHP† Indicated‡ He Leakage (scc/sec)
(OMS/RCS) [Orbital Maneuvering System/Reaction Control System]		
All welded/brazed joints	1×10^{-4}	1×10^{-7}
All Mechanical joints	1×10^{-4}	1×10^{-7}
All He and propellant test points (capped)	1×10^{-4}	1×10^{-7}
All All vent ports (capped)	1×10^{-4}	1×10^{-7}
All He and propellant fill ports (capped)	1×10^{-4}	1×10^{-7}
MPS [Main Propulsion System]		
All cryogenic mechanical, welded and brazed joints	1×10^{-4}	1×10^{-7}
Tank door seals, flange seals, and swaged joints	5×10^{-3}	5×10^{-6}
Engine interface seal	1×10^{-4}	1×10^{-7}
Pneumatic joints less than 1/2 inch in diameter	1×10^{-4}	1×10^{-7}
Pneumatic joints greater than or equal to 1/2 inch in diameter	1×10^{-3}	1×10^{-6}
EPS [Electrical Power System]—Cryo system		
All welded and brazed joints in H_2 and O_2 systems	1×10^{-4}	1×10^{-7}
Quick Disconnects uncapped H_2 and O_2 system	NA	NA
Quick Disconnects capped H_2 and O_2 system	NA	NA
Valve packs in H_2 system	1×10^{-4}	1×10^{-7}
Relief valve in H_2 system	NA	NA
Cryo tanks metallurgical joints H_2 system	1×10^{-4}	1×10^{-7}
All dynatube connections or other mechanical joints	1×10^{-4}	1×10^{-7}
EPG [Electical Power Generator]-Fuel cell system		
All welded and brazed joints	1×10^{-4}	1×10^{-7}
All dynatube connections or other mechanical joints	1×10^{-4}	1×10^{-7}
ECLSS [Environmental Control and Life Support System]		
All welded and brazed or solder joints	1×10^{-4}	1×10^{-7}
All Quick Disconnects in ECLSS (capped or mated) (unless otherwise specified)	1×10^{-4}	1×10^{-7}
All dynatube connections or other mechanical joints	1×10^{-4}	1×10^{-7}
All test ports and instrumentation ports (capped)	1×10^{-4}	1×10^{-7}
Window/Windshield Cavity Conditioning System		
All welded/brazed joints	1×10^{-4}	1×10^{-7}
All mechanical joints	1×10^{-4}	1×10^{-7}
Quick Disconnects capped	1×10^{-4}	1×10^{-7}
Bulkhead seals	5×10^{-3}	5×10^{-6}
All test ports and instrumentation ports (capped)	1×10^{-4}	1×10^{-7}

† MSHP=Mass Spectrometer Hand Probe
‡All leakage indication are "Single Point" leaks. These acceptance criteria, when using the Ultratest M, M2, or F system shall be corrected with the appropriate factors ... to provide the maxiumum indicated reading which may be accepted.

described in this OMI. They inform me that the value of the permeation ratio for Quick Test systems they test is nomrally between 300 and 400, and that the value 400 is typical.

In the case when the sampling system is of branched conduit type, one may relate the actual-to-indicated leak rate ratio to the external and internal supply-to-sample flowrate ratios as follows

$$\underbrace{\frac{(p\dot{\Delta})_{\text{He, test object}}}{(p\dot{\Delta})_{\text{He, analysis cell}}}}_{\substack{\text{actual-to-indicated}\\ \text{leakrate ratio}}} = \underbrace{\frac{(p\dot{\Delta})_{\text{He, test object}}}{(p\dot{\Delta})_{\text{He, probe}}}}_{\substack{\text{external}\\ \text{supply-to-sample}\\ \text{flowrate ratio}}} \times \underbrace{\frac{(p\dot{\Delta})_{\text{He, probe}}}{(p\dot{\Delta})_{\text{He, analysis cell}}}}_{\substack{\text{internal}\\ \text{supply-to-sample}\\ \text{flowrate ratio}}} . \tag{1.16}$$

1.4.4 DIFFICULTIES IN THE INTERPRETATION OF ACCEPTABILITY CRITERIA. With the aid of equation (1.16), one may bring into sharper focus the difficulties of interpreting the Rockwell Leak Point Matrix requirements (Table 1-3) in the case when the sniffer probe on the leak detector is used in conjunction with a sampling system of branched conduit type. To fix ideas, suppose that a probe operator inspects a test object with a Leybold-Hereaus Ultratest F leak detector equipped with a Quick Test sampling system. Now the Quick Test sampling system is of branched conduit type (as opposed to direct conduit type), so one may not assume automatically that the external supply-to-sample flowrate ratio is on the order 1,000 (as in the Boeing data reported in Table 1-2). If one makes this assumption anyway, and assumes further that the internal supply-to-sample flowrate ratio is 400, then a numerical computation of the actual-to-indicated leak rate ratio is, according to (1.16),

$$\underbrace{400{,}000}_{\substack{\text{actual-to-indicated}\\ \text{leakrate ratio}}} = \underbrace{1{,}000}_{\substack{\text{external}\\ \text{supply-to-sample}\\ \text{flowrate ratio}}} \times \underbrace{400}_{\substack{\text{internal}\\ \text{supply-to-sample}\\ \text{flowrate ratio}}} . \tag{1.17}$$

Of course, the value 400,000 for the actual-to-indicated leak rate ratio is much larger than the value 1,000 indicated in the Rockwell Leak Point Matrix (Table 1-3). Two hypotheses for this apparent inconsistency come to mind:

<u>Hypothesis I.</u> *The Quick Test probe acts on the air in its neighborhood as a mass sink. Transport of helium toward the probe in consequence of this sink flow dominates over transport away from the leak source in consequence of molecular diffusion and lateral transport in consequence of ambient crossflow.* The value 1,000 for the external supply-to-sample flowrate ratio, which was assumed in the derivation of (1.17), was based on the 1968 Boeing data (Table 1-2). These data were taken with sampling systems of direct type. But the Quick Test system is of branched type. The sampling system used to collect the data in the Boeing study more nearly resembles the Leybold Hereaus *Standard Probe*, a sampling system of the direct conduit type which may be fit to the Leybold Hereaus Leak Detector as alternative to the Quick Test attachment. The rate at which this direct conduit system draws in atmospheric air is about 1×10^{-3} atm·(cm)^3/sec. For such a device, one may conjecture that the mechanism that dominates the transfer of helium from the leak in the test object to the probe tip is molecular diffusion. Now molecular diffusion transfers helium outward from the source in all directions and the element of solid angle subtended by the probe tip is small compared with the total solid angle of a celestial sphere centered on the source. The effectiveness of such diffusive transferis low at best and may be hampered further by ambient crossflow at the test site. Some such ambient crossflow will always be present owing to deliberate circulation of environmental air in the test area and bouyant convection from temperature nonuniformities of different objects in the test area (to name only two causes). One may surmise that the actual transfer of helium from the leak source to the tip of a Standard Probe with a draw rate of 1×10^{-3} atm·(cm)^3/sec is an inefficient process which is easily

disrupted by unavoidable conditions in the test environment. The high value of the external supply-to-sample ratio observed by the Boeing investigators of Ref. 7 (*i.e.* the value 1,000) is, according to this argument, a reflection of the inefficiency of this diffusive transfer process.

Consider now the Quick Test system. The rate at which the Quick Test system draws in atmospheric air is about 1.5 atm$\cdot$(cm)^3/sec. The flowrate of air through the Quick Test probe is therefore about 1,500 times the corresponding flowrate through the Standard Probe. The flowrate through the Quick Test probe is, moreover, much larger than the actual helium leakage of any marginally acceptable leak indicated in the Rockwell Leak Point Matrix (Table 1-3). One might conjecture that the Quick Test, unlike the Standard probe, exertes an action on the leaked gas analogous to the action of a vacuum cleaner. In particular, the flow of air exterior to the probe tip generated by the sucking of fluid into it is what fluid dynamicists call a *point sink*. The tranfer of helium outward from the leak source due to molecular diffusion and lateral tranfer due to ambient crossflow is counteracted to a greater or lesser extent by the transfer of helium toward the probe tip due to the sink flow. By making the sink flow large enough, one may arrange that the helium transfer in the neightborhood of the probe tip is *dominated* by the sink flow. One may surmise that the actual transfer from the leak source to the tip of a Quick Test Probe with a draw rate of 1.5 atm$\cdot$(cm)^3/sec is a more efficienty process than the transfer in the case of the Standard Probe (whose draw rate is three orders of magnitude smaller). A value of the external supply-to-sample ratio that is conspicuously lower than the one observed by the Boeing investigators of Ref. 7 would reflect the greater efficiency of the Quick Test system in transferring helium. If, for example, one assumes an external supply-to-sample ratio of 2.5 then, under Hypothesis I, a computation corresponding to (1.16) would be of the form

$$\underbrace{1,000}_{\substack{\text{actual-to-indicated}\\ \text{leakrate ratio}}} = \underbrace{2.5}_{\substack{\text{external}\\ \text{supply-to-sample}\\ \text{flowrate ratio}}} \times \underbrace{400}_{\substack{\text{internal}\\ \text{supply-to-sample}\\ \text{flowrate ratio}}} . \tag{1.18}$$

Hypothesis II. *When probe-type sampling systems are used, the term Indicated Leak Rate is ambiguous unless preceded by an appropriate adjective. Thus, Uncorrected Indicated Leak Rate refers to the flowrate of helium through the* analysis cell *and Corrected Indicated Leak Rate refers to the flowrate of helium through the* probe. *The numbers under the heading* Indicated Leak Rate *in the Rockwell Leak Point Matrix (Table 1-3) are to be interpreted as Corrected Indicated Leak Rates.* Under Hypothesis II, one may relate the Corrected Indicated Leak Rate to the Uncorrected Indicated Leak rate by an identity of the form

$$\underbrace{(p\dot{\Delta})_{\text{He, probe}}}_{\substack{\text{Corrected Indicated}\\ \text{Leak Rate}}} = \underbrace{\frac{(p\dot{\Delta})_{\text{He, probe}}}{(p\dot{\Delta})_{\text{He, analysis cell}}}}_{\substack{\text{internal}\\ \text{supply-to-sample}\\ \text{flowrate ratio}}} \times \underbrace{(p\dot{\Delta})_{\text{He, analysis cell}}}_{\substack{\text{Uncorrected Indicated}\\ \text{Leak Rate}}} . \tag{1.19}$$

The Correction Factor, as used under Hypothesis II, equals the internal supply-to-sample flowrate ratio (*a.k.a.* the permeation ratio). One may compute the ratio of actual leak rate to corrected indicated leak rate as follows:

$$\frac{(p\dot{\Delta})_{\text{He, test object}}}{(p\dot{\Delta})_{\text{He, probe}}} = \frac{(p\dot{\Delta})_{\text{He, test object}}}{\dfrac{(p\dot{\Delta})_{\text{He, probe}}}{(p\dot{\Delta})_{\text{He, analysis cell}}} \times (p\dot{\Delta})_{\text{He, analysis cell}}} , \tag{1.20}$$

or

$$\underbrace{\frac{(p\dot{\Delta})_{\text{He, test object}}}{(p\dot{\Delta})_{\text{He, probe}}}}_{\substack{\text{Ratio of Actual Leak}\\ \text{Rate to Corrected}\\ \text{Indicated Leak Rate}}} = \underbrace{\left(\frac{(p\dot{\Delta})_{\text{He, test object}}}{(p\dot{\Delta})_{\text{He, analysis cell}}}\right)}_{\substack{\text{end-to-end}\\ \text{supply-to-sample}\\ \text{flowrate ratio}}} \div \underbrace{\left(\frac{(p\dot{\Delta})_{\text{He, probe}}}{(p\dot{\Delta})_{\text{He, analysis cell}}}\right)}_{\substack{\text{internal}\\ \text{supply-to-sample}\\ \text{flowrate ratio}}} \tag{1.21}$$

which is just a rearrangement of equation (1.16) above. If one assumes an end-to-end supply-to-sample flowrate ratio of 400,000 and an internal supply-to-sample flowrate ratio of 400, then a computation corresponding to (1.21) is

$$\underbrace{1{,}000}_{\begin{array}{c}\text{Ratio of Actual Leak}\\ \text{Rate to Corrected}\\ \text{Indicated Leak Rate}\end{array}} = \underbrace{400{,}000}_{\begin{array}{c}\text{end-to-end}\\ \text{supply-to-sample}\\ \text{flowrate ratio}\end{array}} \div \underbrace{400}_{\begin{array}{c}\text{internal}\\ \text{supply-to-sample}\\ \text{flowrate ratio}\end{array}}, \tag{1.22}$$

which is a rearrangement of (1.17).

Now Hypothesis II is the more conservative of the two hypothesis in that it is more likely to overestimate the rate of leakage from a test object. The information in the Rockwell Specification (Ref. 3) is not clear as to which of the above two hypotheses it requires. The relevent passages in the text of the specification read as follows:

> 3.2.1 ...The Leybold-Hereaus Ultratest M, M2, or F with Quick Test atttachment shall have operating instructions attached that shall include the necessary ratio and entrainment information so that its indicated readings may be converted to actual leak figures.
>
> 3.2.2 The permeability ratio shall be determined using the volume method for obtaining the indicated leak rate and shall be 600 or less before the leak detector, with hose and probe, may be used (Ref. Calibration Procedure 344-2027 [titled "Calibration/Service Procedure for the Leybold Hereaus Mass Spectrometer Leak Detector"]).

One should also recall one of the footnotes to the Leak Point Matrix reproduced in Table 1-3 above. That footnote reads

> All leakage indications are "Single Point" leaks. These acceptance criteria, when using the Ultratest M, M2, and F system shall be corrected with the appropriate factors (ref. 3.2.1) to provide the maximum indicated reading which may be accepted.

The quoted passage from §3.2.1 seems to require that the "operating instructions attached" to the Quick Test contain not merely the Permeation Ratio (*i.e.* internal supply-so-sample flowrate ratio), but rather the full end-to-end supply-to-sample flowrate ratio (including the poorly understood factor due to the *external* supply-to-sample flowrate ratio). In this respect, the Rockwell Specification seems to ask a great deal of the personnel reponsible for preparing the machine for use. I have read "Calibration Procedure 344-2027" cited in §3.2.2 above* and am satisfied that the procedure for determining the Permeation Ratio is equivalent to the one now in use (*cf.* Operations and Maintenance Instruction OMI V6H31, Revision A-1 (Ref. 8)).

The footnote to the Leak Point Matrix just quoted above seems to suggest the use of some kind of correction factor to relate Indicated Leak Rate to Corrected Indicated Leak Rate. Parts of the specification that mention corrections for helium background effects also allude to such a correction factor. Thus, in §3.5.1.1 one reads

> 3.5.1.1 ...Normally/ideally the background should be low enough that the allowable leak (to be determined) is at least several times the MDL [Minimum Detectable Leak] . However, the MDL versus the allowable lleak is the limiting factor. The background reading shall not exceed the Indicated Acceptance Criteria in Appendix I [the Leak Point Matrix, *cf.* Table 1-3 above], or for the Ultratest M, M21, or F the corrected Indicated Acceptance Critera (see note 2 of Appendix I).

"Note 2" cited above reads as follows:

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* I am indebted to Dr. William Van Dusen of Rockwell (ZK36) for furnishing me with a copy of this document. I gather that Dr. Van Dusen, in turn, obtained his copy from Mr. Rustin Van Dyke of Lockheed Training (LSO-155). Special thanks are therefore due to Mr. Van Dyke.

2) When using the Ametek or Ultratest M, M2, or F, if the indicated leak rate reading does not exceed the corrected indicated acceptance criteria, a background reading need not be taken.

Here again is an allusion to a correction factor. The last excerpt that provides any clue about indicated leak rate is a definition in §3.5.1.4 (h), *viz*

(h) The indicated leakage rate shall not exceed the leakage criteria of Appendix I [the Leak Point Matrix of Table 1-3] and shall be calculated using any mathematical tool that will produce the same results as the following formula:

$$\begin{matrix}\text{indicated}\\ \text{leak rate}\end{matrix} = \left(\begin{matrix}\text{leak detector}\\ \text{sensitivity}\end{matrix}\right) \times (\text{total - background meter readings})$$

(DuPont, or Veeco machines [of direct conduit type]) [or]

$$\begin{matrix}\text{indicated}\\ \text{leak rate}\end{matrix} = \text{corrected - background meter readings}$$

(Ametek and Leybold-Hereaus Machine [of branched conduit type]) only if the total exceeds 1×10^{-7} [no units given in original].

Again, there is an allusion to a correction factor. Lacking clearer information, one is inclinded to suppose that the intentions of the Rockwell specification are more accurately reflected by Hypothesis II (which involves the use of a correction factor and in which the external supply-to-sample ratio is assumed to be 1,000) than by Hypothesis I (which involves no correction factor and in which the external supply to-sample flowrate ratio is assumed to be approximately 2.5).

In the mean time, Operations and Maintenance Instruction V1009.005 (Ref. 1) covers the procedures used by probe operators inspecting parts of the hydrogen transport system for the shuttle orbiter. These instructions do not make any use of a correction factor. They appear, therefore, to be more consistent with Hypothesis I. There would seem to be a need to reconcile the requirements of References 1 and 3.

SECTION TWO

CAPILLARY FLOW

2.1 THE EQUATIONS OF MOTION OF A VISCOUS GAS IN CYLIDRICAL COORDINATES

Let $\{\hat{\mathbf{e}}_r, \hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_z\}$ be a right handed orthogonal triad of unit vectors belonging to a cylindrical coordinate system with radial coordinate r, azimuth angle θ, and axial coordinate z. Let (u_r, u_θ, u_z) be the scalar components of the fluid velocity vector $\mathbf{u}$ belonging to these coordinates. Let ρ be the mass desity of the fluid (a quantity with the dimensions mass per unit volume). Then one may write the differential equation of conservation of mass in the form

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (r\rho u_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho u_\theta)}{\partial \theta} + \frac{\partial (\rho u_z)}{\partial z} = 0 \,. \tag{2.1}$$

The differential equation for the rate of change of translational momentum is a vector equation. In a three dimensional space, this vector equation has three scalar components. Let p denote the local instantaneous mechancial pressure in the fluid, and let (g_r, g_θ, g_z) be the scalar components of the local gravitational force per unit mass $\mathbf{g}$ belonging to the cylindrical coordinate system introduced above. Let μ be the coefficient of shear viscosity in the fluid (a quantity with the dimensions of stress divided by strain rate, or, equivalently, force times time divided by area).

If one assumes that there are no volumetric forces exerted on the gas other than gravity and makes the usual assumption that the gas is linearly viscous, then the three scalar components of the momentum equation are as follows. The radial component of the momentum equation is

$$\begin{aligned}\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} + u_z\frac{\partial u_r}{\partial z} - \frac{(u_\theta)^2}{r}\right) &= -\frac{\partial p}{\partial r} - \frac{2\mu}{r}\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} - \frac{\operatorname{div}\mathbf{u}}{3}\right) \\ &+ \frac{1}{r}\frac{\partial}{\partial r}\left[r\,2\mu\left(\frac{\partial u_r}{\partial r} - \frac{\operatorname{div}\mathbf{u}}{3}\right)\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)\right] + \rho g_r \,,\end{aligned} \tag{2.2}$$

the azimuthal component of the momentum equation is

$$\begin{aligned}\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + u_z\frac{\partial u_\theta}{\partial z} + \frac{u_\theta u_r}{r}\right) &= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{\mu}{r}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r}\right) \\ &+ \frac{1}{r}\frac{\partial}{\partial r}\left[r\,\mu\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r}\right)\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[2\mu\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} - \frac{\operatorname{div}\mathbf{u}}{3}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right)\right] + \rho g_\theta \,,\end{aligned} \tag{2.3}$$

and the axial component of the momentum equation is

$$\begin{aligned}\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right) &= -\frac{\partial p}{\partial z} \\ &+ \frac{1}{r}\frac{\partial}{\partial r}\left[r\,\mu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right)\right] + \frac{\partial}{\partial z}\left[2\mu\left(\frac{\partial u_z}{\partial z} - \frac{\operatorname{div}\mathbf{u}}{3}\right)\right] + \rho g_z \,,\end{aligned} \tag{2.4}$$

in which $\operatorname{div}\mathbf{u}$ is an abbreviation for the divergence of the velocity field, *i.e.*

$$\operatorname{div}\mathbf{u} \equiv \frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} \,. \tag{2.5}$$

2.2 SIMPLIFICATIONS

2.2.1 NEGLECT OF THE GRAVITY TERMS. I will assume, here and elsewhere, that the gas in the capillary is ideal. Thus,

$$p = \rho RT\,, \tag{2.6}$$

in which the mass specific gas constant R is related to the universal gas constant $\mathcal{R}$ by

$$r = \frac{\mathcal{R}}{M}\,, \tag{2.7}$$

in which M is the molecular mass. Now the elements of the list

$$\left(\frac{\partial p}{\partial r}, \frac{1}{r}\frac{\partial p}{\partial \theta}, \frac{\partial p}{\partial z}\right) \tag{2.8}$$

are the three scalar components of the pressure gradient vector ∇p and the elements of the list

$$(\rho g_r, \rho g_\theta, \rho g_z) \tag{2.9}$$

are the three scalar components of the gravitational force per unit mass $\rho\mathbf{g}$. In view of (2.6), we have

$$\frac{|\rho\mathbf{g}|}{|\nabla p|} = \frac{p}{RT}\frac{|\mathbf{g}|}{|\nabla p|}\,. \tag{2.10}$$

If ℓ is the length of the capillary, then an order of magnitude estimate of the quotient $p/|\nabla p|$ is

$$\frac{p}{|\nabla p|} \sim \ell\,. \tag{2.11}$$

The expression $RT/|\mathbf{g}|$ represents a length which is sometimes called the *scale height of the atmosphere*. To be specific, if an isothermal atmosphere is in static equilibrium under its own weight, then the scale height is the altitude increase within which the pressure decreases by one power of e. For air at $T = 296°\mathrm{K}$, the scale height is about 8.84 kilometers. If one denotes the scale height by H, then (2.10) and (2.11) imply that

$$\frac{|\rho\mathbf{g}|}{|\nabla p|} = \frac{p}{|\nabla p|}\frac{1}{H} \sim \frac{\ell}{H}\,. \tag{2.12}$$

I will assume, here and elsewhere, that $\ell \ll H$ and, accordingly, neglect those contributions to the momentum equations (2.2)–(2.4) due to $\rho\mathbf{g}$ (*cf.* the list (2.9)) in comparison with the contributions due to ∇p (*cf.* the list (2.8)).

2.2.2 THE NEGLECT OF TIME DEPENDENCE. I restrict the present analysis to flows with zero time dependence. Accordingly, all of the terms in the system (2.1)–(2.4) involving the time derivative operator $\partial(\ )/\partial t$ will be equated to zero.

2.2.3 THE ASSUMPTION OF AXISYMMETRIC FLOW WITHOUT SWIRL. I restrict the present analysis to flows in which none of the flow variables depend upon the azimuth angle θ and the swirl component of the velocity, u_θ, is identically zero. Under the assumptions listed thus far, the system of equations (2.1)–(2.5) reduces to

$$\frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r} + \frac{\partial(\rho u_z)}{\partial z} = 0\,. \tag{2.13}$$

$$\rho\left(u_r\frac{\partial u_r}{\partial r} + u_z\frac{\partial u_r}{\partial z}\right) = -\frac{\partial p}{\partial r} - \frac{2\mu}{r}\left(\frac{u_r}{r} - \frac{\operatorname{div}\mathbf{u}}{3}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left[r\,2\mu\left(\frac{\partial u_r}{\partial r} - \frac{\operatorname{div}\mathbf{u}}{3}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)\right]\,, \tag{2.14}$$

$$0 = 0\,, \tag{2.15}$$

$$\rho\left(u_r\frac{\partial u_z}{\partial r} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left[r\,\mu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)\right] + \frac{\partial}{\partial z}\left[2\mu\left(\frac{\partial u_z}{\partial z} - \frac{\operatorname{div}\mathbf{u}}{3}\right)\right]\,, \tag{2.16}$$

and

$$\operatorname{div}\mathbf{u} \equiv \frac{1}{r}\frac{\partial(r u_r)}{\partial r} + \frac{\partial u_z}{\partial z}\,, \tag{2.17}$$

respectively.

2.2.4 USE OF THE ASSUMPTION THAT THE CAPILLARY IS SLENDER. Let ℓ be the inside diameter of the capilllary. If the capillary wall is impermeable, one may solve (2.13) for u_r as follows. If one regards ρ, u_r, & u_z as functions of the argument list (r, z), then one may write (2.13) in the form

$$\frac{\partial}{\partial r}\left[r\rho(r,z)u_r(r,z)\right] = -r\frac{\partial}{\partial z}\left[\rho(r,z)u_z(r,z)\right] \ . \tag{2.18}$$

An equivalent equation written in different notation is

$$\frac{\partial}{\partial \xi}\left[\xi\rho(\xi,z)u_r(\xi,z)\right] = -\xi\frac{\partial}{\partial z}\left[\rho(\xi,z)u_z(\xi,z)\right] \ . \tag{2.19}$$

If one integrates (2.19) with respect to ξ from $\xi = d/2$ to $\xi = r$ and applies an impermeable-wall boundary condition $(u_r)_{r=d/2} = 0$, one obtains

$$r\rho(r,z)u_r(r,z) = -\int_{d/2}^{r}\xi\frac{\partial}{\partial z}\left[\rho(\xi,z)u_r(\xi,z)\right]d\xi \ , \tag{2.20}$$

or

$$u_r(r,z) = \frac{-1}{r\rho(r,z)}\int_{d/2}^{r}\xi\frac{\partial}{\partial z}\left[\rho(\xi,z)u_r(\xi,z)\right]d\xi \ . \tag{2.21}$$

Let U_r & U_z be velocity scales typical of the velocity components u_r and u_z, respectively. Then (2.21) implies that U_r is related to U_z by the following order-of-magnitude balance

$$U_r \sim U_z\frac{d}{\ell} \ , \tag{2.22}$$

or

$$U_r \sim U_z\epsilon \ , \tag{2.23}$$

in which

$$\epsilon \equiv d/\ell \tag{2.24}$$

is a *slenderness parameter* for the capillary. I will assume in what follows that $\epsilon \ll 1$.

Of the various viscous terms in the right member of the axial momentum equation (2.16), one may show that the term

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u_z}{\partial r}\right) \tag{2.25}$$

dominates over the others as follows. Let μ_0 be a viscosity scale typical of the viscosity of the gas in the capillary. Then

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u_z}{\partial r}\right) \sim \frac{\mu_0 U_z}{d^2} \ , \tag{2.26}$$

and

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u_r}{\partial r}\right) \sim \frac{\mu_0 U_r}{d\ell} \sim \frac{\mu_0\epsilon U_z}{d\ell} = \frac{\mu_0\epsilon^2 U_z}{d^2} \ , \tag{2.27}$$

in which the last equality follows from (2.24). Moreover,

$$\frac{\partial}{\partial z}\left(2\mu\frac{\partial u_z}{\partial z}\right) \sim \frac{\mu_0 U_z}{\ell^2} = \frac{\mu U_z}{d^2}\epsilon^2 \ . \tag{2.28}$$

Now

$$\operatorname{div}\mathbf{u} = \frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{\partial u_z}{\partial z} \sim \frac{U_r}{d} + \frac{U_z}{\ell} \sim \frac{\epsilon U_z}{d} + \frac{U_z\epsilon}{d} \ ,$$

or

$$\operatorname{div}\mathbf{u} \sim \frac{U_z\epsilon}{d} \,. \tag{2.29}$$

Thus,

$$\frac{\partial}{\partial z}\left[2\mu\left(-\frac{\operatorname{div}\mathbf{u}}{3}\right)\right] \sim \frac{\mu_0}{\ell}\frac{U_z\epsilon}{d} = \frac{\mu_0 U_z}{d^2}\epsilon^2 \,. \tag{2.30}$$

The viscous terms (2.27), (2.28), & (2.30) are therefore of the order ϵ^2 times the viscous term (2.26). If one neglects these higher order terms, the axial momentum equation (2.16) reduces to

$$\rho\left(u_r\frac{\partial u_z}{\partial r} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u_z}{\partial r}\right) \,. \tag{2.31}$$

If one carries out estimates of the order-of-magnitude of the various terms in the *radial* momentum equation (2.14), one finds that the largest of the various terms is of the order ϵ times the largest of the viscous terms in the axial equation (2.16). By the same token, the ratio of the inertial terms in the radial equation to the inertia terms in the axial equation is also of the order ϵ. Thus, the terms that balance the radial pressure gradient in (2.14) are of the order ϵ times the terms that balance the axial pressure gradient in (2.16) and one concludes that $\partial p/\partial r$ is of the order ϵ times $\partial p/\partial z$. One is then led to treat p as a function of the axial coordinate z only and to write (2.31) in the form

$$\rho\left(u_r\frac{\partial u_z}{\partial r} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{dp}{dz} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u_z}{\partial r}\right) \,. \tag{2.32}$$

2.2.5 USE OF AN ASSUMPTION INVOLVING THE CAPILLARY REYNOLDS NUMBER. Let ρ_0 be a density scale typical of values of ρ in the capillary. One may estimate the terms in the left member of (2.31) as follows

$$\rho u_r\frac{\partial u_z}{\partial r} \sim \rho_0\frac{U_rU_z}{d} \sim \frac{\rho_0\epsilon U_z^2}{d} \,, \tag{2.33}$$

$$\rho u_z\frac{\partial u_z}{\partial z} \sim \rho_0\frac{U_z^2}{\ell} \sim \rho_0\frac{U_z^2}{d}\epsilon \,. \tag{2.34}$$

The ratio of the order of magnitude of the inertia terms in the left member of (2.32) to the order of magnitude of the viscous term in the right member is therefore

$$\frac{\left(\rho_0\frac{U_z^2}{d}\epsilon\right)}{\left(\frac{\mu_0 U_z}{d^2}\right)} = \left(\frac{\rho_0 U_z d}{\mu_0}\right)\epsilon \,. \tag{2.35}$$

The expression $\rho_0 U_z d/\mu_0$ is the *Reynolds number* based on the capillary diameter and axial velocity scale. I will assume in what follows that

$$\left(\frac{\rho_0 U_z d}{\mu_0}\right)\epsilon \ll 1 \,, \tag{2.36}$$

which allows one to approximate (2.32) by the simpler equation

$$\frac{dp}{dz} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u_z}{\partial r}\right) \,. \tag{2.37}$$

2.3 DEDUCTIONS FROM THE SIMPLIFIED EQUATIONS

2.3.1 SOLUTION FOR THE AXIAL VELOCITY DISTRIBUTION. If one assumes that the viscosity μ does not vary significantly across the cross section of the capillary, one may rewrite (2.37) in the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) = \frac{1}{\mu}\frac{dp}{dz}\,. \tag{2.38}$$

One may write the general solution of the differential equation (2.38) in the form

$$u_z = \frac{r^2}{4}\frac{1}{\mu}\frac{dp}{dz} + A\ln\left(\frac{r}{d/2}\right) + B\,, \tag{2.39}$$

in which the parameters μ, p, A, and B are all independent of r. If one imposes the condition that the velocity be finite on the axis $r = 0$, one deduces that $A = 0$. If, moreover, one applies the *no-slip boundary condition*

$$(u_z)_{r=d/2} = 0\,, \tag{2.40}$$

one may compute the value of B. The resulting formula for u_z is

$$u_z = \left[r^2 - \left(\frac{d}{2}\right)^2\right]\frac{1}{4\mu}\frac{dp}{dz}\,. \tag{2.41}$$

Equation (2.41), or some equation of equivalent meaning, appears in nearly every textbook on fluid dynamics and is normally associated with the names HAGEN and POISEUILLE. All of the standard derivations of (2.41) with which I am familar, however, assume that the fluid is incompressible. The present derivation, which makes no incompressibility assumption *per se* generalizes the result (2.41) to gas flow.

If one evaluates (2.47) on the centerline $r = 0$, one obtains

$$(u_z)_{\max} = -\left(\frac{d}{2}\right)^2\frac{1}{4\mu}\frac{dp}{dz}\,. \tag{2.42}$$

If one orients the axes so that the positive z-axis points in the direction of the flow, then $(u_z)_{\max} > 0$ by construction. It follows from (2.42) that $dp/dz < 0$, *i.e.* that *pressure decreases in the direction of the flow* (as expected!).

2.3.2 THE RATE OF TRANSPORT OF FLUID VOLUME. In (1.1), let $\mathcal{S}$ be a circular cross section (normal to the axis of the capillary), in which $0 \le r \le d/2$ and $0 \le \theta \le 2\pi$. Let $d\mathbf{A} = rd\theta dr\hat{\mathbf{e}}_z$ and $\mathbf{u} = u_z\hat{\mathbf{e}}_z$. Then $\mathbf{u}{\cdot}d\mathbf{A} = u_z r d\theta dr$ and (1.1) becomes

$$\dot{\Delta} = \int_{\theta=0}^{2\pi}\int_{r=0}^{d/2} u_z r d\theta dr = \frac{1}{4\mu}\frac{dp}{dz}2\pi\int_{r=0}^{d/2}\left[r^2 - \left(\frac{d}{2}\right)^2\right]rdr = -\frac{\pi}{8\mu}\frac{dp}{dz}\left(\frac{d}{2}\right)^4\,. \tag{2.43}$$

2.3.3 COMPUTATION OF THE THROUGHPUT $p\dot{\Delta}$. USE OF AN ISOTHERMAL ASSUMPTION. STREAMWISE VARIATION OF THE PRESSURE. If one multiplies the two members of (2.43) by the mass density ρ, one obtains

$$\rho\dot{\Delta} = -\frac{\pi}{8\mu}\rho\frac{dp}{dz}\left(\frac{d}{2}\right)^4\,. \tag{2.44}$$

According to the ideal gas law $p = \rho RT$. One may therefore substitute $p/(RT)$ for ρ in the right member of (2.44) to obtain

$$\rho\dot{\Delta} = -\frac{\pi}{8\mu}\frac{p}{RT}\frac{dp}{dz}\left(\frac{d}{2}\right)^4 - \frac{\pi}{8\mu}\frac{1}{RT}\left(\frac{d}{2}\right)^4\frac{d}{dz}\left(\frac{p^2}{2}\right) . \tag{2.45}$$

Now $\rho\dot{\Delta}$ is the rate of transport of *mass* through the capillary and is therefore independent of the streamwise coordinate z. If one assumes, moreover, that the absolute temperature is effectively independent of z, then one may integrate (2.45) with respect to z to obtain

$$(\rho\dot{\Delta})(z - z_0) = -\frac{\pi}{8\mu}\frac{1}{RT}\left(\frac{d}{2}\right)^4\left(\frac{p^2}{2} - \frac{p_0^2}{2}\right) , \tag{2.46}$$

in which the constant of integration has been fixed by the condition

$$(p)_{z=z_0} = p_0 . \tag{2.47}$$

If one muliplies (2.46) by $RT/(z - z_0)$ and recalls that $RT\rho = p$, one obtains

$$p\dot{\Delta} = -\frac{\pi}{16\mu}\left(\frac{d}{2}\right)^4\frac{p^2 - p_0^2}{z - z_0} . \tag{2.48}$$

If one evaluates (2.48) at a station where

$$(p)_{z=z_1} = p_1 , \tag{2.49}$$

then (2.48) implies that

$$p\dot{\Delta} = -\frac{\pi}{16\mu}\left(\frac{d}{2}\right)^4\frac{p_1^2 - p_0^2}{z_1 - z_0} . \tag{2.50}$$

One may apply (2.50), for example, in the case when station zero is the inlet (at, say, atmospheric pressure) and station one is at the outlet (where the pressure is some given lower pressure). In this case, of course, $z_1 - z_0$ represents the length of the capillary, ℓ.

If one eliminates $p\dot{\Delta}$ between (2.48) and (2.50), one obtains, after simplification,

$$\frac{p^2 - p_0^2}{z - z_0} = \frac{p_1^2 - p_0^2}{z_1 - z_0} . \tag{2.51}$$

One may intepret (2.51) as an interpolation formula for the local pressure $p(z)$ at an arbitrary station z in the range $z_0 < z < z_1$. If one solves (2.51) for p, one obtains

$$p = \sqrt{p_0^2 + (p_1^2 - p_0^2)\left(\frac{z - z_0}{z_1 - z_0}\right)} . \tag{2.52}$$

which shows that in the case of gas flow, unlike the case of liquid flow, the pressure depends nonlinearly upon the axial coordinate.

2.3.3 THE TIME FOR A FLUID PARTICLE TO TRAVEL THE LENGTH OF A CAPILLARY. Consider a reference frame attached to a hypothetical material point which moves with the gas velocity $\mathbf{u}$. Since the material point moves with the fluid and the pressure decreases in the downstream direction, the pressure to which the material point is subject must be time dependent. If one denotes the time derivative of p relative to the material point by Dp/Dt, one may show from the chain rule and the kinematic definitions of the velocity components in cylindrical coordinates that

$$\frac{Dp}{Dt} = \frac{\partial p}{\partial t} + u_r \frac{\partial p}{\partial r} + \frac{u_\theta}{r}\frac{\partial p}{\partial \theta} + u_z \frac{\partial p}{\partial z} . \tag{2.53}$$

The assumption that the flow is steady (*cf.* §2.2.2) implies that $\partial p/\partial t = 0$. The assumption that the flow is axisymmetric (*cf.* §2.2.3) implies that $\partial p/\partial \theta = 0$. The assumption that the flow lacks swirl (*cf.* §2.2.3) implies that $u_\theta = 0$. The assumption that the capillary is slender (*cf.* §2.2.4) implies that $u_r \ll u_z$ and $\partial p/\partial r \ll \partial p/\partial z$. Equation (2.53) thus reduces to

$$\frac{Dp}{Dt} = u_z \frac{dp}{dz} \tag{2.54}$$

under assumptions and idealizations already introduced. If one multiplies (2.54) by p^2, one obtains

$$p^2 \frac{Dp}{Dt} = (pu_z)\left(p\frac{dp}{dz}\right) . \tag{2.55}$$

Consider a cylindrical surface where u_z equals the cross sectionally averaged fluid velocity $\overline{u}_z$ defined by

$$\overline{u}_z \equiv \frac{\dot{\Delta}}{\pi(d/2)^2} . \tag{2.56}$$

On a streamline where $u_z = \overline{u}_z$, (2.55) becomes

$$\left(p^2 \frac{Dp}{Dt}\right)_{u_z=\overline{u}_z} = \frac{p\dot{\Delta}}{\pi(d/2)^2} p \frac{dp}{dz} . \tag{2.57}$$

If one eliminates dp/dz from the right member by menas of (2.43), one obtains

$$\left(p^2 \frac{Dp}{Dt}\right)_{u_z=\overline{u}_z} = \frac{p\dot{\Delta}}{\pi(d/2)^2} p \left(-\frac{8\mu}{\pi}\frac{\dot{\Delta}}{(d/2)^4}\right) ,$$

or

$$\left[\frac{D}{Dt}\left(\frac{p^3}{3}\right)\right]_{u_z=\overline{u}_z} = -\frac{8\mu}{\pi^2(d/2)^6}(p\dot{\Delta})^2 . \tag{2.58}$$

The right member is independent of the axial coordinate z. It follows that the right member is independet of t in a reference frame moving with the velocity $u_z = \overline{u}_z$. One may thus integrate (2.58) with respect to time subject to the conditions

$$(p)_{t=t_0} = p_0 \quad , \quad (p)_{t=t_1} = p_1 \tag{2.59}$$

to obtain

$$\frac{p_1^3 - p_0^3}{3} = -\frac{8\mu}{\pi^2(d/2)^6}(p\dot{\Delta})^2(t_1 - t_0) . \tag{2.60}$$

The two equations (2.50) and (2.60) relate the inlet and the outlet pressures to three other fixed parameters for the capillary, namely the capillary diameter, d, the throughput, $p\dot{\Delta}$, and the transit time, $t_1 - t_0$. One may eliminate any one of these latter three parameters between (2.50) and (2.60) to obtain a formula relating the remaining two. If, for example, one eliminates the throughput $p\dot{\Delta}$ from (2.60) by means of (2.50), one obtains

$$\frac{p_1^3 - p_0^3}{3} = -\frac{8\mu}{\pi^2(d/2)^6}\left[-\frac{\pi}{16\mu}\left(\frac{d}{2}\right)^4 \frac{p_1^2 - p_0^2}{z_1 - z_0}\right]^2 (t_1 - t_0) = -\frac{8}{(2\cdot 8)^2}\frac{1}{\mu}\left(\frac{d}{2}\right)^2 \frac{(p_1^2 - p_0^2)^2}{(z_1 - z_0)^2}(t_1 - t_0) , \tag{2.61}$$

whence

$$t_1 - t_0 = -\frac{p_1^3 - p_0^3}{(p_1^2 - p_0^2)^2}\frac{32}{3}\mu\frac{(z_1 - z_0)^2}{(d/2)^2} \tag{2.62}$$

The result (2.62) expresses the time of transit $t_1 - t_0$ in terms of the inlet and outlet pressures, the fluid viscosity, and the ratio $d/(z_1 - z_0)$ (which is the slenderness parameter ϵ introduced in §2.2.4).

2.4 DIMENSIONS OF THE QUICK TEST CONDUIT. A ONE-CAPILLARY MODEL FOR PREDICTING $p\dot{\Delta}$ AND $t_1 - t_0$. COMPARISONS WITH OBSERVATION.

The Quick Test attachment to the Leybold Hereaus Ultratest F mass spectrometer leak detector may be operated with hoses of various internal diameters. Thus, a standard red hose has an internal diameter $d = d_r = 0.75$ mm, a standard green hose has an internal diameter $d = d_g = 0.90$ mm, and a standard blue hose has an internal diameter $d = d_b = 1.15$ mm*. During maintenance of the machine some combination of red and green hose is chosen by trial and error to ensure that the probe throughput $p\dot{\Delta}$ has the nominal value 1.5 atm·(cm)^3/sec. In a typical configuration, 25 feet of red hose is placed upstream of 75 feet of green hose. Upstream of the hoses is a metal capillary whose length is typically 8 cm and whose inside diameter is $d = d_c = 0.5$ mm.

Suppose one idealizes the problem by pretending that a single green hose of length 100 feet constitutes the whole conduit. Then in (2.50), one may take $z_1 - z_0 = \ell = 100$ ft $= 3048$ cm and $d/2 = d_g/2 = 0.045$ cm. For air at room temperature, say $T = 295°\mathrm{K} = 71.33°\mathrm{F}$, one may take $\mu = 1.8122 \times 10^{-4}$ dyne·sec/(cm)^2. If, moreover, one assumes that $(p_1/p_0)^2 \ll 1$, then (2.50) takes the approximate form

$$p\dot{\Delta} \approx \frac{\pi}{16\mu}\left(\frac{d_g}{2}\right)^4 \frac{p_0^2}{\ell}, \tag{2.63}$$

in which $p_0 = 1$ atm $= 1.01325 \times 10^6$ dyne·sec/(cm)^2. Thus,

$$p\dot{\Delta} \approx \frac{\pi(0.045\ \mathrm{cm})^4[(1.013 \times 10^6\ \mathrm{dyne/(cm)^2}]^2}{16[1.8122 \times 10^{-4}\ \mathrm{dyne\cdot sec/(cm)^2}](3048\ \mathrm{cm})} = 1.496 \times 10^6\ \frac{\mathrm{dynes}}{\mathrm{(cm)^2}}\frac{\mathrm{(cm)^3}}{\mathrm{sec}}. \tag{2.64}$$

If one multiplies by

$$1 = \frac{1\ \mathrm{atm}}{1.013 \times 10^6\ \mathrm{dyne/(cm)^2}}, \tag{2.65}$$

one obtains

$$p\dot{\Delta} \approx 1.477\ \frac{\mathrm{atm\cdot(cm)^3}}{\mathrm{sec}} \quad \text{(all green hose)}. \tag{2.66}$$

This value agrees well with the target value $p\dot{\Delta} = 1.5$ atm·(cm)^3/sec.

If one repeats the calculation of $p\dot{\Delta}$ but for an all red hose with $d = d_r = 0.75$ mm, instead of a green one (with $d = d_g = 0.90$ mm), one obtains

$$p\dot{\Delta} \approx 0.712\ \frac{\mathrm{atm\cdot(cm)^3}}{\mathrm{sec}} \quad \text{(all red hose)}. \tag{2.67}$$

Thus a reduction of hose diameter from $d_g = 0.9$ mm to $d_r = 0.75$ mm roughly halves the throughput.

If one substitutes the same values of the parameters into formula (2.62) for the transit time $t_1 - t_0$, one obtains

$$t_1 - t_0 \approx \frac{32}{3}\frac{\mu\ell^2}{p_0(d/2)^2}. \tag{2.68}$$

For the all green hose, (2.68) gives

$$t_1 - t_0 \approx \frac{32}{3}\frac{[1.8122 \times 10^{-4}\ \mathrm{dyne\cdot sec/(cm)^2}](3048\ \mathrm{cm})^2}{[1.013 \times 10^6\ \mathrm{dyne/(cm)^2}](0.045\ \mathrm{cm})^2} = 8.754\ \mathrm{sec} \quad \text{(all green hose)}. \tag{2.69}$$

The corresponding calculation for the all red hose gives

$$t_1 - t_0 = 12.61\ \mathrm{sec} \quad \text{(all red hose)}. \tag{2.70}$$

The response time for a typical Quick Test configuration is about 8 or 9 seconds. Once again, the idealization that the hose is all green agrss well with nominal performance data for the Quick Test.

* I am indebted to Messes Kelvin R. Polk and J. Perry Godfrey of the Mass Spectrometer Lab of the OPF Shops (LSO-417) for furnishing me with this information.

SECTION THREE

A CONCEPT FOR A MICROENCLOSURE, OR SPONGE-TIPPED PROBE

The difficulty of obtaining repeatable measurements for the external supply-to-sample flowrate ratio is vexing. This problem becomes moot if one can construct an alternative probe design that simulates an enclosure, analogous to the brush head of a vacuum cleaner, rather than a probe end exposed to the free air in the test area.

I envisage a probe with a flat metal disk welded to the end (punctured at the center, so the probe capillary is not plugged). The diameter of the metal disk must be large compared to the capillary diameter. If the capillary diameter is 0.5 mm then the disk diameter may be as small as 0.5 cm. A porus sponge or foam material adheres to the disk and allows the user to place the probe flat against the test surface.

If the draw rate of air into the probe is large compared to the rate of discharge of helium from the leak one is trying to measure (as will be the case if the probe is attached to a typical branched conduit system), then there is reason to believe that external supply-to-sample flowrate ratio may be reduced to a level on the order of, say, 1.1 or less.

SECTION FOUR

RECOMMENDATIONS AND CONCLUSION

Recommendations:

1. Determine (by consultation with the authors, if necessary) whether the indicated-to-actual leak rate ratio in the Rockwell specification MF0001-003 should be interpreted as: (I) the end-to-end supply-to-sample flowrate ratio (*i.e.* the product of the internal and external supply-to-sample flowrate ratios); or, (II) the external supply-to-sample flowrate ratio only. If hypothesis (II) is correct then the Operations and Maintenance Instruction V1009.005 (and, hence, current practice at KSC) is in conflict with the Rockwell specification.
2. Initiate a program to develop a microenclosure probe, preferably a program that would lead to the fabrication and testing of a prototype device at KSC within the next twelve months.

Conclusion:

The observed flow rate $p\dot{\Delta}$ and the time $t_1 - t_0$ for a gas particle to travel the length of a capillary are predicted well by the theory of capillary flow of a gas developed in the written report for this project (provided it is applied under its stated assumptions).

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