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Feasibility of Detecting Aircraft Wake Vortices Using Passive Microwave Radiometers

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ABSTRACT

The feasibility of detecting the cold core of the wake vortex from the wingtips of an aircraft using a passive microwave radiometer was investigated. It was determined that there is a possibility that a cold core whose physical temperature drop is 10°C or greater and which has a diameter of 5 m or greater can be detected by a microwave radiometer. The radiometer would be a noise injection balanced Dicke radiometer operating at a center frequency of 60 GHz. It would require a noise figure of 5 dB, a predetection bandwidth of 6 GHz, and an integration time of 2 seconds resulting in a radiometric sensitivity of 0.018 K. However, three additional studies are required. The first would determine what are the fluctuations in the radiometric antenna temperature due to short term fluctuations in atmospheric pressure, temperature and relative humidity. Second, what is the effect of the pressure and temperature drop within the cold core of the wake vortex on its opacity. The third area concerns the possibility of developing a 60 GHz radiometer with a radiometric sensitivity an order of magnitude improvement over the existing state-of-the-art.

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INTRODUCTION

The feasibility of using a passive microwave radiometer to detect the presence of a wake vortex generated by an aircraft during approach and landing at an airport was investigated. Wake vortices are a hazard to aircraft operations and therefore have an impact on terminal area productivity. NASA's Langley Research Center has undertaken a program to investigate measurement techniques for the detection of wake vortices. These include active and passive techniques in the microwave and infrared regions of the electromagnetic spectrum.

Wake vortices[1] are a pair of counter-rotating vortices generated at an aircraft's wing tips. These vortices are approximately circular in shape with a core whose diameter varies from 0.5 to 5 meters. These vortices extend behind the aircraft, since they travel at approximately one-half the velocity of the aircraft. The length of time they persist and the distance they exist behind the generating aircraft is variable and is a function of many factors. Data has shown that they can exist up to a distance of 6 kilometers behind the generating aircraft.

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There is a pressure drop within the core of the wake vortex which results in a drop of the physical temperature somewhere between 1° C and 10° C. The measurement of the presence of this cold core for a short period of time by a remote sensing temperature measuring instrument is one of the candidates for detecting the presence of a wake vortex.

RADIOMETRIC MEASUREMENTS OF PHYSICAL TEMPERATURE

A radiometer is an instrument that measures the power level of the electromagnetic radiation received. All matter, that is in thermal equilibrium at a physical temperature above absolute zero, radiates electromagnetic energy at all frequencies. If the material is a blackbody, the amount of electromagnetic energy radiated can be determined from Planck's Radiation Law [2]

$$B_f = \frac{2hf^3}{c^2}\left(\frac{1}{e^{\frac{hf}{kT}} - 1}\right) \qquad \frac{W}{m^2\,sr\,Hz} \qquad (1)$$

where:

   B_f = Blackbody spectral brightness, $Wm^{-2}sr^{-1}Hz^{-1}$

   h = Planck's constant = 6.63×10^{-34} joule-sec

   f = frequency, Hz.

   k = Boltzmann's constant = 1.38×10^{-23} joule K^{-1}

   T = absolute temperature, K

   c = velocity of light = 3×10^{8} ms^{-1}

In the microwave frequency region where $hf/kt \ll 1$, Planck's Radiation Law can be approximated by the Rayleigh-Jeans Law[2]

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$$B_f = \frac{2kT}{\lambda^2} \qquad \frac{W}{m^2\,sr\,Hz} \qquad\qquad (2)$$

where:

λ = wavelength = c/f , m

The spectral brightness can be related to the total power received by the radiometer's receiving antenna by[2]

$$P = \frac{1}{2} A_r \int_f^{f+\Delta f} \iint_{4\pi} B_f(\theta,\phi)\, F_n(\theta,\phi)\, d\Omega\, df \qquad W \qquad\qquad (3)$$

where:

A_r = effective receiving area of the receiving antenna, m^2

$F_n(\theta,\phi)$ = normalized radiation pattern of the receiving

antenna

If the blackbody material that is radiating is at a constant uniform temperature and completely fills the receiving antenna beam, assuming the Rayleigh-Jeans approximation is valid, the following substitutions can be made to determine the power.

$$P_{bb} = kT\Delta f \frac{A_r}{\lambda^2} \iint_{4\pi} F_n(\theta,\phi)\, d\Omega \qquad W \qquad\qquad (4)$$

The integral in the above equation is the solid angle of the receiving antenna, or[2]:

$$\iint_{4\pi} F_n(\theta,\phi)\, d\Omega = \Omega_p \qquad sr \qquad\qquad (5)$$

The solid angle is related to the effective receiving area by[2]:

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$$\Omega_p = \frac{\lambda^2}{A_r} \qquad \text{sr} \qquad\qquad (6)$$

Substituting (5) and (6) into (4) yields the power received in the bandwidth of Δf by a radiometer from a blackbody at a physical temperature T

$$P_{bb} = kT\Delta f \qquad W \qquad\qquad (7)$$

Since most materials are grey bodies and not blackbodies, they emit less radiation than a blackbody and do not absorb all the electromagnetic energy incident upon them.

In the microwave region, we define the **brightness** B_{bb} of a blackbody at a physical temperature T as[2]:

$$B_{bb} = \frac{2kT}{\lambda^2} \Delta f \qquad \frac{W}{m^2 sr} \qquad\qquad (8)$$

In order to account for the grey body radiation, a new term is defined, **BLACKBODY EQUIVALENT RADIOMETRIC TEMPERATURE OR BRIGHTNESS TEMPERATURE**[2].

$$B(\theta,\phi) = \frac{2k}{\lambda^2} T_B(\theta,\phi) \Delta f \qquad \frac{W}{m^2 sr} \qquad\qquad (9)$$

where $T_B(\theta,\phi)$ is the brightness temperature in Kelvin.

The **EMISSIVITY** $e(\theta,\phi)$ is defined as the ratio of the brightness temperature to the physical temperature[2]. In other words, the brightness temperature is the physical temperature that a blackbody must be at to radiate the same electromagnetic energy as the greybody radiates. Therefore the brightness temperature must always be equal to or less than the physical temperature. The emissivity is:

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$$e(\theta,\phi) = \frac{T_B(\theta,\phi)}{T} \tag{10}$$

Therefore the brightness temperature, which is what a radiometer measures, is the product of the emissivity and the physical temperature; or

$$T_B(\theta,\phi) = e(\theta,\phi)\,T \qquad K \tag{11}$$

The power received by the radiometer's receiving antenna can be determined by substituting (9) into (3)

$$P = \frac{1}{2}A_r \int\limits_f^{f+\Delta f} \iint\limits_{4\pi} \frac{2k}{\lambda^2} T_B(\theta,\phi)\, F_n(\theta,\phi)\, d\Omega\, df \qquad W \tag{12}$$

If the brightness temperature is constant over the frequency band Δf, then

$$\int\limits_f^{f+\Delta f} df = \Delta f \qquad Hz \tag{13}$$

and (12) becomes

$$P = \frac{1}{2}A_r \iint\limits_{4\pi} \frac{2k}{\lambda^2} T_B(\theta,\phi)\, \Delta f\, F_n(\theta,\phi)\, d\Omega \qquad W \tag{14}$$

where P is the power delivered by the radiometer's receiving antenna. Within the microwave region and using (7), the power delivered by the radiometer's receiving antenna when it is immersed in a blackbody at a physical temperature equal to T_A is[2]

$$P = kT_A\Delta f \qquad W \tag{15}$$

where T_A is defined as the **ANTENNA RADIOMETRIC TEMPERATURE**.

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Equating (14) and (15) yields

$$T_A = \frac{A_r}{\lambda^2} \iint_{4\pi} T_B(\theta, \phi) \; F_n(\theta, \phi) \, d\Omega \qquad K \qquad\qquad (16)$$

Substituting (5) and (6) into (16) yields

$$T_A = \frac{\displaystyle\iint_{4\pi} T_B(\theta, \phi) \; F_n(\theta, \phi) \, d\Omega}{\displaystyle\iint_{4\pi} F_n(\theta, \phi) \, d\Omega} \qquad K \qquad\qquad (17)$$

Examination of (17) points out two important facts concerning radiometric measurements. First, the radiometer measures the integrated power received over the entire 4π sterradians by the radiometer's receiving antenna. The determination of the actual brightness temperature from a specific direction θ, ϕ requires a detailed knowledge of the antenna radiation pattern $F_n(\theta, \phi)$ and a complex mathematical inversion algorithm. These algorithms, known as antenna pattern correction algorithms have been used only with limited success. Secondly, the use of the Rayleigh-Jeans approximation, to relate a radiometric antenna temperature to the actual power measured produces an error in the temperature value. This error is[3]

$$\varepsilon_T = 2.4 \times 10^{-2} f - 1.9 \times 10^{-4} \frac{f^2}{T} \qquad K \qquad\qquad (18)$$

where

    f = frequency in GHz

    T = radiometric temperature in K

Solving (18) at 60 GHz and 300 K results in an error of 1.44 K.

This is a significant error, however if only a relative measurement and not a absolute measurement is made, then this error can be neglected.

The measurement of the physical temperature of a body requires the knowledge of the its' emissivity. Equation (10) can be used to relate the physical temperature T to the radiometric temperature $T_B(\theta,\phi)$ [3].

$$T = \frac{T_B(\theta,\phi)}{e(\theta,\phi)} \quad K \tag{19}$$

Any medium, that is in thermal equilibrium, which can absorb electromagnetic radiation also emits an equal amount of electromagnetic radiation. Therefore the emissivity of the medium is equal to its' absorptivity[3].

$$e(\theta,\phi) = \alpha(\theta,\phi) \tag{20}$$

where:

$e(\theta,\phi)$ = emissivity of the medium

$\alpha(\theta,\phi)$ = absorptivity of the medium

Therefore a semi-infinite medium whose absorptivity is $\alpha(\theta,\phi)$ will radiate a brightness temperature given by

$$T_B(\theta,\phi) = \alpha(\theta,\phi) T \quad K \tag{21}$$

where T is the physical temperature of the medium.

RADIOMETRIC MEASUREMENT OF ATMOSPHERIC RADIATION

The constituents of the atmosphere emit electromagnetic

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radiation in the microwave frequency region. This is based on (21) since the atmospheric constituents absorb electromagnetic radiation. The principal constituents involved in this process include oxygen molecules, water vapor, liquid water, ice and snow. The absorption coefficient $\alpha_a(z)$ is the summation of all the contributions from the constituents, which have a contribution in the microwave region, and is given by[3]

$$\alpha_a(z) = \alpha_{O_2}(z) + \alpha_{wv}(z) + \alpha_{lw}(z) + \alpha_{ice}(z) + \alpha_{snow}(z) \qquad (22)$$

where:

$\alpha_{O_2}(z)$ = absorption coefficient due to oxygen

$\alpha_{wv}(z)$ = absorption coefficient due to water vapor

$\alpha_{lv}(z)$ = absorption coefficient due to liquid water

$\alpha_{ice}(z)$ = absorption coefficient due to ice

$\alpha_{snow}(z)$ = absorption coefficient due to snow

Two equations can be derived from the radiative transfer equation, one for the case of a radiometer on the surface of the Earth looking up through the atmosphere and the second one for the case of a radiometer on a satellite looking down through the atmosphere at the Earth's surface.

The equation for the brightness temperature $T_{B_{DN}}(\theta)$ incident upon the upward looking radiometer receiving antenna, neglecting scattering losses in the atmosphere is[3]

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$$T_{B_{DN}}(\theta) = T_{\infty} e^{-\sec\theta \int_0^{\infty} \alpha_a(z)\, dz} + \int_0^{\infty} \alpha_a(z)\, T(z)\, e^{-\sec\theta \int_0^{z} \alpha_a(z')\, dz'} \sec\theta\, dz \qquad (23)$$

where:

T_{∞} = extraterrestrial electromagnetic radiation

$\alpha_a(z)$ = absorption coefficient as a function of altitude(z)

in the atmosphere from all sources

θ = angle between the radiometer's antenna beam and nadir

$T(z)$ = physical temperature as a function of altitude(z) of

the atmosphere

The first term in the equation is the radiation from outside the Earth's atmosphere attenuated by its' propagation through the losses in the atmosphere. The second term is the electromagnetic radiation from the atmosphere. The atmosphere is divided into a large number of layers, each with a thickness of Δz. The radiation from each layer is equal to the product of the absorption coefficient of that layer $\alpha_a(z)$ and the physical temperature of that layer $T(z)$. As the electromagnetic radiation from that layer propagates toward the radiometer on the Earth's surface, it is attenuated by the absorption coefficient of the layers between it and the radiometer. The total radiation received by the radiometer is the summation of all layers, each of which is attenuated by the layers between it and the radiometer. This is the second term of (20) where dz is the limit of Δz as z approaches zero and the summation is replaced by an integration over all altitudes.

The equation for the brightness temperature $T_{B_{UP}}(\theta)$ of a

downward looking radiometer located on a satellite outside the Earth's atmosphere is[3]

$$T_{B_{Up}}(\theta) = T_{B_s}(0)\, \varepsilon^{-\sec\theta \int_0^{h_{sat}} \alpha_a(z)\,dz} + \sec\theta \int_0^{h_{sat}} \alpha_a(z)\, T(z)\, \varepsilon^{-\sec\theta \int_s^{h_{sat}} \alpha_a(z')\,dz'}\, dz \qquad (24)$$

where:

$T_{B_s}(0)$ = brightness temperature from the Earth's surface within the radiometer's antenna beam incident on the surface

h_{sat} = altitude of the satellite

The brightness temperature from the Earth's surface $T_{B_s}(0)$ is determined by[3]:

$$T_{B_s}(0) = e(\theta,\phi)\, T(0^-) + (1 - e(\theta,\phi))\, T_{B_{DW}} \qquad (25)$$

where:

$T(0^-)$ = physical temperature of the Earth's surface layer

The first term is the electromagnetic radiation emitted by the Earth's surface and the second term is the reflection of the downwelling electromagnetic radiation $T_{B_{DW}}(\theta)$ back towards the radiometer on the spacecraft. The reflection coefficient of a semi-infinite medium is equal to one minus the emissivity[3].

The first term of (24) is the electromagnetic radiation from the Earth's surface attenuated by the atmosphere and is incident on the radiometer receiving antenna on the spacecraft. The second term of (24) is the upwelling radiation from the atmosphere

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received by the radiometer on the spacecraft. It is similar to the downwelling atmospheric radiation in that the atmosphere can be considered as a large number of layers, each Δz in thickness. Each layer radiates due to the product of the absorption coefficient of that layer and its' physical temperature. This radiation is then attenuated by the layers between the radiating layer and the spacecraft. Again, as in (23), the summation is replaced by an integration.

The detection of an aircraft wake vortex by an upward looking radiometer would be a case similar to that of (23). The first term of (23) can be neglected. The extraterrestrial radiation has a value less than 3 K. This is attenuated by the entire Earth's atmosphere. At the frequencies of interest to this problem, 50 to 70 GHz, the amount of extraterrestrial radiation reaching a radiometer on the Earth's surface is insignificant.

The wake vortex radiometer would be looking at nadir, therefore the sec θ would be equal to 1. Therefore (23) reduces to

$$T_{B_{DW}} = \int_0^\infty \alpha_a(z) \, T(z) \, e^{-\int_0^z \alpha_a(z')dz'} \, dz \qquad (26)$$

ATMOSPHERIC ATTENUATION COEFFICIENT

The attenuation of an electromagnetic signal as it propagates through the Earth's atmosphere is the subject of a vast amount of

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research. The propagation constant, which accounts both for the attenuation constant and phase constant, is significant to many different systems. Also, the velocity of propagation, which is a function of the variable index of refraction of the atmosphere is a significant factor in spaceborne altimeters and position location systems like the Global Positioning System (GPS).

Dr. Hans J. Liebe of the Institute for Telecommunication Services has developed an atmospheric millimeter-wave propagation model known as MPM89 [4]. The abstract of [4] describes this model.

"A broadband model for complex refractivity is presented to predict propagation effects of loss and delay for the neutral atmosphere at frequencies up to 1000 GHz. Contributions from dry air, water vapor, suspended water droplets (haze, fog, cloud), and rain are addressed. For clear air, the local line base (44 O_2 plus 30 H_2O lines) is complemented by an empirical water-vapor continuum. Input variables are barometric pressure, temperature, relative humidity, suspended water droplet concentration, and rainfall rate."

Dr. Liebe has continued his research in this area and has produced a revision to the model known as MPM92 [5]. Figure 1 presents the attenuation and delay for moist air plus fog at sea level for a physical temperature of 15°C. This data was computed from MPM92 over a frequency range from 1 to 350 GHz. A radiometer which would be used to detect the physical temperature drop within a wake vortex would operate somewhere within the frequency range of the oxygen absorption lines near 60 GHz. Figure 2 shows the

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attenuation, both from measurements and predicted from MPM92 for the frequency range from 52 to 68 GHz. This data is for dry air at 6°C as a function of altitude from 0 km to 18 km in 3 km steps. Both Figs 1 and 2 are from ref [5]. Dr. R.W. McMillan of Georgia Institute of Technology computed, using MPM92, the attenuation coefficient for moist air and fog. This is shown in Fig. 3 for three levels of humidity and three levels of non-precipating fog. This figure is from a presentation by Dr. McMillan to the Antenna and Microwave Research Branch, NASA-Langley.

Any attenuation coefficient between 1 dB/km and 15 dB/km could be used in the design of a wake vortex detecting radiometer. The desired value would be achieved by the proper selection of an operating frequency as illustrated in Fig. 3.

RADIOMETRIC SENSITIVITY

The electromagnetic radiation emitted by a medium at a physical temperature based on Planck's Law is a gaussian random noise signal with zero mean. When this noise signal is received by a radiometer, it is added to the internally generated noise within the radiometer which is also a gaussian random noise signal with zero mean. The noise powers of these two signals add linearly within the radiometer. A square law detector within the radiometer converts the noise power level to a dc voltage level which fluctuates with the fluctuations in the noise power. When the incoming noise power, referred to as the antenna radiometric

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temperature, increases by an increment ΔT_A and the resulting dc output voltage increases by an amount equal to the RMS value of the fluctuations in the dc voltage output; this is the **radiometric sensitivity.** This is the minimum change in the antenna radiometric temperature which can be detected by a radiometer.

The radiometric sensitivity is a function of the design of the radiometer. A total power radiometer[6] has the best sensitivity if it can be calibrated fairly often. Existing total power spaceborne radiometers, such as SSM/I, are calibrated every 2 seconds. They spend 2/3 of the time in calibration and only 1/3 of the time making measurements. This is a result of mechanically scanning where they can only view the Earth's surface 1/3 of the time in each rotation of the radiometer.

The type of radiometer best suited for an application where it is fixed at one position making continuous measurements, such as a wake vortex radiometer, is a balanced Dicke radiometer using the noise injection technique[6]. This radiometer needs calibration somewhere between once a day and once a year depending upon the stability of the calibration noise diodes. The radiometric sensitivity of a noise injection radiometer is[6]

$$\Delta T_A = \frac{2(T_{ref} + T_{rad})}{\sqrt{B\tau}} \quad K \qquad (27)$$

where:

T_{ref} = reference load temperature typically 310 K

T_{rad} = effective input noise temperature of the radiometer

B = predetection bandwidth of the radiometer

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τ = integration time after square law detection

The state-of-the-art in 60 GHz low noise amplifiers(LNA) is such that a 5 dB noise figure is available. Assuming 0.5 dB losses in front of the LNA results in an effective input noise temperature of 739 K. A predetection bandwidth of 400 MHz and an integration time of 1 second results in a radiometric sensitivity of 0.1 K. This is a realistic sensitivity to use for a 60 GHz wake vortex radiometer.

RADIOMETRY OF LAYERED MEDIUMS

A model of a passive microwave radiometer detecting a wake vortex involves layered media. Therefore it is desirable to convert (26) into a form directly applicable to layered mediums.

Figure 4 will be used to establish some definitions and concepts. An electromagnetic wave is propagating through free space and is incident upon a lossy medium. The incident power P_i is partially reflected by the boundary. The reflected power is denoted by P_r. The remaining power passes through the medium and some of it is absorbed by the medium, P_a. That portion which is transmitted into free space on the other side of the medium is denoted P_t. The following must be true.

$$P_i = P_r + P_a + P_t \qquad (28)$$

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Dividing (28) by P_i

$$1 = \frac{P_r}{P_i} + \frac{P_a}{P_i} + \frac{P_r}{P_i} \qquad (29)$$

Defining reflectivity Γ as the ratio of reflected power to the incident power, absorptivity a as the ratio of absorbed power to incident power, and transmissivity Υ as the ratio of transmitted power to incident power, (29) can be written as

$$1 = \Gamma + a + \Upsilon \qquad (30)$$

Since any medium in thermal equilibrium must have an emissivity equal to its' absorptivity, then the medium will emit an electromagnetic radiation equal to the emissivity times its' physical temperature. This is illustrated in Fig. 4 as P_e.

The electromagnetic wave propagating through the medium in Fig. 4 is attenuated by the attenuation constant α of the medium. Since the attenuation constant is usually a function of distance z through the medium, a new term **opacity**[2] is defined as

$$\tau = \int_{z_1}^{z_2} \alpha(z)\,dz \qquad (31)$$

where $\alpha(z)$ is the attenuation constant as a function of distance z through the medium. The transmissivity is related to the opacity by[2]

$$\Upsilon = e^{-\tau} \qquad (32)$$

Next, assuming that there are no reflections at the boundary of the medium, $\Gamma = 0$, then

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$$a = 1 - \Upsilon = 1 - \varepsilon^{-\tau}$$

<div align="right">(33)</div>

The radiometric layered medium configuration is illustrated in figure 5. The radiometric brightness temperature T_{in} represents the power of the electromagnetic wave incident upon the medium at z_1. The power absorbed by the medium is T_a. The physical temperature of the medium is T_p. The power emitted by the medium is the emissivity, e, times the physical temperature or eT_p. At z_2 there are two outputs from the medium, the portion of T_{in} transmitted through the medium and the self emissions from the medium. Since the emissivity is equal to the absorptivity and using (32) and (33), the electromagnetic radiation from the medium at z_2 expressed as a radiometric brightness temperature is

$$T_{out} = \varepsilon^{-\tau} T_{in} + (1 - \varepsilon^{-\tau}) T_p$$

<div align="right">(34)</div>

It should be noted that as $\tau \rightarrow 0$, then $T_{out} \rightarrow T_{in}$ and as $\tau \rightarrow \infty$, then $T_{out} \rightarrow T_p$. As an example if the opacity is 40 dB, the input temperature is 0 K and the physical temperature is 300 K; the output temperature would be 299.97 K. The opacity at 60 GHz of the atmosphere looking straight up towards the zenith is greater than 100 dB. Therefore any zenith viewing radiometer, operating near 60 GHz, would measure the physical temperature of the lower portion of the atmosphere.

WAKE VORTEX RADIOMETRIC MEASUREMENT SCENARIO

A scenario for the detection of a wake vortex with a 60 GHz

microwave radiometer is illustrated in figure 6. A zenith viewing radiometer is located on the extended centerline of the runway. The beamwidth of the radiometer Ω is selected such that the beam covers the width of the runway at the altitude, h, of the glideslope located above the radiometer. A runway width of 45.7 m (150 ft) is assumed. The diameter of the cold core of the wake is assumed to be 5 m. The altitude of the wake vortex above the radiometer varies from 50 m to 200 m depending upon the distance from the runway to the location of the radiometer. The radiometer is assumed to have a perfect antenna, one in which the normalized radiation pattern $F_n(\theta,\phi) = 1$ over the solid angle Ω and $F_n(\theta,\phi) = 0$ elsewhere. As can be seen in figure 6, the 5 meter diameter wake vortex occupies 13.93% of the radiometer beam.

Several assumptions have been made in this measurement scenario, most of which are optimistic so as to enhance the probability of detecting the wake vortex. First, it is assumed that the lower 200 m of the atmosphere is homogeneous, that is the physical temperature, pressure, and humidity is constant over the altitude range from the surface to 205 m. This says that the attenuation coefficient and physical temperature is constant over this altitude range. Second, the downwelling brightness temperature, $T_{B_{DN}}(h + t)$, is equal to the ambient physical temperature of the atmosphere above the wake vortex. Third, the ambient atmosphere short term fluctuations (period of one second or less) in the physical temperature, barometric pressure and relative

humidity is such that the fluctuations in the downwelling radiometric temperature is less than the sensitivity of the radiometer and therefore not detectable. Long term fluctuations, periods greater than the time duration of the wake vortex, will not have an effect since the radiometer is making a differential measurement over the duration time of the wake vortex.

The output of the radiometer will be a summation of the brightness received in an incremental beam, $\Delta\Omega$, summed over the total antenna beam. As shown in figure 6, only 13.93% of the incremental beams will see the wake vortex. The remaining 86.07% of the beam will receive a brightness equal to the physical temperature of the atmosphere.

Figure 7 is the radiometric model of the wake vortex layered medium for the 13.93% of the antenna beam which sees the wake vortex. The opacity of the wake vortex is τ_1. The opacity of the atmosphere between the wake vortex and the radiometer on the surface is τ_2. Since the attenuation coefficient is assumed to be the same in both regions, then

$$\tau_2 = \frac{h}{t}\tau_1 \tag{35}$$

The radiometric brightness temperature, T_{B_0}, is equal to the ambient physical temperature of the atmosphere surrounding the wake vortex, T_{amb}. The physical temperature of the cold core of the wake vortex is lower than the ambient physical temperature of the atmosphere by an amount ΔT. The radiometric brightness temperature from the cold core of the wake vortex can be determined from (34)

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$$T_{B_1} = \varepsilon^{-\tau_1} T_{amb} + (1 - \varepsilon^{-\tau_1})(T_{amb} - \Delta T) \qquad (36)$$

The electromagnetic radiation is propagated along with the radiation from the atmosphere between the wake vortex and the surface to the wake vortex radiometer located on the surface. The radiometric brightness temperature arriving at the radiometer in the incremental beam $\Delta\Omega$ can also be determined by (34)

$$T_{B_2} = \varepsilon^{-\tau_2} T_{B_1} + (1 - \varepsilon^{-\tau_2}) T_{amb} \qquad (37)$$

Since the wake vortex only partially fills the radiometer antenna beam, define a factor f_w, which is the fraction of the total beam that the wake vortex occupies. Therefore the antenna radiometric temperature, assuming a perfect antenna, where the cold core of the wake vortex is partially in the beam is

$$T_{A_w} = (1 - f_w) T_{B_0} + f_w T_{B_2} = (1 - f_w) T_{amb} + f_w T_{B_2} \qquad (38)$$

where T_{A_w} is the antenna radiometric temperature when the cold core of the wake vortex is partially in the beam. Let T_{A_0} represent the antenna radiometric temperature when the wake is not in the beam. Therefore

$$T_{A_0} = T_{B_0} = T_{amb} \qquad (39)$$

Equation (36) can be rearranged as

$$T_{B_1} = (T_{amb} - \Delta T) + \Delta T e^{-\tau_1} \tag{40}$$

Substituting (40) into (37) and rearranging yields

$$T_{B_2} = T_{amb} - \Delta T(e^{-\tau_2} - e^{-\tau_1}e^{-\tau_2}) \tag{41}$$

Substituting (41) into (38) and rearranging yields

$$T_{A_v} = T_{amb} - f_w \Delta T(e^{-\tau_2} - e^{-\tau_1}e^{-\tau_2}) \tag{42}$$

The difference between the antenna radiometric temperature when the wake vortex is not in the antenna beam T_{A_0} and when the wake vortex is in the antenna beam T_{A_v} is

$$T_{A_0} - T_{A_v} = T_{amb} - \left[T_{amb} - f_w \Delta T(e^{-\tau_2} - e^{-\tau_1}e^{-\tau_2}) \right] \tag{43}$$

or

$$T_{A_0} - T_{A_v} = f_w \Delta T \left(\frac{1}{e^{\tau_2}} - \frac{1}{e^{\tau_1}e^{\tau_2}} \right) = f_w \Delta T \left[\frac{(e^{\tau_1} - 1)}{e^{(\tau_1 + \tau_2)}} \right] \tag{44}$$

Defining the ratio of the change in the physical temperature of the cold core of the wake vortex to the change in the antenna radiometric temperature in the presence of the wake vortex as K or

$$K = \frac{\Delta T}{T_{A_0} - T_{A_v}} = \frac{1}{f_w} \left[\frac{e^{(\tau_1 + \tau_2)}}{(e^{\tau_1} - 1)} \right] \tag{45}$$

The value of K represents how much greater the change in the physical temperature of the cold core of the wake vortex must be

over the radiometer's sensitivity. As an example, if the expected change in the physical temperature of the wake vortex is 10°C and the radiometer's sensitivity is 0.1 K, then K has a value of 100. The ideal value of K is 1. When (45) is solved, K must be equal to or less than 100 for the example cited.

Let n be the ratio of the altitude, h, to the diameter of the wake vortex, t. Therefore (35) becomes

$$\tau_2 = \left(\frac{h}{t}\right)\tau_1 = n\tau_1 \tag{46}$$

Substituting (46) into (45) yields

$$K = \frac{1}{f_w}\left[\frac{e^{(n+1)\tau_1}}{(e^{\tau_1} - 1)}\right] \tag{47}$$

Examination of (47) will show that K becomes large and approaches infinity when τ_1 either approaches zero or infinity. Therefore at some value of τ_1, K will be a minimum. K must be equal to or less than 100 for the detection of a 10°C change in the wake vortex with a 0.1 K radiometer sensitivity. Equation (47) can be simplified as τ_1 becomes large.

$$K = \frac{e^{n\tau_1}}{f_w} \tag{48}$$

Since n has values between 10 and 40 for this scenario and f_w is 0.1393, K approaches infinity very rapidly as τ_1 becomes large. Equation (47) can be simplified through the use of a series expansion of the exponential as τ_1 becomes small.

$$K = \frac{1}{\tau_1} + (n+1) \qquad\qquad \textbf{(49)}$$

Again as τ_1 approaches zero, K approaches infinity.

This result is logical. The larger the opacity, the greater the reduction in electromagnetic radiation from the cold core of the wake vortex. However, the higher the opacity, the more attenuation exists between the wake vortex and radiometer on the surface. Therefore there is an optimum value of opacity which minimizes the value of K.

The range of available opacities range from 0.1 dB/km to 15 dB/km as illustrated in Fig. 2 and 3. The diameter of the wake vortex was taken as 5 meters and the altitude range of 50 to 200 meters was chosen for this study. This gives a range of n from 10 to 40. The value of f_w is 0.1393. K must have a value of 100 or less for the detection of the wake vortex in this scenario.

A computer program for (47) was written which determined the minimum value of K as a function of n with opacity of the wake vortex τ_1 as the variable. The results of this analysis are presented in Table 1. The variable n, which is the ratio of the altitude of the wake vortex above the radiometer to the diameter of the wake vortex, was varied from 2 to 200. The corresponding value of altitude for a 5 m wake vortex diameter is given in second column. The minimum value of K for that value of n is listed in the third column. The required drop in the physical temperature of the cold core of the wake vortex if the sensitivity of the radiometer is 0.1 K is given in the fourth column. The total

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opacity, τ_1, within the 5 m diameter core of the wake vortex required to achieve the minimum value of K by (47) is given in the fifth column. The minimum value was determined by iterative solutions of (47) as a function of opacity until the minimum value of K was obtained. The atmospheric attenuation coefficient required to produce the opacity required within the core of the wake vortex is tabulated in the sixth column.

The required drop in the physical temperature of the cold core of the wake vortex as a function of n and altitude h is shown in figure 8. Examination of figure 8 shows that for the predicted physical temperature of 10°C in a 5 m diameter cold core, the maximum height of the wake vortex above the radiometer on the surface is 24 m. Taking an absurd extreme case, if the cold core dropped in temperature from 30°C to absolute zero (-273°C), the maximum altitude is approximately 700 m.

The required attenuation coefficient as a function of n and altitude h is shown in figure 9. Examination of figure 9 shows that the required attenuation coefficient exceeds the maximum available attenuation coefficient (15 dB/km) for all altitudes below approximately 300 m. The required attenuation coefficient at 24 m, the maximum altitude where a ΔT of no more than 10°C can be detected, is approximately 160 dB/km. At an altitude of 300 m (n = 60), the required drop in the physical temperature of the cold core is greater than 118°C for a 5 m diameter cold core and a radiometric sensitivity of 0.1 K.

From (47), there is an optimum value of opacity τ_1 for a given

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value of n. However, the required drop in physical temperature ΔT decreases with increasing opacity. Therefore the best condition to detect a wake vortex is to operate at the frequency which results in maximum opacity. The maximum available opacity is at 60 GHz and has a value of 0.0165. Substituting $\tau_1 = 0.0165$, $k_w = 0.1393$, radiometric sensitivity of 0.1 K, and $t = 5$ m; (47) becomes

$$\Delta T = 43.15\,e^{(3.3 \times 10^{-3})h} \tag{50}$$

where ΔT is the required drop in the physical temperature of the cold core of the wake vortex. The required ΔT is proportional to altitude. Therefore it is desirable to keep the wake vortex as close as possible to the upward viewing radiometer. Figure 10 is a plot of (50). It can be seen that between an altitude of 10 to 100 m, the required ΔT changes only from 44.60°C to 60.02°C. For this case, the required ΔT approaches 43.15°C, as the altitude approaches zero. If the ΔT in the cold core was approximately 51°C, then the radiometer could detect the presence of the wake vortex.

The next logical question is how good a radiometric sensitivity is required to detect a ΔT of 10°C in a 5 m diameter cold core of a wake vortex. Solving (47) for a opacity τ_1 of 0.0165, a k_w of 0.1393, a t of 5 m, and a ΔT of 10°C yields

$$\Delta T_{rad} = (2.3175 \times 10^{-2})\,e^{-(3.3 \times 10^{-3})h} \tag{51}$$

Figure 11 is a plot of (51). Examination of figure 11 shows that if a radiometric sensitivity of 0.01 K could be achieved, a cold core of the wake vortex could be detected up to an altitude of

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250 m. A radiometric sensitivity of 0.02 K would allow detection up to an altitude of 50 m.

The radiometric sensitivity can be improved by either increasing the predetection bandwidth or the integration time or both(see equation 27). The proposed 60 GHz radiometer could have the predetection bandwidth increased from 400 MHz to 6 GHz. This is possible since the attenuation coefficient of the oxygen line from 57 to 63 GHz is greater than 10 dB/km. If the integration time can be increased to 2 seconds, then the 60 GHz radiometer would have a sensitivity of 0.018 K. This sensitivity would allow detection of a 10°C cold core wake vortex up to an altitude of 75 meters.

There are potentially two problems which need further study. First, the short term fluctuations in the pressure, temperature and humidity in the atmosphere between the radiometer and the wake vortex might create fluctuations in the opacity of the atmosphere which would result in fluctuations greater than 0.01 K in the antenna radiometric temperature without the presence of a wake vortex. A more detailed theoretical study and a possible field experiments would be required to answer this question.

The second problem is the question of the opacity within the wake vortex. The attenuation coefficient of the oxygen molecule is a function of both the pressure and the temperature of the atmosphere. The attenuation coefficient is directly proportional to the pressure and inversely proportional to the cube of the physical temperature. Therefore the drop of the physical

temperature of the cold core due to a pressure drop should result in an increase in the opacity of the core without increasing the opacity of the atmosphere outside the core. This would enhance the capability of the radiometer to detect the presence of the wake vortex. A study using MPM92, which has been requested and should be available at NASA-Langley shortly, should be undertaken to answer this question.

Another factor which would enhance the ability to detect the wake vortex is to use antenna beam shaping to improve the beam fill factor, f_w, from 0.1393 to 0.5. This would result in a 3.6 times improvement in the detectability of the wake vortex. As an example, a elongonated rectangular shaped beam pattern in which both wingtip wake vortices could exist would improve the detectability.

SUMMARY AND CONCLUSIONS

The feasibility of detecting the cold core of an aircraft's wake vortex using a passive microwave radiometer was the objective of this investigation. The radiometer was an upward looking instrument located on the Earth's surface which viewed the aircraft as it approached a runway for landing. Each wingtip of the aircraft produces a vortex in which there is a drop in physical temperature due to a drop in the atmospheric pressure. Due to the opacity of the atmosphere near the oxygen absorption line, the drop in physical temperature will cause a corresponding drop in the electromagnetic radiation from the cold core of the wake vortex. This electromagnetic radiation, although attenuated by the

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atmosphere between the wake vortex and the radiometer, will cause a drop in the antenna radiometric temperature received at the radiometer.

A model of the measurement scenario was derived. A factor K was defined as the ratio of the drop in physical temperature within the cold core of the wake vortex to the drop in radiometric antenna temperature at the input of the radiometer. The factor K is a function of the opacity of the atmosphere, the altitude of the wake vortex above the radiometer, and the diameter of the wake vortex. The ideal value of K is unity, and it is desirable to have K be as small as possible. K must lie between 1 and infinity. The factor K approaches infinity as opacity approaches either zero or infinity. Therefore there is a value of opacity for which K will be a minimum.

A computer analysis to determine the minimum value of K as a function of altitude and opacity was performed. The results of this analysis showed that maximum available opacity is needed to minimize K. This occurs at the peak of the oxygen absorption line at 60 GHz. Next two analyses were performed, one to determine the required drop in physical temperature in the cold core if the radiometer sensitivity was 0.1 K and second to determine the required radiometric sensitivity if the drop in the physical temperature of the cold core was 10°C.

Based on the analyses performed, it was determined that a radiometer with a radiometric sensitivity of 0.02 K could detect up to an altitude of 50 m a cold core of a wake vortex in which the

drop in physical temperature was 10°C and the diameter was 5 m. It was also determined that a noise injection balanced Dicke radiometer with a noise figure of 5 dB, a predetection bandwidth of 6 GHz, and an integration time of 2 seconds could achieve a radiometric sensitivity of 0.018 K.

Therefore it was determined that there is a possibility that a 60 GHz passive microwave radiometer could detect the cold core of a wake vortex. However, there are potentially three problems which need further study.

First, the short term fluctuations in the pressure, temperature and humidity in the atmosphere between the radiometer and the wake vortex might create fluctuations in the opacity of the atmosphere which would result in fluctuations in the antenna radiometric temperature without the wake vortex greater than 0.01 K. A more detailed theoretical study and a possible field experiments would be required to answer this question.

The second problem is the question of the opacity within the wake vortex. The attenuation coefficient of the oxygen molecule is a function of both the pressure and the temperature of the atmosphere. The attenuation coefficient is directly proportional to the pressure and inversely proportional to the cube of the physical temperature. Therefore the drop of the physical temperature of the cold core due to a pressure drop should result in an increase in the opacity of the core without increasing the opacity of the atmosphere outside the core. This would enhance the capability of the radiometer to detect the presence of the wake

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vortex. A study using MPM92 should be undertaken to answer this question.

The third area concerns the possibility of developing a 60 GHz radiometer with a radiometric sensitivity an order of magnitude improvement over the existing state-of-the-art.

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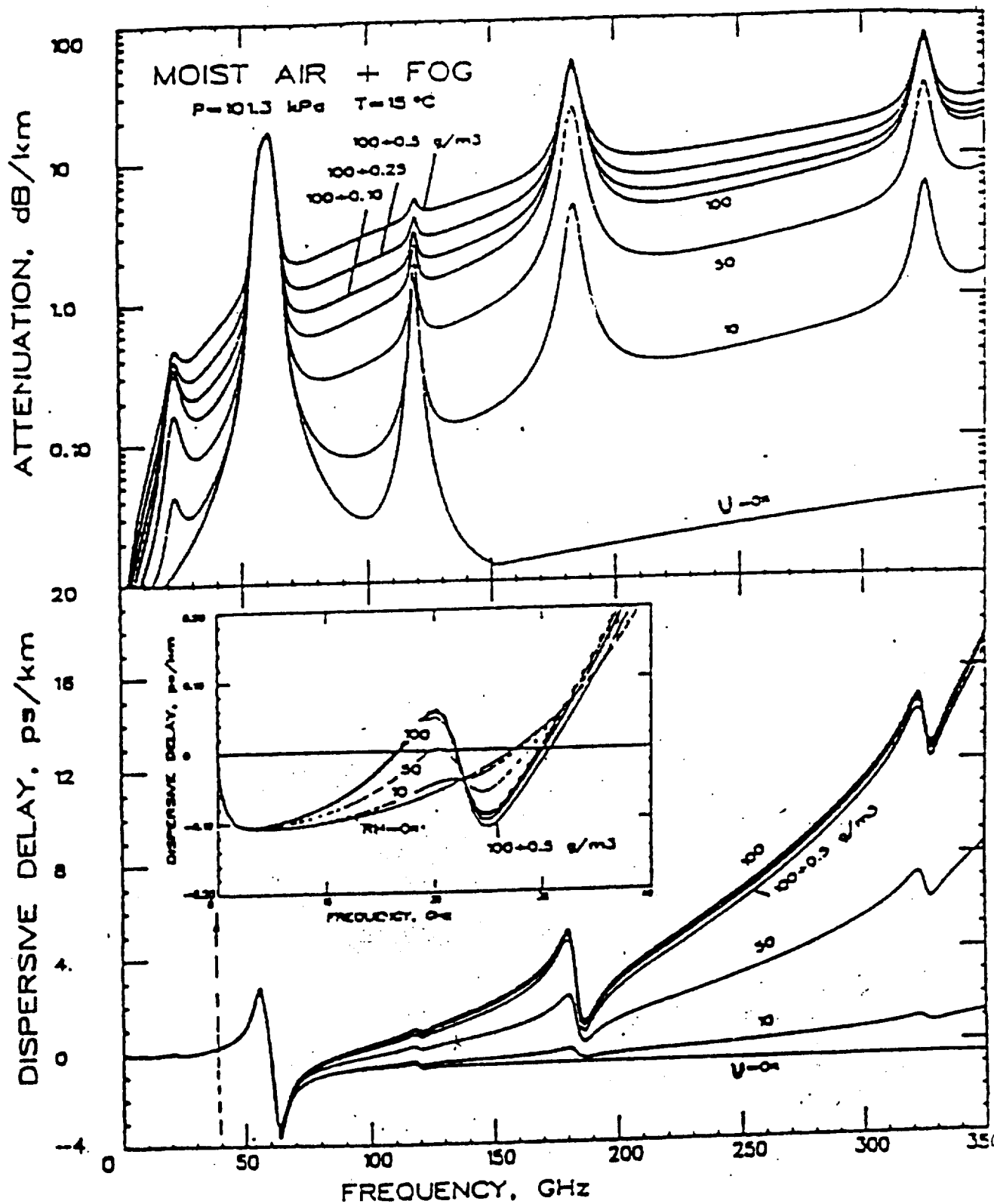
Fig. 1. MPM-predicted attenuation and delay rates from 1 to 350 GHz due to moist air and fog at sea level conditions.
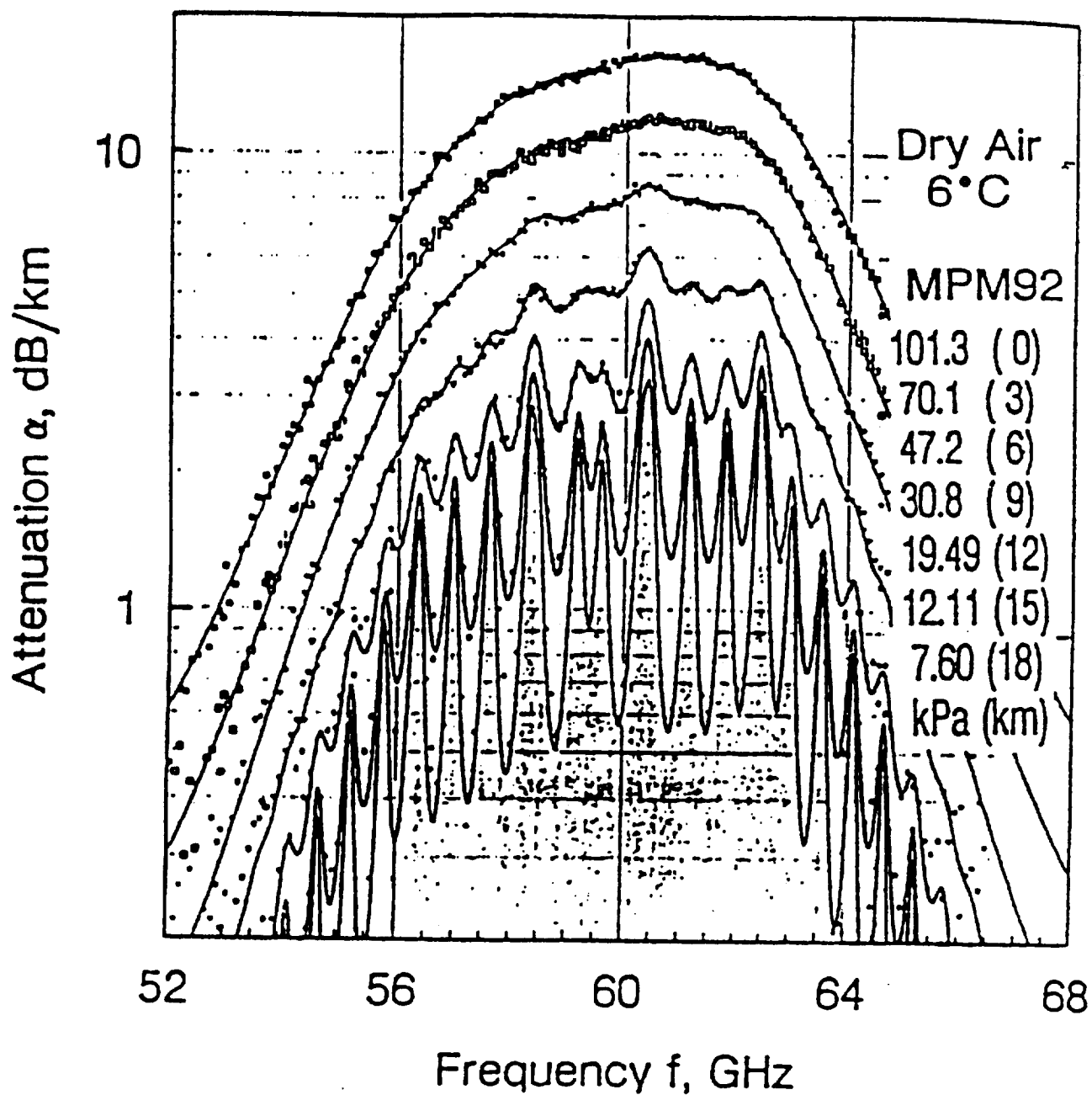
Fig. 2. Attenuation measurements of dry air between 52 and 68 GHz at 6°C for seven pressures (101.3 - 7.6 kPa). Curves are computed from MPM92
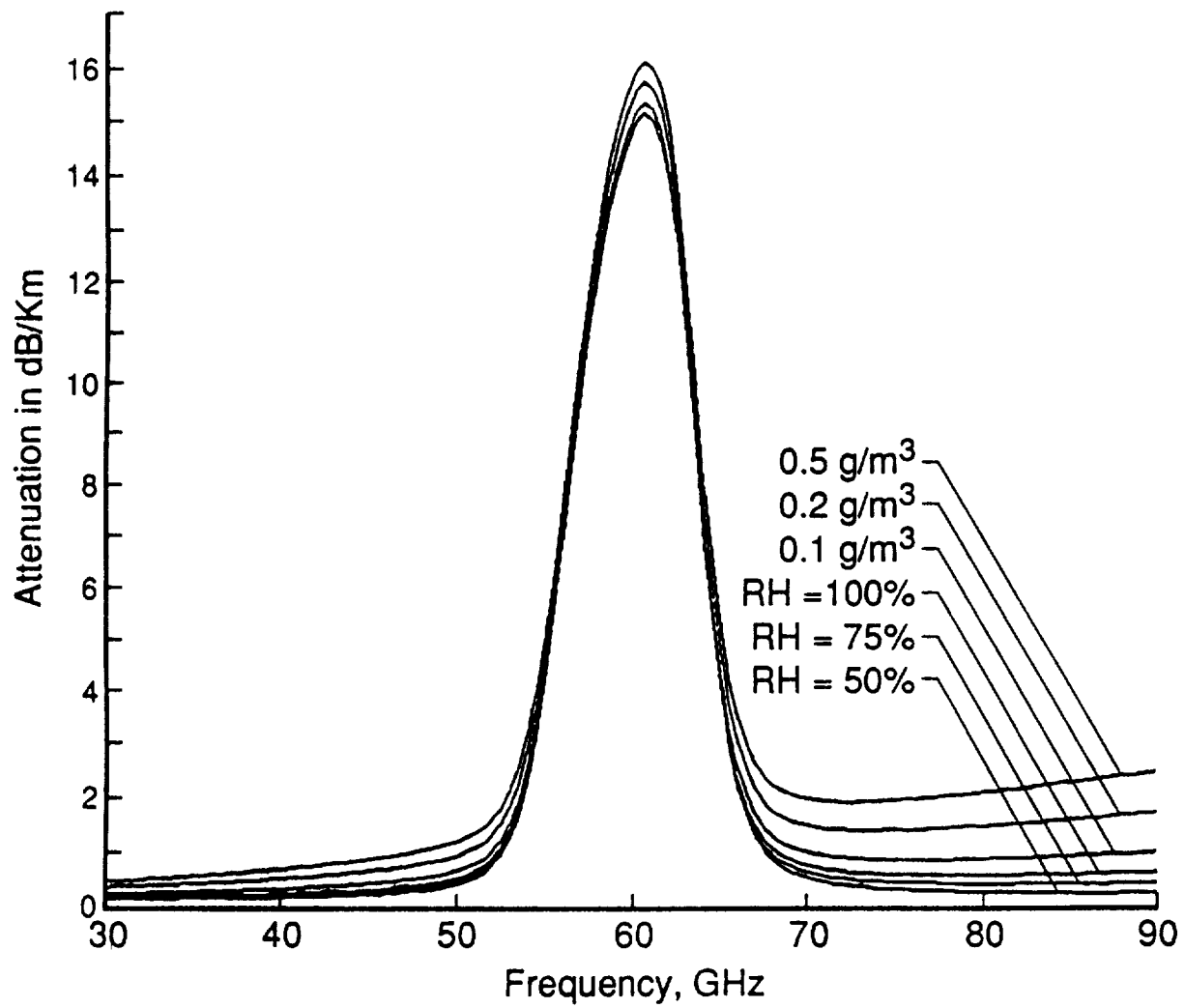
Figure 3. Atmospheric attenuation at the 60GHz oxygen line as a
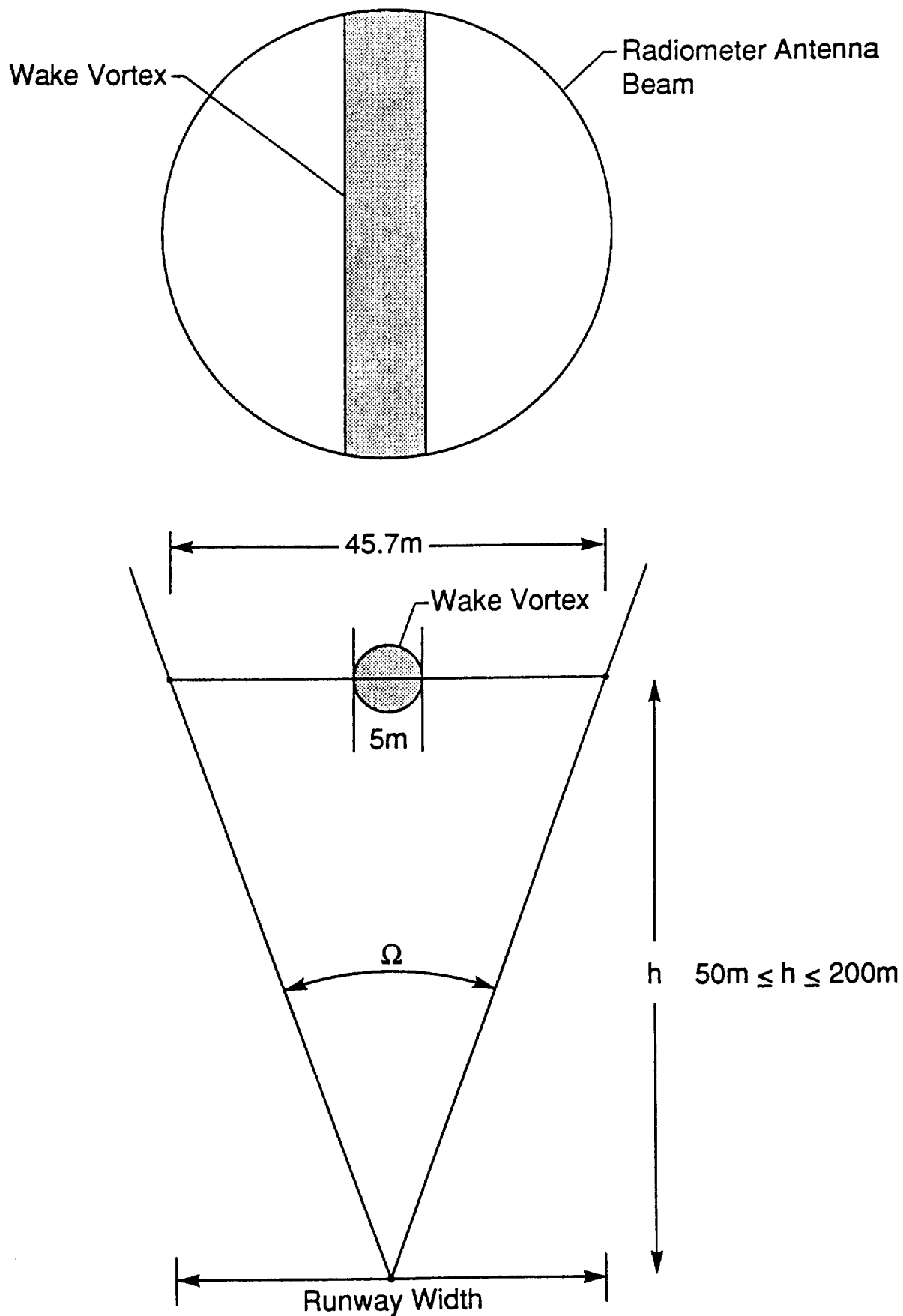function of relative humidity and fog.

Figure 4. Layered medium.

T_{IN}

$\mathcal{E}^{-\tau}T_{IN}$

T_{out}

T_a

$(1-\mathcal{E}^{-\tau})T_p$

eT_P

z_1 z_2

+z
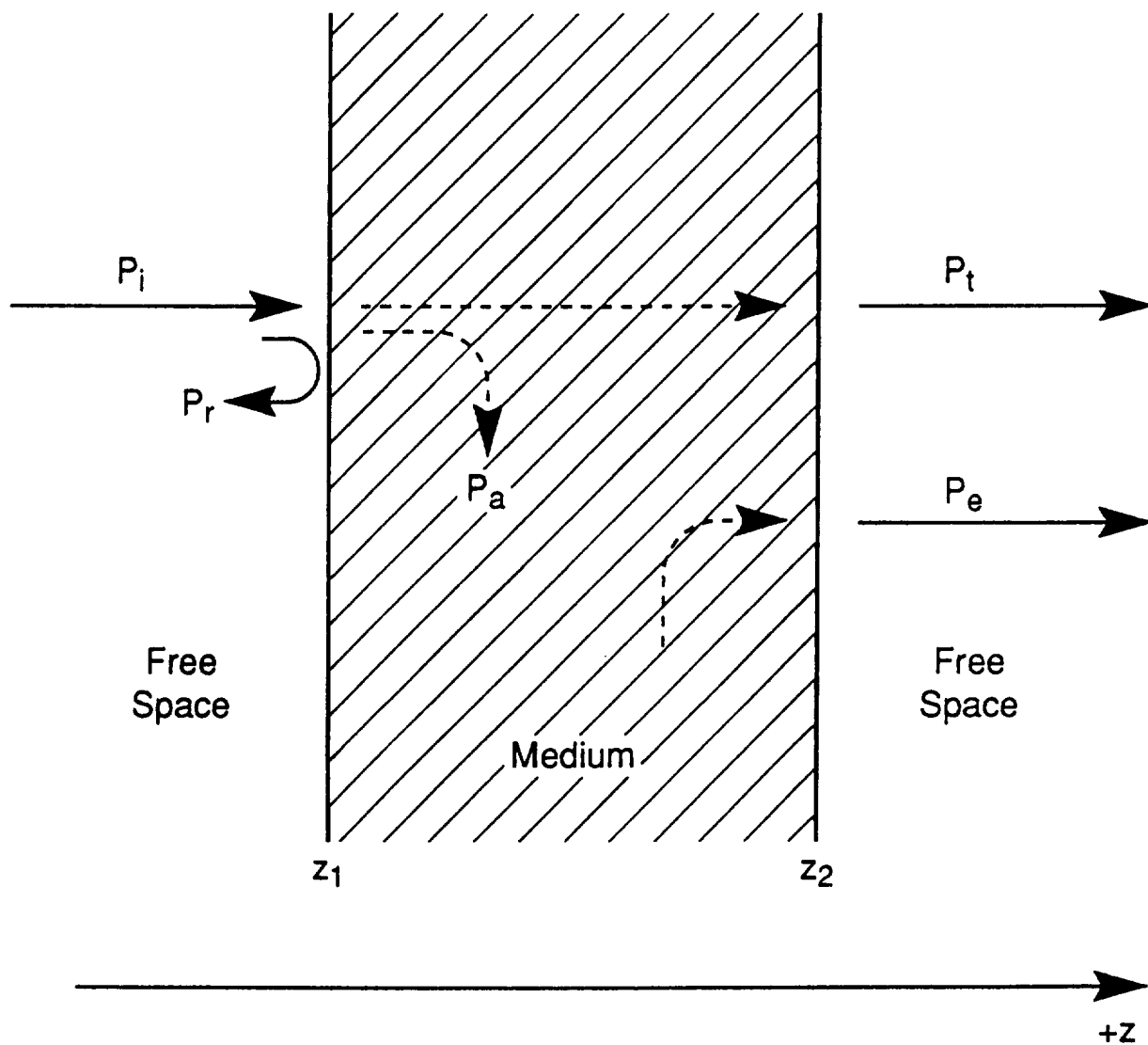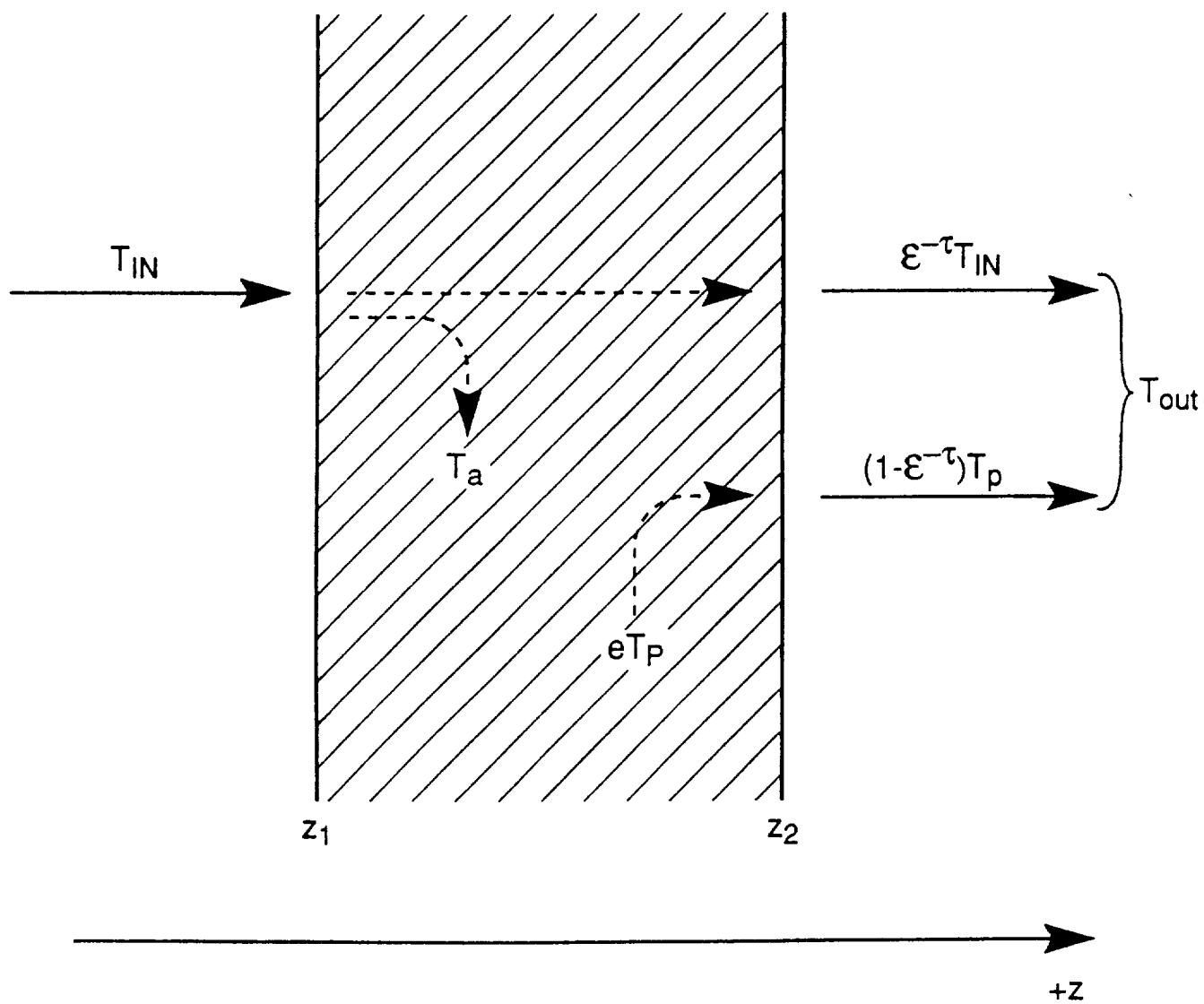
Figure 5. Radiometer layered medium.

Figure 6. Wake vortex measurement scenario.

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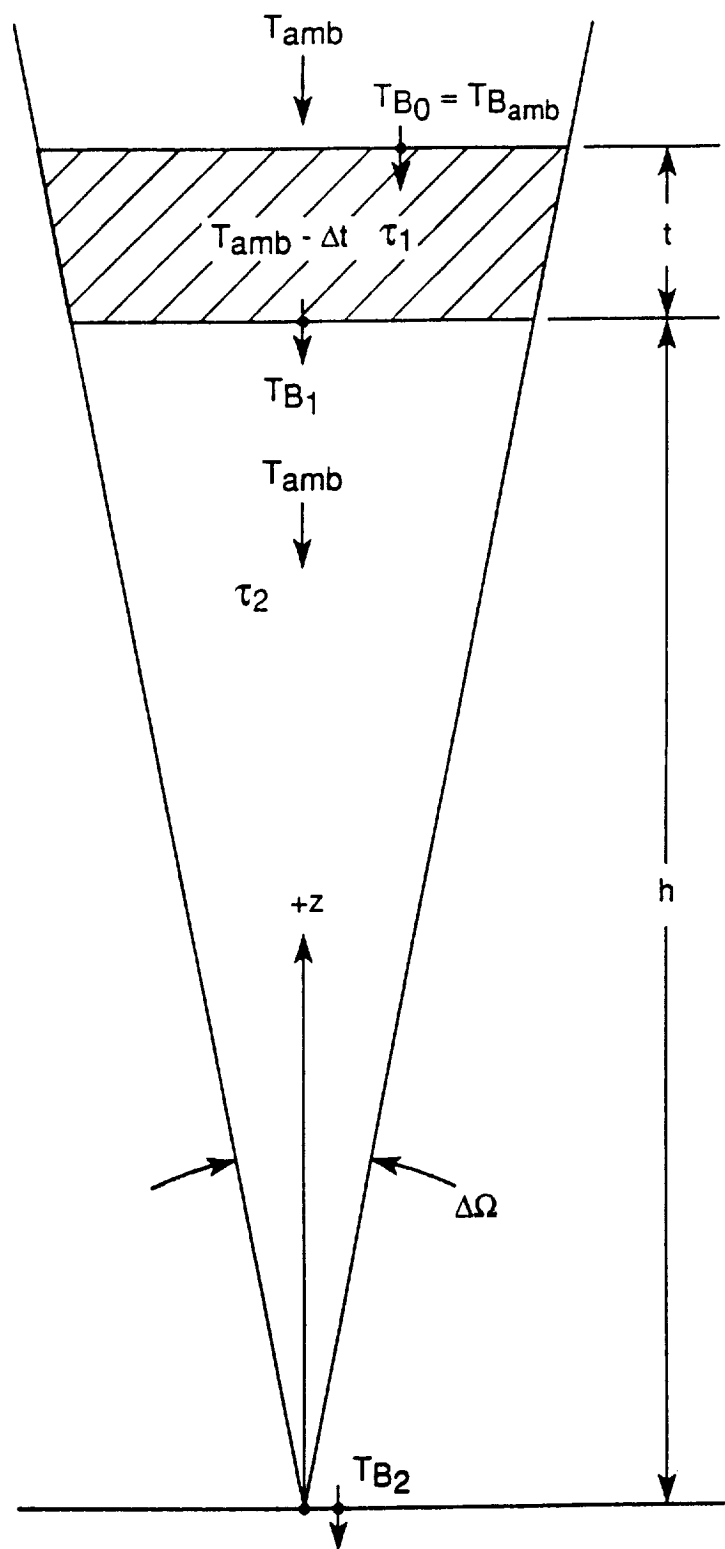
Figure 7. Radiometer wake vortex layered medium.
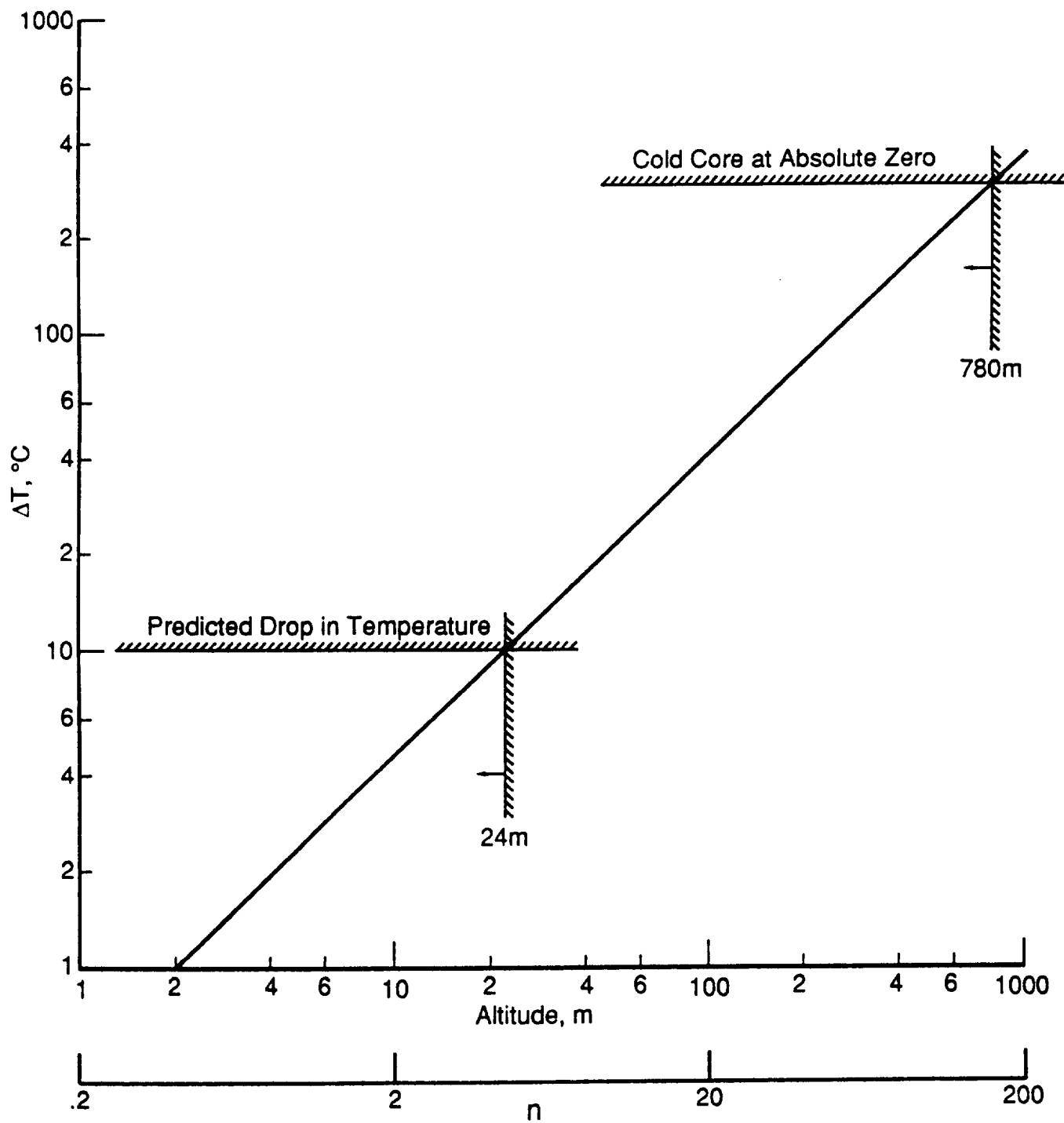
Figure 8. Required drop in physical temperature as a function
of altitude of wake vortex.

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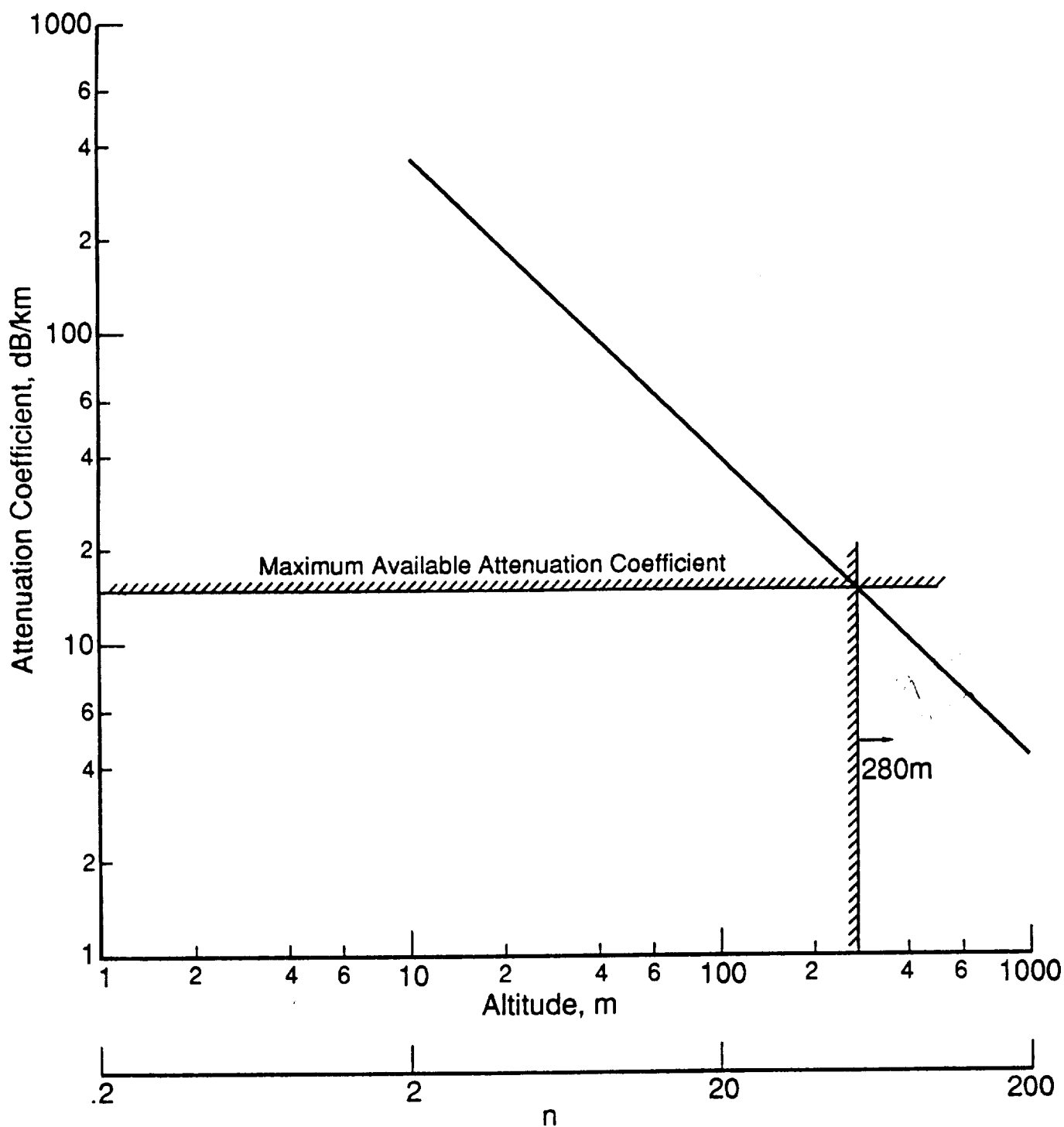
Figure 9. Required attenuation coefficient as a function
of altitude of wake vortex.

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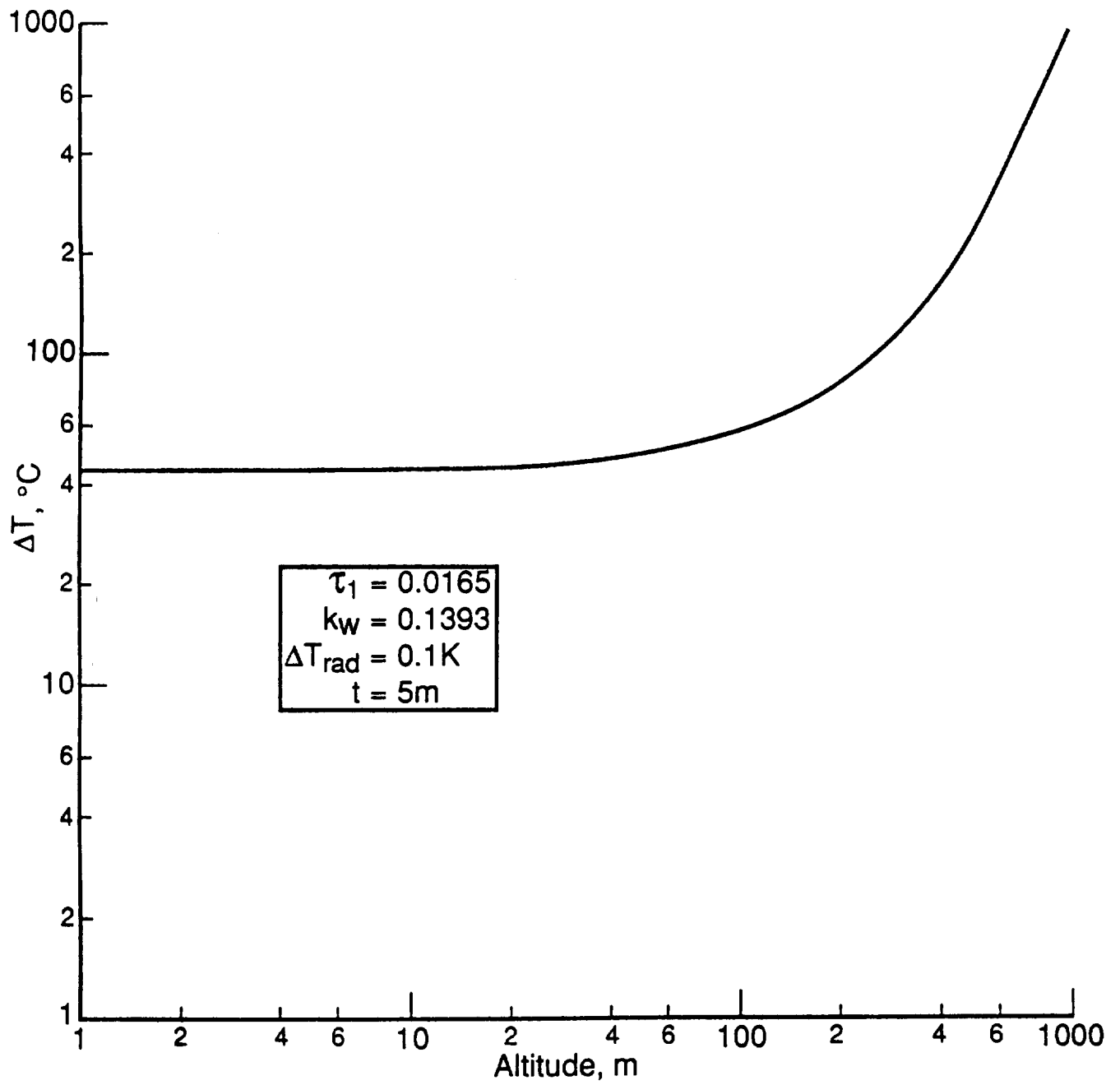
Figure 10. Required drop in physical temperature of cold core of wake vortex.
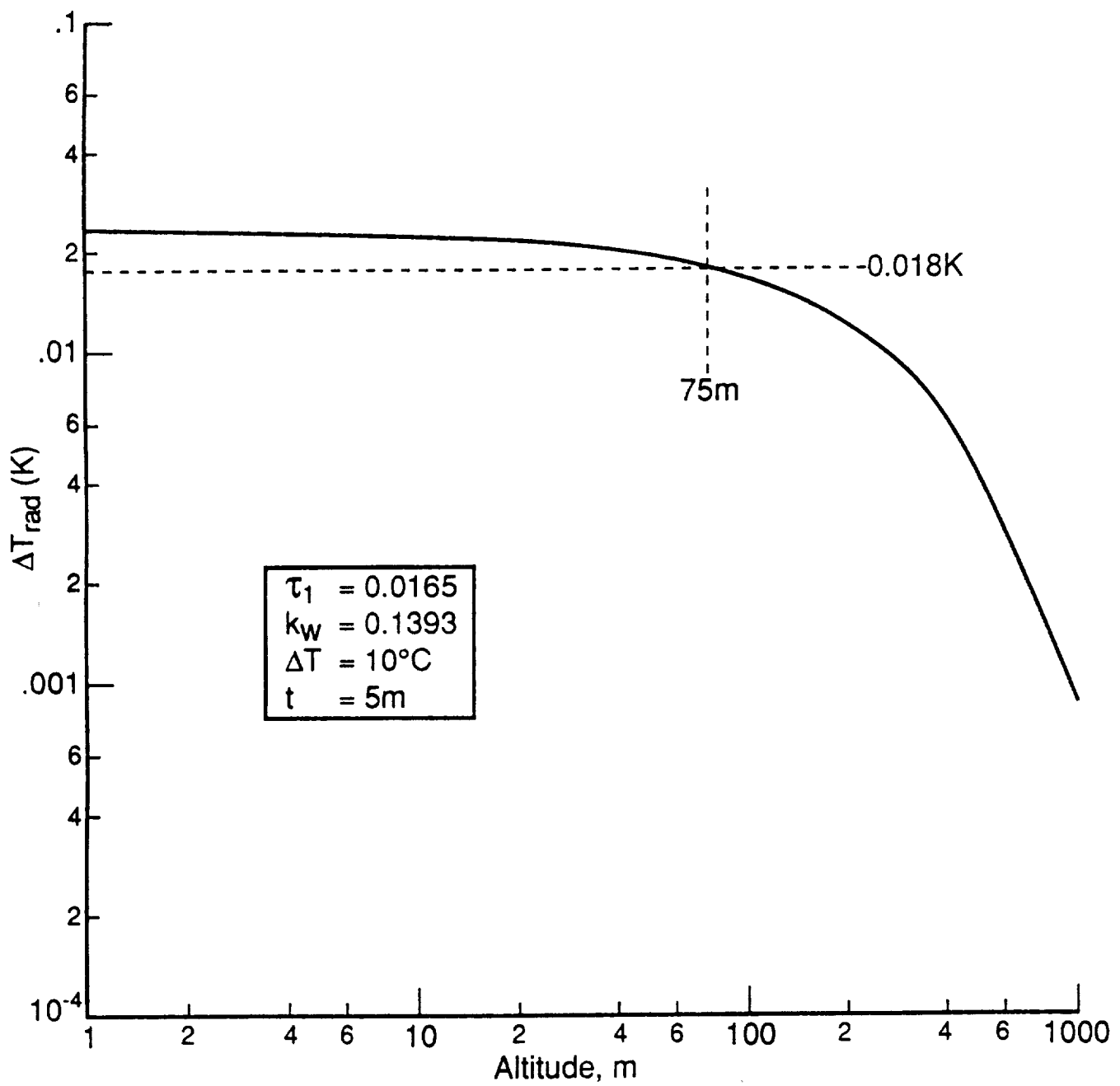
Figure 11. Required radiometer radiometric sensitivity as a function of altitude.

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Table I. Computer Analysis of Equation (47)

n	h (m)	K_{min}	ΔT (°C)	τ_1	α_a (dB/km)
200	1000	3912.6	391.3	.0050	4.34
100	500	1961.2	196.1	.0100	8.69
60	300	1180.6	118.1	.0165	14.33
50	250	985.5	98.6	.0200	17.37
40	200	790.3	79.0	.0250	21.72
30	150	595.2	59.5	.033	28.66
20	100	400.0	40.0	.049	42.56
10	50	204.8	20.5	.095	85.52
5	25	107.2	10.7	.182	158.09
4	20	87.6	8.8	.223	193.70
2	10	48.5	4.9	.405	351.78

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13. ABSTRACT *(Maximum 200 words)*

The feasibility of detecting the cold core of the wake vortex from the wingtips of an aircraft using a passive microwave radiometer was investigated. It was determined that there is a possibility that a cold core whose physical temperature drop is 10°C or greater and which has a diameter of 5 m or greater can be detected by a microwave radiometer. The radiometer would be a noise injection balanced Dicke radiometer operating at a center frequency of 60 GHz. It would require a noise figure of 5 dB, a predetection bandwidth of 6 GHz, and an integration time of 2 seconds resulting in a radiometric sensitivity of 0.018 K. However, three additional studies are required. The first would determine what are the fluctuations in the radiometric antenna temperature due to short-term fluctuations in atmospheric pressure, temperature, and relative humidity. Second, what is the effect of the pressure and temperature drop within the cold core of the wake vortex on its opacity. The third area concerns the possibility of developing a 60 GHz radiometer with a radiometric sensitivity an order of magnitude improvement over the existing state of the art.

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