Material Data Representation of Hysteresis Loops for Hastelloy X Using Artificial Neural Networks

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SUMMARY

The artificial neural network (ANN) model proposed by Rumelhart, Hinton, and Williams is applied to develop a functional approximation of material data in the form of hysteresis loops from a nickel-base superalloy, Hastelloy X. Several different ANN configurations are used to model hysteresis loops at different cycles for this alloy. The ANN models were successful in reproducing the hysteresis loops used for its training. However, because of sharp bends at the two ends of hysteresis loops, a drift occurs at the corners of the loops where loading changes to unloading and vice versa (the sharp bends occurred when the stress-strain curves were reproduced by adding stress increments to the preceding values of the stresses). Therefore, it is possible only to reproduce half of the loading path. The generalization capability of the network was tested by using additional data for two other hysteresis loops at different cycles. The results were in good agreement. Also, the use of ANN led to a data compression ratio of approximately 22:1.

INTRODUCTION

The design of high-performance aircraft engines requires the development of new materials that can withstand extreme temperature and loading conditions. To fulfill this need, new materials are fabricated and extensively tested to assess characteristics such as strength, fatigue/fracture behavior, high-temperature creep, and relaxation response. The test results are used to develop constitutive equations (ref. 1) that can be employed to perform structural analysis simulations of aircraft engine components (ref. 2).

In its original form, the material data from extensive testing provides general guidelines about the material behavior. In a strict sense, these data are applicable only to the test conditions. Although some generalized conclusions can be drawn, it is necessary to develop more general theories about the material behavior that can reproduce the experimental behavior accurately; also, it is necessary to be able to make predictions about the material behavior under conditions for which experimental data are not available. These necessities have led to the development of different constitutive modeling theories ranging from the continuum models to the consideration of the micromechanics of material behavior. Three such theories, discussed in reference 1, are complex and require an understanding of the physics underlying the problems of material characterization.

The testing of materials for behavior under extreme conditions results in data files containing several thousands of data points. A simplified approach is to develop a functional approximation of the available test data using nonlinear regression analysis or a similar method. This approach has restricted applicability when compared with that of the general constitutive theories. The approach will not allow a deeper understanding of the material behavior but can be used effectively to calculate general material parameters such as Young's
modulus, tangent modulus, and creep parameters for a stress analysis simulation package. In this approach, the general pattern of the material behavior can be captured without dwelling too much on the reasons which cause it.

As shown in reference 3, artificial neural networks (ANN) can be used to develop approximate functional mappings similar to nonlinear regression analyses. The ANN training can be a time-consuming process; however, its main advantage lies in its compact representation of the functions that could be used for interpolation. The training can also very easily accommodate the mapping of any number of independent variables to a given set of dependent variables. Ken-Ichi Funahasi (ref. 4) proved mathematically that any continuous mapping can be approximated by multilayer neural networks with at least one hidden layer. This work was further extended by Hornik et al. (ref. 5) to include other types of squashing functions used in simulating the ANN. They also provided the mathematical proof (ref. 6) that these types of ANN are also capable of approximating arbitrary functions, including their derivatives. Further refinement of this work can be found in reference 7. These mathematical proofs provide a sound theoretical basis for using the multilayer feed-forward networks with a continuous squashing function (such as the sigmoid) to create approximate compact functional mappings using a backpropagation algorithm (ref. 8).

The ANN has been successfully utilized in solving certain classes of problems in computational structures technology (CST). A comprehensive list of such applications can be found in an unpublished manuscript (L. Berke and J. Alam, "Application of Artificial Neural Networks in Structures Technology (An Overview)," NASA Lewis Research Center, Cleveland, Ohio, 1993). These applications illustrate the research community’s considerable interest in experimenting with the ANN approach to tackle the problems when a best-fit function is needed for the available data. However, these applications do not establish guidelines for creating an appropriate network configuration or for training the networks for any particular data to be represented in compact functional form.

OBJECTIVE AND SCOPE OF STUDY

The present study was conducted to experiment with using the ANN approach to develop approximate functional representations as applied to material deformation behavior. This study utilizes the experimental hysteresis loop data taken from a deformation study on a nickel-base superalloy, Hastelloy X (Mike Castelli, July 1991, Cleveland, Ohio, personal communication). One objective of the study was to choose different configurations of the ANN for network training and assess the accuracy of the associative recall process utilized by the trained networks with the data used for training. The testing of the generalization capability of the ANN, generally known as functional interpolation, was also part of the study. A different data set of hysteresis loops not used for ANN training was employed for this purpose. Another objective was to try to keep the ANN configuration as small as possible and to evaluate the extent of data compression possible using the ANN methodology.

ANN CONFIGURATION AND MODELING

Standard configurations of feed-forward networks were utilized for this study. As shown in figure 1, they include an input layer, an output layer, and a hidden layer. The computer program NETS (ref. 9) was used for all the network training and the associative recall process. It has an implementation of backpropagation algorithm, as described in reference 8; it was written at the NASA Lyndon B. Johnson Space Center (ref. 9). The stresses and strains for the hysteresis loops are dependent upon the loading and unloading of the test specimen. Therefore, to distinguish between these two conditions, a similar procedure suggested in reference 10 was used to decide the number of input and output processing units in a network. The number of processing units in the hidden layer was established by first using an arbitrary selection based on prior experience with the ANN modeling (ref. 3) and then by assessing the accuracy of the trained network model.
RESULTS AND DISCUSSION

ANN Training

Figures 2 and 3 are plots of five hysteresis loops for Hastelloy X obtained by experiments at the NASA Lewis Research Center (Mike Castelli, July 1991, Cleveland, Ohio, personal communication). These data comprised stress and strain values for five hysteresis loops at progressively increasing numbers of fatigue cycles obtained at a constant strain rate of 0.0001 s\(^{-1}\) and a temperature of 1100 °F. Two thousand five hundred values of the stress and strain pairs are available (i.e., five hundred pairs per loop). The hysteresis loop for cycle 1 was selected to assess the accuracy of the ANN results and to choose an appropriate network configuration. Table I shows the configurations of different networks used in this study. A training data set containing 247 stress-strain values was created by choosing approximately alternate values of stress-strain from the available 500 for the entire hysteresis loop for cycle 1. The training data were normalized between 0.1 and 0.9 for all the ANN models.

For the case of five input processing units, the inputs are \(\epsilon_1, \sigma_1, \epsilon_2, \sigma_2, \) and \(\Delta\epsilon_2\) respectively, where \(\Delta\epsilon_2\) is defined by equation (1):

\[
\Delta\epsilon_2 = \epsilon_3 - \epsilon_2
\]

where \(\epsilon\) and \(\sigma\) are strain and stress, respectively. The single output processing unit is \(\Delta\sigma_2\), given by equation (2):

\[
\Delta\sigma_2 = \sigma_3 - \sigma_2
\]

Figure 4 shows these relationships graphically and provides further explanation of the chosen path. Table I also shows the number of iterations needed for training along with the maximum (max) and root-mean-square (rms) errors obtained after training. With the exception of ANN model 5-6-1, all networks converged to a chosen accuracy of 0.04 rms error. Most of the error was reduced in the first thousand iterations. Further training leads to a very slow reduction in error. The training data set was propagated through the network to obtain the results for \(\Delta\sigma_2\). Since they are in the finite-difference form, the previous stress value \(\sigma_2\) was added to find the next value of the stress \(\sigma_3\) on the path. In figure 5 the results from ANN models 5-5-1 and 5-7-1 are plotted along with the original data. Both results are fairly accurate. However, the 5-7-1 ANN produced results closer to the original data. From these plots it can be concluded that the 5-7-1 network produced the best results among all the models considered herein. Another advantage shown by these plots is the compactness of the ANN model: only seven hidden processing units are employed for the hidden layer. This leads to a significant data reduction because only a small file containing 42 weight values and 7 bias values is saved for the network recall process. All the essential characteristics of the original hysteresis loop are captured in this small file. A quick calculation shows that, if a floating point number is stored using 4 bytes, then the file containing weights and biases will require 196 bytes. A file which stores all the data points for the hysteresis loop requires 2000 bytes, leading to a data compression ratio of 10.2:1. A 2-byte integer representation was used to store the experimental results.

Table II shows the neural network configurations used to develop an approximate functional representation for the three hysteresis loops shown in figure 2. The nomenclature to specify the network is the same as that used in table I except that one more input processing unit is added at the beginning to include the cycle number \(n\) of the hysteresis loop. Thus, the number of input processing units is increased from five to six. The output processing unit is the same \(\Delta\sigma_2\) as used before. The 743 stress-strain values in the training data set were chosen from 1500 values available from the three hysteresis loops; the selection was made by choosing approximately alternate stress-strain pairs. Table II also includes the number of iterations used for training and the maximum and rms errors. In this table the 6-15-1 ANN model failed to converge; the 6-7-2-1 network used a very high
number of iterations to converge (it was the only network model which used two hidden layers). In monitoring the errors during the training process, it was observed that most of the error reduction was achieved in the first 100 iterations used for training the ANN model. Further training led to a very slow reduction in the errors; however, it was needed to achieve the desired accuracy.

**ANN Recall**

The training data set was used for the ANN associative recall. The predicted results from the network models were computed from the same scheme used earlier for the single hysteresis loop. These results for cycles 1, 100, and 5000 from the ANN models listed in table II along with the original data are plotted in figures 6, 7, and 8, respectively. For cycle 1, the results from the 6-7-2-1 net are in close agreement with the original data. The other two networks are close at the start of the propagation; however, a drift occurs from the original data as one proceeds along the loop. This drift becomes significant at the ends because of the cumulative nature of the error in retracing the original loading path. These types of problems require a higher accuracy compared with other problems where the functional approximation is restricted to only one point at a time. A similar trend can be observed from figures 7 and 8 for the hysteresis loops for cycles 100 and 5000, respectively. Overall, it can be concluded that for all the cycles the predicted results from the 6-7-2-1 network are within the allowable engineering accuracy of the 20 percent desired from the functional approximation. In the case of this ANN model, a file containing 58 weight values and 9 biases is stored and requires 268 bytes, assuming that 4 bytes are needed to represent a floating point number. A 2-byte integer representation was used to store the experimental data; hence, to store all the hysteresis loops in a file will require 6000 bytes, leading to a data compression ratio of 22.4:1 when the ANN model is used.

**Interpolated Prediction**

To test the generalization capability of the trained ANN model, the data for additional hysteresis loops for intermediate cycles 10 and 1000 (shown in fig. 4) are used. For each cycle, 500 pairs of stress and strain data were available. Of these, 166 pairs were used for each cycle, and data were presented to ANN model 6-7-2-1 for \( n, \varepsilon_1, \sigma_1, \varepsilon_2, \sigma_2, \) and \( \Delta\varepsilon_2, \) respectively. The output from the ANN model is \( \Delta\sigma_2, \) which is used to obtain \( \sigma_3 \) from the following equation:

\[
\sigma_3 = \sigma_2 + \Delta\sigma_2
\]

(3)

This procedure provides the value for the new pair \( \varepsilon_3, \sigma_3 \) and is repeated for the 166 points for both of the hysteresis loops. The results are plotted in figures 9 and 10, respectively. For both the hysteresis loops, the results are in close agreement with the original experimental data, which illustrates that the ANN model can be used for functionally interpolating the data for which it was not trained. Also possible is finding results at intermediate values by using the generalization capability of the ANN model.

**CONCLUSIONS**

The artificial neural network model (ANN) was able to provide an approximate functional representation of a single hysteresis loop. It provided rough guidelines for selecting the network configuration to develop the functional approximation for three hysteresis loops at different loading cycles considered simultaneously. One of the objectives of the study, to develop compact ANN configurations, was achieved by having an ANN configuration as small as a 6-7-2-1 net. One of the additional benefits of the compactness of a net configuration is that it leads to a data compression ratio of approximately 22 because in this approach a very small file containing the
weights and biases is saved to re-create the original function. This file captures all the essential characteristics of
the material data used for training.

The ANN model was also successful in predicting the stress values (at intermediate loading cycles) which
were not used for training the network. There is a significant drift at the ends of the predicted hysteresis loops.
This drift can be attributed to the sharp bends at the corners of the hysteresis loops. It is anticipated that more
accurate solutions can be obtained by separately training the ANN models for loading and unloading and by
combining these two ANN models in one to make it transparent to the end user. This approach to ANN training
will circumvent the two mathematical singular points located at the ends of the hysteresis loops.

RECOMMENDATION FOR FUTURE WORK

There are no established guidelines for configuring an appropriate network for a specific problem. Similarly,
it is not possible to predict a priori the number of iterations needed for training the ANN model. There is a need
to establish some useful guidelines concerning these questions. The ANN training is a time-consuming process.
The newer developments in parallel computation and ANN paradigms, such as fast backpropagation algorithm,
should be explored to exploit the inherent parallelism in the ANN method to reduce the training time in future
work.

REFERENCES

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### TABLE I—NEURAL NETWORK CONFIGURATIONS FOR SINGLE HYSTERESIS LOOP (CYCLE 1)

<table>
<thead>
<tr>
<th>ANN configuration</th>
<th>Error</th>
<th>Number of iterations</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Maximum</td>
<td>Root mean square</td>
</tr>
<tr>
<td>5-5-1</td>
<td>0.1596</td>
<td>0.0387</td>
</tr>
<tr>
<td>5-6-1</td>
<td>(b)</td>
<td>(b)</td>
</tr>
<tr>
<td>5-7-1</td>
<td>0.159</td>
<td>0.039</td>
</tr>
</tbody>
</table>

*Configuration nomenclature: number of input processing units—total number of processing units in hidden layer—total number of output units.

*Failed to converge.

### TABLE II—NEURAL NETWORK CONFIGURATIONS FOR THREE HYSTERESIS LOOPS

[From fig. 2.]

<table>
<thead>
<tr>
<th>ANN configuration</th>
<th>Error</th>
<th>Number of iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum</td>
<td>Root mean square</td>
</tr>
<tr>
<td>6-7-1</td>
<td>0.185</td>
<td>0.045</td>
</tr>
<tr>
<td>6-9-1</td>
<td>0.193</td>
<td>0.043</td>
</tr>
<tr>
<td>6-7-2-1</td>
<td>0.154</td>
<td>0.277</td>
</tr>
<tr>
<td>6-15-1</td>
<td>(c)</td>
<td>(c)</td>
</tr>
</tbody>
</table>

*Configuration nomenclature: number of input processing units—total number of processing units in hidden layer—total number of output units.

*bThe only network model to use two intermediate (hidden) layers.

*Failed to converge.
Figure 1.—Configuration of a typical neural network.

Figure 2.—Hastelloy X experimental hysteresis loops used in artificial neural network (ANN) training. Temperature, 1100 °F; strain rate, 0.0001 s⁻¹.

Figure 3.—Hastelloy X experimental hysteresis loops used for testing the ANN generalization capability. Temperature, 1100 °F; strain rate, 0.0001 s⁻¹.

Figure 4.—Stress-strain behavior.
Figure 5.—Comparison of results from ANN with data for Hastelloy X hysteresis loop.

Figure 6.—Comparison of results from ANN with data for Hastelloy X hysteresis loop (cycle 1).

Figure 7.—Comparison of results from ANN with data for Hastelloy X hysteresis loop (cycle 100).

Figure 8.—Comparison of results from ANN with data for Hastelloy X hysteresis loop (cycle 5000).
Figure 9.—Comparison of results from ANN (6-7-2-1) with data for Hastelloy X hysteresis loop (cycle 10).

Figure 10.—Comparison of results from ANN (6-7-2-1) with data for Hastelloy X hysteresis loop (cycle 1000).
The artificial neural network (ANN) model proposed by Rumelhart, Hinton, and Williams is applied to develop a functional approximation of material data in the form of hysteresis loops from a nickel-base superalloy, Hastelloy X. Several different ANN configurations are used to model hysteresis loops at different cycles for this alloy. The ANN models were successful in reproducing the hysteresis loops used for its training. However, because of sharp bends at the two ends of hysteresis loops, a drift occurs at the corners of the loops where loading changes to unloading and vice versa (the sharp bends occurred when the stress-strain curves were reproduced by adding stress increments to the preceding values of the stresses). Therefore, it is possible only to reproduce half of the loading path. The generalization capability of the network was tested by using additional data for two other hysteresis loops at different cycles. The results were in good agreement. Also, the use of ANN led to a data compression ratio of approximately 22:1.