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PREDICTION OF THREE SIGMA MAXIMUM DISPERSED DENSITY FOR AEROSPACE APPLICATIONS

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SUMMARY

Free molecular heating (FMH) is caused by the transfer of energy during collisions between the upper atmosphere molecules and a space vehicle. The dispersed free molecular heating on a surface is an important constraint for space vehicle thermal analyses since it can be a significant source of heating. To reduce FMH to a spacecraft, the parking orbit is often designed to a higher altitude at a the expense of payload capability. Dispersed FMH is a function of both space vehicle velocity and atmospheric density; however, the space vehicle velocity variations are insignificant when compared to the atmospheric density variations. The density of the upper atmosphere molecules is a function of altitude, but also varies with other environmental factors, such as solar activity, geomagnetic activity, location, and time.

A method has been developed to predict three sigma maximum dispersed density for up to 15 years into the future. This method uses a state-of-the-art atmospheric density code, MSIS 86, along with 50 years of solar data, NASA and NOAA solar activity predictions for the next 15 years, and an Aerospace Corporation correlation to account for density code inaccuracies to generate dispersed maximum density ratios denoted as 'K-factors'. The calculated K-factors can be used on a mission unique basis to calculate dispersed density, and hence dispersed free molecular heating rates. These more accurate K-factors can allow lower parking orbit altitudes, resulting in increased payload capability.

FREE MOLECULAR HEATING

Free molecular heating (FMH) is caused by the kinetic transfer of energy during the collisions between the rarefied upper atmosphere molecules and a space vehicle traveling at high velocities. The dispersed free molecular heating on a surface normal to the velocity vector can be a significant portion of the space heating environment (Figure 1). Therefore, the FMH during the period between payload fairing (PLF) jettison and spacecraft separation is an important constraint for space vehicle thermal analysis.

Nominal FMH is predicted by the following equation which relates the free molecular heat flux, q_{fmh}, to the atmospheric density, ρ, and to the vehicle velocity relative to the atmosphere velocity, v_{rel}:

$$q_{fmh} = 1/2\, \rho_{nom}\, v_{rel}^3 \qquad (1).$$

Units conversion factors for Equation 1 are listed in Table 1 for various standard English units of density and velocity. Dispersed free molecular heating is predicted by using the dispersed density, ρ_{disp}, in Equation 1:

$$q_{fmh,disp} = 1/2\, \rho_{disp}\, v_{rel}^3 \qquad (2).$$

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Table 1. Units Conversion Factors for Free Molecular Heating Equation (1)

Common Density Units	Common Velocity Units	Common FMH Units	Conversion Factor Value [1.]	Conversion Factor Units
$slug/ft^3$	ft/sec	$BTU/hr\text{-}ft^2$	4.62268	$\frac{BTU/hr\text{-}ft^2}{(slug/ft^3)(ft^3/sec^3)}$
lb_m/ft^3	ft/sec	$BTU/hr\text{-}ft^2$	0.143677	$\frac{BTU/hr\text{-}ft^2}{(lb_m/ft^3)(ft^3/sec^3)}$
$lb_f\text{-}sec^2/ft^4$	ft/sec	$BTU/hr\text{-}ft^2$	4.62268	$\frac{BTU/hr\text{-}ft^2}{(lb_f\text{-}sec^2/ft^4)(ft^3/sec^3)}$

Note 1. The following values were used to determine the conversion factors:
J = mechanical equivalent of heat, 778.770 $ft\text{-}lb_f/BTU$
g_c = proportionality constant between mass and force, 32.174 $lb_m\text{-}ft/lb_f\text{-}sec^2$

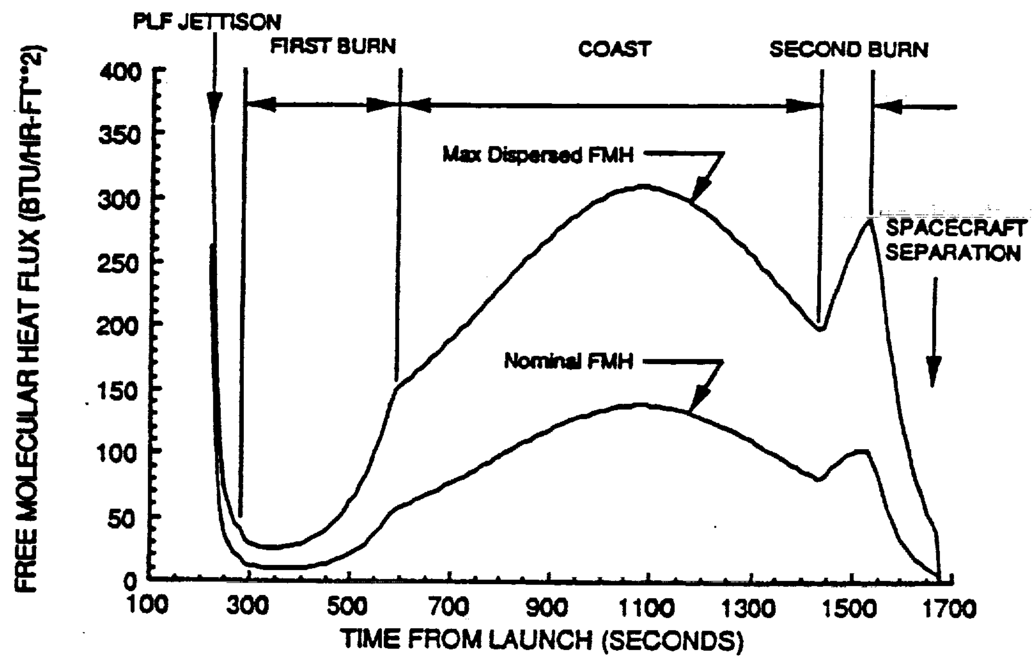


Figure 1 Example of Nominal and Dispersed Free Molecular Heating Fluxes for a Standard Atlas/Centaur Launch Vehicle Parking Orbit

The dispersed density, ρ_{disp}, can be calculated from the Equation 3 which relates the dispersed density to the U.S. Standard 1976 nominal density (Reference 1), ρ_{76}, by a density dispersion factor, K, also called a 'K-factor':

$$\rho_{disp} = K\,\rho_{76} \qquad (3).$$

Both the density and the density dispersion factor are functions of altitude. The U.S. Standard 1976 density is a nominal, mid-solar cycle density which is tabulated as a function of altitude. The density dispersion factor 'K' is a function of several parameters, but is affected primarily by altitude and level of solar activity.

Free Molecular Flow Regime

In the upper atmosphere, the density of the gas molecules is so low that the intermolecular collisions are negligible. This is the free molecule flow regime where the kinetic theory of gasses takes over from the continuum fluid mechanics of the lower atmosphere. The basic free molecular flow assumption is that the flow of atmospheric gas molecules incident on a body is undisturbed by the presence of that body. Thus, there is no boundary layer and no flow around a body, and collisions between the body and the gas molecules only take place on surfaces which are normal to the velocity vector (Reference 2).

The transition between the free molecule and continuum flow regimes depends on whether or not the molecules' collisions with each other affect their collisions with the body. In other words, the free molecule regime is characterized by single collisions between the gas molecules and the body, which means that the mean-free path of the molecules has to be much greater than the characteristic length of the body. The ratio of the mean-free path in the gas, λ, to the characteristic dimension, d, is called the Knudsen number, Kn:

$$Kn = \lambda / d \qquad (4).$$

Free molecule flow is assumed for $Kn \geq 10$. The mean-free path varies with density and is a function of altitude. Assuming a space vehicle characteristic dimension of 3.0 m, the vehicle will be fully in free molecule flow when the molecules have a mean-free path greater than 10 x 3.0 m or 30 m. This mean-free path length has been measured at about 130 km (70 nmi or 425 kft, Reference 1), which is below the 80 to 100 nmi typical low Earth parking orbits.

Free Molecular Heat Flux

Free molecular heating occurs when the incident rarefied molecules are reflected from a surface. These reflections are characterized as diffuse, specular, or some combination of these two extremes. A diffuse reflection is one in which the molecules are momentarily absorbed by the surface and then emitted according to a Maxwell velocity distribution corresponding to the surface temperature. A specular reflection is one in which the nominal velocity of the molecules is reversed and the tangential velocity remains the same. Diffuse reflections are very common for most surfaces, whereas, specular reflections are only common for very highly polished, non-oxidized metals or for very small grazing angles (Reference 3).

The thermal accommodation coefficient, α, is the measure of how close the actual energy transfer to the wall is to the purely diffuse energy transfer. For purely diffuse reflections, α=1.0 and for purely specular reflections, α=0.0. The accommodation coefficient increases with increasing oxidation, with increasing surface roughness, and with increasing wall temperature (Reference 4). For most cases involving space vehicles, diffuse reflections are predominant and accommodation coefficients of at least 0.9 are appropriate (Reference 3). An accommodation coefficient of 1.0 is often used for a maximum heating analysis and an accommodation coefficient of 0.0 is often used for a minimum heating analysis (Reference 5). The heat flux absorbed by a surface normal to the flow is calculated by

$$q_{fmh,absorbed} = \alpha \left(^1/_2 \, \rho_{disp} \, v_{rel}^3 \right) \qquad (5).$$

The velocity and altitude are determined from the trajectory. The dispersed density is determined from the altitude and the appropriate density dispersion K-factors (Equation 3). The derivation of this equation from the kinetic theory of gasses is described in Reference 4 and requires many simplifying assumptions. The main assumptions are:

1) free molecular flows, $Kn \geq 10$
2) surfaces are normal to the flow
3) vehicle velocities much greater than molecular velocities
4) surface temperatures much lower than gas stagnation temperatures.

Nominal Atmospheric Density

The nominal free molecular heating from Equation 1 is calculated using a nominal atmospheric density from a standard atmosphere model. The most widely used nominal atmosphere model is the U.S. Standard Atmosphere, 1976 (Reference 1). This atmosphere model is a hypothetical, constant, vertical distribution of atmospheric temperature, pressure and density which, by international agreement, is roughly representative of year-round, mid-latitude conditions for the range of solar activity that occurs between sunspot minimum and sunspot maximum. In the model, the gasses of the atmosphere are assumed to obey the perfect gas law, $P = \rho R T$, and hydrostatic equation, $dP/dh = -\rho g c$, which, when taken together, relate temperature, pressure and density with altitude. The U.S. Standard 1976 Atmosphere is the base atmosphere on which the density dispersion factors (K-factors) are calculated.

These nominal atmosphere models are designed to fit the available atmospheric data. From 50 to 90 km, the atmospheric measurements of temperature, density, and pressure were made almost exclusively with rocket-borne instruments, mass spectrometers and density gauges. From 140 to 1000 km, these values were determined from satellite-related observations (satellite drag calculations and satellite-borne instruments) and radar incoherent scatter techniques (Reference 1).

Atmospheric densities calculated from both composition measurements made with rocket-borne mass spectrometers and from values inferred from satellite drag calculations have been compared over the range from 100 to 200 km (54 to 108 nmi). The mass spectrometer-calculated densities were usually lower than satellite drag-calculated densities (Reference 1). The U.S. Standard 1976 atmosphere (Reference 1) relies principally on the satellite drag data.

Dispersed Atmospheric Density

The atmospheric density, though principally a function of altitude, is dynamic and variations in it occur frequently. The range of density variation which occurs is illustrated in Figure 2. Several types of variations occur which are attributed to: different levels of solar activity, fluctuations in geomagnetic activity, geographical location, seasonal variations, and variations between day and night levels. Low Earth orbiting satellites passing through the upper atmosphere are significantly affected by these density variations due to increased satellite drag and increased free molecular heating. Changes in satellite drag due to density variations can cause satellite lifetimes to vary by a factor of ten between minimum to maximum conditions (Reference 6). The sudden density variations from solar storms can significantly change satellite orbits, which requires relocation and reacquisition of the satellites and uses on-board propellant to correct deviations (Reference 7). For launch vehicles, the main effect of this dispersed density is from increased free molecular heating on vehicle and payload surfaces.

Solar Cycle Effects

The largest atmospheric density dispersion is from the variation in the solar flux received by Earth's atmosphere. The waning and waxing of the solar flux, known as the solar cycle, lasts approximately 11 years (Figure 3). At the peak of the solar cycle, large energetic events occur on the Sun which greatly increase the extreme ultraviolet (EUV) radiation arriving at the Earth. All of the EUV is absorbed in the Earth's lower thermosphere, a layer which is located approximately between 83 to 120 km (45 to 65 nmi) in altitude. The lower thermosphere layer absorbs the EUV energy and expands outward, decreasing the density at the very lowest altitudes but greatly increasing the density in the higher altitudes where the low earth parking orbits are (from 148 to 185 km) and above. Above 296 km (160 nmi) the density can vary by an order of magnitude due to solar cycle effects (Figure 2).

Because none of the EUV radiation reaches the ground, the EUV cannot be monitored using ground-based techniques. Measurements have been made by a few specially equipped satellites over relatively short periods. The solar 10.7 cm (radio wavelength) flux is measurable from Earth and varies closely with the EUV flux as can be seen in Figure 4, which compares satellite EUV measurements with coincident ground measurements of 10.7 cm flux. Therefore, the F10.7 index, which is a measure of the 10.7 cm radio flux levels with units of 10^{-22} Watts m^{-2} Hz^{-1}, is used as a proxy index in atmospheric density models in place of the EUV flux levels (Reference 7).

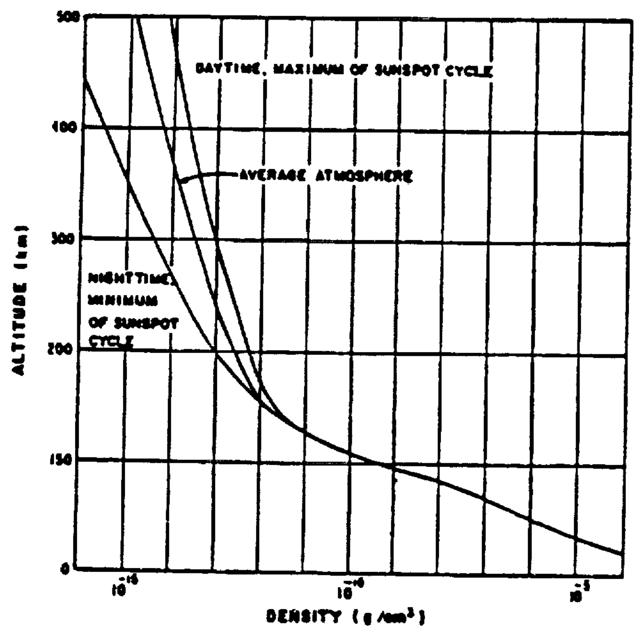


Figure 2. Atmospheric Density Variation: Nominal, Minimum and Maximum (Reference 8)

The solar cycle, as recorded from the 10.7 cm flux, shows significant daily variation (Figure 5). The trend is difficult to see in this data; therefore, a 13-month running mean, using data from both sides of the given day, is used to eliminate the daily fluctuations. Use of this mean allows easier tracking of the solar cycle progress and easier comparison from one solar cycle to another. The 10.7 cm flux also correlates well with the historical method of recording solar cycles by the number of appearances of sunspots (used in Figure 3). This correlation is clearly seen over the last few solar cycles in Figure 6. There are significant differences between the activity levels during different solar cycles, with Cycle 19 having the highest activity level recorded to date.

The National Oceanic and Atmospheric Administration (NOAA) provides regular updates on solar 10.7 cm radio flux activity and also provides a nominal prediction of the F10.7

index with 90% bounds for a full solar cycle (approximately 11 years) which is used in dispersed density predictions. The F10.7 index over the last few solar cycles is shown in Figure 7 along with the NOAA prediction for the current solar cycle.

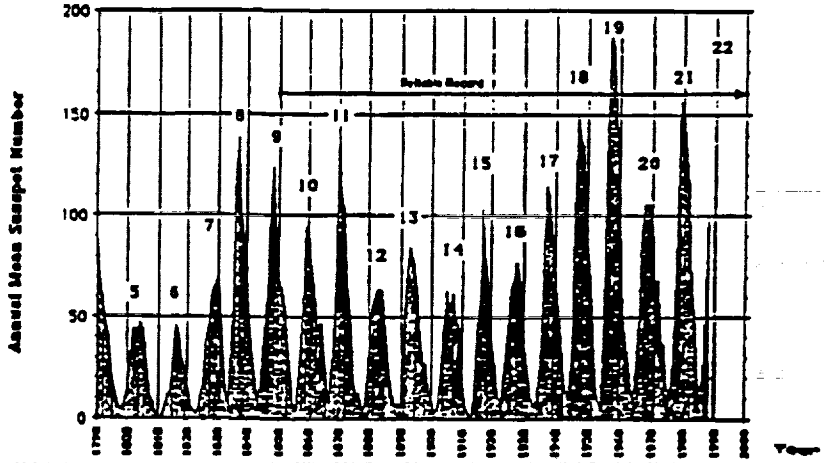


Figure 3. Solar Cycles from 1790 to 1988 (Reference 9)

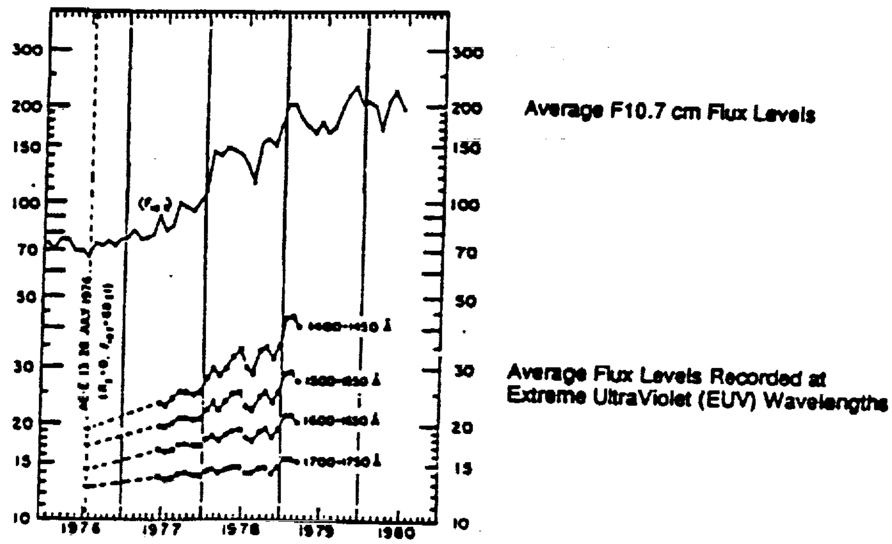


Figure 4. Comparison for Solar 10.7 cm Flux Levels Recorded on Earth to EUV Flux Levels Recorded by the AE-E Satellite During Solar Cycle 21 (Reference 7).

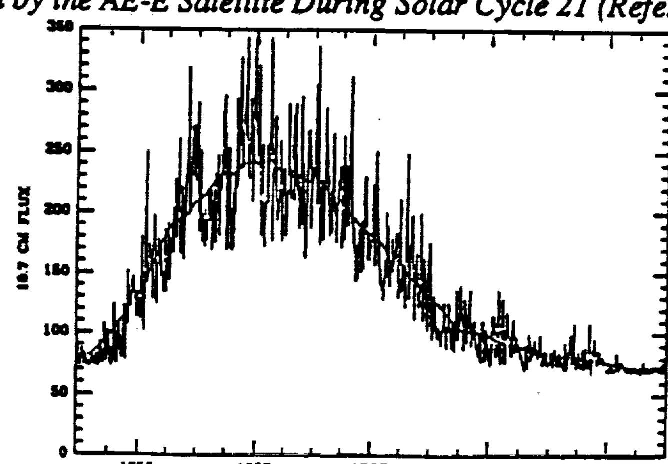


Figure 5. Smoothed and Daily 10.7 cm Flux for Solar Cycle 19 (Reference 9)

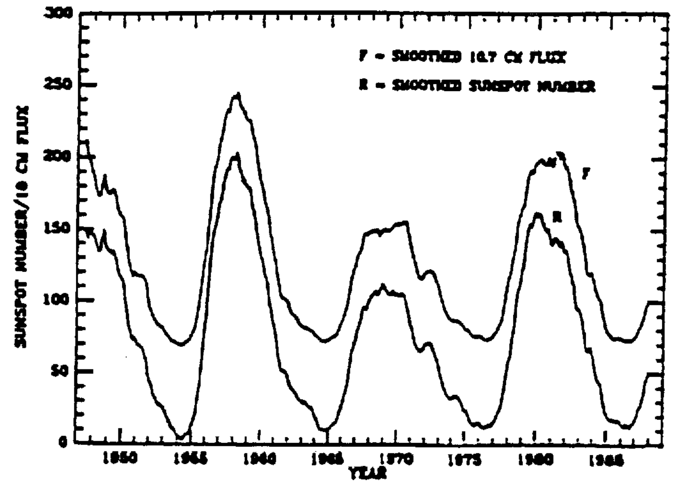
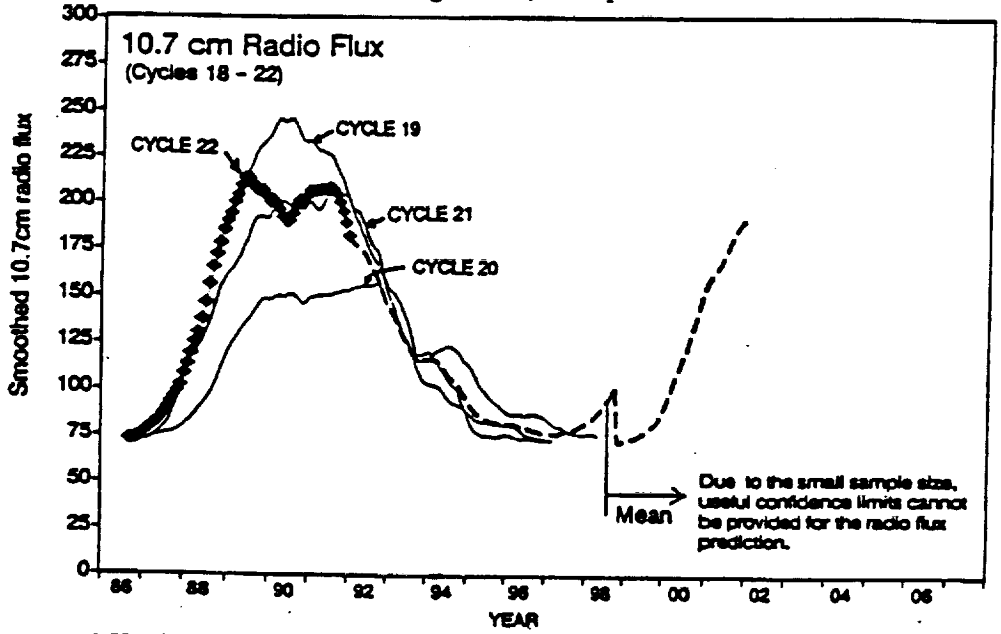


Figure 6. Smoothed (13-month Running Mean) Sunspot Number R and Solar Flux (Ref. 9)



* 90% Prediction interval or 90% confidence limit of the mean.

Figure 7. Smoothed Solar 10.7 cm Flux for Past and Predicted Cycles (Reference 10)

Geomagnetic Activity Effects

Geomagnetic storms are caused by bursts of charged particles from the sun colliding with the Earth's magnetosphere. Large amounts of energy are deposited at high latitudes during these storms and can significantly increase the atmospheric density at low Earth parking orbit altitudes in a matter of hours. Unlike the 10.7 cm radio flux, the geomagnetic activity has a relatively short term effect of only a few days. The geomagnetic activity effects are a strong function of latitude and are much stronger near the poles than at the equator (Reference 7).

The Ap index is a measure of the intensity of atmospheric activity due to geomagnetic storms. It is a derived index which is unitless and varies from 0 (quiet) to 400 (maximum storm activity). There is a pronounced maximum magnetic storm frequency during the declining phase of the solar cycle, which is illustrated in Figure 11. Increased geomagnetic activity begins about halfway between the solar maximum and the solar minimum and continues until the solar cycle minimum is reached (Reference 7).

The Ap level is determined by mapping the Kp level, which varies from 0 to 9, onto the Ap range of 0 to 400. The Kp index is a quasi-logarithmic measure of the amplitude of geomagnetic disturbances, where the change from 1 to 2 represents a small increase in amplitude while the change from 8 to 9 represents a major increase in amplitude. The Kp index is a 24-hour combination of the eight 3-hour K indices reported from twelve observatories around the world. The K index varies from 0 to 9 like the Kp index and is a measure of the local magnetic character for each 3-hour period recorded at each location (Reference 8). Marshall Space Flight Center (MSFC) produces a monthly update on the geomagnetic activity and provides a nominal Ap prediction with 97% bounds for a full solar cycle.

Diurnal, Seasonal, and Geographical Variations

There is a diurnal or daily variation in atmospheric density which also occurs. The time at which the maximum occurs is around 1400 local solar time and varies with altitude (see Appendix). The amount of variation in density levels is latitude- and altitude-dependent. At the higher altitudes, above 200 km (108 nmi), the density during the day can be up to eight times higher than the density at night (References 11 and 12). However, the effects are not as pronounced at the lower parking orbit altitudes.

There is a semiannual variation in the atmospheric density levels. This variation is clearly shown in the figures in the Appendix. There is a primary minimum in July, followed by a primary maximum in October, a secondary minimum in January, followed by a secondary maximum in April (References 11 and 12). This variation depends principally on solar activity levels.

The atmospheric density also varies depending on the latitude and longitude. This variation extends down to about 150 km (81 nmi) and is shown in the Appendix. Some of the latitude variation is due to the geomagnetic activity having a more pronounced effect nearer the poles (Reference 12).

Atmospheric Density Codes

There are two principal atmospheric density calculation codes available for altitudes above 90 km (50 nmi). These are the Jacchia model, developed by L. G. Jacchia from satellite drag data, and the MSIS model, developed by A. E. Hedin from mass spectrometer and incoherent scattering measurements made from satellites and ground stations respectively. Both have been shown to have an estimated error of around 15%. These codes require the user to input values for all of the principle density variations mechanisms: altitude, F10.7, Ap, time of day, day of year, latitude, and longitude. Based on these inputs, these atmospheric models calculate nominal atmospheric densities. The Free Molecular Heating K-Factor Calculation Section discusses the use of the MSIS code to calculate K-factors.

K-Factors

To calculate a three-sigma dispersed density, the nominal atmospheric densities have to be increased to account for the solar activity and other dispersion mechanisms. The atmospheric density calculation codes, with suitably dispersed input parameters and suitably applied error estimates, could be used in each calculation of dispersed density; however, an easier approach is generally used. Atmospheric density dispersion factors called "K-factors" have been developed to disperse the nominal densities, for different altitudes and for different times in the solar cycle. These K-factors can be conservatively calculated to bound a given time period by dividing the three sigma dispersed density calculated from the density codes by the nominal density.

The dispersed density is then regenerated by multiplying the nominal density at a given altitude by the corresponding K-factor for that altitude and for the range of years desired as in Equation 3. The method for deriving K-factors is discussed in the following section.

FREE MOLECULAR HEATING K-FACTOR CALCULATION

Free molecular heating (FMH) is calculated by cubing the relative velocity and multiplying it by one half the atmospheric density (Equation 1). Given nominal atmospheric density and velocity, nominal FMH fluxes can be calculated. However, standard engineering practices require additional conservatism to account for possible dispersions. These dispersions are accounted for by multiplying the nominal density by a dispersion factor or K-factor .

To calculate dispersed atmospheric density, four primary sources are used: 1) Atmospheric density model, which predicts nominal atmospheric density given the atmospheric location with respect to the Earth, and the solar, and geomagnetic index parameters; 2) Forecasts of solar activity; 3) Measured daily solar and geomagnetic index data collected over the past 59 years; and 4) Atmospheric model uncertainty equation used to disperse nominal predicted densities. Using a Monte Carlo statistical technique and the U.S. Standard 1976 atmospheric density (Reference 1), K-factors can be determined. To automate this process a computer program has been developed by the authors.

Atmospheric Density Model (MSIS-86)

Due to the significant effect that the upper atmospheric density has on space vehicles and satellites, a great deal of work has been done in developing a method for determining atmospheric densities and temperatures. Through the years, several mathematical models have been developed (Figure 8) based on atmospheric calculations from satellite drag data, incoherent scatter data from ground sites, and measured atmospheric conditions from mass spectrometer measurements on numerous satellites. Not only can these models provide estimates of density, but atmospheric composition, temperature, molecular mass, pressure scale height, and density scale height for altitudes between 85 and 2500 km (46 and 1350 nmi).

The MSIS-86 (Mass Spectrometer and Incoherent Scatter) model (Reference 13) is one of the latest in the series of empirical models developed. This model uses user-provided values of day of year, time of day, altitude, latitude, longitude, local solar time, geomagnetic index, predicted three month average F10.7 cm radio solar flux (F10.7), and daily F10.7 solar flux. The model predicts atmospheric temperature, total density, and densities of N2, O2, O, N, He, Ar, and H. This model is based on atmospheric composition and temperature data from eight scientific satellites (OGO-6, San Marc-3, Aeros-A, AE-C, -D, -E, ESRO-4, and DE-2) and numerous rocket probes, as well as five ground-based incoherent scatter stations (Millstone Hill, St. Santin, Arecibo, Jicamarca, and Malvern) (Reference 14).

GDSS has adopted the MSIS-86 atmospheric code to predict nominal atmospheric densities as a function of altitude and year. This decision was based on the Reference 14 comparison study, identifying the MSIS-86 model as the most up-to-date atmospheric model. However, an additional study (Reference 15) was performed that showed discrepancies in the MSIS-86 predicted density results for latitudes near the north pole. To overcome this discrepancy, atmospheric densities calculated with latitudes greater than 60 degrees were changed to reflect the southern hemisphere values.

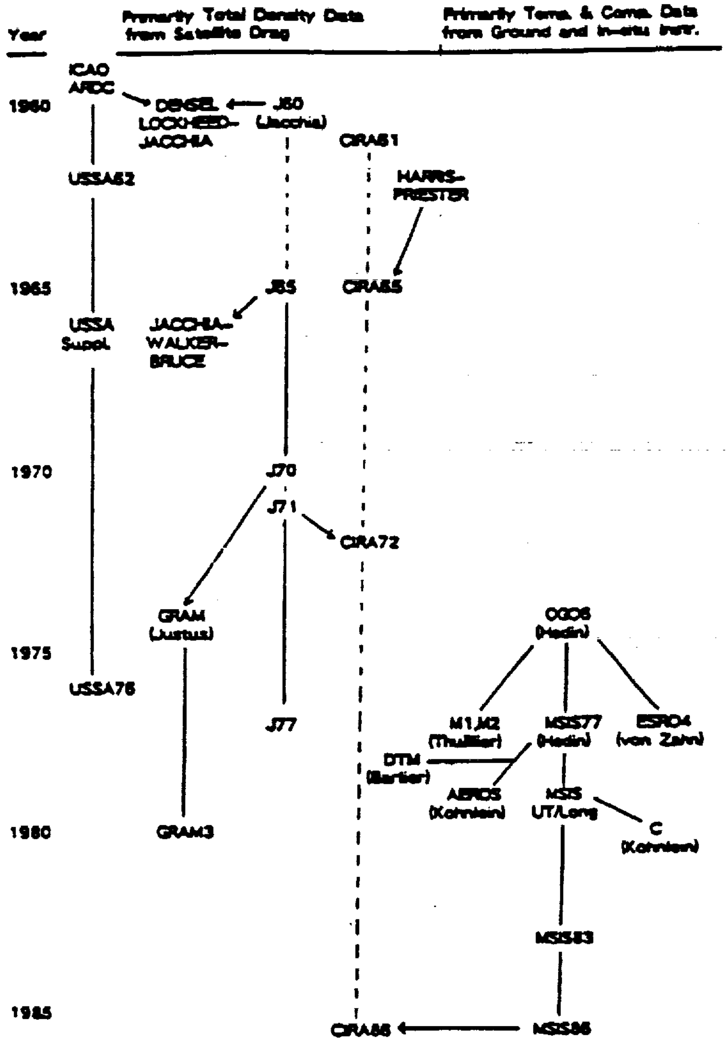


Figure 8. Historical Development of Empirical Thermoshpere Models (Reference 14).

Solar and Geomagnetic Activity Forecast

Upper atmospheric density fluctuations are strongly influenced by changing levels of solar activity. The F10.7 cm solar flux and geomagnetic index inputs are ground measurements used to indicate the level of solar activity. This solar activity data is collected and summarized in monthly publications provided by NASA-Marshall Space Fight Center (MSFC) (Reference 16) and National Oceanic and Atmospheric Administration (NOAA) (Reference 10). This data is used as input data to upper atmospheric models to predict upper atmospheric conditions.

The MSFC solar flux (F10.7) and geomagnetic index (Ap) long range statistical estimates are based upon historical F10.7 cm solar flux and geomagnetic index measurements. These estimates (1992 to 2008) are based on the statistical data of solar cycles 9 through 22 (1850 to 1992). Figure 9 illustrates the MSFC 13-month average F10.7 cm solar flux predictions. As shown in Figure 9, nominal predictions as well as 2-sigma dispersed high and low predictions are provided. The nominal curve represents the most likely curve that is expected in the future based on the most recent observed data. To provide additional conservatism to the calculation of FMH, the 2-sigma (97.7%) high 13-month F10.7 solar flux average predictions were used.

Due to the variations in daily F10.7 cm solar flux and geomagnetic index, these parameters cannot be projected into the future with any acceptable degree of statistical confidence. Because these parameters are required input to the MSIS-86 program, alternate techniques of determining these values have been developed (see Measured Solar and Geomagnetic Index Data and Computer Program Sections).

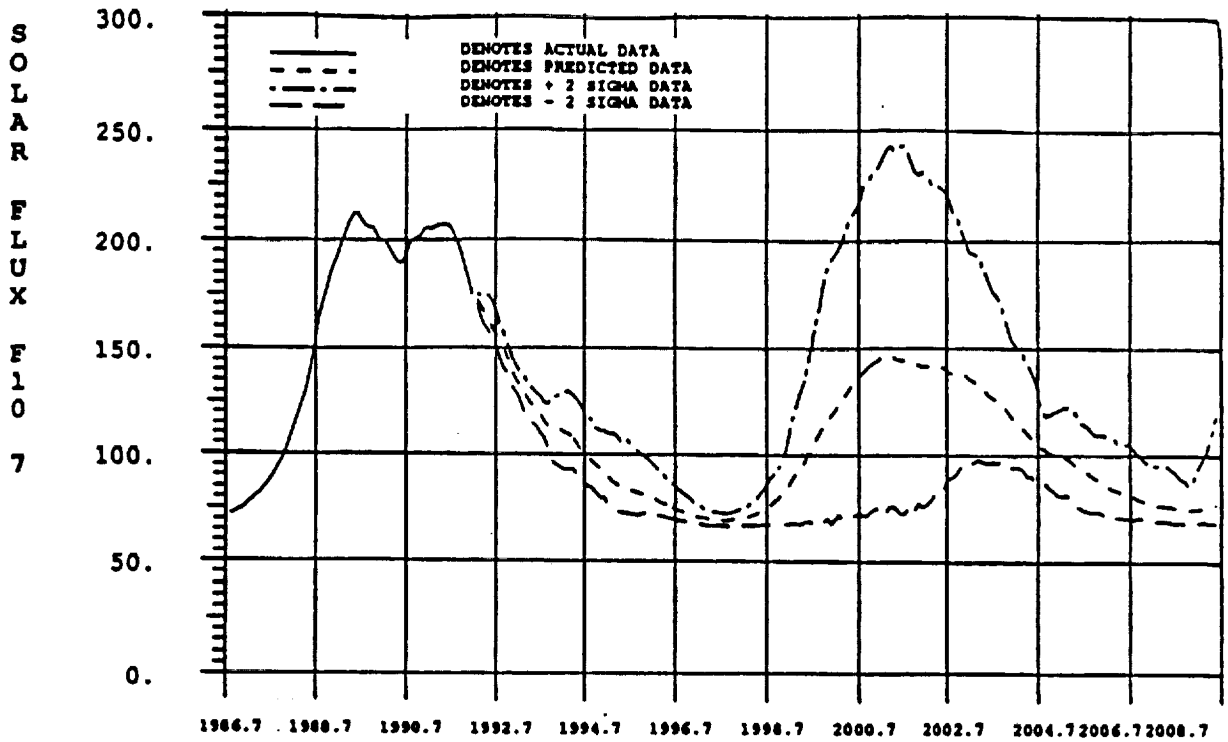


Figure 9. MSFC (Ref. 18) Long Range Estimates of Solar Activity for Solar Cycles 22 and 23.

Measured Solar and Geomagnetic Index Data

Measured daily geomagnetic and solar activity summaries were obtained through the National Geophysical Data Center (Reference 17). The available data contains daily summary records from January 1, 1932 through June 30, 1990. The planetary geomagnetic index (ap) is provided every three hours throughout the day and represents the planetary average based on 13 observations between 46 degrees north and 63 degrees south latitudes. The daily Ap average is determined from the arithmetic mean of the day's eight ap values. The daily Ap index is used in the calculation of atmospheric density. The daily F10.7 cm solar flux represents the value measured in Ottawa, Canada at 1700 GMT. The F10.7 cm solar flux measurements began on February 14, 1947 and are expressed in units of 10^{-22} Watts m^{-2} Hz^{-1}. Figures 10 and 11 illustrate the measured daily F10.7 cm solar flux and Ap values for solar cycles 18 through 22.

Based on this measured data, a new data file was generated which consists of the date, daily average Ap and daily F10.7 cm solar flux. Additionally, for each day, the 3-month and 13-month F10.7 cm solar flux averages were calculated and added to the data file. Because the F10.7 predictions are only available in 13-month averages and the MSIS-86 program requires 3-month F10.7 averages as input, it was necessary to generate these 3-month averages. The 3-month F10.7 cm average solar flux for a given day was calculated by averaging the preceding and following 45 daily F10.7 cm solar flux measurements (90 total values). Similarly the 13-month F10.7 cm solar flux averages were calculated by averaging the previous and following 195 daily F10.7 cm solar flux measurements (390 total values). Figure 12 illustrates the resulting 3 and 13-month averages for solar cycle 19.

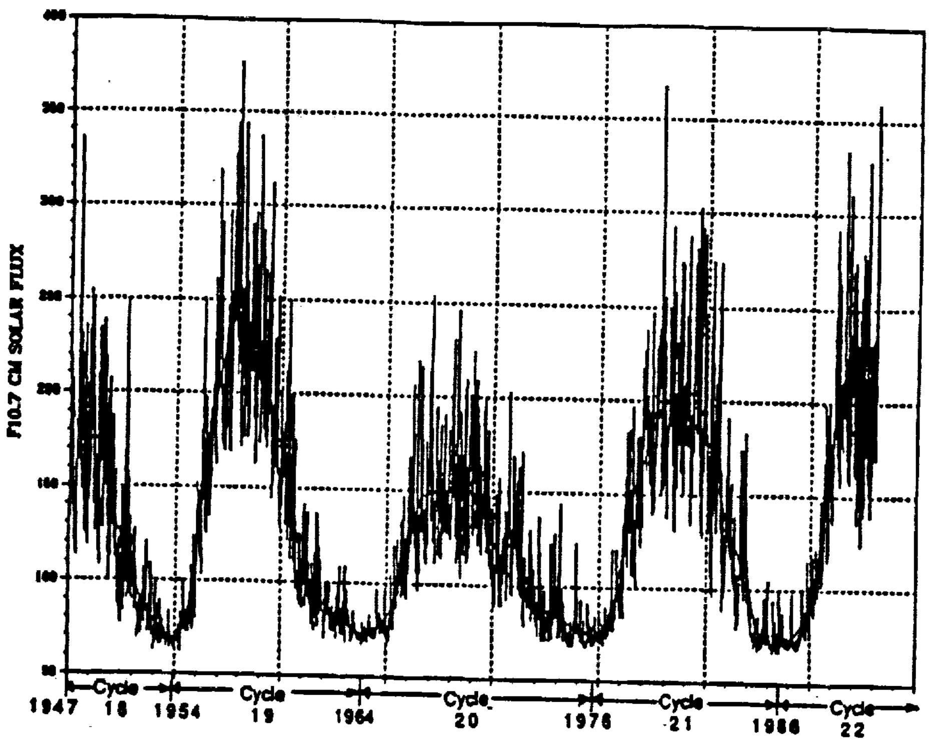


Figure 10. Measured Daily F10.7 cm Solar Flux for Solar Cycles 18 through 22 (Ref. 17).

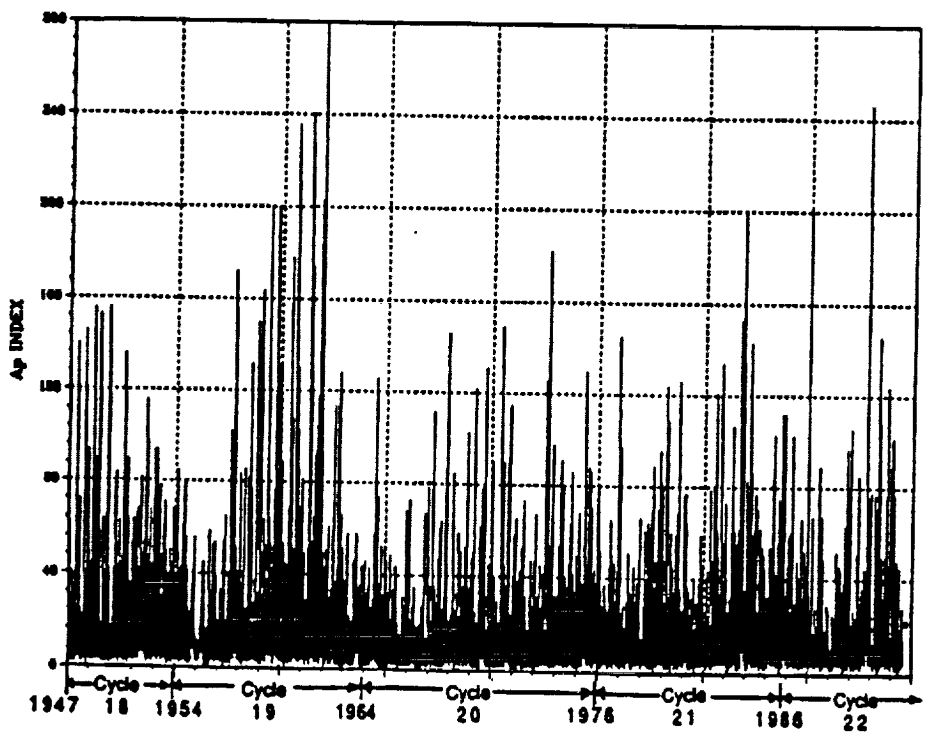


Figure 11. Measured Daily Geomagnetic Index (Ap) for Solar Cycles 18 through 22 (Ref. 17).

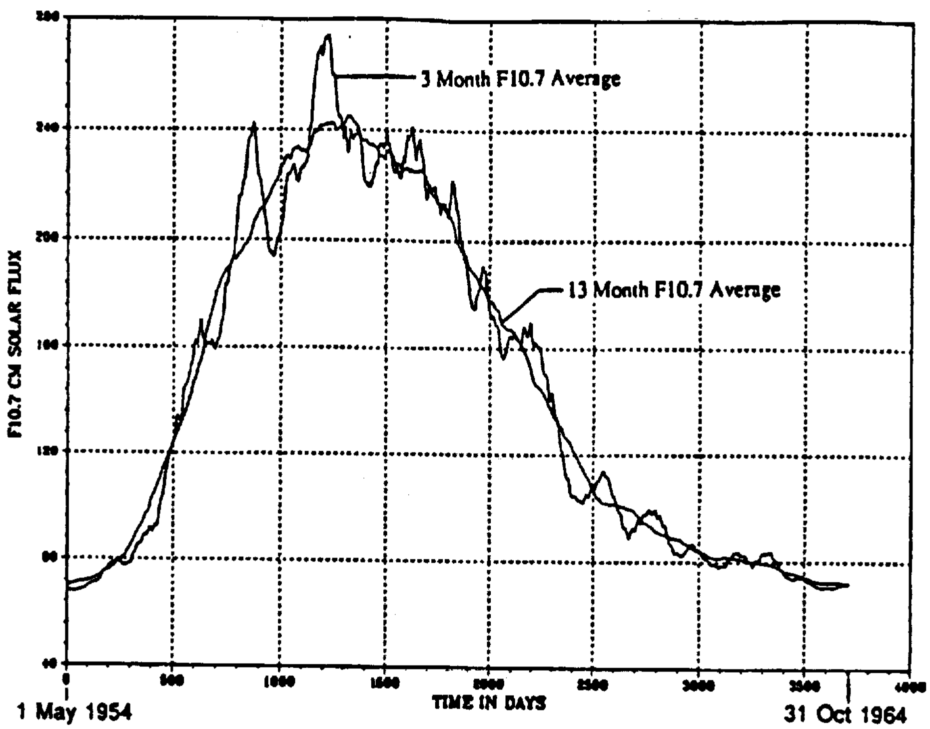


Figure 12. Calculated 3 and 13-month F10.7 Solar Flux Averages for Solar Cycle 19.

Atmospheric Model Uncertainty

Even though the inputs to MSIS-86 are selected so as to maximize density, the atmospheric densities calculated by MSIS-86 represent the best approximation of density and are considered nominal values. To account for MSIS model prediction uncertainties another dispersion factor was incorporated. D. Kayser, Aerospace Corp., developed analytical functions to predict dispersed atmospheric density within 99.865% (approximately 3-sigma) confidence level relative to MSIS-83 atmospheric model (Reference 15). Predictions from the MSIS-83 model were compared to the available Aerospace density data base to derive these equations. Knowing that the differences in predicted atmospheric density between the MSIS-83 and MSIS-86 models are small between latitudes -90 and 60 degrees, the authors applied the model uncertainty equations developed for the MSIS-83 model to the MSIS-86 model.

The Reference 15 derived expression for the calculation of dispersed density as a function of altitude is provided below:

$$r_c(h) = (0.769 - 0.307 \exp\{-[(h-102.5)/126.21]^4\}) \qquad (6)$$

where:

$r_c(h)$ is the 3-sigma confidence level as a function of altitude (h) in kilometers.

$$\rho_{disp} = \rho_{nominal} \exp(r_c(h)) \qquad (7)$$

where:

ρ_{disp} = 3-sigma dispersed atmospheric density

$\rho_{nominal}$ = MSIS-86 predicted density

Monte Carlo Statistical Technique

There have been a large number of calculations of the drag exerted by the atmosphere on satellites and measurements made by density gauges and mass spectrometers aboard rockets and satellites. These calculations and measurements have revealed several different effects other than altitude that result in variations of density, temperature, and composition of the upper atmosphere. The parameters and assumed ranges used as input to the MSIS-86 program are listed below:

IYD - Day of the year (1 through 365 days)
UT - Universal time or GMT (0 to 86400 seconds)
ALT - Altitude (440 to 1000 kft)
LAT - Latitude (user provided)
LONG - Longitude (0 to 360°)
LST - Local solar time (0 to 24 hours)
F107A - Predicted 3-month average F10.7 cm solar flux
F107 - Daily F10.7cm solar flux
Ap - Geomagnetic index

The MSIS-86 input parameters associated with defining solar activity are F107A, F107, and Ap. The remaining input parameters define the Earth's position relative to the sun (IYD, UT), point of interest within the Earth's atmosphere (ALT, LAT, and LONG), and location on Earth with respect to the sun (LST). A parametric analysis was performed to identify the sensitivity of atmospheric density as a function of each parameter. Results from this study are shown in the Appendix.

Since these parameters are dependent on each other, it is not a simple task to identify which combination of parameters will result in maximizing atmospheric density for a given set of solar conditions. To simplify this procedure, a Monte Carlo statistical approach was incorporated to predict maximum dispersed atmospheric density values.

For a given altitude, random values of IYD, UT, LAT, LONG, and LST were selected within their valid ranges. Additionally, daily and 3-month average F10.7 solar fluxes and Ap values for a given day were randomly selected from a generated data file (see Measured Solar and Geomagnetic Index Data Section). Based on these inputs atmospheric density was calculated using the MSIS-86 program. Using this process, 1000 density values were calculated and a 3-sigma dispersed density was determined using the following equation:

$$RHO3SIG = RHOAV + 3\left[\frac{\sum_{i=1,n}(RHO_i - RHOAV)^2}{N-1}\right]^{.5} \quad (8)$$

where:

RHO3SIG = 3-sigma dispersed density
RHOAV = calculated average density
RHO_i = a given density value
N = number of density values calculated

K-Factor Computer Program

To automate the calculation of K-factors, a computer program was developed. Figure 13 illustrates the program flow chart. The only program inputs required are the predicted 13-month F10.7 cm solar flux, and latitude range for which the K-factors are to apply. Variations in latitude were incorporated to allow flexibility for missions which may have unique parking orbit latitudes. Results presented herein were based on latitudes between ±60 degrees which will encompass all typical geosynchronous transfer orbit trajectories and latitudes between ±90 degrees to include polar orbit trajectories. Based on the parametric study results (see the Appendix), K-factors tend to increase as a function of increasing latitude.

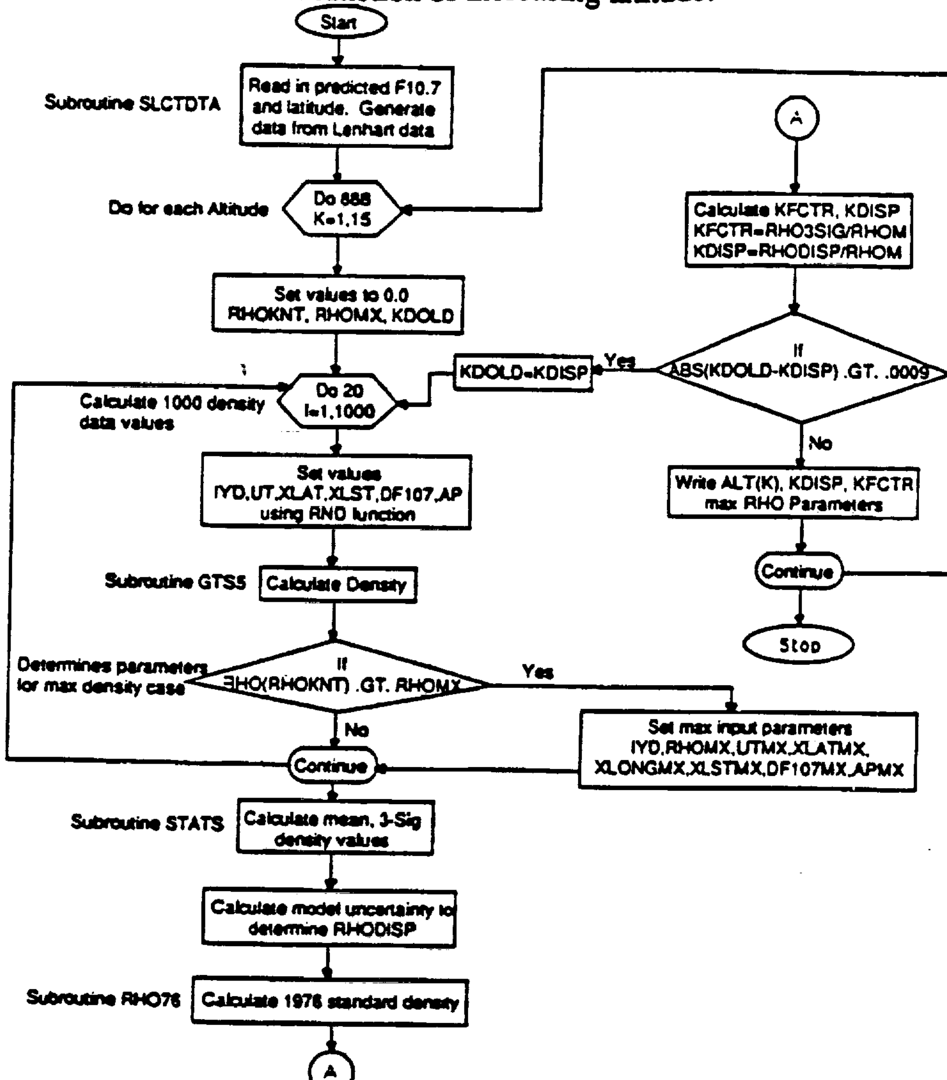


Figure 13. FMH K-Factor Calculation Computer Program Flow Chart

FMH K-factors were calculated at various altitude heights ranging from 134 to 305 km (72 to 165 nmi). For altitudes up to 133 km (72 nmi), 3-sigma dispersed densities calculated specifically for the Atlas/Centaur launch vehicle (Reference 18) are used. For altitudes greater than 305 km (165 nmi) the density is assumed to be negligible.

The entered F10.7 cm solar flux 13-month predicted average is used to determine the data set of daily values of F10.7, Ap, and calculated 3-month F10.7 averages extracted from measured historical data (see Measured Solar and Geomagnetic Index Data Section). The 13-month F10.7 average solar flux value, for a given day, represents the average of the 195 preceding and following F10.7 measured daily values. Using the Reference 17 measured historical data, a data set of daily F10.7 and corresponding Ap values can be determined which are representative of the predicted 13-month F10.7 solar flux value (user provided). When the predicted 13-month F10.7 value intersects the measured 13-month F10.7 value for past solar cycles, the previous and following daily measured F10.7 and Ap values are written to a data file for each intersection of measured 13-month F10.7 values. Additionally, because the MSIS-86 atmospheric model requires 3-month F10.7 averages as input, these values are calculated and included in this data set. Figure 14 illustrates the methodology of this data selection process.

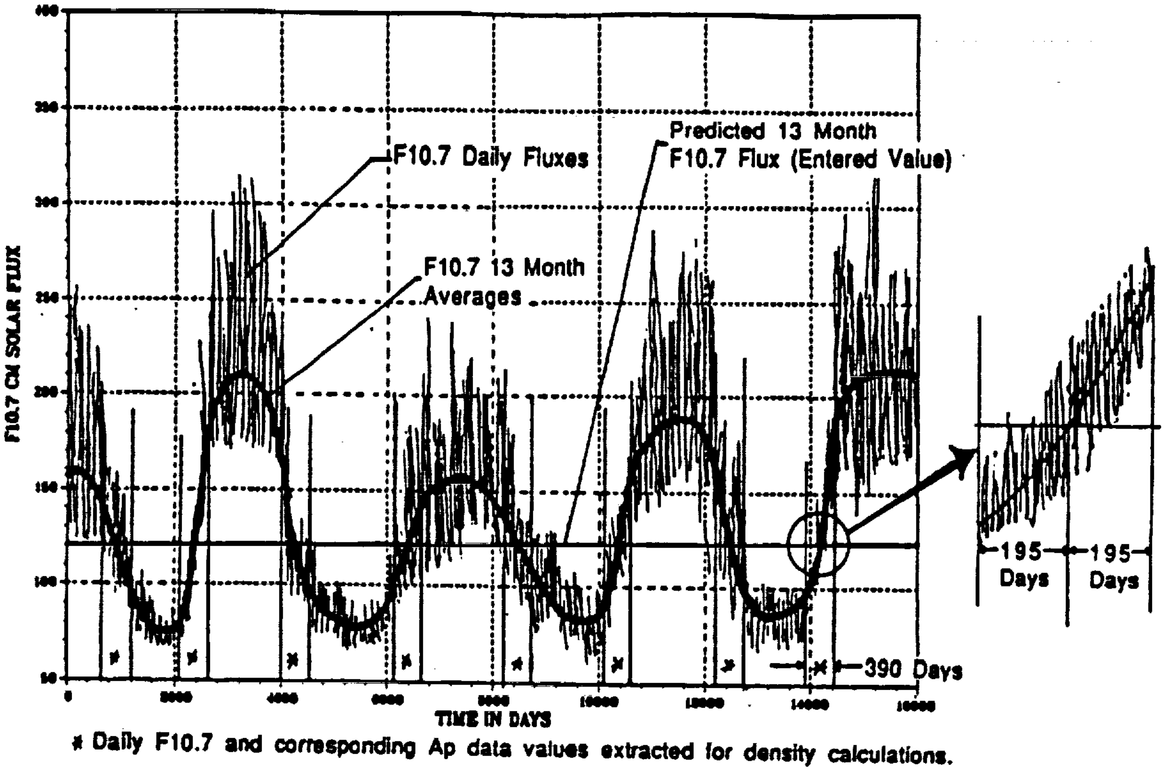


Figure 14. Illustration of the Data Selection Methodology in Determining Daily F10.7 and Ap Measured Values for the Calculation of FMH K-Factors.

Values of Ap and F10.7 daily and 3-month averages are randomly selected from this data file in performing the Monte Carlo statistical method (see Monte Carlo Statistical Technique Section). After performing this operation 1000 times, the 3-sigma dispersed atmospheric density is calculated using Equations 7 and 8.

After another 1000 density values are calculated and added to the data set, a new 3-sigma dispersed density is determined and compared to the original calculated 3-sigma density value. If the difference between these two values is less than the convergence criterion (set to 0.0009) it is assumed that a solution has been determined. If the convergence criterion has not been satisfied, the process is repeated. Once the convergence criterion has been satisfied, the FMH K-factor for a given altitude is calculated by dividing the 3-sigma dispersed density by the U.S. Standard 1976 atmospheric density (Reference 1).

K-Factor Results

Tables 2 and 3 list the calculated 3-sigma K-factors for the maximum recorded solar flux value (worst case) and worst case yearly values from 1992 to 2008. Table 2 lists the K-factors which are valid for missions with low Earth parking orbits circling the Earth between latitudes of ±60 degrees and Table 3 lists K-factors valid for all latitudes (±90 degrees). The predicted K-factor for a given year is based on the Reference 16 largest monthly predicted 2-sigma high 13-month F10.7 cm solar flux average for that year. If the month of a scheduled launch is known, less conservative K-factors could be calculated based on actual monthly values; however, it is recommended that yearly values be used to avoid invalidating analysis results due to a possible launch delay.

Because the K-factors are calculated using atmospheric densities based on random inputs, slight variations in the results can occur. The magnitude of these variations has been observed to be a function of altitude and entered 13-month average F10.7 solar flux with the variations increasing with increasing altitude and F10.7 value. Table 4 illustrates the calculated worst case predicted K-factors from six separate runs using the same input values. For low altitudes (less than 191km (625 kft)) differences in results are approximately ±0.02. For altitudes greater than 191 km (625 kft) the variation in the results were found to be approximately ±0.03.

It is noted here that the results of this analysis are only as good as the F10.7 cm solar flux predictions. The farther into the future the FMH K-factors are calculated, the greater the uncertainty of the prediction. Even though great effort has been made to predict conservative K-factors there have been recorded instances where abnormally high levels of geomagnetic disturbances and solar fluxes have occurred (Reference 19). In the unlikely event this were to happen, the predicted K-factor values would not be bounding. Even though there is a chance (beyond 3-sigma) this could occur, it would be unrealistic to design a space vehicle to withstand these levels.

Table 2. Worst Case Free Molecular Heating K-Factors vs. Altitude as a Function of Year for Latitudes Between ± 60°

Altitude Kft	Altitude nmi	Worst Case (244.0)	1992 (188.9)	1993 (151.0)	1994 (134.0)	1995 (112.3)	1996 (96.6)	1997 (78.5)	1998 (97.4)	1999 (186.6)
440.0	72.42	1.86	1.79	1.73	1.72	1.69	1.68	1.66	1.68	1.78
450.0	74.06	1.91	1.82	1.77	1.75	1.72	1.70	1.68	1.69	1.82
475.0	78.18	2.08	1.95	1.86	1.83	1.78	1.74	1.70	1.75	1.95
500.0	82.29	2.26	2.09	1.97	1.93	1.86	1.81	1.73	1.82	2.09
525.0	86.40	2.49	2.25	2.09	2.03	1.94	1.87	1.77	1.87	2.23
550.0	90.52	2.69	2.41	2.21	2.15	2.03	1.93	1.80	1.95	2.41
575.0	94.63	2.96	2.59	2.36	2.26	2.13	2.00	1.84	2.01	2.59
600.0	98.75	3.24	2.81	2.52	2.44	2.24	2.10	1.87	2.08	2.81
625.0	102.86	3.55	3.06	2.70	2.59	2.38	2.19	1.91	2.20	3.06
650.0	106.98	3.92	3.31	2.89	2.78	2.50	2.29	1.98	2.30	3.31
675.0	111.09	4.31	3.60	3.10	2.95	2.66	2.39	2.03	2.40	3.59
700.0	115.21	4.72	3.96	3.34	3.16	2.80	2.51	2.07	2.53	3.89
800.0	131.66	6.71	5.33	4.34	4.02	3.45	2.95	2.26	2.99	5.32
900.0	148.12	8.93	6.73	5.29	4.77	3.94	3.28	2.32	3.36	6.72
1000.0	164.58	11.15	8.04	5.95	5.31	4.23	3.37	2.26	3.45	7.97

Altitude Kft	Altitude nmi	2000 (231.4)	2001 (243.0)	2002 (231.0)	2003 (200.5)	2004 (154.3)	2005 (122.6)	2006 (109.9)	2007 (96.7)
440.0	72.42	1.84	1.86	1.84	1.79	1.74	1.70	1.69	1.68
450.0	74.06	1.89	1.91	1.88	1.84	1.78	1.73	1.71	1.69
475.0	78.18	2.04	2.07	2.05	1.95	1.87	1.81	1.78	1.75
500.0	82.29	2.22	2.28	2.21	2.09	1.98	1.89	1.85	1.80
525.0	86.40	2.41	2.48	2.42	2.27	2.12	1.99	1.94	1.87
550.0	90.52	2.61	2.69	2.63	2.43	2.26	2.08	2.02	1.93
575.0	94.63	2.87	2.94	2.85	2.62	2.40	2.21	2.12	1.99
600.0	98.75	3.15	3.23	3.13	2.85	2.56	2.34	2.23	2.09
625.0	102.86	3.43	3.56	3.43	3.10	2.76	2.46	2.35	2.19
650.0	106.98	3.78	3.90	3.76	3.34	2.95	2.62	2.49	2.29
675.0	111.09	4.13	4.29	4.12	3.67	3.17	2.80	2.63	2.38
700.0	115.21	4.53	4.75	4.55	3.97	3.44	2.96	2.78	2.50
800.0	131.66	6.46	6.76	6.42	5.44	4.49	3.71	3.40	2.95
900.0	148.12	8.44	8.94	8.43	6.96	5.41	4.35	3.85	3.26
1000.0	164.58	10.42	11.12	10.38	8.31	6.27	4.70	4.15	3.39

Note: Values in parentheses are entered 13-month averaged F10.7 values

Table 3. Worst Case Free Molecular Heating K-Factors vs. Altitude as a Function of Year for Latitudes Between ± 90°

Altitude Kft	Altitude nmi	Worst Case (244.0)	1992 (188.9)	1993 (151.0)	1994 (134.0)	1995 (112.3)	1996 (96.6)	1997 (78.5)	1998 (97.4)	1999 (186.6)
440.0	72.42	1.90	1.82	1.79	1.77	1.74	1.72	1.70	1.73	1.82
450.0	74.06	1.97	1.87	1.83	1.79	1.78	1.75	1.73	1.75	1.87
475.0	78.18	2.14	2.00	1.93	1.90	1.86	1.82	1.77	1.83	2.00
500.0	82.29	2.36	2.16	2.06	1.99	1.94	1.90	1.82	1.89	2.16
525.0	86.40	2.56	2.34	2.19	2.13	2.04	1.98	1.88	1.99	2.33
550.0	90.52	2.78	2.51	2.33	2.26	2.14	2.05	1.93	2.03	2.49
575.0	94.63	3.06	2.71	2.46	2.38	2.26	2.12	1.96	2.14	2.69
600.0	98.75	3.34	2.91	2.66	2.56	2.36	2.22	2.02	2.25	2.93
625.0	102.86	3.70	3.16	2.83	2.72	2.50	2.33	2.08	2.33	3.17
650.0	106.98	4.05	3.46	3.01	2.90	2.64	2.43	2.11	2.44	3.41
675.0	111.09	4.44	3.74	3.24	3.10	2.78	2.53	2.18	2.56	3.72
700.0	115.21	4.88	4.05	3.50	3.29	2.93	2.63	2.20	2.65	4.04
800.0	131.66	6.90	5.43	4.48	4.18	3.55	3.05	2.39	3.09	5.42
900.0	148.12	9.03	6.83	5.35	4.92	4.01	3.31	2.40	3.38	6.74
1000.0	164.58	11.05	7.99	5.96	5.35	4.20	3.38	2.25	3.42	7.94

Altitude Kft	Altitude nmi	2000 (231.4)	2001 (243.0)	2002 (231.0)	2003 (200.5)	2004 (154.3)	2005 (122.6)	2006 (109.9)	2007 (96.7)
440.0	72.42	1.88	1.90	1.87	1.81	1.79	1.76	1.75	1.73
450.0	74.06	1.92	1.97	1.93	1.87	1.82	1.79	1.78	1.75
475.0	78.18	2.09	2.13	2.10	2.00	1.93	1.88	1.85	1.83
500.0	82.29	2.29	2.33	2.27	2.15	2.06	1.98	1.95	1.90
525.0	86.40	2.49	2.57	2.49	2.33	2.22	2.07	2.05	1.98
550.0	90.52	2.69	2.79	2.71	2.51	2.35	2.18	2.14	2.05
575.0	94.63	2.98	3.03	2.96	2.71	2.51	2.32	2.24	2.13
600.0	98.75	3.24	3.36	3.19	2.96	2.69	2.44	2.39	2.21
625 0	102.86	3.54	3.70	3.55	3.20	2.88	2.59	2.49	2.32
650 0	106.98	3.89	4.07	3.90	3.48	3.08	2.75	2.62	2.45
675.0	111.09	4.27	4.41	4.26	3.78	3.30	2.90	2.75	2.50
700.0	115.21	4.68	4.85	4.68	4.14	3.56	3.09	2.91	2.66
800.0	131.66	6.56	6.90	6.57	5.49	4.59	3.82	3.50	3.08
900.0	148.12	8.50	9.03	8.49	7.03	5.54	4.38	3.93	3.32
1000.0	164.58	10.26	10.97	10.30	8.17	6.17	4.71	4.12	3.40

Note: Values in parentheses are entered 13-month averaged F10.7 values

Table 4. Worst Case Variations in the Calculation of FMH K-Factors

Altitude		Calculated K-Factors Assuming Latitude 90 deg and F10.7 = 244.0						Average	Max Delta
Kft	nmi								
440.0	72.42	1.90	1.90	1.90	1.90	1.90	1.89	1.90	0.01
450.0	74.06	1.97	1.96	1.95	1.97	1.96	1.96	1.96	0.01
475.0	78.18	2.14	2.12	2.14	2.13	2.13	2.14	2.13	0.01
500.0	82.29	2.36	2.33	2.34	2.33	2.33	2.36	2.34	0.02
525.0	86.40	2.56	2.57	2.56	2.56	2.55	2.56	2.56	0.01
550.0	90.52	2.78	2.80	2.79	2.80	2.78	2.78	2.79	0.01
575.0	94.63	3.06	3.06	3.04	3.04	3.07	3.05	3.05	0.02
600.0	98.75	3.34	3.36	3.36	3.35	3.35	3.35	3.35	0.01
625.0	102.86	3.70	3.69	3.68	3.68	3.68	3.70	3.69	0.01
650.0	106.98	4.05	4.04	4.05	4.04	4.01	4.04	4.04	0.03
675.0	111.09	4.44	4.42	4.47	4.43	4.45	4.45	4.44	0.03
700.0	115.21	4.88	4.90	4.87	4.88	4.87	4.90	4.88	0.01
800.0	131.66	6.90	6.89	6.88	6.90	6.86	6.89	6.89	0.03
900.0	148.12	9.03	9.01	9.00	9.02	9.02	9.01	9.01	0.02
1000.0	164.58	11.05	11.07	11.03	11.01	11.01	11.04	11.04	0.03

Note: Max Delta = Maximum difference between average and predicted value for a given altitude

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APPENDIX

MSIS-86 Input Parameters Parametric Analysis

To determine the trends and sensitivity each MSIS-86 input parameter has on the calculation of K-factor, a parametric analysis was performed. For this analysis "standard" values were assumed for each input (Table A1). By varying one parameter at a time between its minimum and maximum value, while constraining the remaining inputs, nominal K-factors were calculated as a function of altitude and MSIS-86 inputs. Table A1 lists the assumed standard values and the dispersed ranges for each parameter.

Table A1. MSIS-86 Input Parameters Standard Values and Dispersions Assumed in the Parametric Analysis

PARAMETER	STANDARD VALUE	DISPERSION
Latitude (deg)	-30	-60 to 60
Longitude (deg)	150	0 to 360
Local Solar Time (hr)	13	0 to 24
Universal Time (sec)	25200	0 to 90000
Day of Year (day)	300	0 to 360
Average F10.7 Solar Flux	125	75 to 250
Daily F10.7 Solar Flux	150	75 to 300
Geomagnetic Index	20	5 to 80

The results for each input parameter are illustrated in Figures A1 through A8. Since these parameters are dependent on each other, a different set of standard values can significantly change the results. These results are only presented to show the sensitivity and trends for each input in the calculation of K-factor, and are not to be used as a method to determine K-factors.

MSIS-86 INPUTS
LONG = 150 (deg)
LST = 13 (Hr)
UT = 25200 (sec)
IYD = 300 (day)
AF10.7 = 125
DF10.7 = 150
Ap = 20

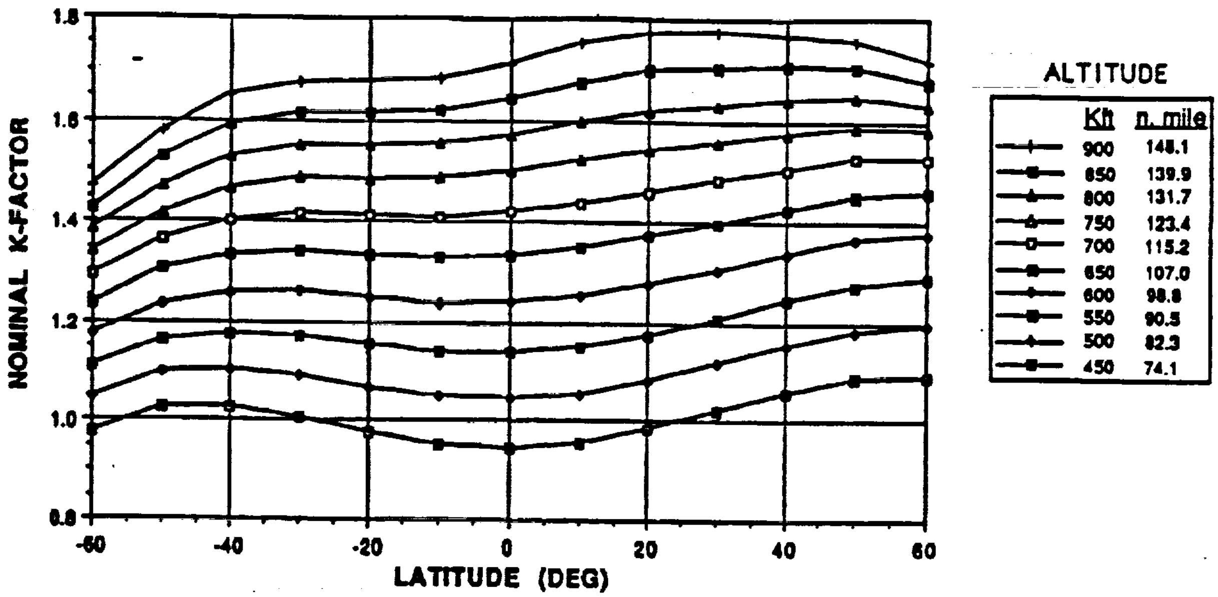

Figure A1. Nominal K-factor Calculation as a funciton of Altitude and Latinude For a Given Set of MSIS-86 Inupt Parameters.

MSIS-86 INPUTS
LAT = -30 (deg)
LST = 13 (Hr)
UT = 25200 (sec)
IYD = 300 (day)
AF10.7 = 125
DF10.7 = 150
Ap = 20

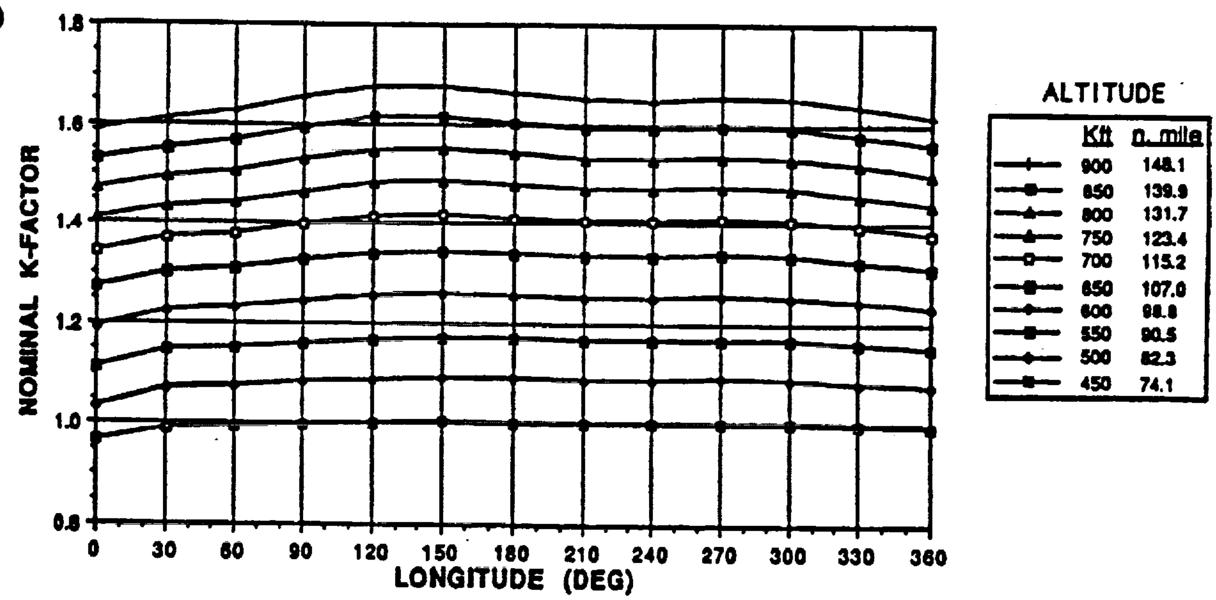

Figure A2 Nominal K-factor Calculation as a funciton of Altitude and Longitude For a Given Set of MSIS-86 Inupt Parameters.

MSIS-86 INPUTS
LAT = -30 (deg)
LONG = 150 (deg)
UT = 25200 (sec)
IYD = 300 (day)
AF10.7 = 125
DF10.7 = 150
Ap = 20

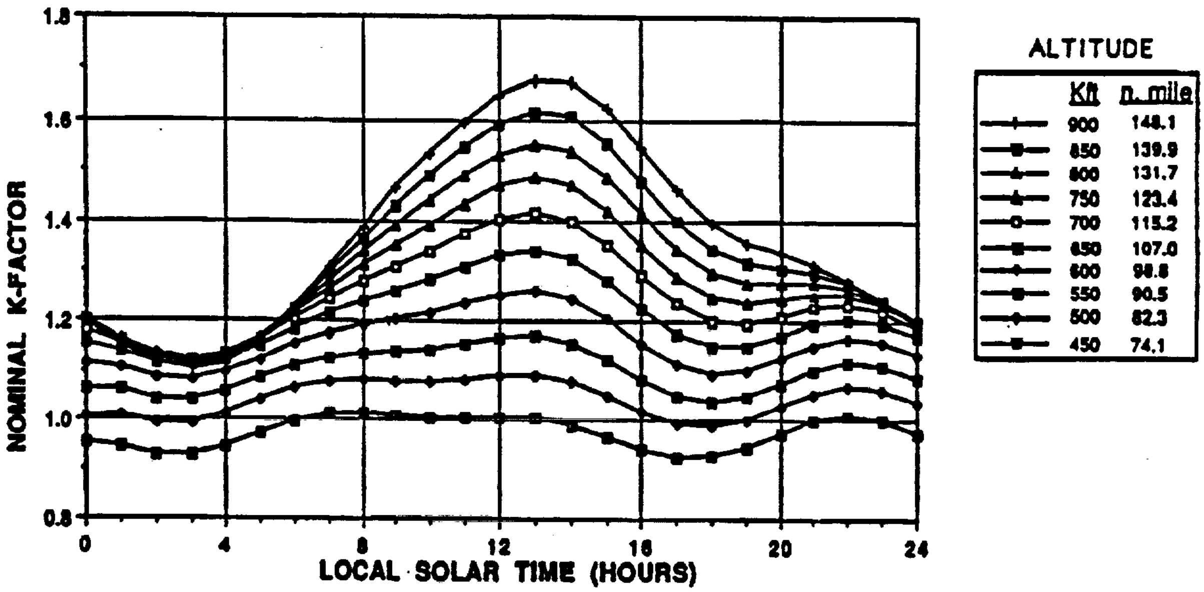

Figure A3. Nominal K-factor Calculation as a funciton of Altitude and Local Solar Time For a Given Set of MSIS-86 Inupt Parameters.

MSIS-86 INPUTS
LAT = -30 (deg)
LONG = 150 (deg)
LST = 13 (Hr)
IYD = 300 (day)
AF10.7 = 125
DF10.7 = 150
Ap = 20

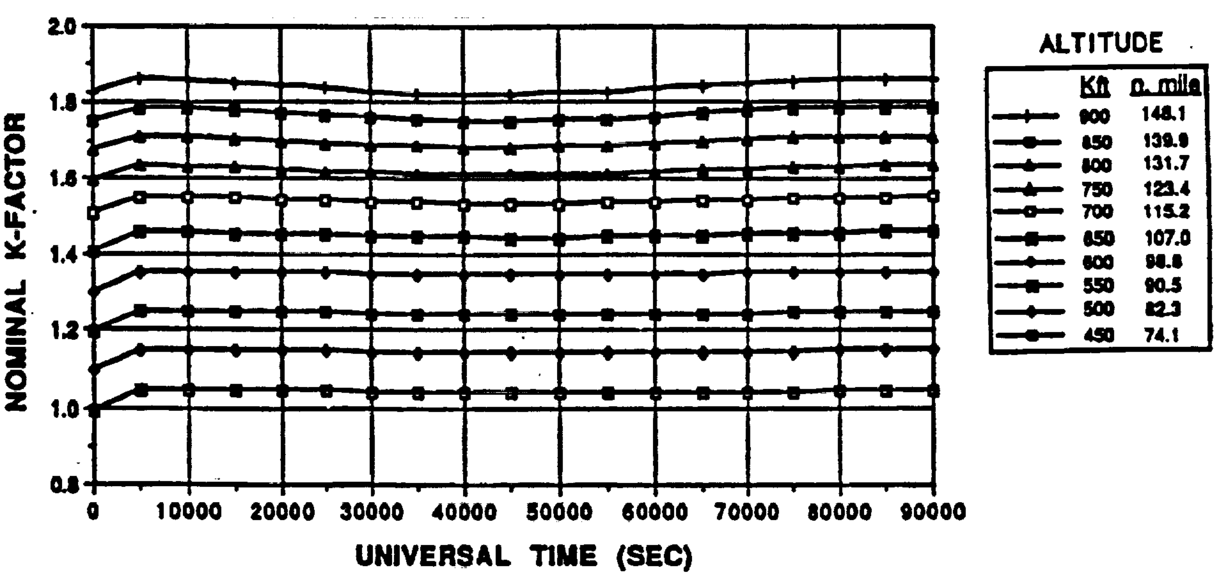

Figure A4. Nominal K-factor Calculation as a funciton of Altitude and Universal Time For a Given Set of MSIS-86 Inupt Parameters.

MSIS-86 INPUTS
LAT = -30 (deg)
LONG = 150 (deg)
LST = 13 (Hr)
UT = 25200 (sec)
AF10.7 = 125
DF10.7 = 150
Ap = 20

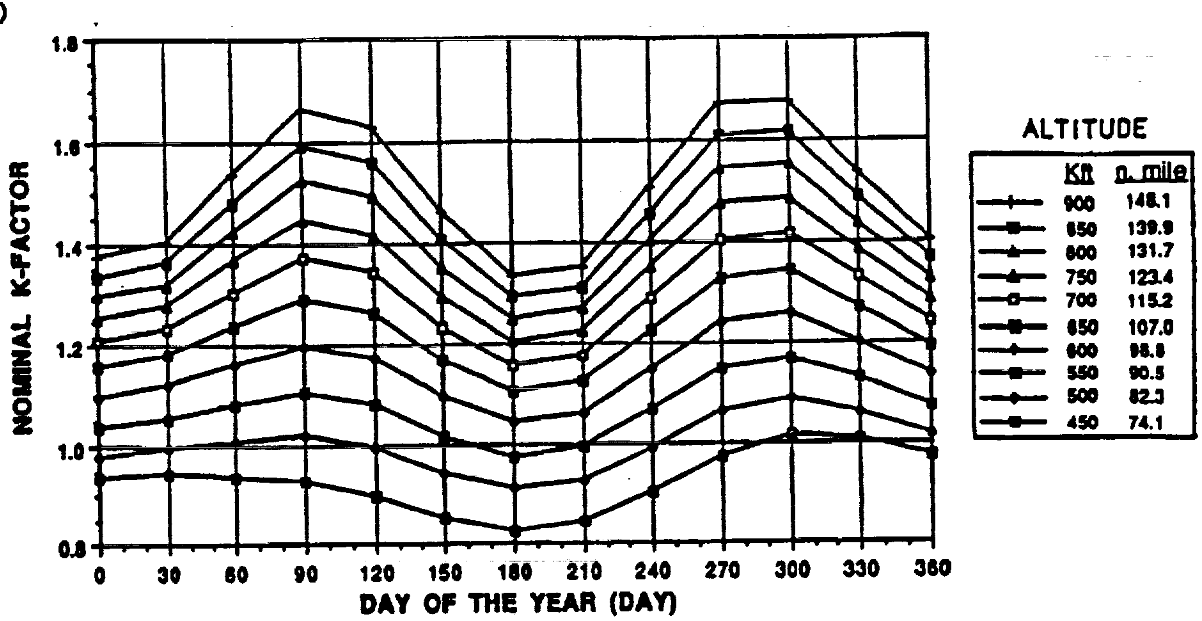

Figure A5. Nominal K-factor Calculation as a funciton of Altitude and Day of Year For a Given Set of MSIS-86 Inupt Parameters.

MSIS-86 INPUTS
LAT = -30 (deg)
LONG = 150 (deg)
LST = 13 (Hr)
UT = 25200 (sec)
IYD = 300 (day)
DF10.7 = 150
Ap = 20

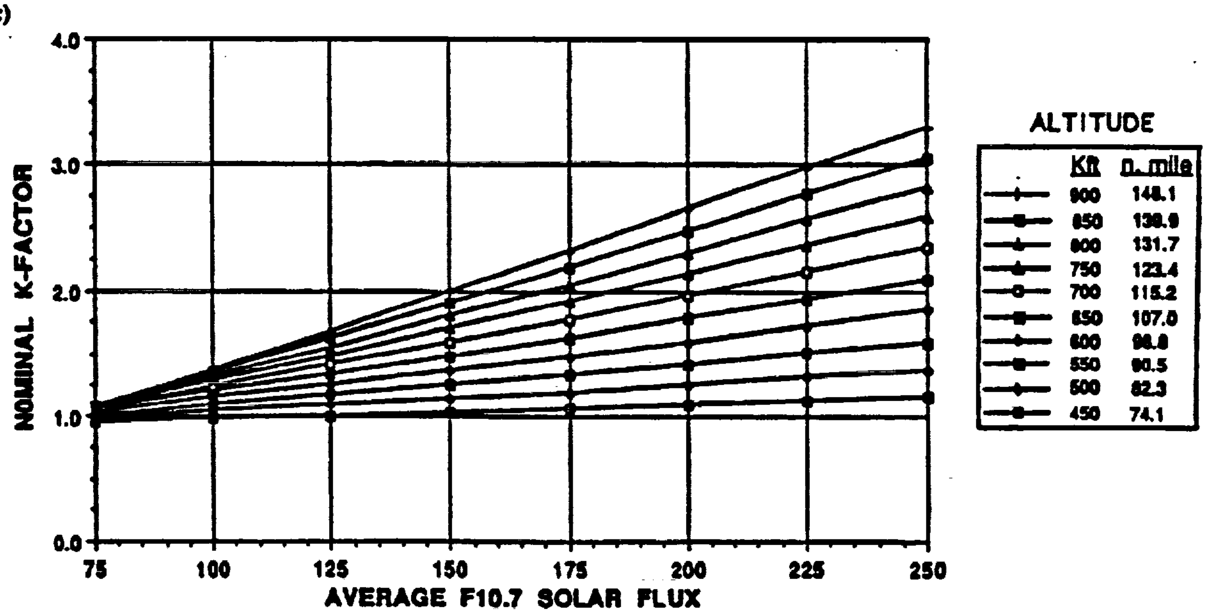

Figure A6. Nominal K-factor Calculation as a funciton of Altitude and Average F10.7 Solar Flux For a Given Set of MSIS-86 Inupt Parameters.

MSIS-86 INPUTS
LAT = -30 (deg)
LONG = 150 (deg)
LST = 13 (Hr)
UT = 25200 (sec)
IYD = 300 (day)
AF10.7 = 125
Ap = 20

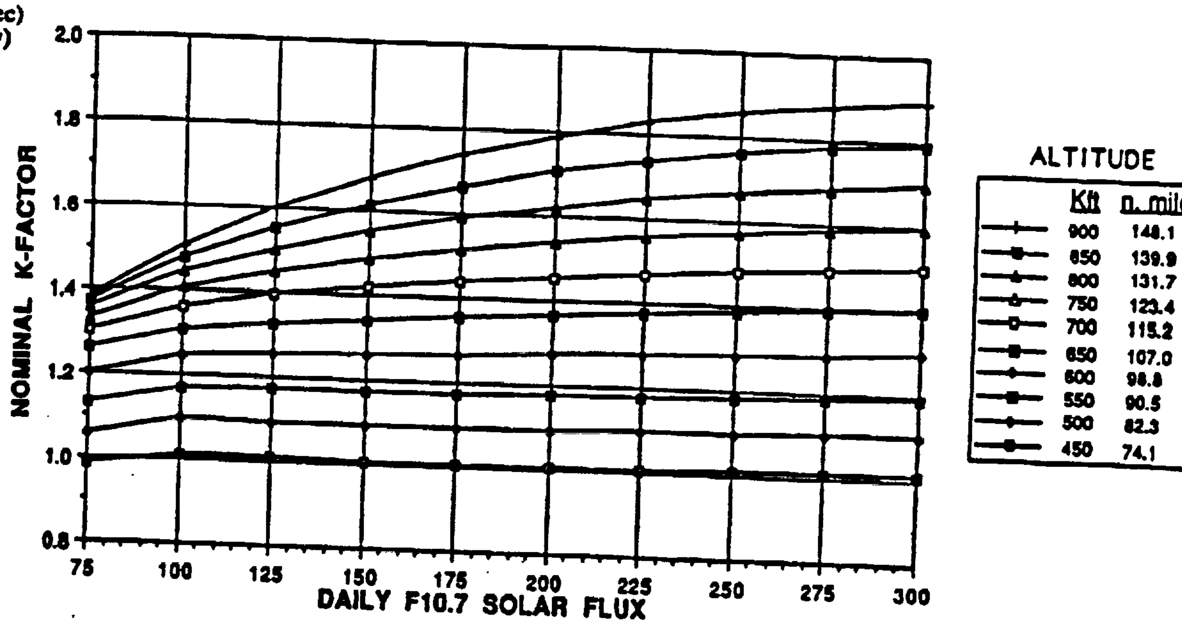

Figure A7. Nominal K-factor Calculation as a funciton of Altitude and Daily F10.7 Solar Flux For a Given Set of MSIS-86 Inupt Parameters.

MSIS-86 INPUTS
LAT = -30 (deg)
LONG = 150 (deg)
LST = 13 (Hr)
UT = 25200 (sec)
IYD = 300 (day)
AF10.7 = 125
DF10.7 = 150

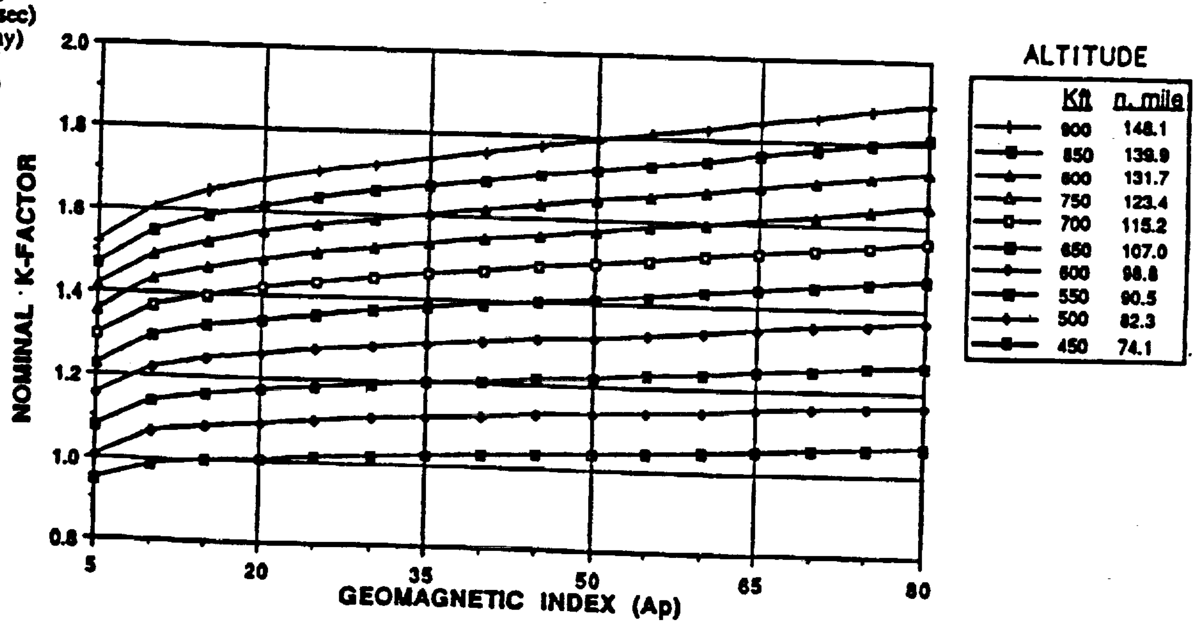

Figure A8. Nominal K-factor Calculation as a funciton of Altitude and Geomagnetic Index For a Given Set of MSIS-86 Inupt Parameters.