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AN ANALYTIC STUDY OF A TWO-PHASE LAMINAR AIRFOIL IN SIMULATED HEAVY RAIN

by

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Nomenclature

English letters

- a = indicates air
- c = chord length
- D = drop
- f = subscript, denotes fog or vapor state
- i = subscript, denotes incident
- L = characteristic length
- t = time
- U_2 = free stream velocity
- u,v = velocity components along x-y-directions respectively
- w = subscript, denotes water
- x,y = physical coordinates

Greek letters

- α = angle of attack, or non-dimensional ration = W_1/ρ_w
- β = droplet incident angle
- ρ = density of the fluid
- ρ_v = density of the vapor
- ρ_1 = density of the liquid
- μ = coefficient of viscosity
- δ = thickness of the boundary layers
- υ = kinematic coefficient of viscosity = μ/ρ

SUMMARY

A mathematical model for a two-phase flow laminar airfoil in simulated heavy rain has been established.

The set of non-linear partial differential equations has been converted into a set of finite difference equations; appropriate initial and boundary conditions are provided. The numerical results are compared with the experimental measurements. They show good agreement in quality.

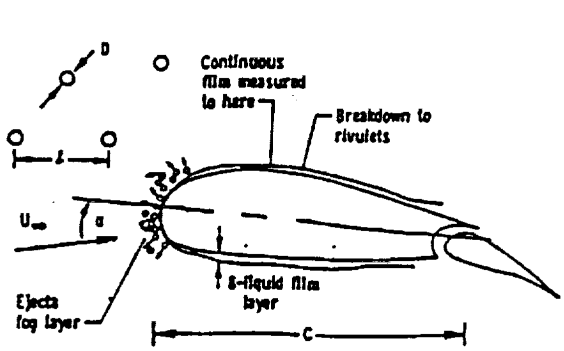
I. INTRODUCTION

This paper investigates the rain effects to the airfoil during landing and take-off. When the heavy rain hits the àirfoil, the fine water drops near to the wing surface form a liquid film while the water vapor above the wing surface establishes a gaseous fog. The former is in liquid state, the latter is in gaseous state. This is the so-called two-phase phenomenon. The thin liquid film close to the wing surface usually forms a laminar boundary layer. However, the heavy rain fall may cause a premature turbulent boundary layer. That is, the heavy rain effects may cause the laminar boundary layer transit into turbulent boundary layer. As a result, the lift is appreciably decreased, while the drag is considerably increased. In a a more serious situation, the decrease in lift and increase in drag may develop into wind shear which causes an airplane catastrophe involving loss of human life and property damage. This important knowledge should keep the pilot well-informed as a precaution of flight safety. A variety of experimental and analytic methods used to investigate the heavy rain effects on airfoil may be referred to in previous works, notably references # 1 through # 13. This paper has established a mathematical model for the airfoil under the simulated heavy rain. The said mathematical model consists of a set of non-linear partial differential equations for which a numerical solution has been obtained. A comparison between theory and experiment has been made and will shed some light on flight safety.

II. FORMULATION

As in the following figure, the two-phase-flow around an airfoil is clearly

RAINDROPS INTERACTING WITH AN AIRFOIL



FIGURE(1)

described. To simplify the problem the following assumptions are made:

1. The fluid flow is non-steady, viscous, and incompressible.
2. There is laminar boundary in the flow region.
3. The airfoil is represented by a flat plate. The physical coordinates are shown in Figure (2).

For a two-dimensional, nonsteady two-phase flow, we obtain the following sets of fundamental equations under boundary layer approximation: For the liquid phase

Equation of Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = \frac{W_L}{\rho_\infty}\,\frac{V_1 \sin\beta}{\delta} \quad \ldots \ldots \qquad (1)$$

Equation of Momentum : $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}+ v\frac{\partial u}{\partial y} = \frac{\mu}{\rho_w}\,\frac{\partial^2 u}{\partial y^2}\;\frac{W_L V_1^2 \sin\beta\cos\beta}{\rho_w \delta} \quad \ldots \ldots \qquad (2)$$

Film thickness: $$\delta = 5\sqrt{\upsilon x / U_2} \quad \ldots \ldots \qquad (3)$$

For the fog phase Equation of Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots \ldots \qquad (4)$$

Equation of Momentum: $$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho_f}\,\frac{\partial^2 u}{\partial y^2} \quad \ldots \ldots \qquad (5)$$

The initial conditions are:

$$\text{at } t = 0,\ u = U_2(x,O),\ V = 0,\ \delta = 0 \quad \ldots \ldots \qquad (6)$$

The boundary conditions are:

y = 0, for the liquid phase
u = 0, v = 0 (no slip condition) } ...... (7)

at y = δ (liquid-fog interface)

$(u)1 = (u)f \qquad (v)1 = (v)f$

$$\left(\mu\frac{\partial u}{\partial y}\right)_1 = \left(\mu\frac{\partial u}{\partial y}\right)_f$$

} ...... (8)

In other words, U, V, and $\partial u/\partial y$ for both liquid and fog must be compatible at the interface, as $y \to \infty$ (for the fog phase), $u = U(x,t)$ ... ... (9)

III. NON-DIMENSIONALIZATION

In order to non-dimensionalize the set of fundamental equations, we introduce the following:

$$\left.\begin{array}{llll} \bar{x} = \dfrac{x}{L} & \bar{y} = \dfrac{y}{L} & \bar{u} = \dfrac{u}{U_c} & \bar{v} = \dfrac{v}{U_c} \\ \bar{\delta} = \dfrac{\delta}{L} & \tau = \dfrac{U_c T}{L} & \bar{V_i} = \dfrac{V_i}{U_c} & \end{array}\right\} \quad \ldots \ldots \qquad (10)$$

where Uc is the reference velocity, and L is the chord length of the airfoil. Substituting the above relations into Equations (1), (2), (3), (4) and (5), and omitting the bar notations, we obtain:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{W_1}{\rho_w}\frac{V_1}{\delta}\sin\beta \ldots\ldots \qquad (11)$$

$$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{(RN)_1}\frac{\partial^2 u}{\partial y^2} + \frac{W_\ell}{\rho_w}\frac{V_1^2 \sin\beta\cos\beta}{\delta} \ldots\ldots \qquad (12)$$

$$\delta = 5\sqrt{\frac{\nu x}{U_2}}/L \qquad \ldots\ \ldots \qquad (13)$$

In Equ. (12), W_1 has the dimension of M/L^3, so does the density ρ. W_1/ρ_w is a non-dimensional quantity, where $(RN)_1 = \rho_w U_2 x/\mu$ Reynolds number of the liquid water. We let $\alpha = W_l/\rho_w =$ non-dimensional ratio of the liquid water content to the water density.

For the fog phase

Equation of Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \ldots\ldots \qquad (14)$$

Equation of momentum

$$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{(RN)_f}\frac{\partial^2 u}{\partial y^2} \quad \dots \dots \qquad (15)$$

where $(RN)_f = \dfrac{\rho_f U_2 L}{\mu}$ = Reynolds number of the fog.

The corresponding initial and boundary conditions are:

$$\left.\begin{array}{l} \text{at } \tau = 0, \quad u = U(x,o), \quad V = 0 \\ \delta = 0 \end{array}\right\} \dots \dots \qquad (6.a)$$

for the liquid phase, at y = 0, u = v=o (No slip condition) . . . . (7.a)

at $y = \delta/L$ (Liquid-vapor interface)

$$\left.\begin{array}{l} (u)_1 = (u)_f \qquad (V)_1 = (V)_f \\ \left(\mu\dfrac{\partial u}{\partial y}\right)_1 = \left(\mu\dfrac{\partial u}{\partial y}\right)_f \end{array}\right\} \dots \dots \qquad (8.a)$$

As $y \to \infty \quad u = U(x,\tau)$ . . . . . . (9.a)

IV. FINITE DIFFERENCE EQUATIONS

The previously derived set of non-dimensional partial differential equations can be transformed into the finite difference equations in the following manner: An explicit method is used. Let U', V' and δ' denote the values of U, V and δ at the end of a time-step. Then the appropriate sets of finite difference equations are:

For the liquid

Equation of Continuity:

$$\frac{U'_{i,j} - U'_{i-1,j}}{\Delta x} + \frac{V'_{i,j} - V'_{i,j-1}}{\Delta y} = \alpha \frac{V_1}{\delta'_{i,j}} \sin\beta \ldots\ldots \quad (11.a)$$

Equation of Momentum:

$$\frac{U'_{i,j} - U_{i,j}}{\Delta\tau} + U_{i,j} \frac{(U'_{i,j} - U'_{i-1,j})}{\Delta x} + V_{i,j} \frac{(U_{i,j+1} - U_{i,j})}{\Delta y}$$

$$= \frac{1}{(RN)_1} \frac{(U_{i,j+1} - 2U_{i,j} + U_{i,j-1})}{(\Delta y)^2} + \frac{\alpha}{\delta'_{i,j}} V_1' \sin\beta\cos\beta \ldots\ldots \quad (12.a)$$

Film thickness:

$$\delta'_{i,j} = 5 \sqrt{\frac{\nu I \Delta x}{U_2}} / L \ldots\ldots \quad (13.a)$$

For the fog

Equation of Continuity:

$$\frac{U'_{i,j} - U'_{i-1,j}}{\Delta x} + \frac{V'_{i,j} - V'_{i,j-1}}{\Delta y} = 0 \quad \ldots\ldots \quad (14.a)$$

Equation of Momentum:

$$\frac{U'_{i,j} - U_{i,j}}{\Delta\tau} + U_{i,j} \frac{(U_{i,j} - U_{i-1,j})}{\Delta x} + V_{i,j} \frac{(U_{i,j+1} - U_{i,j})}{\Delta y}$$

$$= \frac{1}{(RN)_f} \frac{(U_{i,j+1} - 2U_{i,j} + U_{i,j-1})}{(\Delta y)^2} \ldots\ldots\ldots \quad (15.a)$$

The boundary conditions are

$$\left.\begin{array}{llll} \text{at} & \tau = 0, & u = U_2, & v = 0 \\ & & \delta = 0 & \end{array}\right\} \ldots\ \ldots \quad (16)$$

for the liquid phase, $y = 0$, $U=V=0$ .... (17) . At the interface $0 < x < L$, $O < y$
$y < \delta_{max}$. We choose integers M_x and M_y such that

$$\left.\begin{aligned} (V_{o,j})_1 &= (V_{o,j})_f \quad (V_{M_x,j})_1 = (V_{M_x,j})_f \\ (V_{i,o})_1 &= (V_{i,o})_f \quad (V_{i,M_y})_1 = (V_{i,M_y})_f \end{aligned}\right\} \ldots\ldots\ldots \qquad (16.a)$$

$$\left.\begin{aligned} (U_{o,j})_1 &= (U_{o,j})_f \quad (U_{M_x,j})_1 = (U_{M_x,j})_f \\ (U_{i,o})_1 &= (U_{i,o})_f \quad (U_{i,M_y})_1 = (U_{i,M_y}) \end{aligned}\right\} \ldots\ldots\ldots \qquad (16.b)$$

All the points denoted by Equations (16.a) and (16.b) are boundary points at which the values of U and V are already known. Furthermore the shearing force of the interface is denoted by the following relation

$$\left[\mu \frac{(U_{i,j+1} - U_{i,j})}{\Delta y}\right]_1 = \left[\mu \frac{(U_{i,j+1} - U_{i,j})}{\Delta y}\right]_f \ldots\ldots \qquad (17.a)$$

V. THE STABILITY OF THE FINITE-DIFFERENCE EQUATIONS

Since an explicit procedure is used, we wish to know the largest time-step consistent with stability. Equation of continuity is ignored since $\Delta\tau$ does not appear in it. The general terms of the Fourier expansion for U at a time arbitrarily called $\tau=0$ are both $e^{i\alpha x}$ $e^{i\beta y}$, apart from a constant (Here $i = \sqrt{-1}$). At a time τ later, these terms will become

$$U: \ \psi(\tau)\ e^{i\alpha x} \cdot e^{i\beta y}$$

Substituting the above into Equation (12.a), regarding the coefficients U and V as constants over any one step, and denoting the values after time-step by ψ' gives

$$\frac{\psi' - \psi}{\Delta\tau} + U\frac{\psi[1 - e^{-i\alpha\Delta x}]}{\Delta x} + V\frac{\psi[e^{i\beta\Delta y} - 1]}{\Delta y}$$

$$= \frac{1}{RN}\ \frac{2\psi(\tau)[\cos(\beta\Delta y) - 1]}{(\Delta y)^2} + \frac{\alpha}{\delta_{i,j}}\ V_i^2\ \sin\beta\cos\beta e^{-i(\alpha x+\beta y)} \qquad (18)$$

Through a very tedious algebraic manipulation, there obtains the criterion of stability:

$$U \frac{\Delta\tau}{\Delta x} - V \frac{\Delta\tau}{\Delta y} + \frac{2\Delta\tau}{RN(\Delta y)^2} \leq 1 \quad \ldots\ldots\ldots \qquad (19)$$

In the present research, R_N, the Reynolds number, is in the order of 10^6. Equation (19) follows automatically. However, the coefficients U and V, treated as constants over any one time-step, will vary from one time-step to the next in a manner which cannot be predicted as a priori. That is the maximum permissible time-step consistent with stability and is itself variable, but its value can always be checked during computation if necessary.

VI. CALCULATION OF LIFT COEFFICIENT

According to Glauert, the lift coefficient is given by

$$C_L = \pi(\alpha+\varepsilon_0) \quad \ldots \ \ldots \qquad (20)$$

where d is the angle of attack,

$$\varepsilon_o = \int_0^1 \left(\frac{y}{c}\right) f_1\left(\frac{x}{c}\right) d\left(\frac{x}{c}\right) \quad \ldots \ \ldots \qquad (21)$$

$$\text{and } f_1(x/c) = \frac{1}{\pi(1-\frac{x}{c})\sqrt{\frac{x}{c}(1-\frac{x}{c})}} \quad \ldots \ \ldots \qquad (22)$$

The relationship between (x/c) and f (x/c) is :

(x/c)	0.025	0.05	0.10	0.20	0.30	0.40
$f_1(x/c)$	2.090	1.54	1.18	1.00	0.99	1.08
(x/c)	0.50	0.60	0.70	0.80	0.90	0.95
$f_1(x/c)$	1.27	1.62	2.31	3.98	10.60	29.20

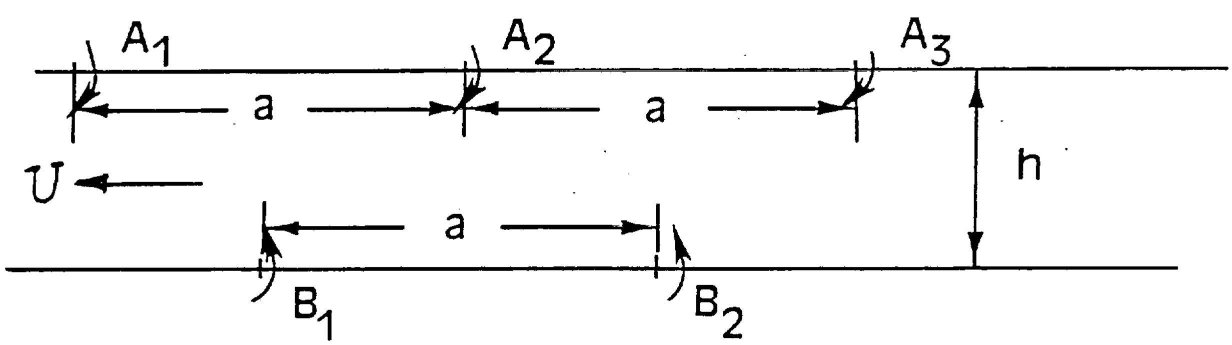
For Wortmann airfoil, the value of (y/c) for each (x/c) can be found. The integration can be performed easily. However, for flat plate airfoil, ε_0 = O, the lift coefficent reduces to $C_L = \pi\alpha$.

VII. CALCULATION OF DRAG COEFFICIENT

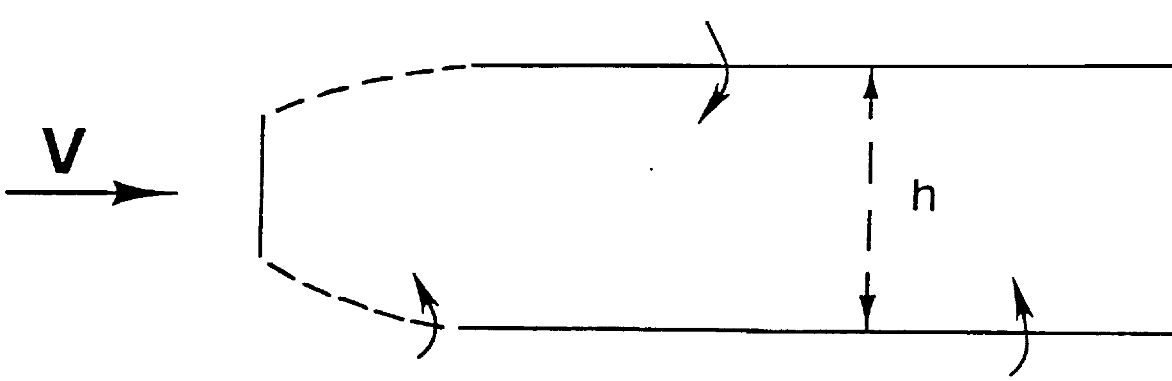
A body of buff form, particularly if it has sharp edges like a flat plate (which is the case here) will shed strong vortices and will have a large form drag. As a first approximation, the form drag coefficient may be written

$$C_D = 2.83\left(\frac{h}{b}\right)\bullet\frac{U}{V} = 0.281\,\frac{K}{bV} \quad \cdots\;\cdots \qquad (23)$$

where $K = 2\sqrt{2}\ a\ U$, is the strength of each point vertex, the so- called Karman vortex street, a is the distance separating the successive vortices of each row, U is the induced velocity, h is the distance between the two rows, i.e. the breadth of the street. A sketch for von Karman vortex street is given as follows :



Two Vortex Rows



Von Karman Vortex Street

Figure (3).

The skin frictional drag coefficient of a flat plate is given by a classical formula:

$$Cd = 1.328/\sqrt{RN} \qquad \ldots\ \ldots \qquad (24)$$

The historical results and the current computation are tabulated as follows:

Reynolds No.	1.14×10^4	0.57×10^4	3×10^5	10^6	7×10^6
Experimental Measurement			0.0057	0.0047	0.0035
Karman			0.0058	0.0045	0.0031
Blasius			0.0024	0.0013	0.0005
Current Computation	0.0124 (Dry)	0.0176 (Wet)			

As shown in the above table, the drag coefficient of the airfoil with rain is higher than that without rain. The computed results are much lower than those measured by Hansman(in references 5,and 6).The reason for this is that the over-simplified mathematical model of an airfoil by a flat plate does not cover the physical reality of a true airfoil which has proper thickness and suitable chord length.

Also the premature transition of laminar flow into turbulent flow may cause higher drag coefficient . This possible turbulent phenomenon is not covered due to our previous assumptions.

The superposition of form drag to the skin frictional drag should improve the agreement in quantity with the experimental measurement. However, the analytic determination of form drag in Eq.(23) remains to be a difficult task for further investigation.

VIII Conclusion and Discussion

The set of finite difference equations has been converted into Fortran language, and a numerical solution is obtained. The complexity and computation time are far beyond the investigator's anticipation. For the numerical results it was found that there is decrease in lift and increase in drag due to the heavy rain effects. The reason for this is that the heavy rain causes roughness on the wet surface of the airfoil. A comparison of the present numerical calculation with Hansman's [6] experimental measurement is shown in Figure (5). According to Hansman, the Wortmann section had the greatest lift degradation: nearly 25%. The computed decrease in lift and increase in drag are both lower than that of the experimental measurements. The reason may be that the over-simplified mathematical model could not cover the physical reality, such as the premature boundary layer transition near the leading edge of the airfoil and the three-dimensional effects.

The laminar layer thickness of both the liquid film and fog is shown in Figure (6). The velocity profile near the surface of the airfoil is shown in Figure (4). It is evident that the U-component of the velocity near the surface of the airfoil is decreased due to the rain-effects.

It requires further investigation for a turbulence model and three-dimensional wing in order to accomplish perfect agreement between analytical investigation and experimental measurements.

IX. Acknowledgements

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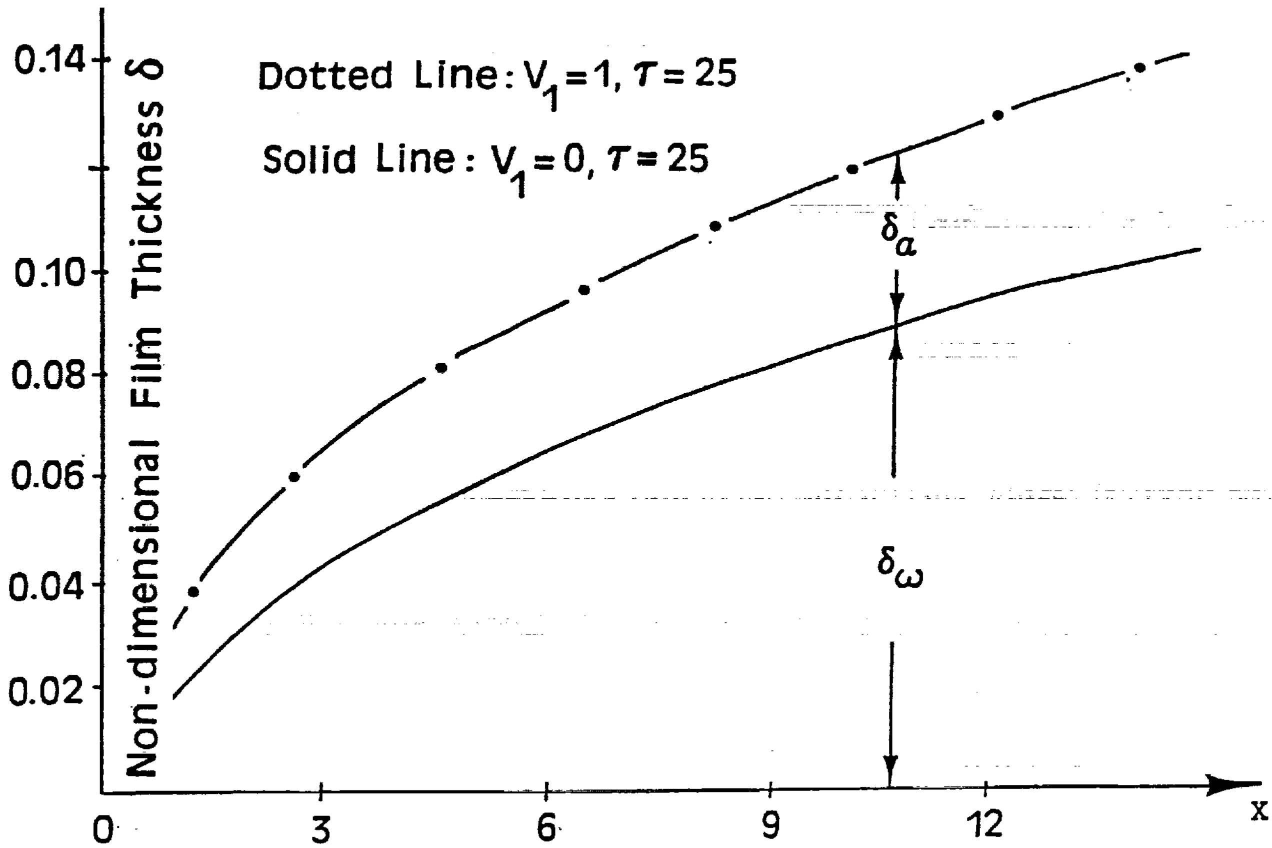


Figure (6). X-chordwise Length.

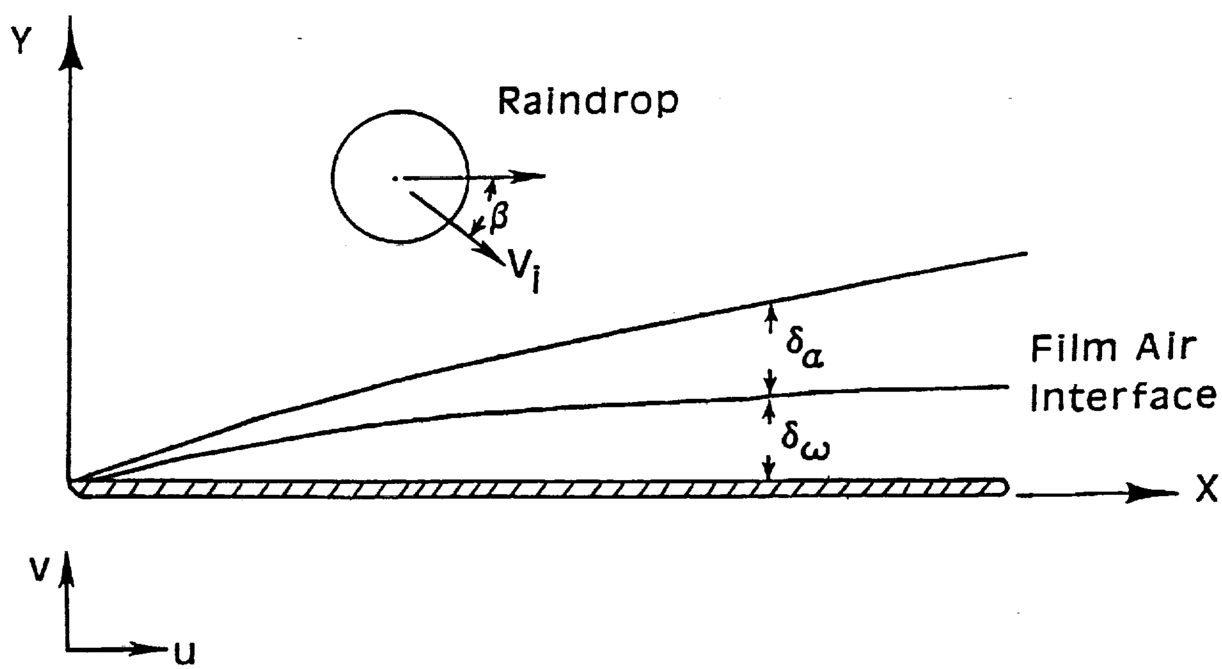
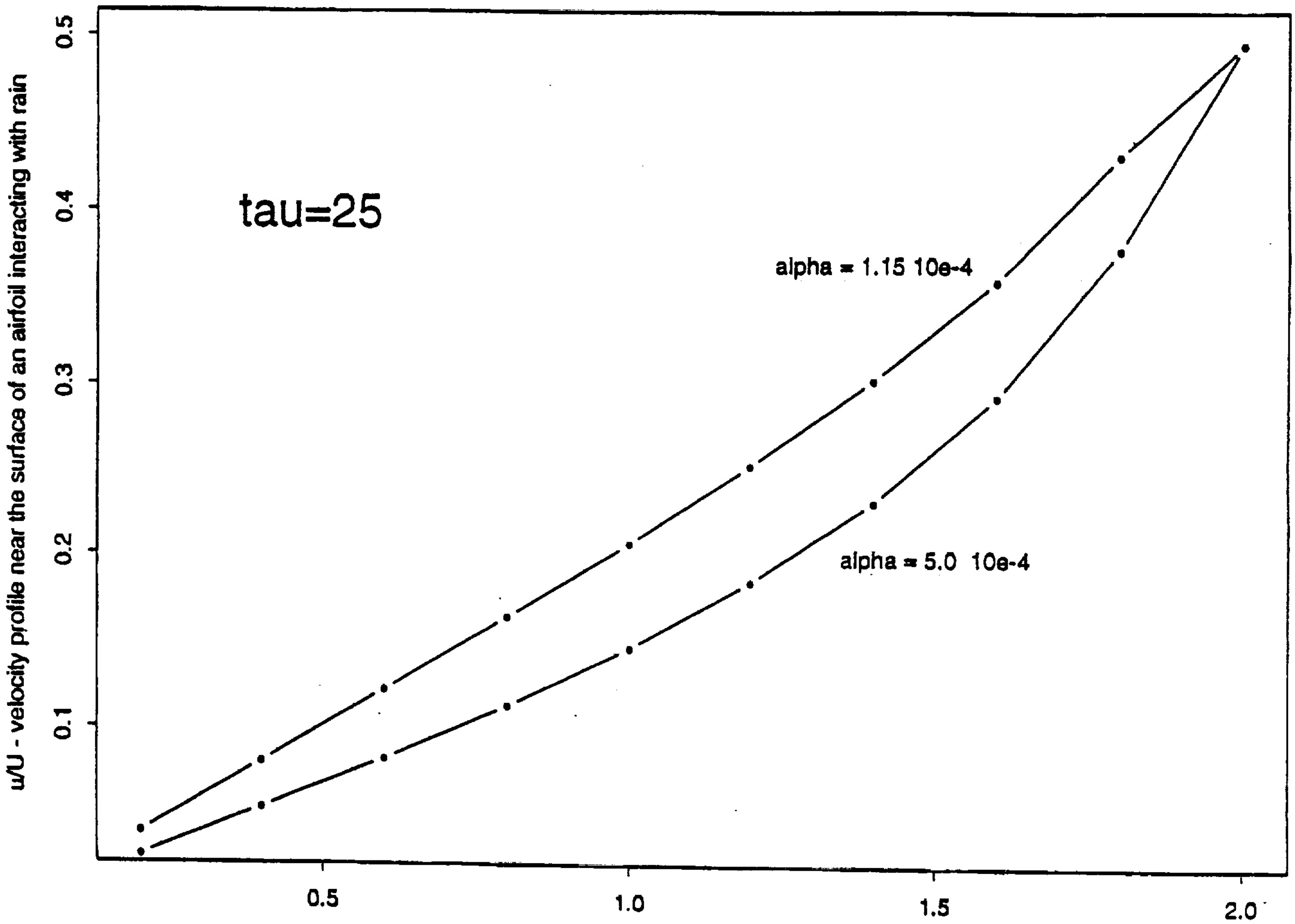


Figure (2). Physical Coordinates of Boundary Layer.



Figure(4) u/U versus y (y-height from the surface)

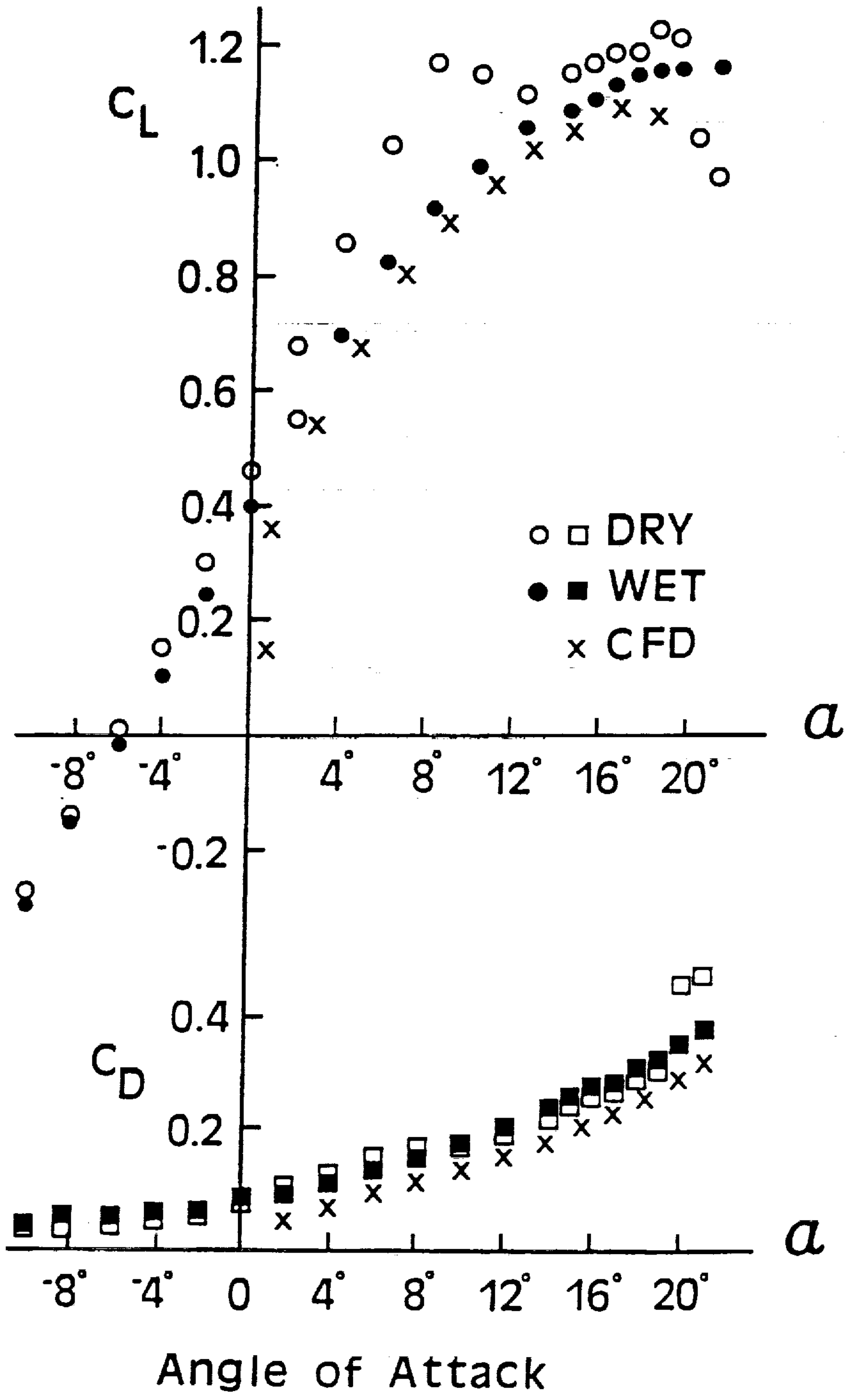


Figure (5). Lift and drag coefficients vs angle of attack for the Wortmann FX67-K170 airfoil in dry and wet conditions.

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