

TRANSIENT STUDIES OF CAPILLARY-INDUCED FLOW

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SUMMARY

This paper presents the numerical and experimental results of a study performed on the transient rise of fluid in a capillary tube. The capillary tube problem provides an excellent mechanism from which to launch an investigation into the transient flow of a fluid in a porous wick structure where capillary forces must balance both adverse gravitational effects and frictional losses. For the study, a capillary tube, initially charged with a small volume of water, was lowered into a pool of water. The behavior of the column of fluid during the transient that followed as more water entered the tube from the pool was both numerically and experimentally studied.

INTRODUCTION

A capillary tube, initially charged with a small volume of distilled water, is lowered into a pool of distilled water to initiate the experiment. The amount of time required for the water within the capillary tube to reach steady-state, the final height of the column at steady-state and the behavior of the column during the transient were all sought. The solution to the problem will be accomplished both numerically and experimentally. The numerical solution will be discussed first. The experimental portion will be used to provide data to verify the numerical solution.

THEORY AND NUMERICAL SOLUTION

One dimensional, unsteady continuity and momentum equations were derived for the capillary tube flow problem. The energy equation was not used since this problem had no measurable transfer of energy taking place (ie., no external heat addition or subtraction).

Consider the cylindrical control volume seen in Figure 1. This control volume encompasses all of the fluid within the capillary structure at any given time. The lower boundary is the only one which mass can flow across and is the interface between the pool of fluid and the bottom of the tube. The upper boundary of the control volume is the bottom of the meniscus which is formed due to surface tension effects between the water in the tube and the tube wall. No mass transfer is allowed to occur across the meniscus. The distance between the bottom of the control volume and the bottom of the meniscus is defined as h . The unsteady, one-dimensional continuity equation applied to this control volume, assuming a constant density fluid, is

$$\frac{dh}{dt} = V_i \quad (1)$$

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where V_i is the velocity of water entering the bottom of the tube.

Figure 2 depicts the same control volume with the appropriate forces and momentum fluxes present. The capillary force ($\sigma 2\pi r \cos \theta$), is depicted as a single force acting on the top of the control volume. This assumption results partly from the work done by Swanson and Herdt (1:434-441), who investigated the evaporating meniscus in a capillary tube, and partly from observations from this work on the behavior of the meniscus during the early portion of the transient. Swanson and Herdt's numerical study showed that the local capillary pressure remained constant for varying degrees of superheat and that the mean curvature of the meniscus asymptotically approached that of a hemispherical meniscus. In the experimental portion of this study, a capillary tube with a small volume of water was lowered into a pool of water *just* until the transient began (ie., fluid began climbing up the capillary tube). From observations of the very early portion of this transient, the column of fluid began to rise immediately after coming in contact with the fluid in the reservoir. This implied that the capillary force was present at the onset of the transient. Close observation of the meniscus revealed that it did not appreciably change shape throughout the entire transient, implying that the magnitude of the capillary force remained relatively constant. Based on these observations, the capillary force was therefore treated as a constant that formed instantaneously at time equal to zero.

The unsteady, one-dimensional momentum equation for the control volume in Figure 2 yields,

$$\frac{d}{dt}(hV_i) - V_i^2 + hg + \frac{2\tau h}{\rho_l r} - \frac{2\sigma \cos \theta}{\rho_l r} = 0 \quad (2)$$

Where g is the acceleration due to gravity, ρ_l is the water density, σ is the water surface tension, θ is the contact angle between the fluid and the tube wall, and r is the capillary tube radius. The shear stress, τ , represents the overall shear between the fluid and the capillary tube. Note at steady-state conditions, the time derivative, velocity and shear stress go to zero, and Eq. [2] reduces to,

$$h = \frac{2\sigma \cos \theta}{\rho_l g r} \quad (3)$$

which is the classic equation given in any physics text, describing the steady-state height rise of a column of water in a capillary tube (2:30).

Combining Eqs. [1] and [2] into the matrix form required by the time integrating scheme used in this work (3:357-372) yields the following forms for the U and F matrices,

$$U = \begin{bmatrix} h \\ hV_i \end{bmatrix}$$

$$F = \begin{bmatrix} V_i \\ V_i^2 - hg - \frac{2\tau h}{\rho_l r} + \frac{2\sigma \cos \theta}{\rho_l r} \end{bmatrix}$$

The integrating scheme was,

$$U^{n+1} = U^n + \Delta t F^n \quad (4)$$

which is a forward-time integrating scheme using values for F at the previous timestep.

The initial conditions of this problem are easily specified. The initial height of fluid in the groove is known, i.e., $h(t=0)$ is a measurable or specifiable quantity, and the velocity of the fluid in the groove is initially zero, $V_i(t=0)=0$. The only thing left to be determined is the shear stress, τ . An expression given in White (4:292) provides a correlation for the *apparent* friction factor for laminar flow in a circular tube,

$$f_{app} Re_D = \frac{3.44}{\xi^{1/2}} + \frac{f Re_D + K_{\infty} / (4\xi) - 3.44 / \xi^{1/2}}{1 + c / \xi^2} \quad (5)$$

where Re_D is the hydraulic Reynold's number, $\xi = (h/D)/Re_D$, $f Re_D = 16.0$ from the Hagen-Poiseuille flow solution, and K_{∞} and c are constants given as 1.25 and 0.000212 respectively. The apparent friction factor is the local friction factor, integrated over the length of channel of interest, taking into account the entrance region effects. The overall shear stress can then be calculated from,

$$\tau = \frac{1}{2} f_{app} \rho_l V_i^2 \quad (6)$$

While Eqs. [5] and [6] provide a compact, closed-form solution to the shear stress, they are somewhat lacking for this work. The reason is that the flow in a capillary tube is not really accelerating tube flow, as is implied by Eq. [5]. In the region of the tube entrance, the equations should apply; however, the fact that the meniscus does not change shape throughout the transient implies that the velocity profile in the vicinity of the meniscus is uniform and equal in magnitude to the average velocity in the tube (from continuity). Boundary layer growth begins at the tube entrance but must disappear by the top of the fluid column. The difficulty is explaining what happens to the velocity profile between the entrance to the tube and the meniscus region.

Consider a shock wave passing over a flat plate through a medium initially at rest. It is known that a boundary layer forms *behind* the shock as it passes over the plate. A second boundary layer can be seen at the leading edge of the plate where the flow is fully developed. If we consider the rise of the meniscus to be similar to the shock as it passes down the plate, then it is reasonable to assume that a boundary layer may form behind the meniscus as it rises in the tube. If this is true, then the discrepancy noted above can be explained. During the initial portion of the transient, a boundary layer is formed as fluid enters the tube. Additionally, as the meniscus rises, another boundary layer forms behind it. These two boundary layers combine to form a kind of diffuser shape within the capillary tube, which basically allows the boundary layer to grow, and then diminish, with approximately uniform velocity profiles at the tube entrance and top of the fluid column.

Assuming merging boundary layers exist, then an approximate expression for the overall shear stress may be derived by dividing the length of the tube into two regions, one where the boundary layer grows from the tube entrance, the other where the boundary layer grows from the advancing meniscus (see Figure 3). The force on each region is given simply as $F_i = \tau_i \pi r h$ ($i = 1$ or 2) and the overall force is given as $F = F_1 + F_2$. If it is further assumed that the two boundary layers grow at the same rate, then $f_{app1} = f_{app2}$ and the overall friction coefficient is simply the Shah friction coefficient, Eq. [5], evaluated at $(h/2)$ instead of h . This method of calculating the shear stress results in an overall increase in the total shear on the fluid element and is referred to as the *modified* Shah friction model.

EXPERIMENTAL SETUP

The experimental equipment consisted of two glass capillary tubes, approximately 0.15 m in length with diameters equal to 0.876 and 1.778 mm (flush cut at each end), a small reservoir full of distilled water, and a mechanism for precisely lowering the capillary tubes into the pool of water. The wetting angle between the glass and water was assumed to be zero radians. A Kodak SP2000 high-speed video motion analyzer (5) was utilized to capture the transient rise of water height within the capillary structure. The SP2000 was capable of capturing from 60 to 2000 frames per second (fps); 60 fps provided adequate resolution of the transient and was utilized in this study. Calibration of the internal clock was accomplished by videotaping a precision stopwatch and comparing the watch movement to the videotape movement at 60 fps. Accuracy of the time measurement was ± 0.01 sec. Length calibration was done by videotaping a precision ruler and comparing the distance between ruler markings on the videotape with the SP2000 crosshair movements. Accuracy of length measurements was ± 0.1 mm.

EXPERIMENTAL PROCEDURE

Prior to insertion into the pool of water, the tubes were thoroughly cleaned with a general cleaning detergent and rinsed with several distilled water baths. The tubes were initially charged with small amounts of distilled water and placed in the lowering mechanism and clamped in place. The video was activated and the capillary tubes lowered into the pool of water until the column of water began to rise, at which point the tube lowering was halted. The video was stopped after the fluid rise transient was over. A quick review of the video after each experimental run indicated whether or not the run was valid. The method of determining a valid run consisted of measuring the height to which the column of water rose in the video and comparing this to the steady-state value calculated from Eq. [3]. If the measured height was equal to the calculated value, the run was deemed valid. If however, the final rise height was lower, then the run was discarded and the video erased. A rise height lower than that predicted by Eq. [3] indicated a fault with the cleaning procedure described above.

RESULTS AND DISCUSSION

Close observation of initial data runs indicated that the formation of the meniscus occurred over a time increment much shorter than what was able to be measured with the equipment at hand. Additionally, the general shape of the meniscus on a macroscopic level, did not change appreciably throughout the transient, whether the fluid was rising or falling within the tube. Therefore, the assumption of a fully-formed meniscus throughout the transient, modeled as a constant capillary force, appeared to be valid.

Figures 4 through 6 show the results of three data runs with the large bore tube, each figure representing a different initial charge. Several general observations are made. For the range of initial charges investigated, the transient was oscillatory in nature, with the behavior essentially damped out after 0.8 seconds. Additionally, the final height rise for all the cases was identical to that predicted by Eqn [3]. This basically answered two of the three questions posed by the problem statement. The last question dealt with how the fluid achieved its final steady-state height. The dashed line indicates the numerical solution using the *modified* Shah friction model. In all three cases, the oscillatory nature of the solution was predicted by the *modified* Shah model; however, it predicted an overshoot of the final rise height slightly greater than what was observed, indicative of insufficient numerical friction. The lower than

predicted rise height during the transient was not believed to be caused by a "dirty" capillary tube, since the final rise height was consistent with that predicted by the steady-state solution. The anomalous behavior was suspected to be caused by either an incorrect assumption in the modeling of the boundary layers or in the omission of additional loss mechanisms. The boundary layer modeling was believed to be accurate due to experimental observations of the meniscus behavior, so additional loss mechanisms were investigated in order to explain the discrepancies.

In all pipe flow systems, there are *major* losses, primarily due to the friction associated with long, straight sections of pipe. These losses are calculated by well-established friction models. However, there are additional losses, termed *minor* losses, that occur due to the presence of valves, bends and obstructions to the flow. In most cases, these losses are small compared to the major losses. The minor losses are generally categorized as to their effect on the recovery of the flow kinetic energy, and are given as head loss coefficients, ϕ . The loss coefficient is a percentage of the incoming kinetic energy that is lost due to the obstruction and is defined as,

$$\phi = \frac{\Delta p}{\frac{1}{2} \rho_1 V_1^2} \quad (7)$$

The loss coefficient is proportional to a friction factor, and for this example, that factor, f_ϕ , is easily shown to be,

$$f_\phi = \frac{\phi r}{2h} \quad (8)$$

This friction factor is then added to the *modified* Shah friction factor calculated previously, and the sum is used in the calculation of the overall shear stress.

The only minor loss present in this experiment is that due to the method in which the flow enters the capillary tube. The entrance of the tube was flush cut; in other words, there was little, if any, rounding of the inlet, which meant the flow had to turn a relatively sharp angle at the entrance to the tube. Based on the level of rounding at the entrance, the loss coefficient could vary anywhere from 0.004 (well-rounded) to 0.500 (no rounding) (2:504-517). Because of the flush cut, this loss coefficient was expected to be closer to 0.5 than to 0.004.

Using a conservative value of $\phi=0.5$, the improvement in the friction model is seen as the solid line in Figures 4 through 6. The trend is obviously in the correct direction; the match with the experimental data was better than with no minor losses included. There was still some discrepancy between this model and the data; however, the difference was well within the experimental scatter.

Figures 7 and 8 show the comparison of the *modified* Shah friction model with the experimental data for the small bore capillary tube. The obvious difference between this and the large bore tube was the exponential shape of the transient curve. In this case, there was no overshoot, but a smooth rise to the final steady-state value predicted by Eq. [3]. The fact that there was no overshoot in this case was most probably due to the fact that because the small bore tube rise height was much greater than the large bore, the overall friction was greater, and the greater friction prevented the overshoot. Including the minor losses improved the match with the experimental data, although the difference between including and not including the minor losses was much less pronounced in the small bore tube as compared to the large bore tube. This was because the losses due to the flush cut acted over a greater percentage of the total rise height in the large bore tube than the small bore, hence their effect was more pronounced.

CONCLUSIONS

Control volume analysis was utilized to numerically study the behavior of a simple, capillary-induced flow. The steady-state rise height, along with the transient behavior of the flow, was accurately modeled with numerical techniques. The nature of the fluid rise in a capillary tube was highly dependant on the tube geometry; the smaller diameter tubes providing higher rise heights and less overshoot during the transient to steady-state. The majority of the frictional losses in capillary tube flow were handled with simple friction models that provided adequate results. Inclusion of minor flow losses in the numerical model improved the correlation between theory and experiment.

The use of high speed video was used to provide transient experimental data on capillary-induced flow. Because of the short transient times in capillary tube flow, a means of capturing the flow behavior was necessary. High speed video was shown to be a viable means of collecting transient data.

Finally, the importance of properly preparing capillary tubes for the experimental portion of the work was perhaps the most valuable lesson learned. Inadequate cleaning of the tubes resulted in several invalid data runs. The only means of verifying proper cleaning of the tubes was an *a priori* knowledge of the steady-state rise height solution for the given tube geometry. Without this, there was no means of determining if the tubes were properly cleaned prior to testing. However, this knowledge did provide a means of choosing a preferred cleaning procedure and once this procedure was followed, few problems were encountered in obtaining valid transient data.

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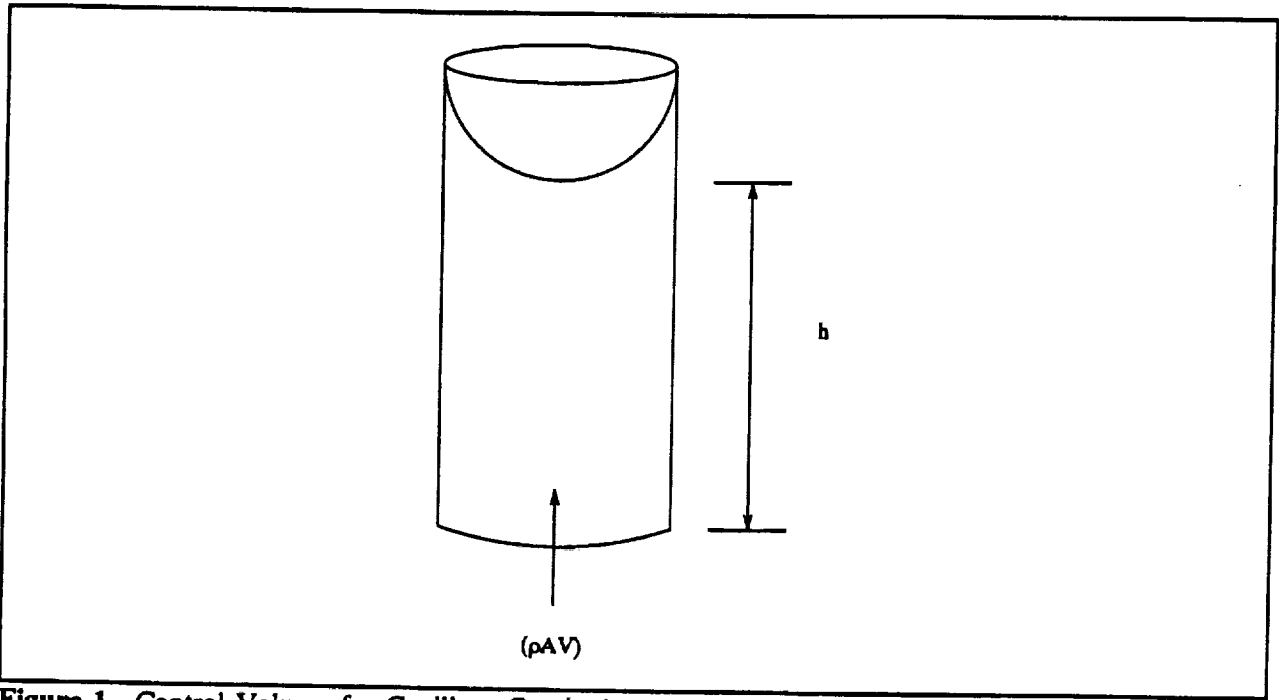


Figure 1. Control Volume for Capillary Continuity Analysis

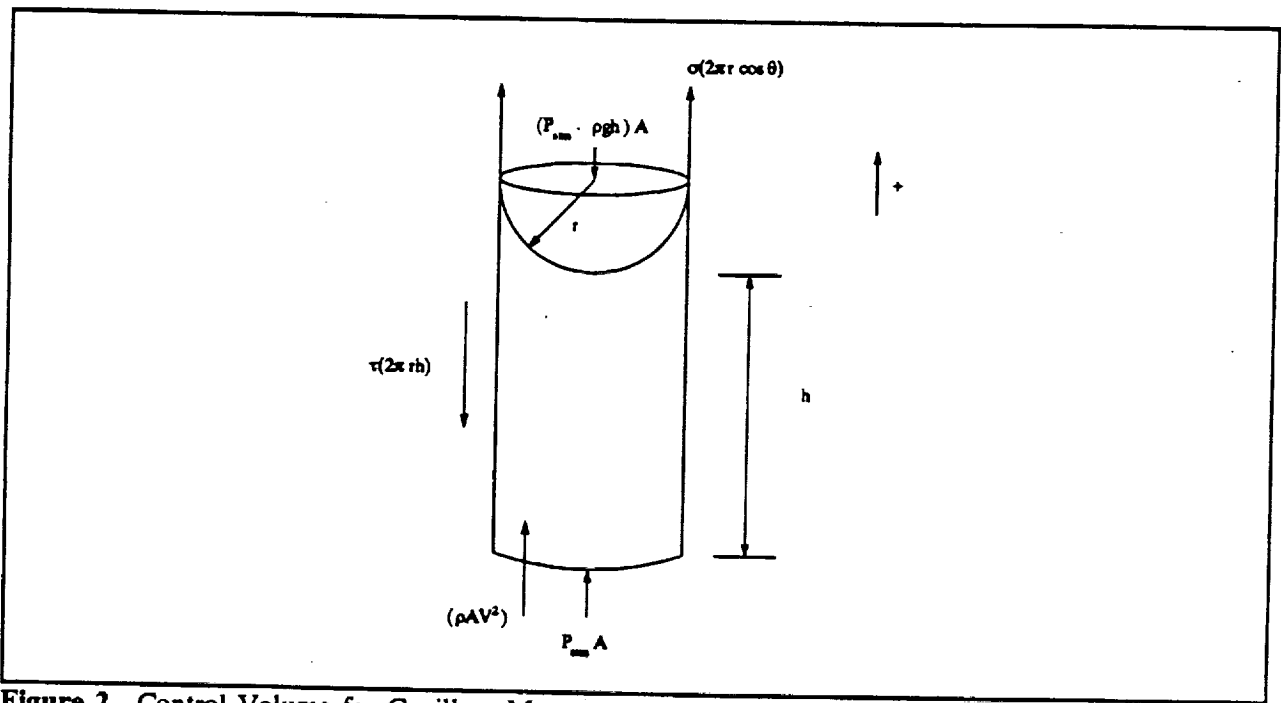


Figure 2. Control Volume for Capillary Momentum Analysis

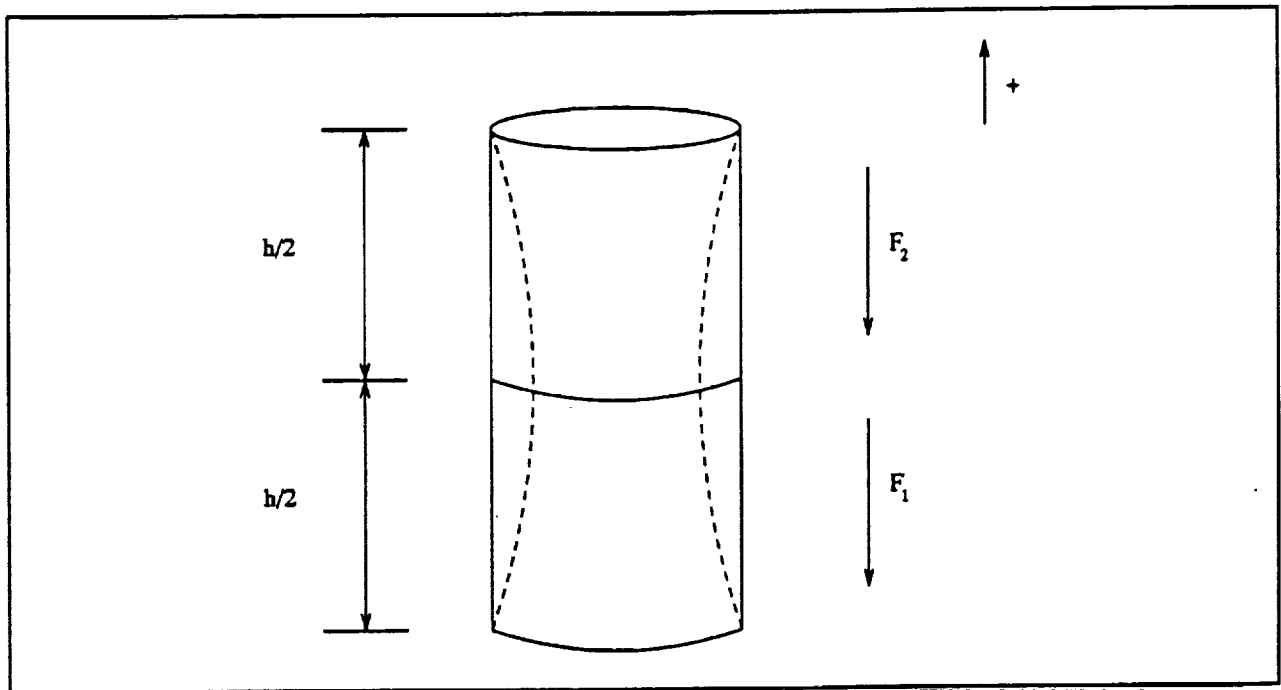


Figure 3. Control Volume for Modified Capillary Shear Analysis

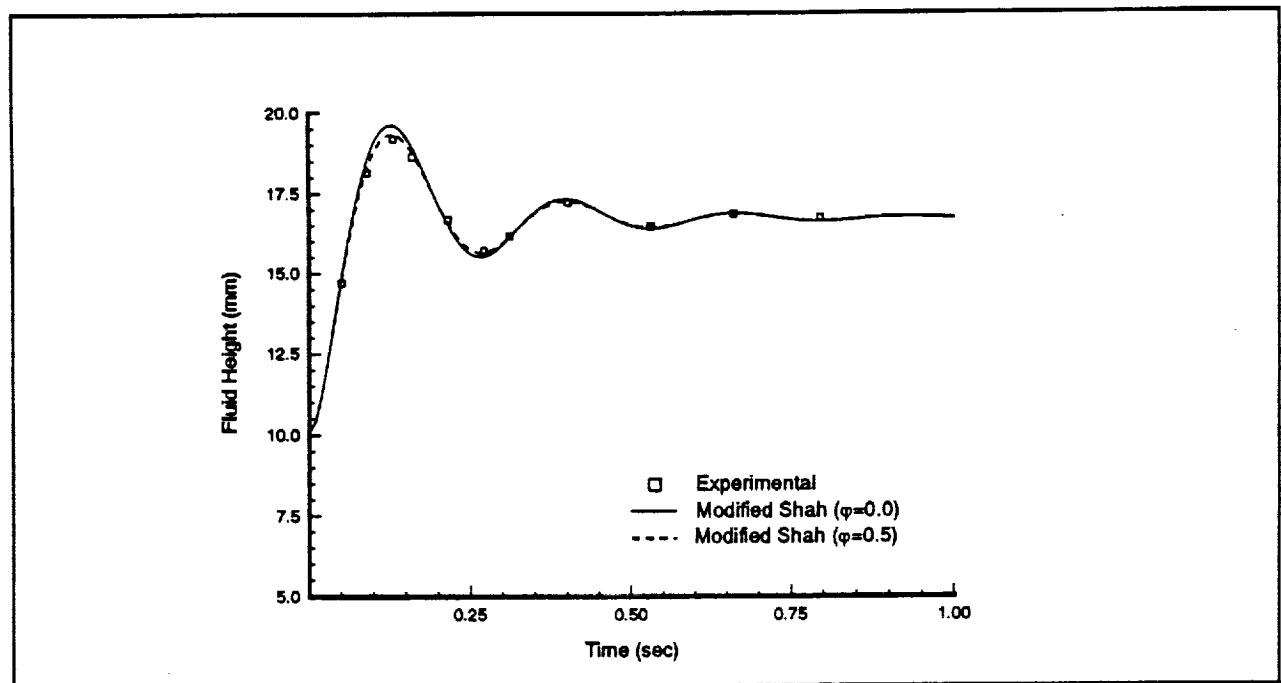


Figure 4. Transient Capillary Rise--Glass/Water $r=0.889\text{mm}$ $h_i=10.073\text{mm}$

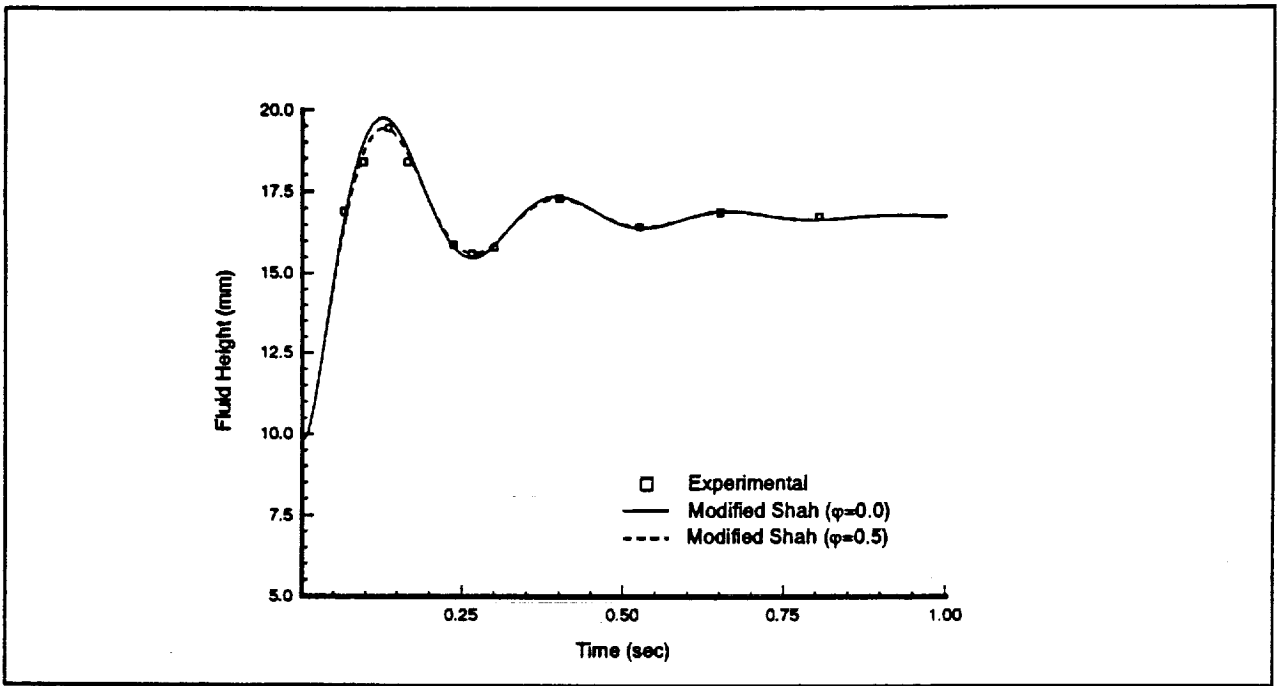


Figure 5. Transient Capillary Rise--Glass/Water $r=0.889\text{mm}$ $h_i=9.709\text{mm}$

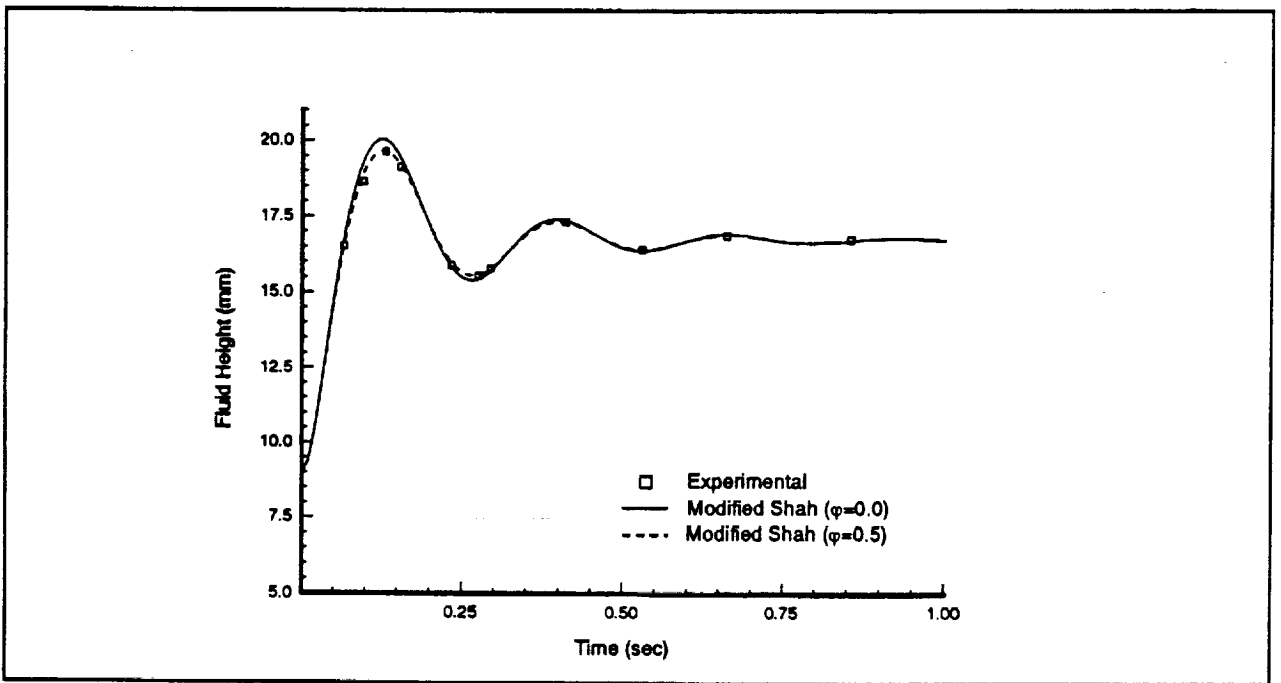


Figure 6. Transient Capillary Rise--Glass/Water $r=0.889\text{mm}$ $h_i=8.932\text{mm}$

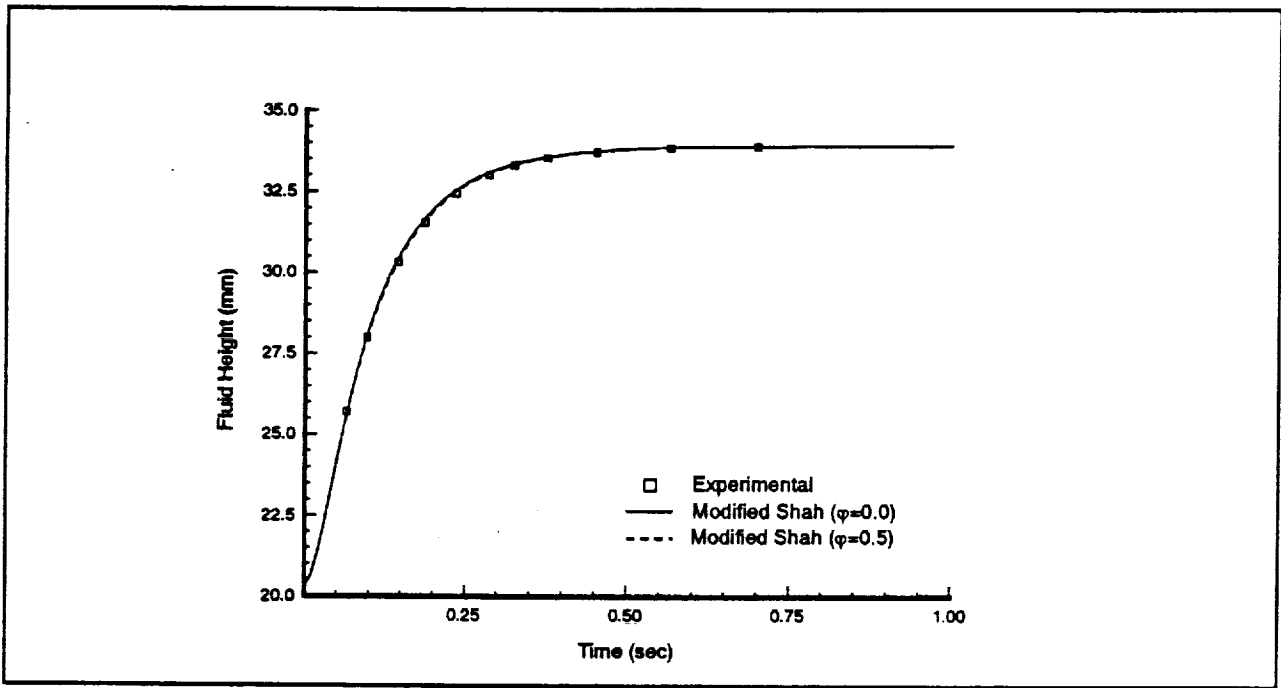


Figure 7. Transient Capillary Rise--Glass/Water $r=0.438\text{mm}$ $h_i=20.32\text{mm}$

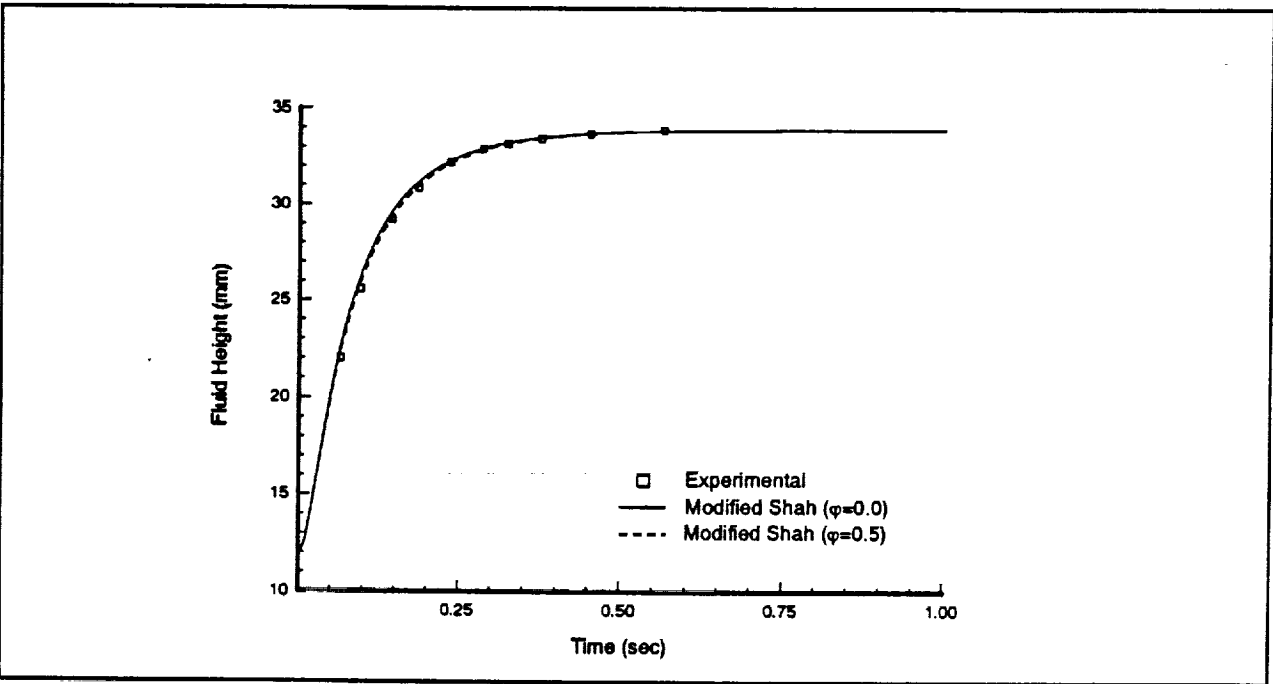


Figure 8. Transient Capillary Rise--Glass/Water $r=0.438\text{mm}$ $h_i=11.73\text{mm}$