

# ON THE DETERMINATION OF LOCAL INSTANTANEOUS AVERAGES IN PARTICULATE FLOW MEASUREMENTS

R. E. Van de Wall and S. L. Soo  
 Department of Mechanical and Industrial Engineering  
 University of Illinois at Urbana-Champaign  
 1206 West Green Street  
 Urbana, IL 61801-2906

## SUMMARY

Determination of instantaneous local average particle density of a gas-particle suspension requires satisfying both the time scale relation and the volume scale relation or its continuum counter part of time averaging. This procedure was validated by comparing simultaneous velocity and mass flux measurements and the laser phase Doppler measurements.

## INTRODUCTION

In an earlier discussion, the corresponding scale relations of both volume- and time-averaging were identified (Soo, 1991). It was noted that almost all measuring techniques in particulate flows are based on time-averaging. Recent work on laser phase Doppler particle analyzer (PDPA) (Aerometrics, 1987) raised the question of probing volume or characteristic dimension of probe in relation to the particle size. This is because the collecting volume of laser beams of the dimension of 100  $\mu\text{m}$  was used to determine the density from mass flux measurement of particles of similar diameters. The long time average density thus determined, with some careful consideration of the effective flow area (Saffman, 1987), is accurate when the flow is nearly fully developed. The same is true in the use of an electrical conductance probe of diameter small than that of bubbles in liquid (Nassos and Bankoff, 1963). This is true as long as the criterion of time-scale relation based on passage of interfaces at an observation point is satisfied according to (Soo, 1991):

$$U_s / (dU_s / dt) > T > \Delta t_k \quad (1)$$

where  $U_s$  is the velocity of an interface,  $t$  is the time, such that the acceleration time be longer than the averaging time  $T$ , and  $\Delta t_k$  is the passage time of a phase. There is no question that Eq. (1) is sufficient when dealing with one-dimensional motion. It is also sufficient for determining long time averages, such as isokinetic sampling of the average mass flux of particles of a suspension. However, no provision was made for determining the local instantaneous density.

## EFFECT OF DIMENSION OF PROBES

Previous experience has been that most of the characteristic dimensions of probes for particle mass flux such as isokinetic sampling (Soo et al., 1969) and electrostatic ball probe (Cheng and Soo, 1970; Zhu and Soo, 1992) were much larger than the size of

particles. Therefore, the concept of volume- or area- averaging given in Soo (1989) were satisfied first of all, or

$$L > v^{1/3} > v_k^{1/3} \quad (2)$$

for volume-averaging of local instantaneous flow properties, where  $L$  is the characteristic dimension of the physical system,  $v$  is the control volume, and  $v_k$  is the characteristic volume of phase  $k$ . It was taken for granted that the inlet opening (diameter  $D$ ) of the isokinetic sampling probe be larger than the particle (diameter  $d$ ). Fig. 1(a) shows that this condition is conceptually significant in measurements based on time-averaging, with mass flux of particles of various sizes determined by the area  $\pi D^2/4$ . However, when  $d$  is large compared to  $D$ , as illustrated in Fig. 1(b), the accuracy of measured mass flux and the deduced density from average velocity of particles become questionable when the former is based on the probe diameter. This is because many large particles which are within the projected area of the area of exclusion of the probe and the particle would be excluded.

When the electrostatic probe is calibrated by the isokinetic sampling probe, the above large (compared to probe diameter) particle effect are not readily corrected from the results of one to the other when dealing with long time averages of density or mass flux. By using a larger probe diameter than the particle diameter, one effectively satisfy the scale relation in Eq. (2) to average the electrostatic charge transfer of a number of particles over the projected area of the probe. The electrostatic probe thus calibrated can be used to determine the instantaneous mass flux of particles based on the projected area  $\eta \pi D^2/4$ ;  $\eta$  being the fraction impacted based on the projected area of exclusion of particles (see for instance, Soo, 1989). This relation is shown in Fig. 2(a) for  $D \gg d$ . Instantaneous mass flux is given by the probe current due to simultaneous collision of particles of average number  $N_c$  given by:

$$N_c = \lambda_B / n^{-1/3} \quad (3)$$

where  $\lambda_B$  is the collision free path of the probe by particles,  $n$  is the number density of particles, and  $n^{-1/3}$  is the mean interparticle spacing (Zhu and Soo, 1992). This number is of order 10 for the present example of 2.4 mm diameter probe and 44-62  $\mu\text{m}$  glass particles at a mass flux to give nearly 1 kg particles/kg of air, or an inter-particle spacing of 10 particle diameters.

A different situation arises when the particle size is large when compared to the diameter of an electrostatic probe (Fig. 2); the effective projected probe area would be  $\eta \pi (D+d)^2/4$ . This illustrates the significance of probe dimension in comparison to the particle size in the above calibration procedure. One also has to rely on the the average from a number of longitudinal spacings of particles to attain a corresponding averaging time to the area average for the instantaneous mass flux.

## FLUCTUATIONS OF DENSITY FROM PDPA OUTPUT

To improve the theoretical basis of determining fluctuating density of particles by phase Doppler particle analyzer (PDPA), we have to extend the scaling relation of volume averaging (see Soo, 1989), given by Eq. (2). Because the collecting volume had a characteristic length of, say, 102  $\mu\text{m}$ , it does not satisfy the criterion of volume averaging (Eq. 2) for particles of 44-62  $\mu\text{m}$  diameter. Its counterpart in local instantaneous time averaging is seen to be over  $\Delta t$  with (Fig. 3):

$$\Delta t_i < \Delta t < T \quad (4)$$

where  $\Delta t_i$  is the duration allotted to the passage of a particle  $i$ ,  $\Delta t$  is the duration corresponding to a "continuum" sampling frequency over a number of particles in say, ms, and  $T$  is the overall averaging time in seconds. In Fig. 3, the phase difference between two detectors in PDPA gives the particle size, and the width indicating passage of a particle  $i$  gives its velocity  $U_{pi}$ ;  $\Delta t_i U_{pi} A_i$  gives its averaging volume.  $A_i$  is the corresponding cross-section of the path of particles and is a function of a given particle size  $d_i$  as shown in Fig. 4 (Saffman, 1987). Correspondence to the case in Fig. 2(b) is noted. The average density given by the procedure of Aerometrics (1987), for a total of  $N$  particle over the duration  $T$ :

$$\langle \rho_p \rangle = (1/T) \sum_i m_{pi} (N_i / \langle U_p \rangle) (L/v)_i = (1/T) \sum_i m_{pi} (N_i / \langle U_p \rangle A_i) \quad (5)$$

where  $m_{pi}$  is the mass of particle of diameter  $d_i$  ( $m_{pi} = \pi \rho_p d_i^3 / 6$ ,  $\rho_p$  is density of particle material),  $L_i$  is the characteristic path length in the laser collecting volume  $v$ . The local instantaneous density is given by, over the duration  $\Delta t$  corresponding to (schematically shown in Fig. 3):

$$\Delta t = \sum_{i=1}^{i=n} \Delta t_i \quad (6)$$

$$\rho_p = \langle \rho_p \rangle + \rho_p' = \sum_{i=1}^{i=n} (N_i m_{pi} / \Delta t_i U_{pi}) (L/v)_i = \sum_{i=1}^{i=n} (N_i m_{pi} / \Delta t_i U_{pi} A_i) \quad (7)$$

Fluctuation in density of particles affects the transport processes in a suspension.

## EXPERIMENTAL PROCEDURE

A complete description of the cyclone-standpipe recirculating pipe flow loop system for a dense suspension of particles and metering is given in Chapter 9 of Roco (1993), Plumpe et al. (1993), or Zhu and Soo (1992a). The Lexel laser facility (5 W two-color argon laser) for laser Doppler (LDV) and phase Doppler particle analyzer (PDPA) (Liljegren and Vlachos, 1990) has been modified for vertical traverse in a horizontal pipe of 127 mm diameter (in a cyclone-standpipe recirculating test loop). The LDV has the capability of measuring fluctuating velocity in two dimensions (longitudinal and transverse).

This system, in combination with an electrostatic probe (Fig. 5), was used in the determination of local velocity and density fluctuations by Slaughter in his thesis (1992) and in Soo et al. (1993). This provides a check for the PDPA measurements of instantaneous local density. The use of a 2.4 mm diameter electrostatic probe gave area average in determining mass flow or density fluctuations with time.

The PDPA has been placed for forward scattering as shown in Fig. 6. The Aerometric software has been modified to give density fluctuation besides average density according to the theory of continuum counterpart of time averaging following Eq. (7).

## EXPERIMENTAL RESULTS

The PDPA gives density fluctuations from a continuum approximation in time corresponding to sampling time in the electrostatic probe. These results on glass particles of 44-62  $\mu\text{m}$  diameter at a mean air velocity of 15 m/s and mass flow ratio  $m^*$  of 1.5 kg particles/kg of air are shown in Fig. 7. Fig. 7(a) gives the long time average density as determined by PDPA, the data for large mass flow ratio  $m^*$  of 1.5 did not extend to positions close to the bottom of the pipe because of interference of the bottom dense layer of particles with the PDPA position in Fig. 6. Fig. 7(b) compares the results of PDPA with the electrostatic probe and LDV measurements, showing general agreement for a sufficient averaging time  $\Delta t$  (1/256 s) in Eq. (7) for averaging, while  $\Delta t_i$  (Fig. 3) may be less than 1/10-th of that. The frequencies of fluctuations of the two measurements are comparable. Fig. 8 shows the mean particle size distribution for various  $m^*$  over the height of the vertical diameter as measured by PDPA; large particle tend to sink toward the bottom of the pipe as expected. Table 1 compares the average particle density and RMS fluctuation of density as determined by the PDPA and the LDV - electrostatic (ball) probe at two averaging times of 1/256 s and 1/128 s for  $m^* = 0.4$  and 1.5, showing that the continuum scale relation was satisfied.

Table 1 Comparison of the average particle density  $\langle \rho_p \rangle$  and RMS density fluctuation  $\langle \rho_p' \rho_p' \rangle^{1/2}$  as determined by PDPA

$m^* = 0.4$			
Method	$\langle \rho_p \rangle$ , kg/m <sup>3</sup>	$\langle \rho_p' \rho_p' \rangle^{1/2}$ , kg/m <sup>3</sup>	Averaging time, s
PDPA	0.95	1.02	1/256
LDV- Ball Probe	0.95	0.62	1/256
PDPA	0.95	0.98	1/128
LDV- Ball Probe	0.95	0.48	1/128
$m^* = 1.5$			
PDPA	1.68	1.14	1/256
LDV- Ball Probe	1.70	1.12	1/256
PDPA	1.68	0.95	1/128
LDV- Ball Probe	1.70	0.99	1/128

## CONCLUSIONS

Measurements of instantaneous local particle density calls for satisfying the criteria of both time and volume averaging or its continuum counter part.

The PDPA provides a means for determining the instantaneous local particle density when the soft ware satisfies the criterion of continuum counter part of time averaging.

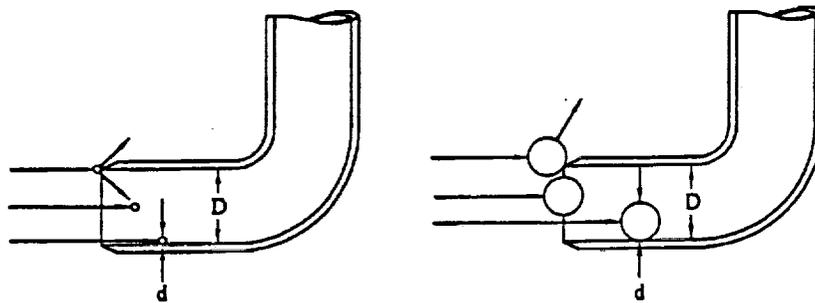
The present procedure as applied to PDPA provides a primary standard for particle density measurement.

## ACKNOWLEDGEMENT

This study was supported by the National Science Foundation under Grant CTS-9014914 of the Fluid, Particulate and Hydraulic System Program directed by Dr. M. C. Roco.

## REFERENCES

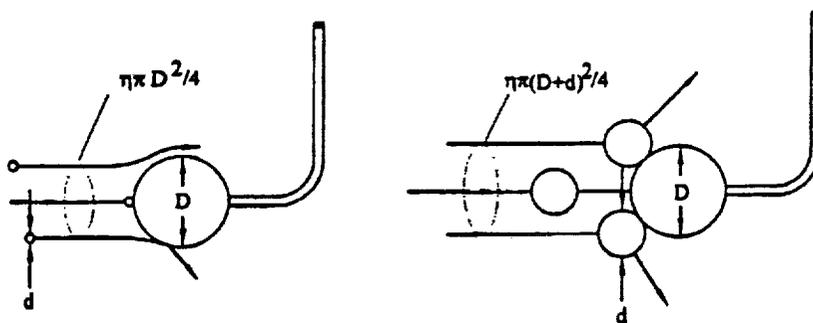
- Aerometrics Corp., 1987, PDPA Operations Manual Release 3.0.
- Cheng, C., and Soo, S. L., 1970, "Charging of Dust Particles by Impact," *J. Appl. Phys.*, Vol. 41, 585-591.
- Liljegren, L. M., and Vlachos, N. S., 1990, "Laser Velocimetry Measurements in a Horizontal Gas-Solid Pipe Flow," *Experiments in Fluids*, Vol. 9, 205-212.
- Nassos, G. P., and S. G. Bankoff, 1967, "Local Resistivity probe for study of Point Properties of Gas-liquid Flows," *The Canadian J. Chem. Eng.*, Vol. 45, pp. 271-274.
- Roco, M. C., 1993, *Particulate Two-phase Flow*, Butterworth-Heinemann, Boston.
- Saffman, M., 1987, "Automatic Calibration of LDA Measurement Volume Size," *Applied Optics*, Vol. 26, (13), pp. 2592-2597.
- Slaughter, M. C., 1992, *Experiments on Particle-Gas Suspension Flow*, M. S. Thesis, University of Illinois at Urbana-Champaign, Urbana, IL 61801.
- Slaughter, M. C., C. Zhu, and S. L. Soo, 1992, "Measurement of Local Statistical Properties of Particle Motion in a Dense Gas-solid Suspension," *Advanced Powder Tech.*, Vol. 3, pp.1-10.
- Soo, S. L., 1989, *Particulates and Continuum - Multiphase Fluid Dynamics*, Hemisphere, DC.
- Soo, S. L. 1991, "Comparisons of Formulations of Multiphase Flow," *Powder Technology*, Vol. 66(1): 1-7.
- Soo, S. L., J. J. Stukel, and J. M. Hughes, 1969, "Measurement of Mass Flow and Density of Aerosols in Transfer," *Environmental Sci. and Tech., Ind. Engr. Chem.*, Vol. 3(4), 386-393.
- Soo, S. L., M. C. Slaughter, and J. G. Plumpe, 1993, "Instrumentation for Flow Properties of Gas-Solid Suspensions," *ASME Symposium Recent Advances in Mechanics of Structured Continua, II*, Charlottesville, VA, June 6-9, 1993.
- Zhu, C., and S. L. Soo, 1992, "A Modified Theory for Electrostatic Probe Measurements of Particle Mass Flows in Dense Gas-Solid Suspensions," *J. Appl. Phys.*, Vol. 72(5), pp. 2060-2062.



(a)  $D \gg d$ ,  $\pi D^2/4$  gives sampling flow area.

(b)  $D \sim d$ , meaningful sampling flow area  $< \pi D^2/4$   
(many large particles are lost).

Fig. 1 Influence of probe size in isokinetic sampling.



(a)  $D \gg d$ ,  $\eta \pi D^2/4$  gives sampling flow area.

(b)  $D \sim d$ , meaningful sampling area  $> \eta \pi D^2/4$ .

Fig. 2 Influence of size of electrostatic ball probe.

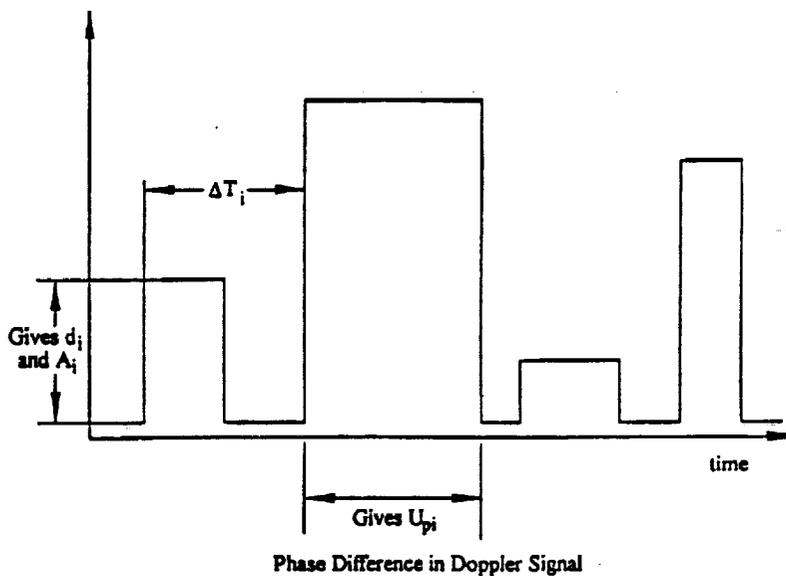
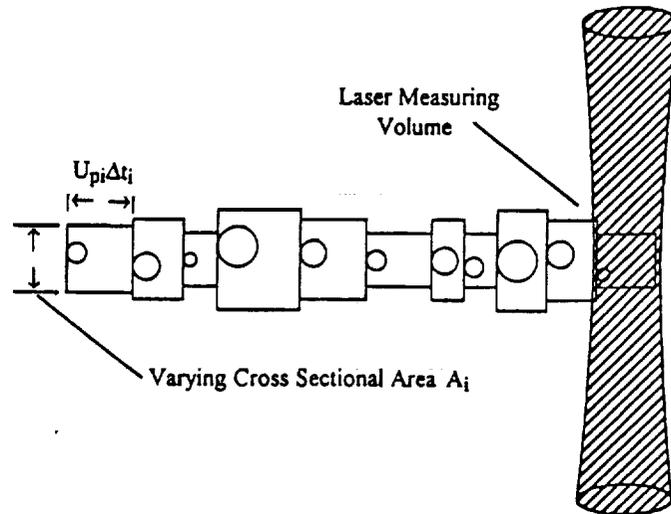
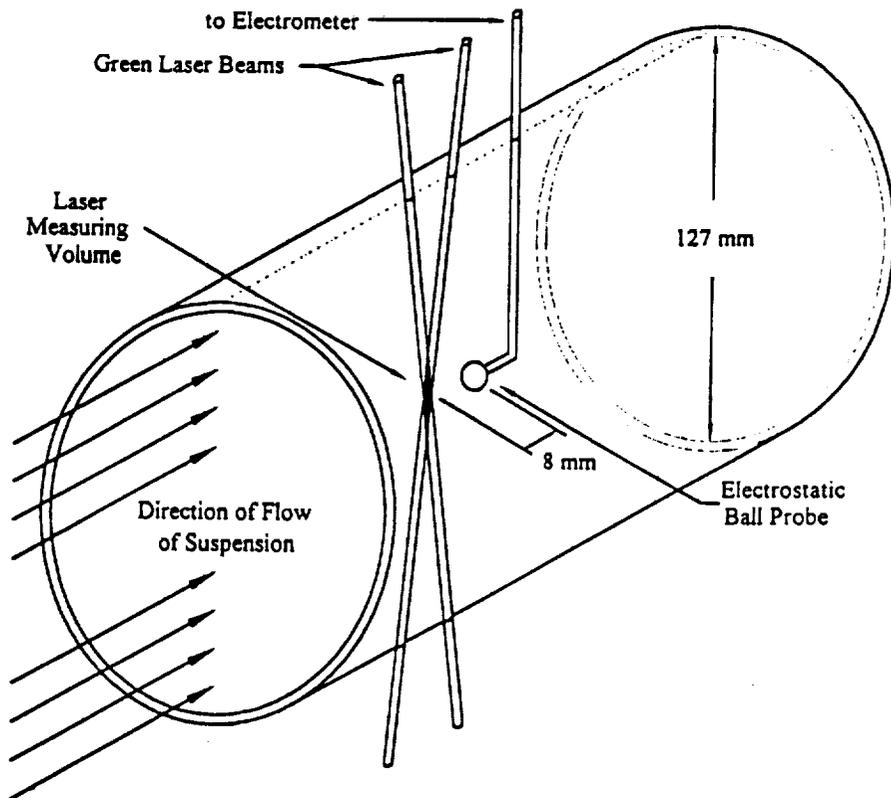


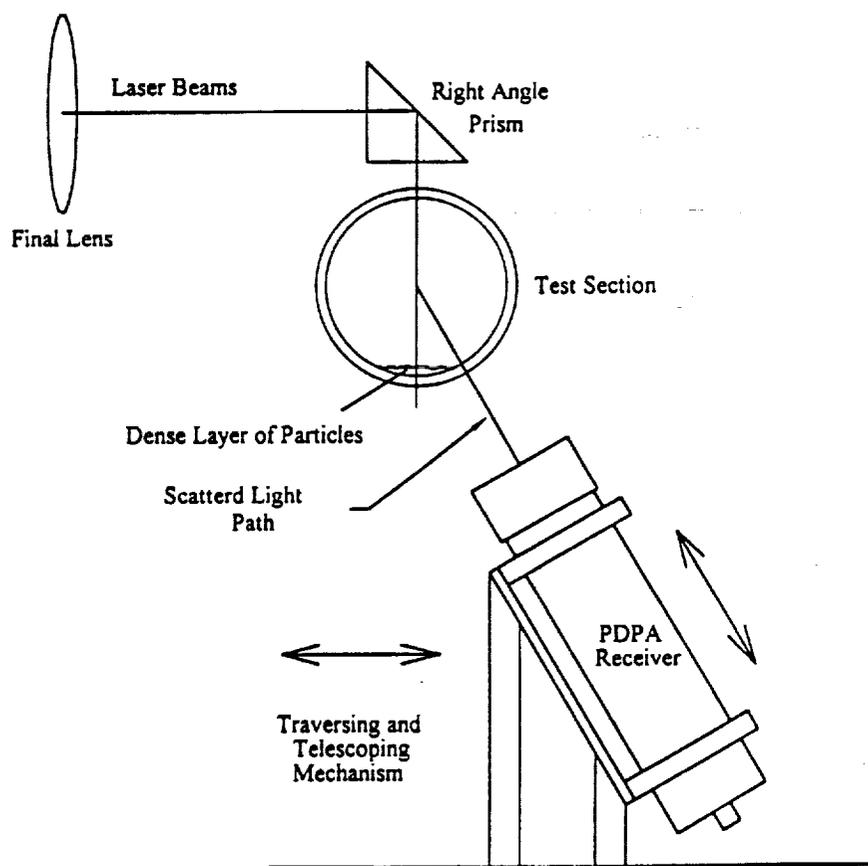
Fig. 3 Time scale relation in determining local density from phase Doppler particle analyzer output.



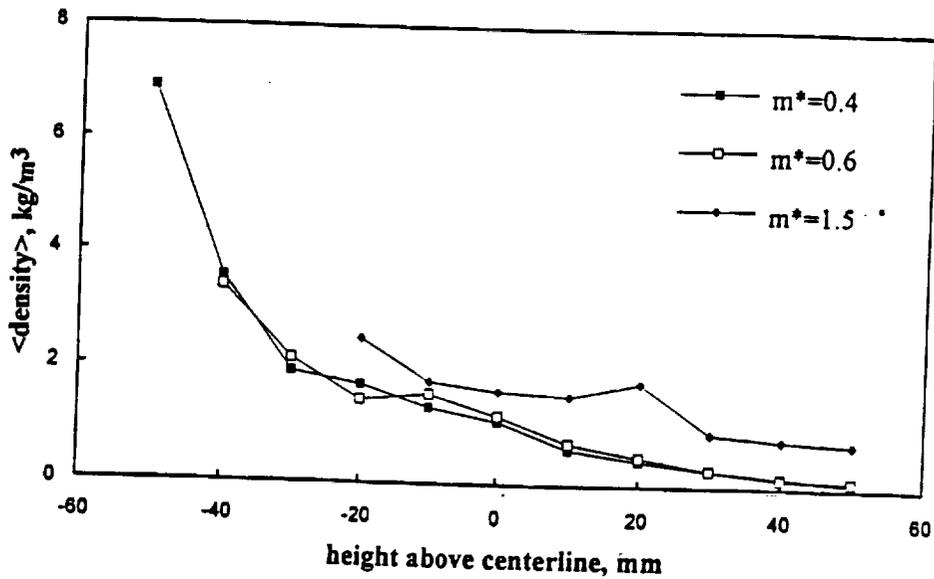
**Fig. 4** Passage of particles through collecting area of beam and the equivalent flow areas.



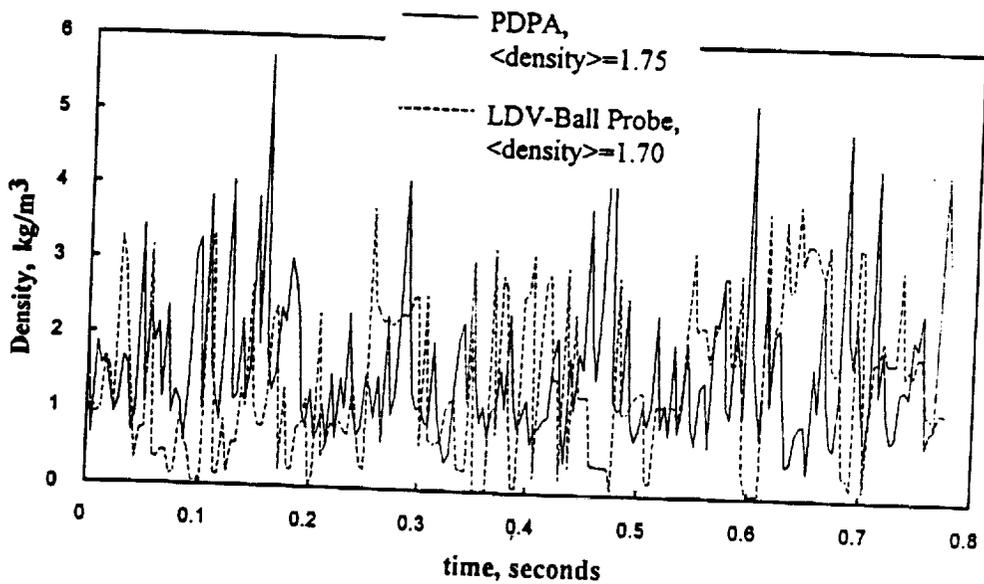
**Fig. 5** LDV and electrostatic ball probe immediately behind the LDV collecting volume in a 127 mm diameter horizontal pipe.



**Fig. 6 PDPA system oriented for forward scattering measurement of particle density along vertical diameter.**



(a) Average particle density over the pipe height at various mass flow ratios as given by PDPA.



(b) Density fluctuation as given by the PDPA at 1/256 s averaging time and the electrostatic ball probe at mass flow ratio of 1.5 along the center line of the pipe.

Fig. 7 Particle density distribution  $\langle \rho_p \rangle$  at 15 m/s mean air velocity and glass particles of 44-62  $\mu\text{m}$  diameter in 127 mm diameter pipe.

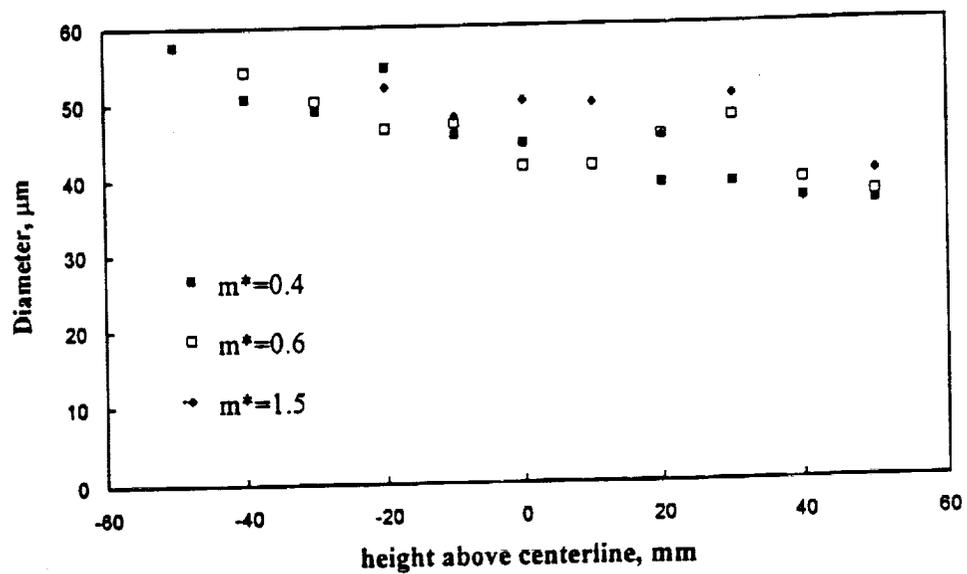


Fig. 8 Mean particle diameter at various heights along the vertical diameter as determined by PDPA for particle size from 44-62 μm and 15 m/s mean air velocity at various mass flow ratio of solid to air  $m^*$ .