Boundary layer receptivity and control

By D. C. Hill

1. Motivation and objectives

Receptivity processes initiate natural instabilities in a boundary layer. The instabilities grow and eventually break down to turbulence. Consequently, receptivity questions are a critical element of the analysis of the transition process. Success in modeling the physics of receptivity processes thus has a direct bearing on technological issues of drag reduction. The means by which transitional flows can be controlled is also a major concern: questions of control are tied inevitably to those of receptivity.

Adjoint systems provide a highly effective mathematical method for approaching many of the questions associated with both receptivity and control. The reader is referred to Hill (1993) for a detailed description of their use in the receptivity context. The long term objective of this project is to develop adjoint methods to handle increasingly complex receptivity questions, and to find systematic procedures for deducing effective control strategies.

The most elementary receptivity problem is that in which a parallel boundary layer is forced by time-harmonic sources of various types. The characteristics of the response to such forcing form the building blocks for more complex receptivity mechanisms. The first objective of this year's research effort was to investigate how a parallel Blasius boundary layer responds to general direct forcing.

Acoustic disturbances in the freestream can be scattered by flow non-uniformities to produce Tollmien-Schlichting waves. For example, scattering by surface roughness is known to provide an efficient receptivity path. This problem has been investigated previously in a number of different ways. The present effort is directed towards finding a solution by a simple adjoint analysis, because adjoint methods can be extended to more complex problems.

In practice, flows are non-parallel and often three-dimensional. Compressibility may also be significant in some cases. How are receptivity characteristics to be found for such flows? Recent developments in the use of Parabolised Stability Equations (PSE) offer a promising possibility. By formulating and solving a set of adjoint parabolised equations, we have developed a method for mapping the efficiency with which external forcing excites the three-dimensional motions of a non-parallel boundary layer. The method makes use of the same computationally efficient formulation that makes the PSE currently so appealing.

In the area of flow control, adjoint systems offer a powerful insight into the effect of control forces (Hill 1992). One of the simplest control strategies for boundary layers involves the application of localized mean wall suction. Why does it work so well? The adjoint method reveals a very simple flow analogy and a concise description of the effect of mean localized suction.

2. Accomplishments

There are four areas where progress has been made. Firstly, the response of a two-dimensional incompressible parallel (Blasius) boundary layer to direct forcing has been investigated. This defines the elementary receptivity characteristics of a boundary layer. Secondly, a variety of natural forcing problems have been solved in which a scattering agent, such as surface roughness, couples freestream acoustic waves to Tollmien-Schlichting waves. Direct forcing of a non- parallel boundary layer is the third topic: here the adjoint to the Parabolised Stability Equations (PSE) is employed to deal in a computationally efficient manner with the non- parallel aspects of the problem. Finally, in the area of flow control, a new perspective is offered on the controlling effect of localized mean wall suction.

2.1 Direct forcing of the Blasius boundary layer

In last year's annual research brief, it was reported that the eigensolutions of the adjoint Orr-Sommerfeld equation, when suitably normalized, provide a detailed description of the response of a boundary layer to direct forcing. The characteristics of the adjoint to the Tollmien- Schlichting wave have been investigated, thereby developing a picture of the elementary processes by which Tollmien- Schlichting waves are produced most effectively. Software has been developed to determine the necessary normalized eigensolution of the adjoint Orr-Sommerfeld equation.

The most significant features of the adjoint eigensolution, and consequently the physical properties of the boundary layer when subjected to direct time-harmonic forcing (i.e. an external source), are summarized as follows:

1. Over a wide range of frequencies and Reynolds numbers, the adjoint stream function corresponding to the Tollmien-Schlichting eigensolution has a simple maximum, and far from the wall, it decays exponentially.

2. The boundary layer is most sensitive to *streamwise* forcing (a momentum source) in the vicinity of the critical layer — the height above the wall at which the flow speed and the phase speed of the Tollmien-Schlichting wave coincide. The most sensitive y-location is shown in Figure 1 as a function of Reynolds number $R = \sqrt{U_\infty L/\nu}$, and frequency $f = 2\pi f^* \nu / U_\infty^2$. Here, L is the distance from the leading edge of the plate, U_∞ is the flow speed at infinity, ν is the viscosity, and f^* is the frequency in Hertz. The solid line indicates the height above the wall at which streamwise forcing is most effective (the location of the maximum of the adjoint streamwise velocity component). The dashed line defines the position of the critical layer.

3. Forcing in the wall-normal direction is much less effective than forcing in the streamwise direction.

4. At the wall, normal motions create Tollmien-Schlichting waves much more effectively than do streamwise motions.

5. The amplification of the Tollmien-Schlichting waves as they travel through the unstable region dictates that forcing at streamwise positions close to the lower branch leads to the strongest response.

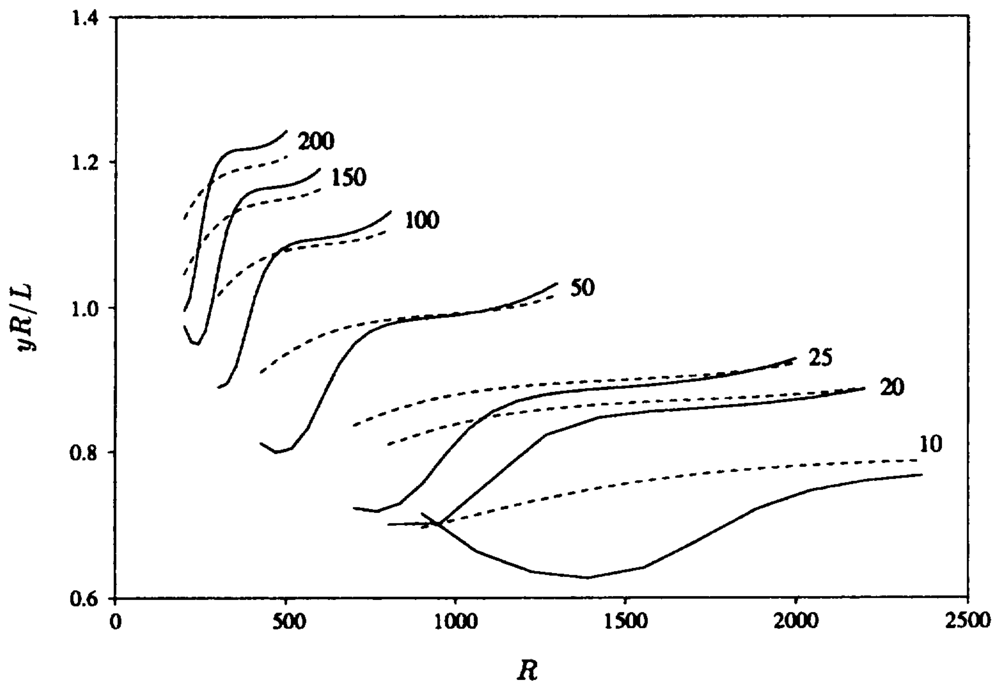

FIGURE 1. Height above wall at which a Blasius boundary layer is most sensitive to direct streamwise forcing, as a function of Reynolds number, for various frequencies $f \times 10^6$. Dashed lines define the location of the corresponding critical layer.

Detailed results are reported in Hill (1993).

2.2 Natural forcing of the Blasius boundary layer

Sound waves in the free stream can be scattered strongly into Tollmien- Schlichting waves if there is even a weak mean flow distortion containing lengthscales commensurate with those of the Tollmien-Schlichting waves. The flow distortion might be caused typically by surface roughness or by mean suction at the wall.

The 'natural response' problem in which a small surface roughness element acts as a scattering agent has been investigated by several researchers. Goldstein (1983) and Ruban (1985) used triple-deck theory to analyze the process in the infinite Reynolds number asymptotic limit. Crouch (1992) and Choudhari & Streett (1992) provide a solution of the incompressible problem at finite Reynolds number. The solutions indicate that the amplitude of the Tollmien-Schlichting wave that is produced by this mechanism is given by the product of an efficiency factor, a geometry factor, and the amplitude of the acoustic wave. The efficiency factor is a complex constant depending on the frequency and Reynolds number, and the geometry factor is the Fourier transform at the Tollmien-Schlichting wavelength of the roughness shape.

In the absence of any roughness, there is a profile $U(y)$ on top of which is superimposed an unsteady motion

$$\underline{v}(\underline{r},t) = u(y)e^{-i\omega t}\hat{x}, \text{ where } u(y) = 1 - e^{-(\omega R/2)^{1/2}(1-i)y}. \tag{1}$$

The planar fluctuations represent a freestream acoustic wave of unit amplitude which has a Stoke's wave signature close to the plate. The frequency ω is defined as f/R, and α is the TS waveumber at that frequency and Reynolds number.

The roughness modifies the mean flow, and the interaction of the Stoke's wave with this mean flow distortion produces a TS-wave. It is *assumed* that, far from the roughness patch, the flow field recovers sufficiently quickly that the scattering takes place in an interaction zone in the vicinity of the roughness.

The efficiency factor for the scattering process is found by examining the solution to the following inhomogeneous adjoint problem for the stream function $\tilde{\Phi}(y)$:

$$-i\alpha U\Big(\frac{d^2}{dy^2} - \alpha^2\Big)\tilde{\Phi} - 2i\alpha\frac{dU}{dy}\frac{d\tilde{\Phi}}{dy} + \frac{1}{R}\Big(\frac{d^2}{dy^2} - \alpha^2\Big)^2\tilde{\Phi} = \alpha^2 u\tilde{v}_{\alpha\omega} - 2i\alpha\tilde{u}_{\alpha\omega}\frac{du}{dy} - i\alpha u\frac{d\tilde{u}_{\alpha\omega}}{dy}, \tag{2}$$

$$\tilde{\Phi} = \frac{d\tilde{\Phi}}{dy} = 0, \text{ on } y = 0, \text{ and as } y \to \infty, \tag{3}$$

where $(\tilde{u}_{\alpha\omega}\hat{x} + \tilde{v}_{\alpha\omega}\hat{y})e^{-i(\alpha x - \omega t)}$ is the normalized adjoint eigensolution corresponding to the TS-wave.

The solution $\tilde{\Phi}$ has some useful properties. The amplitude of the instability induced by the scattering of the freestream disturbance is

$$\Lambda\ \hat{h}(\alpha)\ , \quad \text{where } \hat{h}(\alpha) = \int_{-\infty}^{\infty} h(x)e^{-i\alpha x}\,dx \tag{4}$$

and the efficiency factor

$$\Lambda = -\frac{1}{R}\Big(\frac{dU}{dy}\frac{d^2\tilde{\Phi}}{dy^2} + \frac{du}{dy}\frac{d\tilde{u}_{\alpha\omega}}{dy}\Big)_{y=0}. \tag{5}$$

The equations (2, 3) have been solved numerically, and Λ evaluated. The results are identical with those of Crouch (1992) and Cougar & Streett (1992).

The solution to (2, 3) can also be used directly in the configuration in which mean suction at the wall acts as the scattering agent. Consider a velocity distribution $\underline{V}_s(x)$ representing a suction/blowing distribution on the plate. The amplitude of the Tollmien-Schlichting wave is

$$\Big(\frac{1}{i\alpha R}\frac{d^3\tilde{\Phi}}{dy^3}\hat{y} + \frac{1}{R}\frac{d^2\tilde{\Phi}}{dy^2}\hat{x}\Big)_{y=0}\cdot\int_{-\infty}^{\infty}\underline{V}_s(x)e^{-i\alpha x}\,dx. \tag{6}$$

Surface admittance is defined as the ratio at the wall of the unsteady normal velocity to the unsteady pressure, and thus can be used to represent how the surface responds dynamically to unsteady pressures. Since there is no distortion of the mean flow, in this case the solution $\tilde{\Phi}$ is not employed. Suppose that there are spatially-uniform fluctuations $p_0 e^{-i\omega t}$ in the pressure field. If $\beta_w(x)$ is the surface admittance, a Tollmien-Schlichting wave of amplitude

$$|\tilde{p}_{\alpha\omega}(0)|\, p_0 \int_{-\infty}^{\infty} \beta_w(x) e^{-i\alpha x}\, dx, \tag{7}$$

will be induced. The adjoint pressure at the wall, $\tilde{p}_{\alpha\omega}(0)$, associated with the normalized adjoint eigensolution in this case defines the efficiency factor for the scattering of freestream pressure fluctuations into Tollmien-Schlichting waves.

2.3 Direct forcing of non-parallel flow

There has been considerable development in recent years in modeling transitional flows by the use of Parabolised Stability Equations (Herbert & Bertolotti 1987, Bertolotti 1991). The linear and non-linear dynamics of convectively-unstable disturbances in spatially-evolving boundary layers can be described accurately with little computational effort. The flow can be compressible, and the disturbances three dimensional.

As the name suggests, the method involves "parabolising" the governing equations for boundary layer disturbances. The solution is represented by a disturbance pattern resembling the local eigenfunction, modulated by a spatially-evolving oscillatory factor. Both the disturbance pattern and the wavelength of the oscillation are assumed to evolve slowly with streamwise position. Starting at a chosen streamwise station, the solution is marched downstream; with a single sweep the evolution of the boundary layer disturbance is described.

The formulation and solution of a set of adjoint Parabolised Stability Equations promises to provide a description of the efficiency with which a wide range of boundary layer motions are excited by direct forcing. In contrast with the regular PSE, the adjoint equations are marched *upstream*, starting at the outflow end of the computational domain. In this way, the events within the domain that give rise to a Tollmien-Schlichting disturbance at the outflow are identified. This approach is an extension to non-parallel flows of the results described in Section 2.1. The adjoint PSE can be solved within the same well-established computational framework as the regular PSE. The adjoint solutions are a natural complement to the regular solutions.

Thanks to F. Bertolotti, a copy of the PSE library of subroutines has recently been made available. The following preliminary results have been obtained:

1. The adjoint Parabolised Stability Equations have been formulated for three-dimensional disturbances in a two-dimensional spatially-evolving boundary layer.

2. Solutions for a two-dimensional Tollmien-Schlichting wave in parallel flow have been checked. Figure 2 gives a graphic illustration of the receptivity maximum that

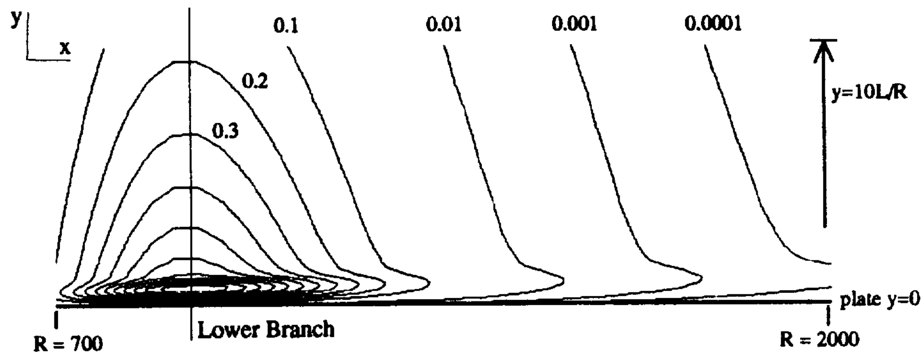

FIGURE 2. Plot of the efficiency with which Tollmien-Schlichting waves are excited by a point source of momentum of unit magnitude, oscillating at a frequency $f = 20 \times 10^{-6}$. Contour values are shown in order to indicate relative magnitudes only.

appears in the Blasius boundary layer (the y-scale has been expanded for purposes of visualization). It is positioned at the lower branch of the neutral stability curve and at a height of about half the displacement thickness from the wall, i.e. at the critical layer. (Figure 2 is a plot of the magnitude of the adjoint velocity and represents the magnitude of the response due to a unit amplitude harmonic momentum source.) The solution has not been normalized, so that contour values do not indicate a physical measure of the response that will arise for unit forcing. However, the relative magnitudes illustrate the zone of high sensitivity.

The effect of non-parallelism is expected to play a larger role for three-dimensional disturbances and compressible flows. This has yet to be investigated.

2.4 Boundary layer control by suction

Small amounts of localized wall suction can reduce dramatically the amplitude of Tollmien-Schlichting waves travelling in a boundary layer. This significant effect has been studied in detail because of its impact on Laminar Flow Control technology (Nayfeh at al. 1986, Saric & Reed 1986, Reynolds & Saric 1986). The numerical perturbation scheme of Reed & Nayfeh (1986) provides a computational analysis of the effect of an arbitrary distribution of suction strips beneath an incompressible boundary layer. Masad & Nayfeh (1992) have developed a scheme for compressible boundary layers.

The effect of suction is to modify the mean flow both upstream and downstream of the slot. A TS-wave that enters this region of the flow grows at a rate which is different from that in the undisturbed flow. Integrating these changes over the entire flow gives the net effect of the suction on the disturbance amplitude.

We construct here an analogous flow, i.e. a flow which has an identical effect upon the TS-wave amplitude. The analogous flow involves a local modification to the boundary layer profile directly above the slot, in proportion to the amount of suction at that streamwise station.

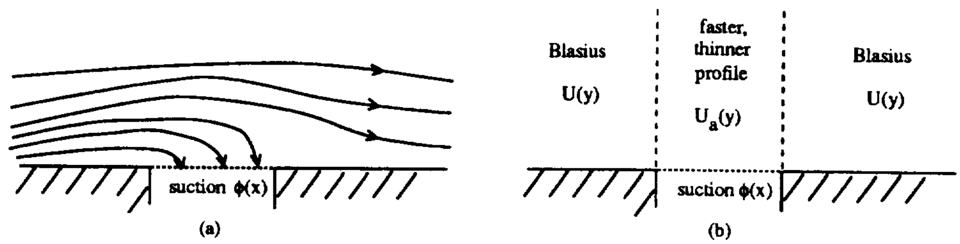

FIGURE 3. Schematic representation of the real flow (a) and analogous flow (b).

The analogous flow replaces the boundary profile by a faster, thinner profile. A simple explicit form has been found for the equivalent profile: Let $\phi(x)$ be the strength of the wall suction velocity (scaled on U_∞) at streamwise location x, with local Reynolds number R. Let $U(y)$ is the form of the profile, y being the distance from the wall. The *net* effect of the suction $\phi(x)$ is *identical* to that of replacing $U(y)$ locally by

$$U_a(y) = U(y) + \phi(x)R\int_0^y (1 - U(y))dy, \tag{8}$$

together with a uniform downflow of strength $\phi(x)$ across the entire boundary layer.

The fractional increase in the effective freestream flow speed is the product of $\phi(x)$ with the local Reynolds number based on displacement thickness.

Figures 3(a) and 3(b) offer a schematic representation of the real flow and the analogous flow, respectively. If the effect of the modification to the local growth of a TS-wave is integrated in each case, the same net change in amplitude will be found.

To understand the effect of suction, we can thus consider the dynamics of Tollmien-Schlichting waves in thinner, faster boundary layers. For frequencies close to the lower branch of the neutral stability curve, the destabilizing influence of an increase in the flow speed is too weak to counter the stabilizing effect of the thinning of the layer. In practice, a disturbance at the frequency which is most "dangerous" from the point of view of transition is controlled by suction applied close to the lower branch. The disturbance amplitude is reduced typically by a large amount. By contrast, for frequencies close to the upper branch (much less "dangerous" from the point of view of transition), the modified profile tends to be *less* stable. Suction leads to an increase in the amplitude of disturbances at these frequencies. However, there is no reported experimental evidence of amplification of higher frequencies due to suction.

3. Future work

The efficiency with which acoustic waves are scattered into TS-waves by surface roughness has already been investigated in detail. The acoustic waves are taken to have infinite wavelength, which reflects the "infinitely" fast speed at which they propagate. Disturbances such as freestream turbulence convect at, or close to, the flow speed and have finite lengthscales associated with them. How efficiently do such motions scatter into TS-waves? I intend to address this question using an extension of the adjoint method employed for the acoustic scattering problem.

Secondary instabilities play a key role in the so-called K- and H-type transition routes: three-dimensional disturbances grow upon a finite amplitude TS-wave until the flow evolves rapidly to turbulence. The following questions will be addressed: What is the most efficient means of exciting the secondary instability? Can it be controlled/suppressed?

Stationary crossflow vortices appear on swept airfoils in response to surface roughness. Secondary instabilities then lead to a breakdown to turbulence. It is important to understand the process by which the crossflow vortices arise and to identify those locations where roughness elements are most important. An investigation of this problem will be made using the adjoint PSE in combination with the classical independence principle.

Acknowledgements

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REFERENCES

BERTOLOTTI, F. P. 1991 Linear and nonlinear stability of boundary layers with streamwise varying properties. *Ph.D. Thesis,* Ohio State University.

CHOUDHARI, M., & STREETT, C. L. 1992 A finite Reynolds number approach for the prediction of boundary-layer receptivity in localized regions. *Phys. Fluids A.* **4,** 2495- 2514.

CROUCH, J. D. 1992 Localized receptivity of boundary layers. *Phys. Fluids A.* **4,** pp.1408-1414.

GOLDSTEIN, M. E. 1983 The evolution of Tollmien-Schlichting waves near a leading edge. *J.Fluid Mech.* **127,** 59-81.

HILL, D. C. 1992 A theoretical approach for analyzing the restabilization of wakes. *AIAA paper No. 92-0067.*

HILL, D. C. 1993 Adjoint systems and their role in the receptivity problem for boundary layers. *CTR Manuscript No. 146.*

HERBERT, TH. & BERTOLOTTI, F. P. 1987 Stability analysis of non-parallel boundary layers. *Bull. Am. Phys. Soc.* **32,** 2079.

MASAD, J. A. & NAYFEH, A. H. 1992 Laminar flow control of subsonic boundary layers by suction and heat transfer strips. *Physics of Fluids A.* **4,** pp.1259-1272.

NAYFEH, A. H., REED, H. L. & RAGAB, S. A. 1986 Flow over bodies with suction through porous strips. *Physics of Fluids.* **29,** 2042-2053.

REED, H. L. & NAYFEH, A. H. 1986 Numerical-perturbation technique for stability of flat-plate boundary layers with suction. *AIAA Journal.* **24**(2), 208-214.

REYNOLDS, G. A. & SARIC, W. S. 1986 Experiments on the stability of the flat-plate boundary layer with suction. *AIAA Journal.* **24**(2), 202-207.

RUBAN, A. I. 1985 On the generation of Tollmien-Schlichting waves by sound. *Fluid Dyn.* **19**, 709 (translation).

SARIC, W. S. & REED, H. L. 1986 Effect of suction and weak mass injection on boundary-layer transition. *AIAA Journal.* **24**(3), 383-389.