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DIMENSIONAL ANALYSIS OF ACOUSTICALLY PROPAGATED SIGNALS

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INTRODUCTION

Traditionally, long term measurements of atmospherically propagated sound signals have consisted of time series of multiminute averages. Only recently have continuous measurements with temporal resolution corresponding to turbulent time scales been available. With modern digital data acquisition systems we now have the capability to simultaneously record both acoustical and meteorological parameters with sufficient temporal resolution to allow us to examine in detail relationships between fluctuating sound and the meteorological variables, particularly wind and temperature, which locally determine the acoustic refractive index.

The atmospheric acoustic propagation medium can be treated as a nonlinear dynamical system, a kind of signal processor whose innards depend on thermodynamic and turbulent processes in the atmosphere. The atmosphere is an inherently nonlinear dynamical system. In fact one simple model of atmospheric convection, the Lorenz system(1), may well be the most widely studied of all dynamical systems. In this paper we report some results of our having applied methods used to characterize nonlinear dynamical systems to study the characteristics of acoustical signals propagated through the atmosphere. For example, we investigate whether or not it is possible to parameterize signal fluctuations in terms of fractal dimensions. For time series one such parameter is the limit capacity dimension. Nicolis and Nicolis were among the first to use the kind of methods we have to study the properties of low dimension global attractors(2).

In this paper we show, for example, that the limit capacity dimensions for atmospherically propagated acoustic signals are greater than those of either the wind speed or the along (propagation) path wind component. Turbulence is the phenomenon which most strongly controls fluctuations in the acoustic refractive index η. Variations in acoustic refractive index are a function of velocity, temperature and, to a lesser extent, humidity fluctuations. Written in terms of the turbulent structure function parameters and neglecting humidity, variations in η are

$$C_{AC}^2 = \alpha C_T^2 + \beta C_V^2 + \gamma C_T^2 C_V^2 \qquad (1)$$

where α and β are constants. Gamma is not a constant but rather a function, in particular, of the stability (heat flux).

Although the use of nonlinear dynamical methods is now rapidly growing, they are not yet nearly so widely known as, e.g., linear Fourier methods[3]. Thus we summarize here the basic analysis method as well as the results of using it.

DIMENSIONAL ANALYSIS OF A TIME SERIES: SOME FUNDAMENTALS

When one is working out of doors it is virtually impossible to measure all of the potentially important environmental variables. Nevertheless it may be possible to extract most of the information necessary to define signal variability by analyzing appropriately combined acoustic and meteorological measurements.

Takens' theorem [4] defines the largest embedding dimension which is needed to analyze a single time series and, thus, to obtain an accurate fractal dimension for the system. The embedding dimension is the state space in which an object can be visualized. For any system having a fractal dimension, e.g. the well known Lorenz attractor, Takens' theorem states that a maximum embedding dimension of $2d+1$ is needed, where d is the fractal dimension rounded to the next higher integer. Thus an embedding dimension of seven should define the Lorenz system, which has a fractal dimension of 2.06. A system might be described in fewer dimensions, but Takens' theorem sets an upper bound for the state space in which the attractor can be embedded.

In the analysis of a time series, if an embedding dimension is used which is less than prescribed by Takens' theorem, the fractal dimension may not be saturated (i.e., reached its peak value). However, as schematically shown in figure 1, if an embedding dimension of higher order is used, little, if any additional information will be gained[5].

Practically it is important to work with the minimum required embedding dimension in order to minimize computational costs.

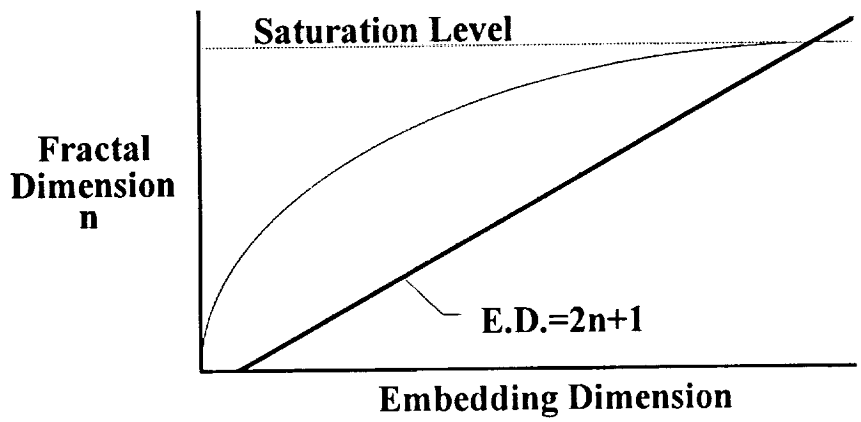

Figure 1. Fractal dimension as a function of embedding dimension

Lagging

In order to extract all information contained within a single time series it is necessary to reconstruct *m* single order equations. Let the single time series X=F(t) be the set of points $(x_1,x_2,x_3, \ldots)$ which are separated by a distance Δx. First we approximate the first derivative of F(t) to be

$$X' = \frac{F(t)}{dx} = \frac{F_{i+1} - F_i}{\Delta x} \quad (2)$$

Actually, there is redundant information in the first derivative, as F_i is the original time series. Therefore, an embedding dimension of two space is created when the original time series is shifted by one time step $(F_i, F_{i+1})^T$. For higher order systems this process is continued until one has created a state space which is large enough so that the attractor can be unfolded.

If the spacing between points in the approximation of the derivative is too small then points will appear to be totally correlated and cannot be considered as independent coordinates[6]. Similarly, if the spacing is too great adjacent points will appear to be unrelated (see figure 2).

In practice, instead of using successive points in the time series to calculate the derivatives, the time series is lagged by a certain number of points. Lagging consists of setting X' = $F'(t)$ equal to the i+l^{th} sample of $F(t)$, (i.e. i, i+l, i+2l, ...), where l is the size of the lag. Lagging the time series allows one to form a matrix as

$$\begin{pmatrix} F_i \\ F_{i+l} \\ F_{i+2l} \end{pmatrix} = \begin{pmatrix} x_1, & x_2, & x_3, & \cdots \\ x_{1+l}, & x_{2+l}, & x_{3+l}, & \cdots \\ x_{1+2l}, & x_{2+2l}, & x_{3+2l}, & \cdots \end{pmatrix} = \begin{pmatrix} X \\ X' \\ X'' \end{pmatrix} \tag{3}$$

where each column of the matrix defines a single point in (2d+1) phase space.

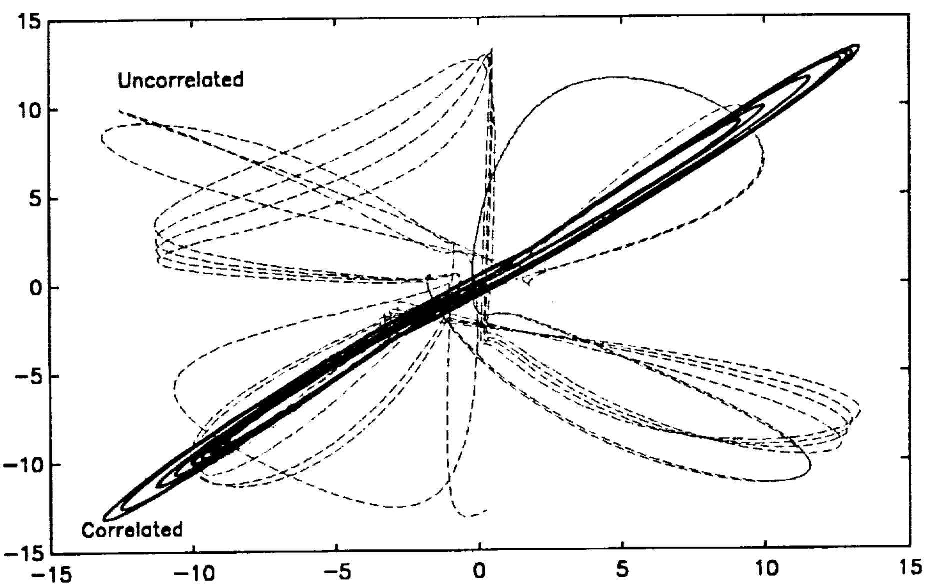

Figure 2. Lorenz attractor with correlated and uncorrelated lags

To estimate an appropriate value of the lag size, three methods are commonly used[7], the autocorrelation time, mutual information, and visualization. For systems having an unknown fractal dimension the autocorrelation method appears to be the conservative approach. To determine the lag there are two possible ways of interpreting a graph of the autocorrelation time scale as shown in figure 3. One is to take the point halfway to the first zero crossing. A second approach is to determine the halfway point to where the autocorrelation curve becomes parallel to the x-axis. If no zero crossing exists this is the only practical method. For the example shown the two methods yield lag sizes of 52 and 35, respectively.

Another method for estimating a proper lag is called mutual information. In this case one increases the lag size until no new information is gained and then defines that point as being the appropriate lag size.

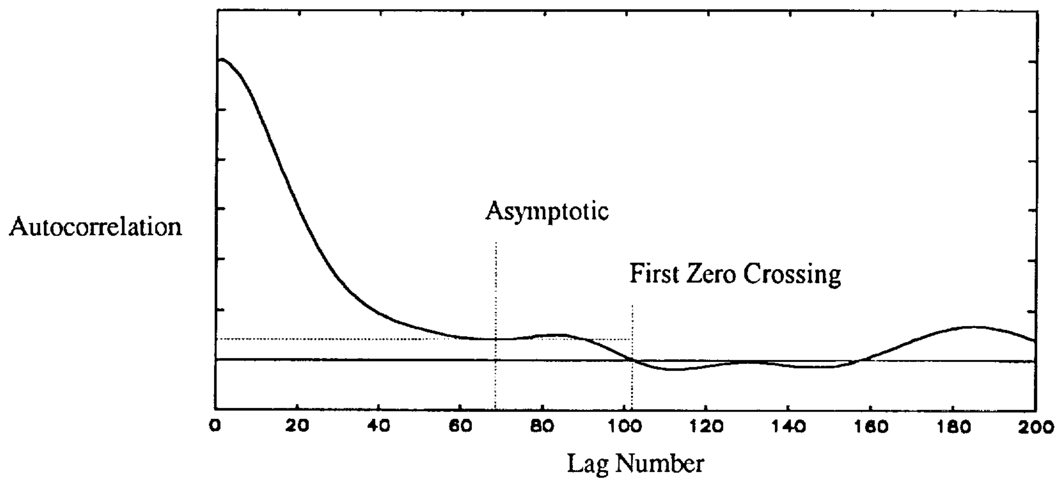

Figure 3. Autocorrelation of the Lorenz attractor

What is called visualization or visual reconstruction may also be used. Visualization is often used in situations where one has some prior knowledge of the system. This method consists of graphically reconstructing the attractor with various lags. If the topology of the attractor is known, e.g., as in the case of the Lorenz attractor, then the lag that appears closest to that for the real system is determined to be the appropriate one.

Limit Capacity Dimension

The limit capacity is one of four commonly used fractal dimensions: capacity, correlation, information, and Lyapunov. Determination of the limit capacity dimension is made as follows[8]. If one lets N(ε) represent the minimum number of m-dimensional cubes of length ε needed to enclose the time series, then as ε decreases one expects N(ε) to increase.

$$N(\varepsilon) \propto \varepsilon^{-d_{cap}} \tag{4}$$

and therefore the capacity dimension is defined as:

$$d_{cap} = \lim_{\varepsilon \to 0} \frac{\log[N(\varepsilon)]}{\log\left[\frac{1}{\varepsilon}\right]} \tag{5}$$

The output of this limit capacity algorithm gives a lower bound to the dimension of the attractor.

Determining the Dimension from Graph

Figure 4 shows the result of applying the limit capacity algorithm to the Lorenz system. In this representation the bin numbers represent distances between points on the attractor. To estimate the limit capacity dimension of a data series a "stable plateau" region must be determined. Definition of the stable plateau may be subjective. Definition of optimal methods for determining the dimension and hence uncertainties in it are still being researched[9].

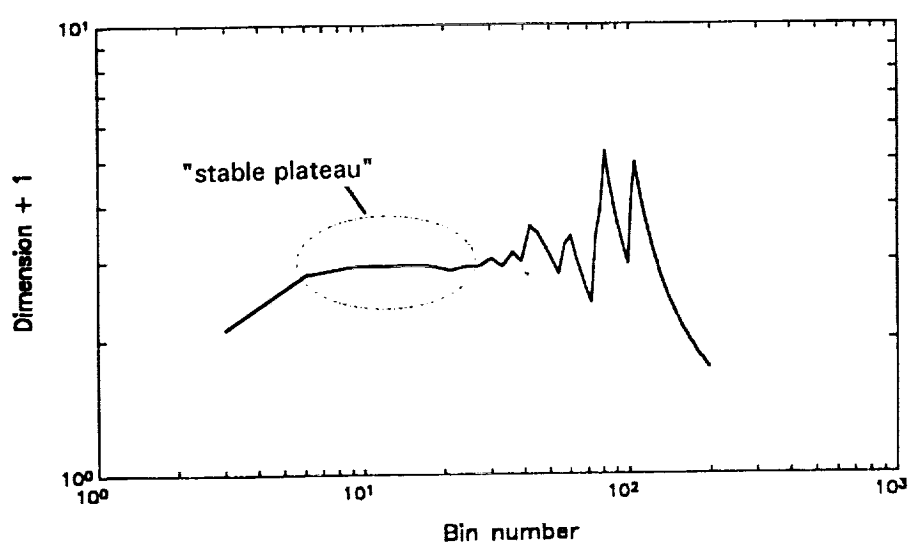

Figure 4. Limit capacity dimension of Lorenz attractor

ACOUSTICAL AND WIND SIGNALS STUDIED

We have applied the same methods as described above for the Lorenz system to hour long recordings of constant frequency sound signals, and to wind speed and the along path component of the wind. These measurements were made as part of the comprehensive Joint Acoustic Propagation Experiment (JAPE) study. Acoustic receivers were located 1 Km from the sound source and spaced logarithmically on a tower to a height of 32 meters. Three tones of 80, 200, and 500 Hz were transmitted. For this analysis the original 2048 samples per second were averaged to one quarter second. The corresponding wind time series had one-tenth second resolution. So far dimensional calculations for only the 80 and 500 Hz tones have been completed.

Lags were determined by calculating the autocorrelation time for each time series. Appropriate lags for the acoustic transmission loss (TL) signals varied between 12.5 and 187.5 seconds; lags for the wind signals ranged from 150 to 400 seconds. Wind speeds were less than 6 meters per second during the recording period. Correlation times of the TL signals measured at 0, 2, and 32 meters decreased both with height and frequency (figure 5).

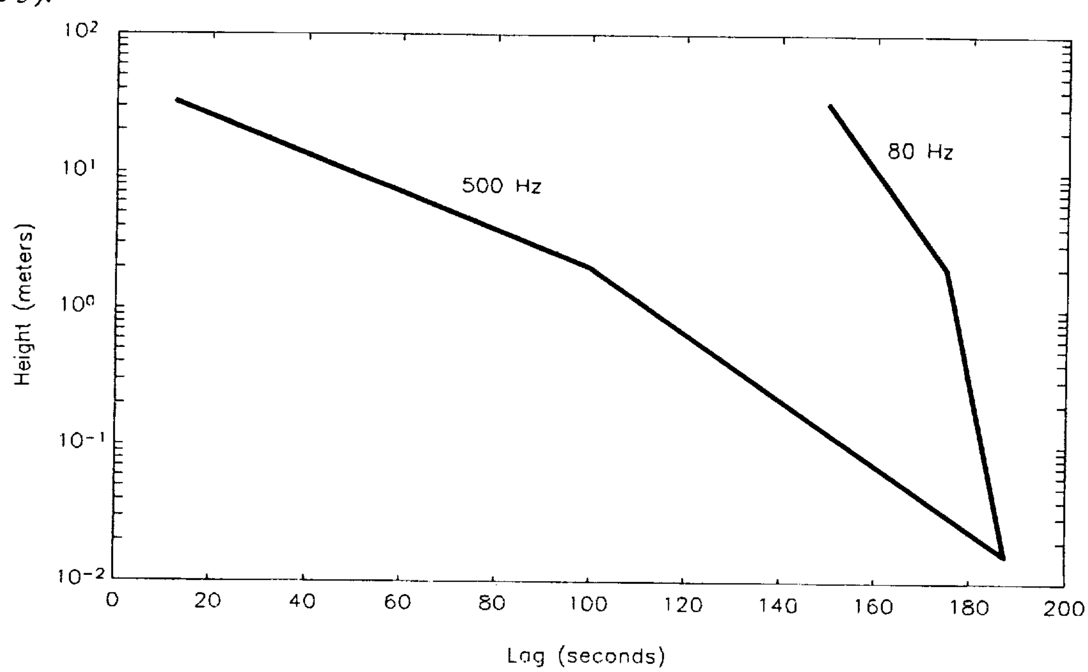

Figure 5. Correlation times vs. height for acoustic signals

Recall that for the Lorenz attractor an embedding dimension of seven was sufficient to unfold the attractor. However, for our acoustic and wind signals the dimension of the attractor was unknown. Thus it was necessary for us to calculate the limit capacity for a number of different embedding dimensions (figure 6). This was done repeatedly until it appeared that the dimensional information had saturated. Figure 6 shows the progression of the limit capacity dimension with increasing embedding dimensions until saturation was reached at roughly an embedding dimension of 12.

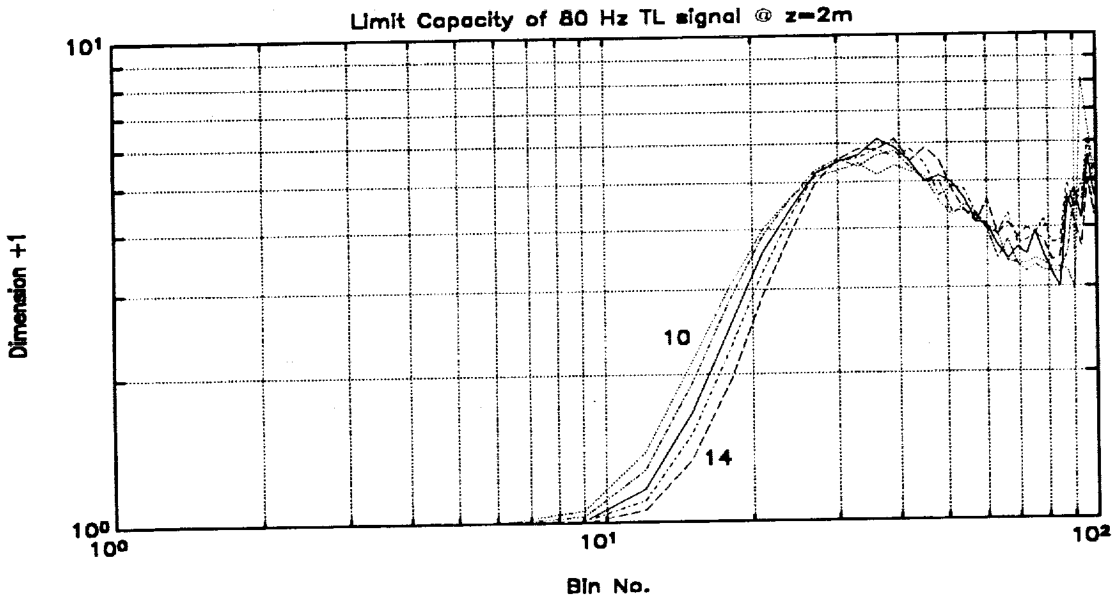

Figure 6: Dimension of 80 Hz tone at 2 meters for various embedding dimensions

Results

Tables I and II, respectively, summarize the calculated limit capacity dimensions for the acoustic signals, the wind speed and the along path component of the wind.

Table I. Limit Capacity Dimension for Acoustic signals

Embedding Dimension	0 m 80 Hz	0 m 500 Hz	2 m 80 Hz	2 m 500 Hz	32 m 80 Hz	32 m 500 Hz
9	4.19	4.69	4.18	4.39	4.42	4.57
10	4.50	4.60	4.31	4.56	4.61	4.82
11	4.34	4.85	4.55	4.64	4.63	4.76
12	4.19	4.87	4.73	4.71	4.81	4.89
13	4.13	4.91	4.68	4.66	4.96	4.95
14	4.44	4.87	4.77	4.53	4.89	4.95

Table II. Limit Capacity Dimension for Wind Speed and Along Path Component Signals

Embedding Dimension	2 m $(U^2+V^2)^{0.5}$	2m $W_{along\ path}$	32 m $(U^2+V^2)^{0.5}$	32m $W_{along\ path}$
9	3.01	3.47	2.58	2.33
10	3.12	3.43	2.33	2.07
11	3.05	3.4	2.21	*
12	3.07	3.35	*	*
13	2.96	3.03	*	*
14	2.97	2.98	*	*

* higher embedding dimensions were not able to be used due to the limited data set and high lag.

Graphical representation of the change in limit capacity dimension with height is shown in figure 7.

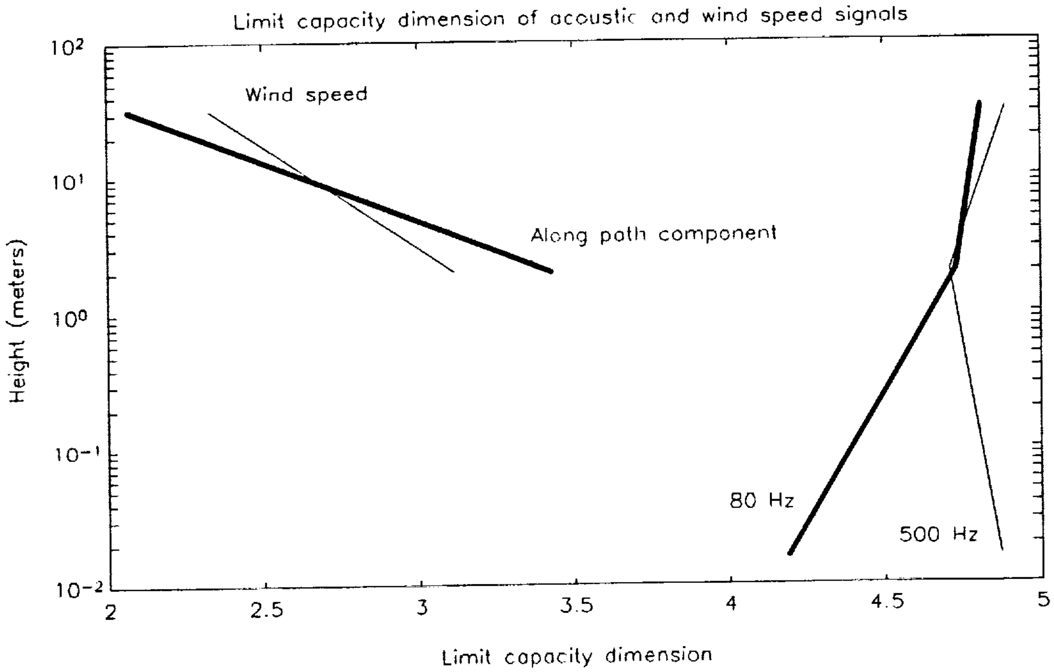

Figure 7. Limit Capacity of acoustic and wind signals

CONCLUSIONS

Low order limit capacity dimensions have been determined to exist for both the acoustic and wind time series. These results confirm the existence of local attractors. The acoustical multivariable dependent signals have higher order attractors than were found for the independent meteorological input variables.

The limit capacity dimension of the acoustic signals appears to increase with height and frequency. We believe that this is due to the role which large eddies (thermals) in the convective boundary layer (CBL) play in controlling intermittent space-time variations in the acoustic refractive index. The properties of propagated sound are sufficiently sensitive to those eddies so that tomographic methods may be used to indirectly measure their properties[10].

We expect that with further dimensional analysis it will be possible to define low order dynamical models that will more precisely define the variability of acoustic signal fluctuations than can be done presently with linear methods. Further studies will require, however, several multihour time series recorded in both stable and unstable boundary layer conditions. The single hour time series recorded during JAPE is of insufficient length. Since large eddies appear to be the dominant signal controlling mechanism, it would also be helpful to have measurements over transmission paths ranging from about 2.5 to 10 km.

REFERENCES

(1) Lorenz, Edward (1963) Deterministic nonperiodic flow. J. Atmospheric Sciences 20, 130 - 141.

(2) Nicolis, C., and Nicolis G.(1980) Is there a climate attractor? , Nature (London) 311, 529-532.

(3) Proceedings of the 1st Experimental Chaos Conference (1992), World Scientific.

(4) Henderson, H., and Wells,R. (1988) Obtaining attractor dimensions from meteorological time series. Advances in Geophysics 30, 205 - 237.

(5) Shirer, H. Nelson, The Pennsylvania State University Professor of Meteorology, personal communication.

(6) Abarbanel, H.D.I., et. al. (Expected Oct 1993) The Analysis of Observed Chaotic Data in Physical Systems., Physical Review.

(7) Thomson, D.W. and Henderson, H.W. (1992) Definitions of local atmospheric attractors using measurements made with surface-based remote sensing systems. Proceedings of the 1st Experimental Chaos Conference, Springer-Verlaag.

(8) Rashband,S.N. (1990) Chaotic dynamics of nonlinear systems. Wiley-Interscience Press, New York.

(9) Wells, R., et al. (1993) Improved Algorithms for Estimating the Correlation Dimension and the Associated Probable Errors. Report AM 114, Pennsylvania State University Department of Mathematics.

(10) Wilson, D.K. (1992) Acoustic Tomographic Monitoring of the Atmospheric Boundary Layer. Ph.D. Dissertation in Acoustics, The Pennsylvania State University, University Park, Pennsylvania.