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TORQUE EQUILIBRIUM ATTITUDES FOR THE SPACE STATION

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Introduction

All spacecraft orbiting in a low earth orbit (LEO) experience external torques due to environmental effects. Examples of these torques include those induced by aerodynamic, gravity-gradient, and solar forces. It is the gravity-gradient and aerodynamic torques that produce the greatest disturbances to the attitude of a spacecraft in LEO, and large asymmetric spacecraft, such as the space station, are affected to a greater degree because the magnitude of the torques will, in general, be larger in proportion to the moments of inertia. If left unchecked, these torques would cause the attitude of the space station to oscillate in a complex manner and the resulting motion would destroy the micro-gravity environment as well as prohibit the orbiter from docking. The application of control torques will maintain the proper attitude, but the controllers have limited momentum capacity. When any controller reaches its limit, propellant must then be used while the device is reset to a zero or negatively-biased momentum state. Consequently, the rate at which momentum is accumulated is a significant factor in the amount of propellant used and the frequency of resupply necessary to operate the station.

A torque profile in which the area under the curve for a positive torque is not equal to the area under the curve for a negative torque is "biased," and the consequent momentum build-up about that axis is defined as secular momentum because it continues to grow with time. Conversely, when the areas are equal, the momentum is cyclic and bounded. A Torque Equilibrium Attitude (TEA) is thus defined as an attitude at which the external torques "balance" each other as much as possible, and which will result in lower momentum growth in the controllers. Ideally, the positive and negative external moments experienced by a spacecraft at the TEA would exactly cancel each other out and small cyclic control torques would be required only for precise attitude control. Over time, the only momentum build-up in the controllers would be due to electro-mechanical losses within the device. However, the atmospheric torques are proportional to the density of the atmosphere and the density varies with the orbital position, time of day, time of year, and the solar cycle. In addition, there are unmodeled disturbances and uncertainties in the mass and inertias. Therefore, there is no constant attitude that will completely balance the environmental torques and the dynamic TEA cannot be solved in closed form. The objective of this research was to determine a method to calculate a dynamic TEA such that the rate of momentum build-up in the controllers would be minimized and to implement this method in the $\mathrm{MATRIX_X}$ simulation software by Integrated Systems, Inc.

Description of Research

Previous methods for calculating TEAs have relied upon approximations of the atmospheric density and have assumed that the atmosphere was constant with respect to the orbital path of the spacecraft. The TEA calculation was reduced to a quasi-closed-form method in which the approximate torques were substituted into the equations of motion, and the resulting system was solved numerically. It was decided to research the possibility of determining dynamic TEAs for the space station while using accurate models of the atmosphere and including all six of the rigid-body degrees-of-freedom (DOF) in the numerical simulations.

A TEA is essentially the "optimal" attitude where the moments required of the controllers are zero-biased, and the research focused on formulating the optimization problem. Although

$MATRIX_X$ has an optimization module available, this feature was not included in the license of the Program Development Office. Consequently, minimization routines for single and multiple variables were adapted from Fortran codes collected by Press et al. (4). The appropriate algorithms were then translated into $MATRIX_X$ executable files.

To determine the feasibility of the optimization approach, a one DOF model was the first case to be tested. The inertia, aerodynamic moment, and gravity-gradient moment coefficients used in the model were taken from space station data so that the numerical results would be of the same order. The aerodynamic moment was given the form

$$M_{aero} = (\alpha - \varepsilon \sin \omega t)\,\theta \qquad [1]$$

to simulate the variable atmosphere. The equation of motion for this system is essentially Mathieu's equation (3) with a constant forcing function and has the form

$$I\ddot{\theta} + [\,(mgr - \alpha) + \varepsilon \sin \omega t\,]\,\theta = mgr\theta_0 \qquad [2]$$

where I is the inertia, $mgr\theta$ is the gravity-gradient moment, and θ_0 is the angle at which the gravity-gradient moment is zero. The cost function used in the optimization algorithm was

$$J = \left|\int M dt\right| \qquad [3]$$

where M is the sum of the environmental torques. This cost function allows the positive moments to cancel the negative moments, but returns a positive-definite value for all possible solutions.

Because this problem can be solved in closed form, the solution from the optimization algorithm could be compared to the analytical solution; the results were very good but also quite surprising. The TEA was successfully calculated with negligible error, but the unexpected result was in the torque profile. A very strong beat phenomenon was displayed where the low frequency component had a period of 20 orbits and the high frequency occurred at the orbital period. Further investigations indicated that the beat is very sensitive to the interaction between the forcing term (the gravity-gradient null position) and the amplitude of the time-varying component of the aerodynamic torques. The beat occurred only when the parameters had a certain proportional value and the range of the proportional constant at which the beat occurred was very small. However, this would seem to indicate that a given spacecraft configuration would exhibit this kind of motion at a certain atmospheric density and this subject will be investigated further.

The next test case was a three DOF model in which the attitude equations were implemented with the simplified gravity-gradient and aerodynamic torques. The environmental torques about each axis had different magnitudes and were completely independent of each other. The attitude dynamics, however, were coupled through Euler's Equations and the equations of motion governing the attitude of the spacecraft (1). With this model, the multi-variable optimization algorithm could be tested with the coupled, nonlinear attitude dynamics but without the complexity of the six DOF simulations. This system could not be solved in closed form, but the attitude at which the torques about each axis are statically balanced could be determined and the TEA would be expected to be somewhere in the neighborhood of this attitude.

The cost function and the $MATRIX_X$ simulation for this system were substantially different from the simple form used in the previous case. The attitude of a spacecraft will vary as the spacecraft reacts to the external torques, but to maintain the micro-gravity environment, a fixed attitude (the TEA) is desired. Therefore, when the actual attitude and the fixed attitude coincide, no control torques are required even though the spacecraft is experiencing external torques at that attitude. When the actual attitude differs from the fixed attitude, the corresponding external torques will differ, and it is this difference that should be zero-biased. The simulation must therefore simultaneously integrate the motion of a spacecraft flying at a fixed attitude and a spacecraft allowed to react to the external moments. The moments are calculated for each spacecraft and the difference is the integrand of the cost function. The cost function is the magnitude of the vector resulting from the integration and is represented mathematically by

$$J = \sum_{i=1}^{3} \left| \int (M_{actual}^{(i)} - M_{fixed}^{(i)})\, dt \right| \qquad [4]$$

where the superscript indicates the i^{th} element of the moment vector.

The optimization algorithm was able to find a TEA that drove the cost function to zero and this TEA was indeed very close to the static equilibrium attitude in the pitch and yaw axes, but differed significantly in the roll axis as shown in Table 1. Additional calculations proved that there was no other TEA in the neighborhood of the static equilibrium attitude and the large roll angle, necessary to obtain the zero-biased torques, was a consequence of the coupling between the axes. The beat phenomenon was again clearly displayed in the torque profiles.

Table 1: TEA for the 3 DOF model

Angles (rad)	Torques Balance Statically	TEA from optimization
Yaw	0.1048	0.0994
Pitch	0.1746	0.1761
Roll	0.0499	0.1899

The next stage of the research was to implement this method of calculating TEAs in the space station simulations. The procedure is essentially the same as that used in the three DOF example. The simulations were changed such that a fixed attitude model was integrated simultaneously with a free-flying model, but the simulations now included all six rigid-body degrees-of-freedom, an accurate atmospheric density model, and detailed atmospheric drag/moment calculations. The cost function remained exactly the same as used in the three DOF model. Examples of the Human Tended Configuration (HTC) and the International Human Tended Configuration (IHTC) were completed.

The results for both configurations were unexpected and were thought, at first, to be in error. Neither configuration had a TEA that resulted in zero-biased torques, and in both cases, the yaw torque was the only one that did not reduce to a zero bias. Additional calculations proved that the result returned from the optimization algorithm was indeed the minimum of the cost function. The explanation for this result is due to the coupling between the axes; an arbitrary body may have an equilibrium condition in which a biased torque about one axis is necessary to produce a zero-biased stable attitude about the other two. The yaw axis is the biased axis because the gravity-gradient torque about the yaw axis is extremely weak.

This type of behavior has been observed in previous studies (2) where yaw-biasing was necessary to provide a stable attitude. Previous attempts to determine the proper yaw bias were accomplished through trial and error methods. A yaw bias was chosen, the optimal attitude was determined for the roll and pitch axes, and the total momentum was calculated. The procedure was repeated for several different yaw angles and the momentum was plotted as a function of the yaw angle. The yaw bias was finally chosen at the point where the momentum was minimized. The calculation of TEAs using the method developed in this research seeks a solution in which the external torques are zero-biased. If such a solution does not exist, however, the optimization algorithm still seeks the minimum bias which in most cases will be the yaw biased attitude.

Conclusions

Calculating TEAs through minimizing the bias of the external torques was shown to be very promising. The method has distinct advantages over quasi-closed-form approaches used in the past because no assumptions about the mathematical behavior of the torques is required. The numerical simulations may contain any degree of complexity in the nonlinear dynamics and calculation of the external torques. The method is very robust, and with the proper optimization routine, can incorporate equality and inequality constraints. Finally, the method will find the zero-bias TEA if such a solution exists, or reduce to the yaw-biased solution. The method was tested on two simple models and several of the space station configurations with excellent result returned in all cases.

References

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