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Steady Induction Effects in Geomagnetism

Part IC: Geomagnetic Estimation of Steady Surficial Core Motions—Application to the Definitive Geomagnetic Reference Field Models

Coerte V. Voorhies
Goddard Space Flight Center
Greenbelt, Maryland

National Aeronautics and Space Administration

Office of Management

Scientific and Technical Information Program

ABSTRACT

In the source-free mantle/frozen-flux core magnetic earth model, the non-linear inverse steady motional induction problem has been solved using the method presented in Part IB. The present paper describes how that method has been applied to estimate steady, broad-scale fluid velocity fields near the top of Earth's core that induce the secular change indicated by the Definitive Geomagnetic Reference Field (DGRF) models from 1945 to 1980. Special attention is given to the derivation of weight matrices for the DGRF models because the weights determine the apparent significance of the residual secular change. The derived weight matrices also enable estimation of the secular change signal-to-noise ratio characterizing the DGRF models.

Two types of weights were derived in 1987-88: radial field weights for fitting the evolution of the broad-scale portion of the radial geomagnetic field component at Earth's surface implied by the DGRFs, and general weights for fitting the evolution of the broad-scale portion of the scalar potential specified by these models. The difference is non-trivial because not all the geomagnetic data represented by the DGRFs constrain the radial field component. For radial field weights (or general weights), a quantitatively acceptable explication of broad-scale secular change relative to the 1980 Magsat epoch must account for 99.94271% (or 99.98784%) of the total weighted variance accumulated therein. Tolerable normalized root-mean-square weighted residuals of 2.394% (or 1.103%) are less than the 7% errors expected in the source-free mantle/frozen-flux core approximation.

1. INTRODUCTION

The work of ELSASSER (1946a,b 1947) and others has led to general acceptance of the importance of induction effects in geomagnetism (see, e.g., JACOBS, 1987). The importance of steady induction effects in geomagnetism, although clearly less, cannot be dismissed simply because slow changes of the geomagnetic field, the secular variation (SV), are not purely linear with time-as evidenced by the inaccuracy of predictions by BOND (1668). Unlike steady magnetic flux diffusion, motional induction by steady fluid flow near the top of the electrically conducting liquid outer core need not imply constant SV. Consider, for example, a high conductivity core threaded by a partly non-axisymmetric magnetic field of internal origin: steady, uniform westward flow of this core relative to a magnetically source-free mantle induces unsteady SV in the mantle reference frame.

1.1 Synopsis

Part IA (VOORHIES, 1992) offered some reasons to consider steady induction effects in geomagnetism. The source-free mantle/frozen-flux core (SFM/FFC) earth model (wherein a rigid, impenetrable, electrically insulating mantle of uniform magnetic permeability surrounds a spherical, inviscid, and perfectly conducting core in anelastic flow) was introduced and used to focus attention upon the fluid motion near the top of the core. The theory underlying some estimates of core surface flow was summarized. Some implications of a few kinematic and dynamic hypotheses were derived. The solution to the forward steady motional induction problem was found to be an example of deterministic chaos. Special attention was given to the long-term geomagnetic effects of persistent, surficially geostrophic flow.

To investigate steady induction effects in a quantitative way, it is useful to treat the supposition of steady surficial core flow as if it were a hypothesis to be tested geomagnetically. Then the idea is to see if such a flow can be designed to induce the secular change signal indicated by geomagnetic data (or broad-scale spherical harmonic models thereof) and, if not, assess the significance of the residual, unmodeled signal. Part IB (VOORHIES, 1993) offered a fairly general method for estimating a steady fluid flow at the top of the core in the context of the SFM/FFC model. The method is based upon iterative solution of the linearized weighted least-squares problem, but admits optional biases favoring surficially geostrophic flow and/or spatially simple flow.

As noted in Part IB, solution of the non-linear inverse problem posed by the hypothesis of (piecewise, statistically) steady core surface flow requires specification of a complete initial geomagnetic condition: the radial field component everywhere at the top of a FFC, $B_r(\mathbf{b},t)$, or, for a SFM, at Earth's surface, $B_r(\mathbf{a},t)$. Such a condition will not be specified by discrete geomagnetic data alone within the forseeable future. Moreover, tests of steady flow are likely embedded in some magnetic earth model-be it the SFM/FFC model or a refinement thereof. The rigor of a purely geomagnetic test is thus limited: it may be that errors in the initial condition or in the earth model, rather than errors in the steady motions hypothesis, lead to unacceptably large residuals. It is thus doubtful that geomagnetic tests of the steady motions hypothesis can be absolutely decisive. Fortunately, we already know that both the SFM/FFC model and the steady motions hypothesis are but conceivably relevant idealizations. The question remains as to what fraction of recent SV cannot be readily attributed to steady induction effects. Such unmodeled signal may contain evidence for many other interesting geomagnetic effects. It turns out that application of the method developed in IB is well suited to answering this question.

1.2 Review of the Theory

For steady flow, the radial component of the magnetic induction equation at the top of a frozen-flux core of radius b is, in spherical polar coordinates (r,θ,ϕ),

$$\partial_t B_{rp}(\mathbf{b},t) + \mathbf{v}_s(\mathbf{b}) \bullet \nabla_s B_{rp}(\mathbf{b},t) = B_{rp}(\mathbf{b},t)\partial_r u(\mathbf{b}) \tag{0}$$

which is a special case of the ROBERTS & SCOTT (1965) equation. With sums running over repeated subscripts, the compact spherical harmonic expansions for the input radial magnetic flux density $B_r(\mathbf{r},t)$ and for the predicted radial magnetic flux density $B_{rp}(\mathbf{r},t)$ are, respectively,

$$B_r(\mathbf{r},t) = g_i(r,t)S_i(\theta,\phi) \tag{1a}$$

$$B_{rp}(\mathbf{r},t) = \gamma_i(r,t)S_i(\theta,\phi). \tag{1b}$$

Here $S_i(\theta,\phi)$ is a Schmidt-normalized spherical harmonic; compact index i is uniquely determined by, and uniquely determines, both the spherical harmonic degree n(i) and the order m(i). Radii a = 6.3712 Mm and b = 3.480 Mm are of interest here, so define: $g_i \equiv g_i(a,t)$; $G_i \equiv g_i(b,t)$; $\gamma_i \equiv \gamma_i(a,t)$; and $\Gamma_i \equiv \gamma_i(b,t)$. For a SFM, $g_i = \Upsilon_{ij}G_j$ and $\gamma_i = \Upsilon_{ij}\Gamma_j$, where $\Upsilon_{ij} = [b/a]^{n(i)+2}\delta_{ij}$ is an element of the diagonal upward continuation matrix. The g_i are the input Schmidt-normalized Gauss coefficients (g_n^m, h_n^m) for the usual internal scalar magnetic potential at reference radius a multiplied by $[n(i) + 1]$.

The hypothetically steady fluid flow at the top of the core

$$\mathbf{v}_s(\mathbf{b}) = \nabla_s T(\mathbf{b}) \times \hat{\mathbf{r}} + \nabla_s U(\mathbf{b}) \tag{2a}$$

is parameterized in terms of the spherical harmonic coefficients for the expansions of the streamfunction -T and velocity potential -U:

$$T(\mathbf{b}) = \alpha_i S_i(\theta,\phi) \qquad U(\mathbf{b}) = \beta_i S_i(\theta,\phi). \tag{2b}$$

Equation (IB.15) follows from substitution of (1a-2b) into (0):

$$\partial_t \Gamma_k = (\Gamma_i X_{ijk})\alpha_j + (\Gamma_i Y_{ijk})\beta_j = P_{kj}\alpha_j + Q_{kj}\beta_j. \tag{3}$$

The (X_{ijk}, Y_{ijk}) are given by (IB.16); (P_{kj}, Q_{kj}) are given by (IB.17). Upward continuation of (3) gives equations (IB.18a-c):

$$\partial_t \gamma_i = \Upsilon_{ik}\partial_t\Gamma_k = \Upsilon_{ik}[(\Gamma_i X_{ijk})\alpha_j + (\Gamma_i Y_{ijk})\beta_j] \tag{4a}$$

$$= \Upsilon_{ik}[P_{kj}\alpha_j + Q_{kj}\beta_j] \tag{4b}$$

$$\equiv p_{ij}\alpha_j + q_{ij}\beta_j \equiv A_{il}\xi_l. \tag{4c}$$

The matrix notation for (4c) is $\partial_t \underline{\gamma} = \underline{\underline{A}}\,\underline{\xi}$, where $\underline{\xi}$ is the extended column vector of streamfunction and velocity potential coefficients and $\underline{\underline{A}}$ is the matrix of normal equations coefficients. The elements A_{il} of $\underline{\underline{A}}$ vary with time due to their dependence upon $B_{rp}(\mathbf{b},t)$, hence upon $\underline{\xi}$ and the initial geomagnetic condition.

1.3 Summary of the Method

The approach to estimating ξ, hence the α_i, the β_i, T, U, and steady flow $\mathbf{v}_s(\mathbf{b})$, described in Part IB is based on minimizing the function

$$4\pi\Delta^2 = 4\pi(\Delta_r^2 + \lambda_g\Delta_g^2 + \lambda_d\Delta_d^2). \tag{5a}$$

In (5a), λ_g and λ_d are adjustable damping parameters. The semi-normalized square-weighted residual, or misfit, is Δ_r^2:

$$\begin{aligned} 4\pi\Delta_r^2 &= \int_{t_o}^{t_f} (\underline{g} - \underline{\gamma})^T \underline{\underline{W}}(\underline{g} - \underline{\gamma})\,dt \\ &= \int_{t_o}^{t_f} \{[\int_{t_o}^{t} (\partial_\tau \underline{g} - \underline{\underline{A}}\,\underline{\xi})\,d\tau]^T \underline{\underline{W}}[\int_{t_o}^{t} (\partial_\tau \underline{g} - \underline{\underline{A}}\,\underline{\xi})\,d\tau]\}\,dt \\ &\equiv \overline{\overline{[\partial_\tau \underline{g} - \underline{\underline{A}}\,\underline{\xi}]}^T \underline{\underline{W}}\,\overline{[\partial_\tau \underline{g} - \underline{\underline{A}}\,\underline{\xi}]}} \end{aligned} \tag{5b}$$

where $\underline{\underline{W}}$ is the time-dependent radial field weight matrix and the initial condition is taken to be $\underline{g}(a,t_o) = \underline{\gamma}(a,t_o)$. For general weights, Ω/f replaces $\underline{\underline{W}}/4\pi$ in (5b). The mean-square ageostrophy of the flow is Δ_g^2:

$$\begin{aligned} 4\pi\lambda_g\Delta_g^2 &= \lambda_g \int_0^{2\pi}\int_0^{\pi} \{\partial_r u(\mathbf{b})\cos\theta + \frac{v(\mathbf{b})}{b}\sin\theta\}^2 \sin\theta\, d\theta\, d\phi \\ &\equiv 4\pi\lambda_g \langle\{\partial_r u\cos\theta + \frac{v}{b}\sin\theta\}^2\rangle \\ &= [\underline{\underline{B}}\,\underline{\xi}]^T \underline{\underline{\Lambda}}_g [\underline{\underline{B}}\,\underline{\xi}]. \end{aligned} \tag{5c}$$

The mean-square radial vorticity plus the mean-square downwelling is just $\Delta_d^2 \equiv \langle[\omega_r(\mathbf{b})]^2 + [\partial_r u(\mathbf{b})]^2\rangle$:

$$4\pi\lambda_d\Delta_d^2 = \underline{\xi}^T \underline{\underline{\Lambda}}_d \underline{\xi}. \tag{5d}$$

The linearized weighted least-squares estimate of the parameters is obtained by treating $\underline{\underline{A}}$ as if independent of $\underline{\xi}$ and minimizing (5a):

$$\underline{\xi} = [\overline{\underline{\underline{\bar{A}}}^T \underline{\underline{W}}\, \underline{\underline{\bar{A}}}} + \underline{\underline{B}}^T \underline{\underline{\Lambda}}_g \underline{\underline{B}} + \underline{\underline{\Lambda}}_d]^{-1} [\overline{\underline{\underline{\bar{A}}}^T \underline{\underline{W}}\, \overline{\partial_\tau \underline{g}}}] \tag{6}$$

which is (IB.32).

Because $\underline{\underline{A}}$ depends on the predicted radial field component $B_{rp}(\mathbf{b},t)$, it depends upon Γ, hence upon $\underline{\xi}$ and the initial condition. Minimization of (5a) is thus a non-linear inverse problem. To solve this problem by iterative solution of the linearized problem, initial matrices $\underline{\underline{A}}(0)$ are computed from the input $\underline{g}(t)$. Then (6) is solved for $\underline{\xi}(1)$ which, in turn, is used to construct $\mathbf{v}_s(\mathbf{b})$ and solve the forward problem (0) from the inital condition at t_o. The resulting predictions $B_{rp}(\mathbf{b},t)$ are used to compute the time-varying predicted radial field coefficients $\underline{\gamma}(1)$, the residuals $\delta \underline{g}(1) = \underline{g} - \underline{\gamma}(1)$, and new matrices $\underline{\underline{A}}(1)$ for the next estimate, $\underline{\xi}(2)$. With $\delta \underline{g}(j) = \underline{g} - \underline{\gamma}(j)$ and $\underline{\xi}(j+1) = \underline{\xi}(j) + \delta\underline{\xi}(j+1)$, the correction to $\underline{\xi}(j)$ is given by deep iteration equation (IB.33a):

$$\begin{aligned}\delta\underline{\xi}(j+1) = {} & [\overline{\underline{\underline{\bar{A}}}(j)^T \underline{\underline{W}}\, \underline{\underline{\bar{A}}}(j)} + \underline{\underline{B}}^T \underline{\underline{\Lambda}}_g \underline{\underline{B}} + \underline{\underline{\Lambda}}_d]^{-1} [\overline{\underline{\underline{\bar{A}}}(j)^T \underline{\underline{W}}\, \delta\overline{\partial_\tau \underline{g}}(j)}] \\ & - [\underline{\underline{B}}^T \underline{\underline{\Lambda}}_g \underline{\underline{B}}][\underline{\xi}(j) - \underline{\xi}_{go}] - \underline{\underline{\Lambda}}_d[\underline{\xi}(j) - \underline{\xi}_{do}].\end{aligned} \tag{7}$$

Shallow iteration is described by equation (IB.33b).

Note that $\underline{\xi}_{go}$ is taken to be $\underline{0}$ in the calculations. The routine prior bias $\underline{\xi}_{do}$ is also taken to be $\underline{0}$; however, some calculations have been performed with a learning algorithm so as to escape from this bias against non-trivial parameters. Then $\underline{\xi}_{do}$ is taken to be $\underline{\xi}(j)$ and $\lambda_d \rightarrow \lambda_d(j+1)$ is selected to keep $\underline{\xi}(j+1)$ close to $\underline{\xi}(j)$ (see IB, section 3.3). The learning algorithm is used to explore the tightness of fit afforded by steady flows which appear so spatially complicated as to inhibit convergence of the iteration scheme (7)–as expected if (5a) has multiple extrema for very small values of λ_d.

2. APPLICATION

The foregoing method has been applied to estimate piecewise steady, optionally geostrophic, and optionally damped, surficial core motions which induce the secular change indicated by the Definitive Geomagnetic Reference Field (DGRF) models. The DGRF models were derived by Working Group 1.1 of the International Association of Geomagnetism and Aeronomy (IAGA, 1988). The DGRFs are slowly varying, broad-scale spherical harmonic models of the magnetic field observed at and near Earth's surface. They include the 120 Gauss coefficients through degree and order 10 at 5-year intervals from 1945 to 1980. Linear interpolation gives the recommended broad-scale geomagnetic field model between DGRF epochs (PEDDIE, 1982); however, the work of ALLDREDGE (1987) and of LANGEL et al. (1988) indicates that such interpolation need not, and indeed should not, be taken too seriously.

The DGRF model for epoch 1980 consists of the Gauss coefficients of the model GSFC (12/83) derived by LANGEL & ESTES (1985) rounded to the nearest nT and truncated to degree and order 10.

PEDDIE (1982) describes the derivation of the DGRF models for epochs 1965, 1970, and 1975. Each of these models is a weighted average of three candidate models derived by different groups of geomagnetists: the GSFC (9/80) model derived by LANGEL et al. (1982); the IGS models derived by BARRACLOUGH et al. (1982); and the USGS models derived by PEDDIE & FABIANO (1982). The weighted averages are rounded to the nearest nT and truncated to degree and order 10. The time-dependent weights assigned to the GSFC, IGS, and USGS models are, respectively, (0.2, 0.6, 0.2) for 1965; (0.25, 0.50, 0.25) for 1970; and (0.3, 0.4, 0.3) for 1975 (PEDDIE, 1982).

The DGRF models for epochs 1945, 1950, 1955 and 1960 were derived by LANGEL et al. (1988). The Gauss coefficients published in that paper are rounded to the nearest 0.1 nT, are accompanied by simultaneously derived SV coefficients, and include uncertainty estimates. The latter estimates depend upon prior assumptions including: (1) the spatial power spectra of the core and crustal fields used to form correlated data weight matrices and (2) the requirement that in regions of poor data coverage the models do not stray too far from a MAGSAT model projected back in time (LANGEL et al., 1988).

Spherical harmonic models of the geomagnetic field have been derived for about 150 years; they represent a vast number of observations quite well. The 8 DGRF models can be (and with general weights are) viewed as 8 points in a 121-dimensional sub-space: 120 dimensions for the coefficients and one for time. The covariance matrices derived for the DGRF models (see section 3.1) describe estimated error ellipsoids around each such point (excluding 1980). My strategy for estimating a single steady flow is to take one of these points (e.g., 1980) as an initial condition and seek a trajectory whose projection onto the 121-dimensional sub-space passes close to the other 7 points. The trajectory is the predicted magnetic field determined by upwardly continuing the results of steady, frozen-flux motional induction at the CMB through a SFM. If a trajectory is found whose projection passes close enough to (e.g., intercepts) all 7 error ellipsoids, then the steady flow hypothesis and the SFM/FFC earth model are compatible with the DGRF models and any geomagnetic data they adequately represent. Indeed, the steady flow hypothesis would then seem to be in the same class as the candidate DGRF models. If no such trajectory is found, then the steady flow hypothesis or the earth model should be suspected of significant error. If no such trajectory exists, then one or more of these suppositions is too poor to provide a quantitatively adequate explication of the indicated secular change.

Application of the method derived in IB (or similar methods) to a particular set of Gauss coefficients (or original data) presents particular technical problems which must be solved either before or during the calculations. The rest of this section describes how these problems have been solved for the case of the DGRFs.

2.1 Initial Conditions

Solutions of the forward steady motional induction problem at the top of a FFC can exhibit an extreme sensitivity to initial conditions common to many problems in deterministic chaos (IA, section 3). Initial conditions should therefore be selected with care. The 1980 DGRF model represents global geomagnetic data gathered by NASA's Magsat satellite as well as other types of data, so it was chosen to constrain the initial conditions by selecting $t_o = 1980$ when fitting either the entire 35-year sequence of DGRF models ($t_f = 1945$) or any sub-interval including epoch 1980. This is a relatively ambitious initial condition because 1980 is an endpoint rather than an interior point of the 35-year time interval, and because errors in the initial condition are expected to grow exponentially with elapsed time $|t - t_o|$ (see equation (IA.14b)). This choice will, however, enjoy an advantage when extending the steady motions hypothesis to make and test predictions for subsequent epochs.

A complete initial condition is required. Though complete in the spatial domain, the DGRFs are truncated spherical harmonic models and are thus incomplete in the Fourier-Legendre spectral domain. The 1980 DGRF model contains no information about Gauss coefficients of degrees greater than 10, so some supposition is needed to complete this model. I have supposed that all Gauss coefficients of degrees greater than 10 are zero at 1980–the expectation value indicated by any probability distribution for these Gauss coefficients which enjoys zero mean. This supposition, along with the SFM/FFC model, limits my ability to falsify the steady flow hypothesis: if this initial condition does not allow derivation of a satisfactory fit to the other 7 DGRF models, there may be a different initial condition with a non-trivial narrow-scale core field which does. Because the 1980 DGRF model is a truncated version of model GSFC (12/83), which includes main field coefficients through degree and order $N_F = 13$, coefficients of degrees 11, 12, and perhaps 13 can be used to test the sensitivity of misfit, estimated velocity fields, and geomagnetic field forecasts to changes in the narrow-scale part of the initial magnetic conditions. Indeed, a test case showed that including non-trivial initial coefficients through degree 12 gave a very slight improvement in fit to the other 7 DGRFs. Effects of still higher degree initial coefficients could be investigated via Monte Carlo inversions of deterministic chaos–a seemingly unenlightening pursuit. Reliance upon non-trivial, narrow-scale initial geomagnetic conditions to provide a satisfactory fit may be possible, but it seems inelegant; indeed, contrivance of such conditions might well be dismissed as a fluke. Of course, with a much more realistic earth model, such design could reveal fine core field structure which is otherwise masked by the crustal field.

2.2 Magnetic Truncation

The spherical harmonic expansions for the input radial field (1a) and the predicted radial field (1b) involve summations over repeated subscript i which ought to run from one to infinity (or to the highest wavenumber describing the macroscopic geomagnetic field). The same is true of the summations required to evaluate the square-weighted residual (5b) and thus to estimate the (similarly infinite dimensional) vector of flow parameters ξ. These sums must be truncated in practice.

The DGRF models do not specify the Gauss coefficients of degrees greater than 10 ($n > N_B = 10$) and thus do not specify the radial field coefficients g_i with $i > N_B(N_B+2) = 120$. This does not imply, nor is it tantamount to assuming, that such high-degree coefficients are zero! It means only that such high-degree coefficients are not known well enough, are hopefully not important enough, and are perhaps too influenced by the crustal field, to include in the DGRF models. Therefore, if the spherical harmonic representation of a predicted magnetic field model contains non-zero radial field coefficients γ_i with $i > 120$, such coefficients must not be directly included in the square-weighted residual. This requirement can be met by truncating both g_i and γ_i in (5b) to length $i_{max} = 120$ when evaluating and Δ_r^2. Such truncation is consistent with the assumption that high-degree DGRF cofficients are unknown. The possibility of using prior beliefs or biases about the evolution of the narrow-scale core field to constrain predicted high-degree coefficients is not pursued here. Such coefficients can be derived by extending spherical harmonic analysis of the predicted field past the 10^{th} degree and they may then be examined in detail. This was not done routinely; however, both the rms predicted radial field component, $\langle [B_{rp}(\mathbf{b},t)]^2 \rangle^{1/2}$, and the rms time rate of change of the predicted radial field, $\langle [\partial_t B_{rp}(\mathbf{b},t)]^2 \rangle^{1/2}$, are routinely calculated to verify that implausible contributions from narrow-scale radial fields at the CMB are in fact not predicted. The former does not exceed a few gauss and the latter typically does not exceed several thousand nT/year.

An alternate approach to the magnetic truncation problem is through the radial field weight function $W(\mathbf{a},t)$ and weight matrices $\underline{\underline{W}}(a,t)$ (or the general weight matrices $\underline{\underline{\Omega}}(a,t)$). The objective is to match the evolution of the broad-scale part of the radial field alone, so the weight function must reflect uncertainties in the broad-scale radial field represented by low-degree coefficients alone. Then (possibly unknown) radial field coefficients of degrees greater than some maximum degree (here 10) must be assigned zero weight. To do so one may set the W_{ij} with i or j (or both) greater than 120 equal to zero. Similarly, if the objective is to match the evolution of the broad-scale scalar potential, then direct contributions to the square-weighted residual from high-degree coefficients must be assigned zero weight. This can be accomplished by truncating each $\underline{\underline{\Omega}}(t)$ to a 120x120 matrix. Truncation of $\underline{\underline{W}}$ (or $\underline{\underline{\Omega}}$) has the same effect on the square-weighted residual as truncation of both $\underline{g}$ and $\underline{\gamma}$. Both approaches avoid including the direct effect of non-trivial predicted high-degree coefficients in the square-weighted residual.

Truncation of weight matrices does not necessarily justify truncation of covariance matrices used to derive them. Ideally, complete covariance matrices would be inverted for complete weight matrices. Truncation of the complete weight matrices would then target the broad-scale objective.

In practice, all g_i above degree 10 are taken to be zero only to start the iteration scheme ($j = 0$ in equation (7)). They are also taken to be zero at time t_o when an initial condition is needed to prime the forward motional induction scheme. The latter is a finite difference representation of equation (0) which induces SV by steady numerical advection of the radial field on a $2^\circ \times 2^\circ$ regular mesh using 10^{-2} year time steps. (The adequacy of a 2° mesh has been confirmed by comparing test results with those obtained using a 1° mesh; the accuracy of the first-order time-stepping has been confirmed using a second-order scheme). The resulting numerical solutions of the forward steady motional induction problem implicitly represent the predicted magnetic field through very high degree: the azimuthal Nyquist wavenumber is 90 on the $2^{o}x2^{o}$ mesh.

This fine resolution allows key effects of the predicted narrow-scale core field on both the solution of the forward problem and the iterative solution of the inverse problem to be retained without having to calculate predicted coefficients above degree 10. Such high-degree coefficients would represent narrow-scale core field structure induced by the interaction of the steady fluid motion with the radial magnetic flux density (VOORHIES, 1984; 1986a; 1986b). Because a steady fluid flow also interacts with the narrow-scale field to induce secular change on all spatial scales, the predicted narrow-scale fields eventually influence the predicted broad-scale field. Specifically, predicted high-degree coefficients eventually influence predicted low-degree coefficients (see untruncated equations IB.15-16b). Because predicted low-degree coefficients determine the square-weighted residual, this indirect effect of predicted high-degree coefficients must be included to accurately measure misfit.

Only the γ_i through degree 10 contribute directly to the square-weighted residual, so only the $\partial_t\gamma_i$ through degree 10 contribute directly to (5b) (see (IB.9c , 11c and 19)). It follows that linearized attempts to minimize the square-weighted residual need only account for the first 120 rows of $\underline{\underline{A}}$ (VOORHIES, 1986b). This is also true for attempts to solve the non-linear inverse problem by iterative solution of the linear problem. However, as noted above, the indirect effects of predicted high-degree coefficients on the predicted low-degree coefficients must be retained when solving the non-linear problem. This requirement can be fairly well, if not strictly, met by several techniques—each of which demands very accurate calculation of the total predicted radial field component needed in (4a-c).

The physical technique adopted tracks $B_{rp}(\mathbf{b},t)$ in the physical domain via finite-difference solution of equation (0) on a fine mesh. Values of $B_{rp}(\mathbf{b},t)$ at the mesh points are periodically stored for use with (4b). Only the low-degree coefficients needed to evaluate the residuals and $\delta\partial_\tau \underline{g}(j)$ are extracted from B_{rp}. The surface integrals giving $\underline{\underline{P}}$ and $\underline{\underline{Q}}$ (IB.17) are evaluated numerically, so both broad and narrow scales in the predicted field contribute to the new $\underline{\underline{A}}(j)$ needed for the next deep iteration (7). One pseudo-spectral technique would solve (0) in the physical domain, but periodic spherical harmonic analysis of $B_{rp}(\mathbf{b},t)$ for the $\Gamma_k(t)$ would extend through very high degree and order (say 90). These Γ_k would then be used in (4a) with X_{ijk} and Y_{ijk} evaluated through extremely high wavenumber k (say $k \le 8,280$), but only $i \le 120$. A purely spectral technique based on (4a) would track evolution of the $\Gamma_k(t)$ through very high degree using (3), the X_{ijk} and Y_{ijk} from (IB.16a-b) evaluated to extremely high wavenumbers i and k, and, say, $\Gamma_k(t+\delta t) \approx \Gamma_k(t) + [\partial_t\Gamma_k(t)]\delta t$ with very small δt. Efficient evaluation of the X_{ijk} and Y_{ijk} has been achieved by K. A. Whaler and D. Winch (1987, personal communication). Spectral methods may be more efficient computationally, but I prefer the simple, direct physical method (which might be readily adapted to an aspherical CMB).

2.3 Velocity Truncation and Damping

The original method of VOORHIES (1984; 1986a) yields steady flows which fit SV models at the CMB under the assumption that SV coefficients above degree N_B are zero. It was found that the streamfunction should include coefficients through degree 12 or more in the case of purely toroidal flow. For combined flows, the preferred solution (C4) included both streamfunction and velocity potential coefficients through $N_T = N_U = N_V = 12$. The second method (VOORHIES, 1986b) yields steady flows which fit SV at Earth's surface without assuming zero high-degree SV; however, omission of non-linear aspects of the inverse problem and the avoidance of damping prohibited meaningful estimation of flow coefficients above degree 5. Estimates with $N_V > 5$ gave smaller linear residuals, but larger genuine, non-linear, residuals obtained via forward solution of the steady motional induction problem, spherical harmonic analysis of $B_{rp}(\mathbf{b},t)$, upward continutation, and comparison with the DGRFs.

After some experimentation with new algorithms, I chose to truncate the velocity field parameterization at degree and order $N_V = 16$ and to use non-zero damping λ_d. Then 288 α_i and 288 β_i, or 576 velocity field parameters ξ_i, are determined; these parameters are not entirely free when either λ_d or λ_g (or both) are non-trivial. Such truncation is satisfactory when equation (7) is solved with large to moderate λ_d and $\underset{\sim}{\xi}_{do} = \underset{\sim}{0}$. The latter selections also yield solutions $\underset{\sim}{\xi}(j)$ which converge in a few iterations. When λ_d is too small, shallow iteration using the initial $\underset{\approx}{A}(j = 0)$ matrices did not appear to yield convergent solutions, indicating a failure of the linearization in the case of vigorous, complicated flow. Moreover, deep iteration with small λ_d sometimes gave $\underset{\sim}{\xi}(j)$ converging towards local minima characterized by greater square-weighted residuals than solutions of lesser roughness. This is to be expected when non-linear aspects of the inverse problem are not overwhelmed by a fixed prior bias $\underset{\sim}{\xi}_{do} = \underset{\sim}{0}$ (towards zero solution norm). Physically, the spectral mapping of a fixed prior bias towards zero norm can inhibit discovery of solutions with (a priori, improbably) tall peaks in the surface kinetic energy density (SKED) spectrum (VOORHIES, 1984, 1986a) that offer a smaller misfit at equal or lesser norm. Linear stability calculations (CHANDRASEKHAR, 1961) lead me to suggest that tall peaks in the SKED spectrum are in fact quite plausible.

The learning algorithm does not suffer so much from this drawback because it allows iterative growth of tall spectral peaks which may be needed to reduce misfit. It was used to derive rougher flows and tighten the upper bounds on the apparent error in the earth model. Examination of SKED spectra indicates $N_V = 16$ is acceptable for flows of moderate roughness derived using the learning algorithm. The learning algorithm need not produce convergent $\underset{\sim}{\xi}(j)$ (unless $\lambda_d(j+1)$ is set to infinity), so increasing N_V could reduce the misfit for the roughest flows explored. For very large N_V, the learning algorithm yields flows with SKED spectra which must in principle still fall off faster than n^{-3} at high n; however, the amplitude of the spectra will depend on both the convergence factors ($\lambda_d(j)$) used in, and the solutions obtained by, all previous iterations. Such hysteresis effects may occur in other applications of the learning algorithm.

2.4 Integrations

The double time integrals in equation (7) can be approximated numerically in many ways. I chose a simple method based on accepting the suggested linear interpolation between DGRF models at face value.

The interval t_o to t_f (e.g., 1945 to 1980) is subdivided into L = 7 sub-intervals. For each sub-interval l, let t_l denote the mid-point, t_{l-} the beginning, t_{l+} the end, and let $\Delta t_l = t_{l+} - t_{l-}$ denote the duration (5 years). The time rate of change of the radial field is approximated by a temporally constant, but spatially variable, SV in each such sub-interval in accord with the suggested linear interpolation. The time-dependent $\underline{\underline{A}}$ matrix is the upward continuation of the concatenated $\underline{\underline{P}}$ and $\underline{\underline{Q}}$ matrices (4c). Elements of $\underline{\underline{P}}$ and $\underline{\underline{Q}}$ (IB.17a-b) are linear in the radial field at the CMB, so the integral of any such element over any sub-interval Δt_l is approximated by the value of that matrix element at the sub-interval midpoint t_l multiplied by Δt_l. These values were estimated by substituting the arithmetic mean of $B_{rp}(\mathbf{b},t)$ at times t_{l+} and t_{l-} into equations (IB.17a-b) and evaluating the integrals over the CMB numerically using a simple double Riemann sum approximation on a 2^ox2^o mesh.

The dummy time integral of the A matrix is thus approximated by its Riemann sum

$$\overline{\underline{\underline{A}}(\tau)}(t) \equiv \int_{t_o}^{t} \underline{\underline{A}}(\tau)d\tau \approx \overline{\underline{\underline{A}}}(t_{l+}) = \int_{t_o}^{t_{l+}} \underline{\underline{A}}(\tau)d\tau \approx \sum_{k=1}^{l} \underline{\underline{A}}(t_k)\Delta t_k \tag{8a}$$

and the elements of $\overline{\delta\partial_\tau \underline{g}}$ at time t are approximated by

$$\overline{\delta\partial_\tau \underline{g}(\tau)}(t) \equiv \int_{t_o}^{t} \delta\partial_\tau \underline{g}(\tau)d\tau \approx \overline{\delta\partial_\tau \underline{g}(\tau)}(t_{l+}) = \int_{t_o}^{t_{l+}} \delta\partial_\tau \underline{g}(\tau)d\tau$$

$$\approx \sum_{k=1}^{l} \delta\partial_\tau \underline{g}(t_k)\Delta t_k \tag{8b}$$

where

$$\delta\partial_t g_i(t_k) = \{[g_i(t_{k+}) - g_i(t_{k-})] - [\gamma_i(t_{k+}) - \gamma_i(t_{k-})]\}(\Delta t_k)^{-1}. \tag{8c}$$

If the initial condition were at 1945, then k would be incremented from 1 (sub-interval 1945 to 1950) to l (sub-interval t_{l-} to t_{l+}). With the initial condition at 1980 (the end of the interval), k is actually decremented from 7 (sub-interval 1975 to 1980) to 1. However, it is convenient (and thermodynamically permissible) to imagine time running backwards in this dissipationless (and thus isentropic) problem. Then relabeling the sub-intervals still allows (8) to be used.

Because $\underline{\underline{A}}$ and $\delta\partial_\tau \underline{g}$ are treated as steady during each sub-interval in the dummy time integrals, it is convenient to further approximate the weight matrix elements W_{ij} as constants during each sub-interval. Then the double time integral of $\underline{\underline{A}}^T\underline{\underline{W}}\underline{\underline{A}}$ in equations (7) is approximated by

$$\overline{\overline{\underline{\underline{A}}(\tau)}^T \underline{\underline{W}}(t)\ \overline{\underline{\underline{A}}(\tau)}} \approx \sum_{l=1}^{L} \{ [\sum_{k=1}^{l} \underline{\underline{A}}(t_k)\Delta t_k]\ \underline{\underline{W}}(t_l)\ [\sum_{k=1}^{l} \underline{\underline{A}}(t_k)\Delta t_k] \} \Delta t_l \tag{9a}$$

and the double time integral of $\underline{\underline{A}}^T \underline{\underline{W}} \delta\partial_\tau \underline{g}$ is approximated by

$$\overline{\overline{\underline{\underline{A}}(\tau)}^T \underline{\underline{W}}(t)\ \overline{\delta\partial_\tau \underline{g}(\tau)}} \approx \sum_{l=1}^{L} \{ [\sum_{k=1}^{l} \underline{\underline{A}}(t_k)\Delta t_k]\ \underline{\underline{W}}(t_l)\ [\sum_{k=1}^{l} \delta\partial_\tau \underline{g}(t_k)\Delta t_k] \} \Delta t_l . \tag{9b}$$

The finite time element approximations (9a-b) seem particularly appropriate for initial iterations ($j = 0$) using the DGRFs–provided the weight matrices are roughly constant. If they are not, it would be better to use a more precise technique (e.g., select $L > 7$). This has not been attempted because the best method for interpolating the DGRF models and their weight matrices is not known. (A more accurate method for evaluating the dummy time integral for $j > 0$ which exploits the results of the forward solution has recently been adopted).

No allowance for the polar caps was made in the spatial numerical integrations used to calculate elements of $\underline{\underline{P}}$ and $\underline{\underline{Q}}$, hence $\underline{\underline{A}}$. These matrix elements are thus accurate to only about two parts in 10^4. Indeed, downward continuation of the 1980 DGRF to the CMB, evaluation of $B_r(\mathbf{b}, 1980)$ on the mesh, spherical harmonic analysis through degree 10 via numerical integration, and subsequent upward continuation yielded rms errors in the radial field of 6.77 nT (about 2 parts in 10^4). This error is due primarily to inexact reproduction of zonal ($i = n^2$) coefficients and to the large axial dipole in particular. Although this error is smaller than the minimum 25 nT rms uncertainty in the pre-1980 DGRF radial field models (see section 3.2), a polar correction was included in the spherical harmonic analysis of the predicted field for the γ_i at all DGRF epochs so as to calculate the misfit precisely. This reduced the error from 6.77 nT to 0.11 nT in the test case.

Small errors resulting from imperfectly exact integrations are unimportant compared with expected errors in the underlying physical assumptions. For example, it is doubtful that the radius of the core is known to better than 1 part in 10^3: one survey of seismic estimates indicated the core radius to be 3485 $\pm$ 5 km (VOORHIES, 1984). Because errors in the core radius δb (and larger errors associated with the omission of core ellipticity) amplify as $[a/(b+\delta b)]^{n+1}$ when B_r is extrapolated from Earth's surface to the CMB, they should exceed the spatial integration errors. Fine tuning of the spatial integration is not too difficult, and may be needed when core asphericity is included. (Indeed, extremely accurate spatial integrations based on a quartrature rule have recently been developed and implemented).

Tests conducted with synthetic field models show that the numerical methods do not contribute significantly to the misfit, Δ_r^2. These tests also reveal that recovery of steady flows used to synthesize test case secular change models, and thus the effective elimination of misfit, requires extremely small damping λ_d. Nevertheless, I can but establish upper bounds on the error in the earth model because of the uncertainty in narrow-scale initial magnetic conditions, the finitude of N_V, and limitations imposed by either fixed bias or learning algorithms.

3. DERIVATION AND IMPLICATIONS OF WEIGHT MATRICES FOR THE DGRF MODELS

3.1 Derivation of the Weight Matrices

Derivation of weight matrices for the 1945, 1950, 1955 and 1960 DGRF models proved straightforward. The total information matrices for these models were generously provided by R. Langel and T. Sabaka (LANGEL et al., 1988). These 200x200 matrices were inverted to obtain the full covariance matrices for these DGRF models. The latter were stripped of main field-secular variation (MF-SV) and SV-SV coefficient covariances because the main field weighting scheme was selected. The square roots of the diagonal elements of the resulting 120x120 main field covariance matrices, $\underline{\underline{V}}(t)$, are the uncertainty estimates for the main field coefficients published by LANGEL et al. (1988). The elements $V_{ij}(t)$ were multiplied by $[n(i)+1][n(j)+1]$ to obtain the epoch-dependent elements of the covariance matrix for the input radial field coefficients, $E_{ij}(t_n)$. These matrices were used to calculate the squared uncertainty estimates for the DGRF models at the mesh points of the 2^ox2^o regular grid using the inverse of equation (IB.10). The resulting $\sigma B_r(\mathbf{a},t_n)$, the square root of the inverse weight functions $W(\mathbf{a},t)^{-1/2}$, are the same as the uncertainty estimates contoured by LANGEL et al. (1988, figures 2c, 3c, 4c, and 5c). As expected, the weight function is large (heavy) in areas of dense data coverage (e.g., Europe) and small (light) where data are sparse (e.g., the SE Pacific Ocean and portions of the Indian Ocean). The weight matrix elements W_{ij} at DGRF epochs 1945, 1950, 1955, and 1960 were evaluated by numerical integration of (IB.12a).

A very different method was used to derive weight matrices for the DGRF models at epochs 1965, 1970, and 1975. Recall that a weighted average of the three candidate models defines the DGRF models at these epochs. The weight factors are supposed to reflect the apparent accuracy of the candidate models (PEDDIE, 1982). This weighted average was taken as defining the expected value of the Gauss coefficients and thus the expected value of functions thereof.

Let the vector of radial field coefficients for candidate model k at epoch t_n be denoted by its elements $g_i^k(t_n)$. Then the radial field coefficients for the DGRF model at epoch t_n are

$$g_i(t_n) \approx \bar{g}_i(t_n) \equiv w_{ij}^k(t_n)\; g_j^k(t_n) \tag{10}$$

where the approximation represents roundoff error. The weight tensor $w_{ij}^k(t_n)$ depends on epoch t_n, candidate model k and the coefficient of interest i; its projection at a given t_n and fixed k is a matrix which is diagonal in the coefficient subscripts i and j. At epochs 1970 and 1975 all candidate models are complete through at least degree 10; w_{ij}^k is then the non-zero constant w^k for $i = j \le 120$ and zero for all other (i,j). Higher degree coefficients, including the GSFC (9/80) coefficients of degrees 11 through 13, are assigned zero weight; this does not imply that they are zero.

The 1965 IGS candidate model includes coefficients only through degree 8 ($i \le 80$). Inspection reveals the 1965 DGRF coefficients of degree 9 and 10 ($81 \le i \le 120$) to be the rounded, equally weighted averages of the GSFC and USGS candidate models. This is in accord with the assignment of zero weight to the undetermined degree 9 and 10 coefficients of the 1965 IGS model. These coefficients are not assumed to be zero here, so the 1965 weight tensor elements depend upon coefficient degree. Note that the assignment of zero weight to certain geomagnetic coefficients is a geophysical judgment; it is not a mathematical result obtained by inverting infinite dimensional information matrices with elements either determined or bounded by either data or prior information.

Equation (10) is here taken as defining the expected value of the DGRF model coefficients. This expected value is a linear combination of the candidate model coefficients: the weighted average with weights inversely related to the uncertainty ascribed to the candidate models. Linear combinations of DGRF coefficients are thus linear combinations of the candidate models, so the expected value of the error of any linear function of DGRF coefficients is zero (apart from roundoff error). For example, the expected broad-scale radial field component

$$E\{B_r(\mathbf{a},t_n)\} = E\{g_i{}^k(a,t_n)S_i(\theta,\phi)\} = [w_{ij}{}^k(t_n)\ g_j{}^k(t_n)]S_i$$
$$= \overline{g}_i(t_n)S_i \approx g_i(t_n)S_i \tag{11a}$$

is that given by the DGRF coefficients (to within roundoff error). The individual coefficient deviations

$$d_i{}^k(t_n) \equiv g_i{}^k(t_n) - g_i(t_n) \tag{11b}$$

have been calculated using the published candidate and DGRF model coefficients. The expected error in the broad-scale radial field

$$E\{\delta B_r(\mathbf{a},t_n)\} = E\{d_i{}^k(a,t_n)S_i(\theta,\phi)\} = [w_{ij}{}^k(t_n)\ d_j{}^k(t_n)]S_i$$
$$= [\overline{g}_i(t_n) - g_i(t_n)]S_i \approx 0 \tag{11c}$$

is indeed zero to within the expected cumulative (rss) roundoff error of 0.7 nT per Gauss coefficient. Note that non-linear functions of DGRF coefficients like the scalar geomagnetic intensity or the absolute flux linking the core (BENTON & VOORHIES, 1987) are non-linear combinations of the candidate models and need not have zero expected error.

The expected value of the squared error in the broad-scale radial field is needed to calculate the weight function (IB.10). This is

$$E\{[\delta B_r(\mathbf{a},t_n)]^2\} = E\{[S_i d_i{}^k][d_j{}^k S_j]\} = S_i E\{[d_i{}^k][d_j{}^k]\}\ S_j$$
$$= S_i\ E_{ij}(t_n)\ S_j = [\sigma B_r(\mathbf{a},t_n)]^2 = W(\mathbf{a},t_n)^{-1}. \tag{12a}$$

Cross correlations between candidate models (different superscripts) and different epochs (t_n) are omitted in accord with (10). For each t_n and each k, introduce the diagonal matrix square root of $w_{ij}{}^k(t_n)$, $\underline{\underline{\omega}}$:

$$\omega_{im}(k;t_n)\ \omega_{mj}(k;t_n) = w_{ij}{}^k(t_n) \tag{12b}$$

with no sum over k or n. The expected value of the squared uncertainty in B_r, $[\sigma B_r(\mathbf{a},t)]^2$, is

$$S_i\ E_{ij}\ S_j = S_i\ \{[\omega_{il}{}^k\ d_l{}^k][\omega_{jm}{}^k\ d_m{}^k]\}\ S_j = W(\mathbf{a},t)^{-1} \qquad (12c)$$

and the covariance matrix for the radial field coefficients is

$$E_{ij} = \Sigma_k\ [\Sigma_l\ \omega_{il}{}^k\ d_l{}^k][\Sigma_m\ \omega_{jm}{}^k\ d_m{}^k] = \Sigma_k\ E_{ij}{}^k \qquad (12d)$$

where the sum on k is performed after the relevant i by j matrices have been formed. The covariance matrix is thus a weighted sum of dyadics. The $\underline{\underline{\omega}}$ matrix allows the coefficient deviations $d_m{}^k$ to be weighted in accord with the DGRF averaging procedure. Because the elements of the weight tensor $\underline{\underline{w}}$ are normalized to unity, both the squared uncertainty in the radial field (12c) and the covariance matrix elements (12d) have the correct units of nT^2.

The radial field covariance matrix elements E_{ij} were calculated using the published Gauss coefficients and (12d). The estimated uncertainty (conditional standard deviation) of the DGRF Gauss coefficients is the ratio of the square root of the diagonal elements of E_{ij} to $n(i)+1$. At epochs 1970 or 1975 the elements of symmetric $\underline{\underline{E}}$ are of the form $E_{ij} = \Sigma_k\ d_i{}^k w^k d_j{}^k$. This is not true at epoch 1965, when the E_{ij} with both $i > 80$ and $j < 80$ reflect both the non-zero probability $(1 - (0.5\text{x}0.2)^{1/2} = 68.4\%)$ that they are unknown and the naive expectation that unknown elements of E_{ij} are zero. Other expectations could well overestimate the uncertainty in the DGRF models. Such poorly determined cross-correlations between different coefficient deviations are assigned reduced weight in my attempts to weight these evolving broad-scale geomagnetic field models.

The squared uncertainty estimates $W(\mathbf{a},t_n)^{-1}$ were evaluated at the mesh points of the regular $2^o\text{x}2^o$ grid using (12c) (the inverse of (IB.10)). This quantity is positive at all such points; however, at a few (4 or 5) points it is less than 1 nT^2, indicating possibly fortuitous outstanding agreement between candidate models. This can be understood in terms of contours of perfect agreement between candidates: contours where the first and second models agree can intersect contours where the second and third models agree. The points of intersection enjoy singular weight. More typically, the weight function again proved to be heavy in regions of dense data coverage (e.g., Europe) where the candidate models should agree, and light in regions of sparse data coverage (e.g., the SE Pacific Ocean and portions of the Indian Ocean) where the candidate models can differ more easily. The weight matrix elements W_{ij} at DGRF epochs 1965, 1970, and 1975 were then evaluated by numerical integration of (IB.12a).

The weight matrices used in equations (9) and thus the solutions of (7) must reflect the typical uncertainty in the broad-scale radial field during the sub-intervals between DGRF epochs. These were estimated by numerical integration of equation (IB.12a) with the weight function taken to be the inverse of the arithmetic mean of the DGRF inverse weight functions at the sub-interval endpoints. Between DGRF epochs the squared uncertainty in the radial field $[\sigma B_r(\mathbf{a},t)]^2$ is thus treated as if it were the (spatially variable) constant equal to the average of its values at the sub-interval endpoints. For the interval 1980-1975, this procedure was used with the additional assumption that the uncertainty in the initial radial field at epoch 1980 is negligible. The estimated rms uncertainty in $B_r(\mathbf{a},t)$ at 1980 for the GSFC 12/83 through degree and order 10 is much smaller than that for the 1975 DGRF (16 nT vs. 58.30 nT), so this is a fair approximation.

For general weights, the $\Omega^*_{ij}(t_n)$ are used in place of the $W_{ij}(t_n)$ described above. The derivation of Ω^* matrices and the effect of using them are described in the Appendix (also see the Appendix of Part IB).

3.2 Weighted Variance in the 1980-1945 DGRF Models

There are many quantities of geophysical interest which can be determined from the DGRF models and their covariance matrices. Of immediate interest are the more easily interpreted scalar quantities which describe the signal and the noise in these models as viewed from the standpoint of one attempting to model geomagnetic secular change relative to the 1980 MAGSAT epoch.

One such quantity is the root-mean-square change in the radial magnetic flux density relative to 1980 averaged over Earth's surface

$$\Delta B_r(a,t) \equiv \langle [B_r(\mathbf{a},t) - B_r(\mathbf{a},1980)]^2 \rangle^{1/2} \tag{13}$$

(VOORHIES, 1986b). Another is the rms uncertainty in the radial field obtained from the DGRF covariance matrices described in section 3.1

$$\sigma B_r(a,t) \equiv \langle [\sigma B_r(\mathbf{a},t)]^2 \rangle^{1/2}. \tag{14a}$$

Such integral measures have many representations; for example, the square of (14a) is

$$[\sigma B_r]^2 = \langle W(\mathbf{a},t)^{-1} \rangle \tag{14b}$$

$$= \langle S_k \, E_{kl} \, S_l \rangle \tag{14c}$$

$$= \sum_{i=1}^{120} \{E_{ii}/[2n(i)+1]\} \tag{14d}$$

$$= \sum_{i=1}^{120} \{(\sigma g_i)^2/[2n(i)+1]\} \tag{14e}$$

$$= \sum_{n=1}^{10} \frac{(n+1)^2}{2n+1} \sum_{m=0}^{n} \{[\sigma g_n^m]^2 + [\sigma h_n^m]^2\} \tag{14f}$$

where (14d) follows from (14c) by orthogonality of the spherical harmonics, the σg_i in (14e) are the estimated uncertainties in the radial field coefficients, and (σg_n^m, σ_n^m) in (14f) are the estimated uncertainties in the Gauss coefficients of standard notation. The ratio $\Delta B_r(a,t)/\sigma B_r(a,t)$ provides a crude dimensionless measure of how significant a change in the radial field is called for by the DGRF model at time t relative to the 1980 DGRF model.

A better measure of the secular change signal in a DGRF radial field model relative to the 1980 DGRF is the weighted version of (13)

$$\delta_0(a,t;t_o) \equiv \langle [B_r(\mathbf{a},t) - B_r(\mathbf{a},1980)]^2 W(\mathbf{a},t)\rangle^{1/2} \tag{15}$$

$$= (\{[g_i(t_k) - g_i(1980)]\ W_{ij}(t_k)\ [g_j(t_k) - g_j(1980)]\}/4\pi)^{1/2}$$

with sums over i and j running from 1 to 120. This quantity measures the secular change signal at time t (relative to 1980) in units of estimated uncertainty (noise). It is the instantaneous rms signal-to-noise ratio. The 4π semi-normalization allows direct comparison of δ_0 to $\Delta B_r/\sigma B_r$. The quantity $4\pi\delta_0{}^2$ is the instantaneous weighted variance in the secular change relative to 1980.

Time integration of the instantaneous weighted variance yields the cumulative weighted variance

$$4\pi[\Delta_0(a,t;t_o)]^2 = 4\pi\int_{t_o}^{t} [\delta_0(a,t;t_o)]^2 dt$$

$$= 4\pi\int_{t_o}^{t} \langle [B_r(\mathbf{a},t) - B_r(\mathbf{a},1980)]^2 W(\mathbf{a},t)\rangle\, dt. \tag{16}$$

At $t = t_f$ this is the total weighted variance in the secular change of the radial field targeted for reduction. This is the square-weighted residual resulting from the absurd hypothesis of no secular change: a constant broad-scale radial geomagnetic component equal to that at 1980.

The method of interpolating both DGRF models and the weight function should be specified to evaluate the time integral in (16). The recommended interpolation is linear, but the working hypothesis indicates interpolation via steady motional induction at the top of a FFC bounded by a SFM. Because linear interpolation is not taken too seriously, and because DGRF models are provided only at discrete epochs, the cumulative weighted variance in the secular change of the radial field called for by the DGRF models is taken to be

$$4\pi[\Delta_0'(a,t_n;t_o)]^2 = 4\pi \sum_{k=1}^{n} [\delta_0'(a,t_k;t_o)]^2 \tag{17}$$

$$= \sum_{k=1}^{n} [g_i(t_k) - g_i(1980)]\ W_{ij}(t_k)\ [g_j(t_k) - g_j(1980)]$$

where t_n and t_k are DGRF epochs. The cumulative weighted variance (17) is conveniently independent of the interpolation method and is dimensionless, as is the total weighted variance $4\pi[\Delta_0'(a,t_n{=}t_f;t_o)]^2$.

When the general weights based on $\Omega^*{}_{ij}(t_n)$ instead of $W_{ij}(t_n)$ are used, the cumulative weighted variance is

$$f[\Delta_0{}^*(a,t_n;t_o)]^2 = f \sum_{k=1}^{n} [\delta_0{}^*(a,t_k;t_o)]^2 \tag{18}$$

$$= \sum_{k=1}^{n} [g_i(t_k) - g_i(1980)]\ \Omega^*_{ij}(t_k)\ [g_j(t_k) - g_j(1980)]$$

where the semi-normalization factor f is taken to be 120 (see Appendix). This choice allows direct comparison between δ_0^* and δ_0' and between Δ_0^* and Δ_0'.

Table 1 lists, as a function of t_n, the rms uncertainty in B_r, σB_r (14); the rms change in B_r, ΔB_r (13); the ratio $\Delta B_r/\sigma B_r$; δ_0' from (15); and the semi-normalized cumulative weighted variance, $\Delta_0'^2$ (17). Table 2 lists $\sigma^* B_r$; ΔB_r; the ratio $\Delta B_r/\sigma^* B_r$; δ_0^*; and Δ_0^{*2} appropriate to general weights. Note $(\sigma^* B_r)^2 = E_{ii}/[2n(i)+1] = (\sigma B_r)^2 + (11.24\text{ nT})^2$ for epochs 1965, 1970, and 1975 (see Appendix).

Table 1 shows that σB_r is about 51 nT for the 3 averaged DGRF models. Though little or no satellite data were available at epoch 1975, the decline of ΔB_r going back from 1975 to 1965 likely reflects the heavier weight ascribed to the IGS candidate model at the earlier epochs. Note σB_r is but 26 nT at 1960–possibly because of improved methods used to derive the 1960 DGRF and its covariance. The rms change in the expected broad-scale radial field ΔB_r between 1945 and 1980 is over 1900 nT. The typical rate of change of B_r was about 55 nT/yr, but was more rapid later in the interval–about 64 nT/yr between 1975 and 1980–in part because of the accelerating decline of the axial dipole. The ratio $\Delta B_r/\sigma B_r$ is always less than δ_0', so the weights increase the signal magnitude (by an rms factor of 1.9). It follows that regions of heavier weight (higher accuracy) correlate with regions of faster secular change (larger ΔB_r). This might be explained by the relatively low areal data density over the Pacific Ocean where secular change is thought to be slow.

The changes in the rms signal magnitude δ_0' as one looks back in time reflect competition between increased change in the radial field due to secular change (increased unweighted signal ΔB_r) and decreased weight (increased noise level σB_r) prior to 1960. The latter is likely due to a sparser data distribution before the International Geophysical Year and the advent of satellite geomagnetometry. Caution in accepting this explanation is advised because the prior information used in deriving the 1945-1960 DGRF might reduce the apparent unweighted signal magnitude in regions of poor data coverage. The semi-normalized total weighted variance is 12,217.8; the square root of this number, 110.5, is the total rms signal-to-noise ratio.

Table 2 shows that the general weights yield much larger values of the rms signal-to-noise ratio and the semi-normalized and cumulative weighted variance than do the radial field weights. This is not too surprising because the different weights correspond to two different objective functions (see Appendix of Part IB). Indeed, radial field weights should indirectly filter out the effects of D, X, Y, and H geomagnetic data which help determine the scalar geomagnetic potential. The semi-normalized total general weighted variance is 57,548.9, indicating a total general rms signal-to-noise ratio of 239.89.

The semi-normalizations allow comparison of weights; they are not intended to be misleading. With $t_o = 1980$ and $t_f = 1945$, the non-normalized total weighted variances are $4\pi[\Delta_0'(a,t_f;t_o)]^2 = 153{,}533$ and $120[\Delta_0^*(a,t_f;t_o)]^2 = 6{,}905{,}832$. The ratio of these numbers is about

TABLE 1

Select Properties of the DGRF Models and Uncertainty Estimates Based on Radial Field Weights

t_n (A.D.)	$\sigma B_r(a,t)$ (nT)	$\Delta B_r(a,t)$ (nT)	$\Delta B_r/\sigma B_r$	$\delta_0'(a,t_n;t_o)$	$\Delta_0'(a,t_n;t_o)^2$
1980	0.0	0.0	-	-	-
1975	58.30	318.9	5.47	13.64	186.0
1970	47.47	612.7	12.91	26.01	862.5
1965	45.42	899.2	19.79	51.96	3562.8
1960	25.67	1157.3	45.08	61.17	7304.4
1955	65.87	1427.2	21.67	64.35	11445.1
1950	127.01	1674.9	13.19	19.24	11815.1
1945	160.89	1929.3	11.99	20.07	12217.8

TABLE 2

Select Properties of the DGRF Models and Uncertainty Estimates Based on General Weights

t_n (A.D.)	$\sigma^* B_r(a,t)$ (nT)	$\Delta B_r(a,t)$ (nT)	$\Delta B_r/\sigma^* B_r$	$\delta_0^*(a,t_n;t_o)$	$\Delta_0^*(a,t_n;t_o)^2$
1980	0.0	0.0	-	-	-
1975	59.37	318.9	5.37	42.09	1771.8
1970	48.78	612.7	12.56	79.95	8164.3
1965	46.79	899.2	19.21	119.17	22366.9
1960	25.67	1157.3	45.08	163.40	49067.6
1955	65.87	1427.2	21.67	72.73	54357.7
1950	127.01	1674.9	13.19	35.69	55631.6
1945	160.89	1929.3	11.99	43.78	57548.6

2.222%; their square roots are 391.833 and 2,627.89; and the square root of their ratio is about 14.91%. This latter figure suggests that the radial field weights filter out 85% of the geomagnetic observations. In this sense it takes 67 of the geomagnetic data modeled to affect the expected radial field in the same way as 10 of the radial field data modeled. This is not too surprising since Z data alone could determine the scalar geomagnetic potential, but are uncommon.

3.3 The Target Residual Variance

The definition of the cumulative weighted variance in the DGRF models (17) requires the cumulative square-weighted residual relative to the DGRF models to be redefined as

$$4\pi[\Delta_r'(a,t_n;t_o)]^2 = 4\pi \sum_{k=1}^{n} [\delta_r'(a,t_k;t_o)]^2$$

$$= \sum_{k=1}^{n} [g_i(t_k) - \gamma_i(t_k)]^T W_{ij}(t_k)[g_j(t_k) - \gamma_j(t_k)] \quad (19)$$

with $i \leq 120$. This measures how well the predicted radial field fits the DGRF models. At $t_n = t_f$ it is the total square-weighted residual. For $t_o = 1980$ and $t_f = 1945$, if the residual at each pre-1980 epoch is one estimated standard deviation ($1\sigma'$), then the semi-normalized square-weighted residual $\delta_r'^2$ is $(1\sigma')^2$ at each epoch and the semi-normalized total square-weighted residual $\Delta_r'(a,t_f;t_o)^2$ is 7.

Replacing the primes with asterisks, W with Ω^*, and 4π with 120 in (19) yields the cumulative generalized square-weighted residual

$$120[\Delta_r^*(a,t_n;t_o)]^2 = 120 \sum_{k=1}^{n} [\delta_r^*(a,t_k;t_o)]^2$$

$$= \sum_{k=1}^{n} [g_i(t_k) - \gamma_i(t_k)]^T \Omega_{ij}(t_k)[g_j(t_k) - \gamma_j(t_k)]. \quad (20)$$

At $t_n = t_f$, this is the total generalized square-weighted residual. For $t_o = 1980$ and $t_f = 1945$, if the residual at each pre-1980 epoch is $1\sigma^*$, then δ_r^{*2} is $(1\sigma^*)^2$ at each epoch and $\Delta_r^*(a,t_f;t_o)^2$ is 7. This is the total square-weighted residual left by any trajectory whose projection onto the DGRF sub-space grazes each of the 7 error ellipsoids. Other trajectories can of course hit the target semi-normalized square-weighted residual of 7 provided residuals greater than $1\sigma^*$ at some epochs are balanced by errors less than $1\sigma^*$ at other epochs.

If one accepts the uncertainty estimates derived for the DGRF models and the radial field weighting, then the target square-weighted residual is 7 out of 12,217.8. Then 99.9427% of the total weighted variance in the evolution of the broad-scale radial field indicated by the DGRF models is signal to be fit. The residual 0.0573% may well be noise and defines the target normalized square-weighted residual. Its square root, 2.394%, is the typical noise-to-signal ratio in the secular change of the radial field described by a DGRF model relative to 1980. The typical signal-to-noise ratio is $(12,217.8/7)^{1/2}$ or 41.78. Typical (rms) residuals of $1\sigma'$, $2\sigma'$, or $3\sigma'$ correspond respectively to total square-weighted residuals of 7, 28, or 63 out of 12,217.8.

If one prefers the general weights, then the target total square-weighted residual is 7 out of 57,548.6. Then 99.9878% of the total generally weighted variance in the evolution of the broad-scale scalar geomagnetic potential indicated by the DGRF models is signal to be fit. The residual 0.0122% is noise and defines the target normalized square-weighted residual. Its square root, 1.103%, is the typical noise-to-signal ratio in the secular change of the scalar geomagnetic potential described by a DGRF model relative to 1980; the typical signal-to-noise ratio is 90.67. This is over twice that indicated by the radial field weights. Typical (rms) residuals of $1\sigma^*$, $2\sigma^*$, or $3\sigma^*$ again correspond respectively to total generalized square-weighted residuals of 7, 28, or 63 out of 57,548.6.

In my opinion, residuals less than 0.674σ suggest the unnecessary fitting of noise; residuals exceeding 1σ suggest a flawed hypothesis. The hypothesis to be tested is that the secular change required by the DGRF models can be explained by a source-free mantle surrounding a frozen-flux core in (at least temporarily) steady surficial flow. However, the SFM/FFC model is but an approximation, the core flow is not strictly steady, the initial conditions are not very well known due to ignorance of the narrow-scale core field, numerical approximations are used in applying the method derived in section 3, and my uncertainty estimates are but estimates. Moreover, normalized residuals of 10% indicate a model of secular change which is accurate to first order. Yet such 10% residuals indicate a typical error of $9.06\sigma^*$ (or $4.18\sigma_r'$) per DGRF epoch.

In my opinion, if the hypotheses of steady flow at the top of a FFC surrounded by a SFM can be used to derive a fluid velocity field which, via steady motional induction, reproduces radial field models to within normalized residuals of 10%, then it has some qualitative merit. Residuals less than $3\sigma'$ ($6.52\sigma^*$) indicate appreciable qualitative merit. Residuals less than $2\sigma'$ ($4.34\sigma^*$) indicate some quantitative merit. Residuals less than $1\sigma'$ ($2.17\sigma^*$) indicate appreciable quantitative merit. Residuals less than $1\sigma^*$ demonstrate adequate quantitative merit. However, any residuals greater than $1\sigma^*$ indicate a need to improve either the earth model (by eliminating one or more of the underlying assumptions), or upon the means used to test it.

4. SUMMARY

Application of the method developed in paper IB to the Definitive Geomagnetic Reference Field (DGRF) models [IAGA, 1988] has been described. The numerical methods used and the derivation of weight matrices for the DGRF models were outlined. The DGRF models are truncated spherical harmonic representations which contain no explicit information on any narrow-scale structure of the geomagnetic field. Attempts to fit such models should not be penalized for predicting the existence of such structure, provided its amplitude is not unreasonably large. The derived weight matrices reflect this fact, were used to assess the information on secular change contained in the DGRF models, and were further used to estimate the tolerable residuals which could be produced by a quantitatively acceptable explication of definitive secular change.

Preliminary results of applying the method have been presented elsewhere (e.g, VOORHIES, 1987b,c; 1988a, 1989b); final results are to be presented in part ID.

Application of the method to the DGRF models immediately encounters a formidable obstacle: incomplete initial magnetic conditions in the Fourier-Legendre spectral domain. The Gauss coefficients above degree 10 were therefore treated as if zero to complete the initial condition—preferably at the 1980 MAGSAT epoch. Effects of non-trivial narrow-scale initial magnetic conditions have been considered and tested. The truncation of DGRF models is not problematic at non-initial epochs—a special feature of the steady flow hypothesis. Such truncation is compatible with the goal of matching the evolution of the broad-scale radial field (or low-degree scalar potential). Non-zero damping and a velocity field truncation level of 16 were selected.

Two kinds of weights were investigated: radial field weights and general weights. For radial field weights, the weighted, optionally constrained, optionally damped iterated linear least-squares method yields a steady fluid velocity that tracks the weighted evolution of the radial magnetic field at Earth's surface indicated by the geomagnetic field models fitted. For general weights, the method estimates the fluid motion that tracks the weighted evolution of the scalar geomagnetic potential at Earth's surface indicated by the geomagnetic field models fitted. Both radial field and general weight matrices were derived from the covariance matrices for the 1945, 1950, 1955, and 1960 DGRF models and from the candidate DGRF models for the 1965, 1970, and 1975 DGRF models. The 1980 DGRF model is treated as comparatively perfect—a fair approximation.

The secular change of the broad-scale radial field relative to 1980 called for by the DGRF models was analyzed. The results for radial field weights are summarized in Table 1. These weights nearly double the apparent signal magnitude. The semi-normalized total weighted variance in the secular change of the broad-scale radial field is 12,217.8; the tolerable semi-normalized square-weighted residual is 7. Some 99.94271% of the total weighted variance is thus regarded as signal to be fit. Normalized root-mean-square residuals of 2.394% or less are tolerable for radial field weights.

The results for general weights are summarized in Table 2. Then the semi-normalized total weighted variance in the secular change of the low-degree scalar geomagnetic potential relative to 1980 called for by the DGRF models radial field is 57,548.6; but the tolerable semi-normalized total square-weighted residual is still 7. Some 99.98784% of the total generally weighted variance is regarded as signal to be fit. Normalized root-mean-square residuals of 1.103% or less are tolerable for the generalized weights.

Both radial field and general weights yield estimated tolerable residuals less than the 7% errors expected in the SFM/FFC approximation. When using radial field weights, the 7% expected error from this approximation will cause a typical misfit of about $2.92\sigma'$ per DGRF epoch. When using general weights, the 7% expected error will cause a typical misfit of $6.35\sigma^*$ per DGRF epoch. Therefore, the combined supposition of a SFM surrounding a FFC in surficially steady flow is expected to be rejectable with appreciable confidence. If this is so, the need for a superior earth model will be clear; steps to develop such a model have been taken.

If application of the method developed in this paper to derive a single steady flow from 1980 back to 1945 yields errors exceeding 7%, then the steady flow hypothesis might reasonably be rejected. If the steady flow hypothesis yields errors of 7% or less, then it should be retested using a superior earth model. In light of the interim upper bound of 7% errors (i.e., at least 99.51% weighted variance reductions) on the steady flow hypothesis, the prospects for resolving time-dependent flow with an improved earth model by, say, seeking piecewise steady flows, may not be too good. The ratio of the time-dependent flow signal strength relative to the noise in the definitive secular change models would then be at most 2.92 per DGRF epoch for radial field weights. But corrections for CMB ellipticity and topography or mantle conductivity might reduce the residuals resulting from the supposition of steady flow, and, more importantly, this ratio is 6.35 per DGRF epoch for general weights. The latter figure seems large enough to hope for resolution of time-dependent core flow with a superior earth model.

APPENDIX

For epochs t_n = 1945, 1950, 1955, and 1960, calculation of $\Omega_{ij}(t_n) = \Omega^*_{ij}(t_n)$ was possible for the DGRF models using the derived $E_{ij}(t_n)$. This was not possible for epochs 1965, 1970, and 1975. This might be due to roundoff error in the published Gauss coefficients, the interpretation of the DGRF weighting procedure used to obtain the E_{ij}(1965-1975), or failure of these E_{ij} to account for properly expected errors (e.g., fortuitous agreement of candidate models introducing a singularity as described in section 3.1). On the chance that random roundoff error was the source of this problem, $(0.5\ \text{nT})^2$ was added to the diagonals of V matrices for 1965, 1970, and 1975. The resulting V^* matrices were used to calculate the generalized weight matrices

$$\underline{\underline{\Omega}}^*(t_n) \equiv \underline{\underline{E}}^*(t_n)^{-1} = \{\underline{\underline{E}}(t_n) + \underline{\underline{R}}[(0.5)^2\underline{\underline{I}}]\underline{\underline{R}}\}^{-1}$$

$$= \{\underline{\underline{R}}[\underline{\underline{V}}(t_n) + 0.25\underline{\underline{I}}]\underline{\underline{R}}\}^{-1} = \{\underline{\underline{R}}\underline{\underline{V}}^*(t_n)\underline{\underline{R}}\}^{-1} \tag{A1}$$

for epochs t_n = 1965, 1970, and 1975. This procedure also eliminates any singularity caused by agreement of the candidate models. The Ω^* matrices are used to calculate the generalized weighted residual variance. The arithmetic mean of these matrices at adjacent epochs is inverted to obtain the generalized mean weight matrices $\Omega^*(t_1)$ which replace $W(t_1)$ in equations (9) when the generalized weights are used.

Use of (A1) assumes additional uncorrelated noise in the large DGRF models at epochs 1965, 1970, and 1975. The expected amplitude of this noise in the radial component

$$\left\{\sum_{n=1}^{10} \frac{(n+1)^2}{2n+1} \sum_{m=0}^{n} (0.5\ \text{nT})^2\right\}^{1/2} = \left\{\sum_{n=1}^{10} (n+1)^2 (0.5\ \text{nT})^2\right\}^{1/2}$$

$$= 11.24\ \text{nT (rms)} \tag{A2}$$

is comparable to that of very long wavelength crustal fields expected by VOORHIES (1984)

$$\{E[B_{rx}^2]\}^{1/2} = \left\{\sum_{n=1}^{10} B_{rx}(n)^2\right\}^{1/2} = \left\{\sum_{n=1}^{10} 11.91(0.9969)^n\right\}^{1/2}$$

$$= 10.82\ \text{nT} \tag{A3}$$

or by LANGEL, ESTES & SABAKA (1989),

$$\{E[B_{rx}^2]\}^{1/2} = \left\{\sum_{n=1}^{10} \frac{(n+1)}{2n+1} R_{nx}\right\}^{1/2} = \left\{\sum_{n=1}^{10} \frac{(n+1)}{2n+1} 20(0.9999387)^n\right\}^{1/2}$$

$$= 10.56\ \text{nT}. \tag{A4}$$

However, the parabolic roundoff spectrum associated with (A2) is much more 'blue' than the nearly 'white' crustal spectra (A3) or (A4).

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13. ABSTRACT *(Maximum 200 words)*

In the source–free mantle/frozen–flux core magnetic earth model, the non–linear inverse steady motional induction problem has been solved using the method presented in Part IB of this series. The present paper describes how that method has been applied to estimate steady, broad–scale fluid change indicated by the Definitive Geomagnetic Reference Field (DGRF) models from 1945 to 1980. Special attention is given to the derivation of weight matrices for the DGRF models because the weights determine the apparent significance of the residual secular change. The derived weight matrices also enable estimation of the secular change signal–to–noise ratio characterizing the DGRF models.

Two types of weights were derived in 1987–88: radial field weights for fitting the evolution of the broad–scale portion of the radial geomagnetic field component at Earth's surface implied by the DGRFs, and general weights for fitting the evolution of the broad–scale portion of the scalar potential specified by these models. The difference is non–trivial because not all the geomagnetic data represented by the DGRFs constrain the radial field component. For radial field weights (or general weights), a quantitatively acceptable explication of broad–scale secular change relative to the 1980 Magsat epoch must account for 99.94271 percent (or 99.98784 percent) of the total weighted variance accumulated therein. Tolerable normalized root–mean–square weighted residuals of 2.394 percent (or 1.103 percent) are less than the 7–percent errors expected in the source–free mantle/frozen–flux core approximation.

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