N94- 25109

X-31 AERODYNAMIC CHARACTERISTICS DETERMINED FROM FLIGHT DATA

ALEX KOKOLIOS

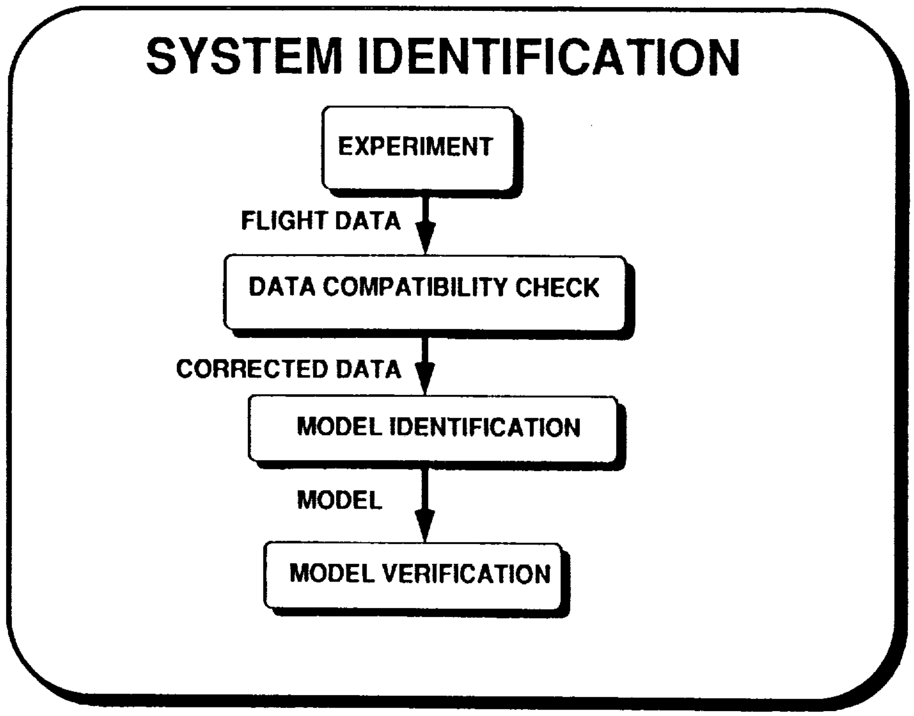
JOINT INSTITUTE FOR THE ADVANCEMENT OF FLIGHT SCIENCES

AIRCRAFT GUIDANCE AND CONTROLS BRANCH

NASA LANGLEY RESEARCH CENTER

Abstract

The lateral aerodynamic characteristics of the X-31 were determined at angles of attack ranging from 20 to 45 degrees. Estimates of the lateral stability and control parameters were obtained by applying two parameter estimation techniques, linear regression and the extended Kalman filter, to flight test data. An attempt to apply maximum likelihood to extract parameters from the flight data was also made but failed for reasons given within. An overview of the System Identification process is given, including a listing of the more important properties of all three estimation techniques that were applied to the data. A comparison is given of results obtained from flight test data and wind tunnel data for four important lateral parameters. Finally, future research to be conducted in this area is discussed.



Overview of System Identification

System Identification is a complex process involving several steps. It begins with the experiment, or flight test, which yields the flight data. The flight data must then be subjected to a data compatibility check to check for the presence of any scale factor or bias errors. Once any such errors have been found and accounted for, the process known as model identification may then be applied to the corrected data. More will be said about this process later, as it is in itself a complex process. Once a model has been determined, it must then be verified either by comparing the estimates to results obtained from other experiments, or by the application of other estimation techniques to the same set of data. When a model has been verified, it may then be used to update the data-base or simulator or to refine the existing control laws of the aircraft.

MODEL IDENTIFICATION

The Process of Model Identification can be separated into two distinct steps:

(1) Model Structure Determination; and

(2) Parameter Estimation.

The Model Identification Process

The identification of a particular model can be broken down into two separate steps: that of determining the structure of the model; and that of estimating the parameters in this particular model. In the case of aircraft aerodynamic analysis the parameters are often the stability and control parameters as is the case in the present study.

MODEL STRUCTURE DETERMINATION

The Model Structure is determined through the use of Stepwise Regression, a generalization of linear regression, as follows:

a) An adequate model is determined from postulated terms (the postulated terms may include linear as well as nonlinear terms);

b) The parameters associated with the selected terms are estimated, using the Least Squares method, by minimizing the following cost function:

$$\boxed{J = \sum_{i=1}^{N} \left(C_{a_i} - \theta_o - \sum_{j=1}^{I} x_{ji}\theta_j \right)^2}$$

Model Structure Determination

For this particular study, the structure of the model was determined through the use of a technique known as stepwise regression, a generalization of linear regression which works as follows. First, an adequate model is determined by choosing the so-called regressors from a pool of postulated terms. These postulated terms may include the states and inputs, as well as any combinations of the two. Thus, the regressors may be linear or nonlinear. Once these regressors have been chosen, the parameters associated with these regressors are estimated using a Least Squares method, which minimizes the given cost function. The cost function minimizes the sum of squares of the difference between the measured aerodynamic force or moment coefficient, C_{a_i}, and the model-predicted coefficient, given by the remaining expression within the parentheses (x represents the chosen regressor and θ the associated parameter to which an estimate is sought; θ_o estimates the steady state value of the coefficient). Note that this summation is carried out over N, the number of data points collected during the maneuver.

PARAMETER ESTIMATION

Three different Parameter Estimation Techniques were applied to the X-31:

1. **Least Squares Method, in the form of Stepwise Regression**

2. **Maximum Likelihood Method**

3. **Extended Kalman Filter Method**

The Parameter Estimation Process

Three different parameter estimation techniques were applied to the X-31 drop model for this particular study: the least squares method in the form of stepwise regression (as discussed in the previous slide); the maximum likelihood method; and the extended Kalman filter method. Results obtained by applying the least squares and extended Kalman filter methods to X-31 drop model flight test data will be presented. First, however, some of the important properties of each of these techniques is discussed. Also, an explanation is given as to why the application of maximum likelihood to flight test did not yield any results.

LINEAR REGRESSION

Has several desirable properties, in that it is:

- **A Simple Linear Estimation Problem**
- **Applied to each Aerodynamic Coefficient separately (thus keeping the number of unknowns small)**
- **Can be applied to an unstable system without difficulty**

However,

- **Parameter Estimates are biased**

Linear Regression Estimation

Parameter estimation through linear regression has several desirable properties. First, linear regression is a simple, linear estimation problem. The regression is applied to each aerodynamic force or moment coefficient individually, thereby keeping the number of unknowns in each equation small. Finally, linear regression can be applied to an unstable system without any difficulties. The drawback to using linear regression however, is that the estimates obtained with this technique are, in general, biased. Thus, the motivation exists to apply a second estimation technique to the flight test data, one that yields unbiased estimates of the parameters. Two such methods are maximum likelihood, and the extended Kalman filter. Generally, the estimates obtained from linear regression are used as the initializations for the second estimation technique.

MAXIMUM LIKELIHOOD METHOD

- Optimizes Parameter Estimates by fitting Outputs predicted by the Model to the measured Outputs
- In the absence of process noise an Output Error Method, but with continual Update of the Measurement Noise Covariance Matrix
- Cost Function: $\boxed{J = \sum_{i=1}^{N} [z_i - y_i(\theta)]^T \hat{R}^{-1} [z_i - y_i(\theta)]}$
- Parameter Estimates are unbiased
- Nonlinear Estimation Technique (thus requiring iterative approach)
- Requires Integration of the Aircraft Equations of Motion (will cause problems if aircraft is unstable)

Maximum Likelihood Estimation

Maximum likelihood optimizes parameter estimates by fitting the model-predicted outputs to the measured outputs. In the absence of any process noise, it is an output error method but more general because it continually updates the measurement noise covariance matrix. In the given cost function, z_i represents the measurement, y_i the prediction (which is a function of θ, the parameters to which an estimate is sought), and R represents the measurement noise covariance matrix. As in the case of the least squares cost function, the maximum likelihood cost function sums over N, the number of data points collected during the maneuver. The estimates obtained in this manner are unbiased. Note that maximum likelihood is a nonlinear estimation technique and thus requires an iterative approach such as the Newton-Raphson method. The difficulty in applying the maximum likelihood method lies in the fact that it requires integration of the aircraft equations of motion, which will cause problems if the aircraft is unstable. Since the X-31 is open-loop laterally unstable at high angles of attack, application of the maximum likelihhod method to X-31 flight data failed to produce reasonable estimates. Thus, another estimation technique had to be found.

EXTENDED KALMAN FILTER

- **A Nonlinear Estimation Problem, with the aircraft model defined as:**

$$\boxed{\dot{x}(t) = f\,[x(t), u(t)] + w(t)}$$

$$\boxed{z_i = h_i\,[x(t_i), u(t_i)] + v_i}$$

$$\boxed{\begin{array}{c} w(t) \sim N\,(0, Q(t)) \\ v_i \sim N\,(0, R_i) \end{array}}$$

Note that x contains the states as well as the parameters (i.e., the stability and control derivatives).

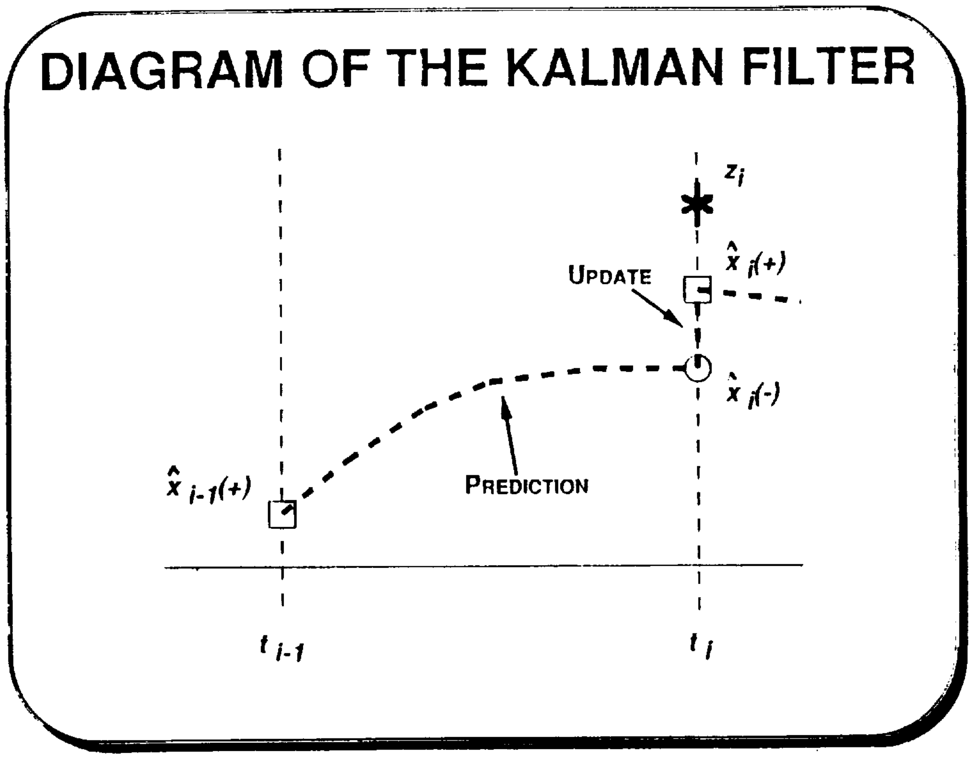
- **Given the model of the aircraft as described above, form an algorithm for calculating the minimum variance estimate of x(t) :**

i.e., minimize:

$$\boxed{J = E\,\{(\hat{x} - x)^T\,(\hat{x} - x)\,\}}$$

The Extended Kalman Filter

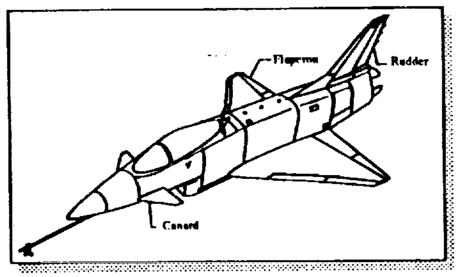
A second method that was used to refine the parameter estimates obtained through linear regression was the extended Kalman Filter. The extended Kalman filter, like maximum likelihood, is also a nonlinear estimation problem, with the aircraft model defined as shown above. The time derivative of the state is assumed to be a function of the states and inputs plus a process noise term, w(t), assumed to be of normal distribution with zero mean and variance given by Q(t). Similarly, the measured output is assumed to be some function of the states and inputs plus a measuremnt noise term, v_i, also assumed to be of normal distribution with zero mean and variance given by R_i. It should be noted that x may contain the states as well as the parameters (i.e., the state vector is augmented with the stability and control parameters to which estimates are sought). Given the model as described above, the extended Kalman filter cost function is formed by determining the minimum variance estimate of the state. Here, x represents the true value of the state, $\hat{x}$ the estimate, and E{ } represents the expected value operator.



Timing Diagram of the Kalman Filter

A timing diagram of the extended Kalman filter is shown, illustrating how an estimate is obtained. Basically, a two-step process is carried out at each data point. Given that an estimate $\hat{x}_{i-1}(+)$ is known at time t_{i-1}. The estimate is then propagated to the next time t_i by simply integrating the equations of motion across one time step. This estimate, $\hat{x}_i(-)$, is then updated by the extended Kalman filter equations, which take into account the new measurement, z_i as well as information about the assumed statistics of the measurement and process noise terms. The updated estimate is denoted by $\hat{x}_i$ (+), the (+) indicating it is the value of the estimate after the update has been carried out (similarly, a (-) indicates the value of an estimate prior to an update). It is this update step which stabilizes the integration scheme where maximum likelihood failed. Note that the updated value of the estimate will always lie between the predicted value and the measured value.

THE X-31 DROP MODEL

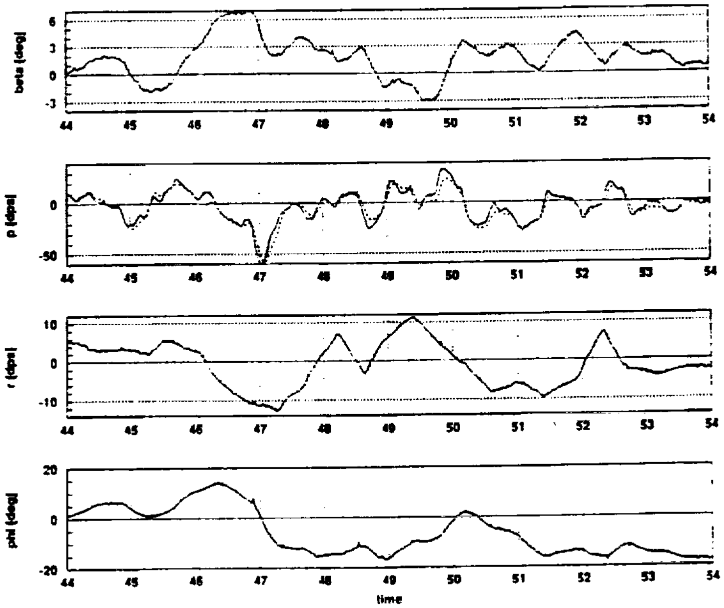


- Delta wing, canard configuration intended to serve as a high-alpha test vehicle
- Unpowered, 27% dynamically-scaled model of the full-scale aircraft.
- Laterally Unstable in high angle-of-attack flight regime.

Application to the X-31 Drop Model

An isometric of the X-31 drop model is shown above. The X-31 is a delta wing, canard configured aircraft intended to demonstrate enhanced maneuverability at high angles of attack. The drop model, currently undergoing flight testing at the Plum Tree test site, is a 27% dynamically-scaled model of the full-scale aircraft. The X-31 is known to be laterally unstable at high angles of attack.

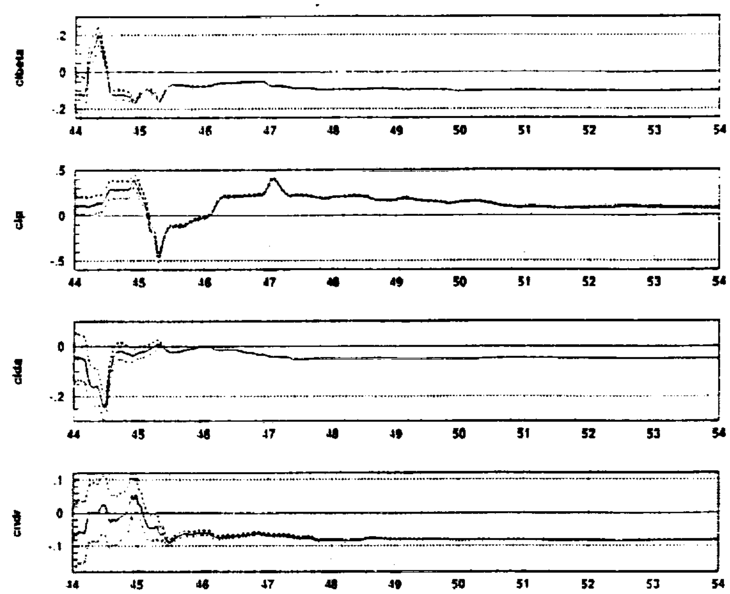
RESULTS OF A TYPICAL MANEUVER



State Estimation

An algorithm using the extended Kalman filter method was applied to X-31 drop model flight test data. The results of the application to one maneuver is shown above. The plots show the time history of four lateral states, including the sideslip angle, the roll-rate, the yaw-rate, and the bank-angle, during a lateral maneuver. The solid lines represent the measured data and the dashed lines represent the estimates as obtained using the extended Kalman filter algorithm. As seen, the algorithm predicts the states very accurately, with the exception of the roll-rate. The exact reason why the algorithm is able to predict all the states accurately with the exception of the roll-rate is not yet fully understood.

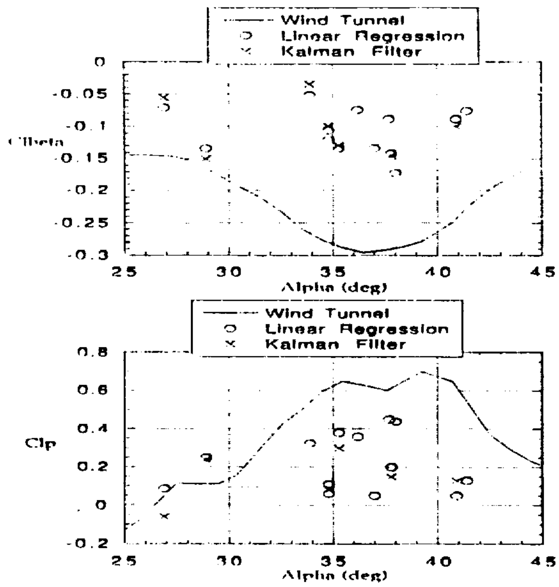
PARAMETER ESTIMATION RESULTS



Parameter Estimation

The above plots show the results for the parameter estimation as obtained by applying the extended Kalman filter algorithm to the same maneuver discussed in the previous slide. Shown are the estimates for the dihedral effect, the roll-damping, the aileron effectiveness, and the rudder effectiveness. The solid lines represent the estimates while the accompanying dotted lines represent the standard errors associated with those estimates. The estimates were initialized at the values obtained from applying linear regression to the same maneuver. As seen from the plots, all four estimates return to values that are close to the initial values, indicating that the extended Kalman filter estimates are in close agreement with the linear regression estimates.

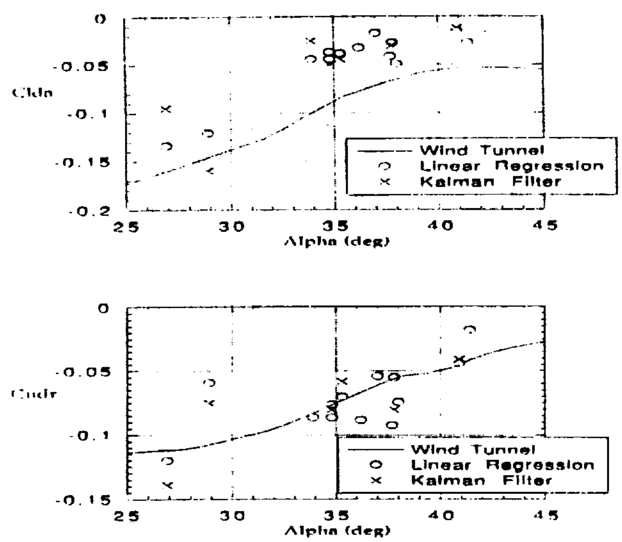
COMPARISON OF RESULTS



Comparison of Results

Shown are the comparison of the estimates obtained for the dihedral effect and the roll-damping as obtained from wind tunnel data and from the application of linear regression and extended Kalman filter methods to flight test data. Notice first that the estimates obtained using linear regression and the extended Kalman filter algorithm agree well with each other for the flight regime under study. The apparent scatter in the estimates may be attributed to the fact that each estimate was obtained from a different maneuver and it ispossible that the estimates are sensitive to the particular maneuver. It appears, however, that there are some significant differences between estimates obtained from wind tunnel data and those obtained from flight data. Several comments may be made about this. First, the estimates obtained from wind tunnel data were obtained from static wind tunnel testing and thus were not subject to the dynamic effects encountered during the flight tests. Thus, the estimates obtained from flight data can be said to embody the dynamics of the aircraft, whereas those obtained from wind tunnel data do not. In addition, for the case of the roll-damping, the wind tunnel estimates are known to be extremely sensitive to the oscillation amplitude as well as the canard setting of the model during the wind tunnel testing. Further investigations to fully explain these differences are currently underway.

COMPARISON OF RESULTS



Comparison of Results

Similar plots shown in the previous slide are shown in this slide for the aileron effectiveness and rudder effectiveness. Note again that the estimates obtained from flight data through linear regression and extended Kalman filter methods are in good agreement with each other. Note also that estimates obtained from wind tunnel data seem to be in better agreement for the two control derivatives than for the previous two stability derivatives. A systematic difference is seen between flight data estimates and wind tunnel estimates for the aileron effectiveness. Preliminary results obtained from X-31 full-scale flight tests seem to favor the estimates obtained from flight data. All estimates seem to be in good agreement for the rudder effectiveness. Note that both the rudder and the aileron have decreased effectiveness with increasing angle of attack, as would be expected.

FUTURE WORK

Future Research will likely include the following:

- Comparison of Wind Tunnel, Drop Model, and Full Scale Aircraft Results.
- Explanation of possible differences in results using various Experiments and/or Estimation Techniques.
- Extension of research to longitudinal data.

Future Research

Future research to be conducted in this area will likely include a comparison of wind tunnel, drop model, and full-scale aircraft results. An attempt will be made to explain any differences that may appear in the results obtained using these various experiments and estimation techniques. And finally, an extension of this research will be made to determine the longitudinal aerodynamic characteristics of the X-31.