N94-25114

Optimal RTLS Abort Trajectories for an HL-20 Personnel Launch Vehicle

Kevin Dutton
Spacecraft Controls Branch

Outline

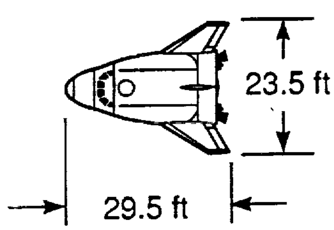
- Objective of study
- HL-20 Vehicle and Mission
- Modelling Information
- Problem Formulation
- Solution Method
- Results
- Concluding Remarks

Objective

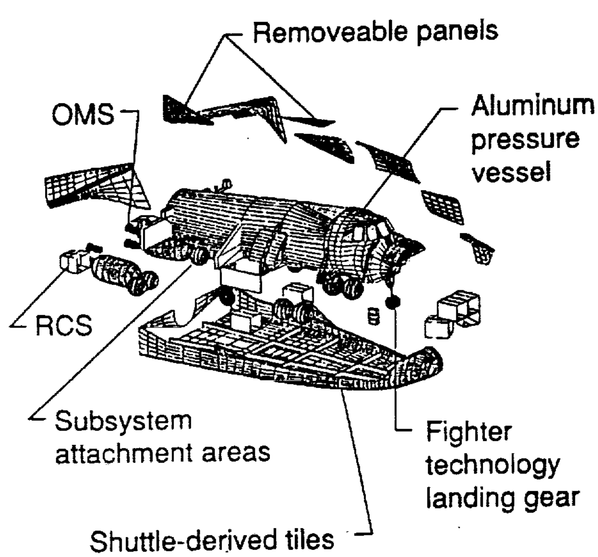
Primary: Determine whether RTLS abort at T seconds along launch trajectory is possible using optimal control theory

Secondary: Assess effects of bank angle constraint, lift coefficient constraint, free and fixed final boundary conditions, etc.

HL-20 PLS BASELINE DESIGN



	Weight, lb
Dry (with 22% margin)	19,777
Landed	22,057
On-Orbit	26,186
Launch Escape System/Adapter	8,420
Gross Launch on NLS	34,607



HL-20 PLS CURRENT TECHNOLOGY DESIGN

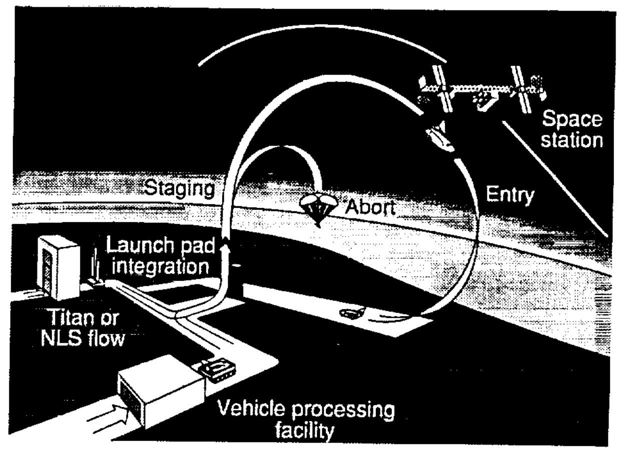
THE PERSONNEL LAUNCH SYSTEM (PLS)

Complementary System to Space Shuttle

- Space Station crew transfer
- Alternate access to/from space for people/priority cargo

Space Station Reference Mission

- Transfer and return up to 8 Space Station personnel and/or priority cargo
- 72-hour mission duration
- 1,100 ft/sec on-orbit propulsive capability
- Placed in orbit by existing or future booster system
- Kennedy Space Center launch/landing site
- Alternate landing site capability



Talay 34

HL-20 Aborts

VAB Analysis

- **On the pad**
- **0-20 sec** **Return to Launchsite (Shuttle landing facility)**
- **20-65 sec** **RTLS (Skid strip)**
- **65-403 sec** **Ocean landing by parachute**
- **403-478 sec** **Transatlantic abort landing**
- **478+ sec** **Abort to orbit**

Vehicle Aerodynamic Model

- Aerodynamic data from Jackson and Cruz
- At each angle of attack and Mach number, find $\delta_E, \delta_L, \delta_U$ that trim vehicle and minimize drag; calculate Cl and Cd here
- For each Mach number, determine coefficients for Cd expression

$$C_D = C_{D_0}(M) + C_{D_1}(M)\, C_L + C_{D_2}(M)\, C_L^2$$

Optimal Control Theory

- Cost $\min_{\bar{u}} J = \Phi[\bar{x}(t_0), \bar{x}(t_f)]$
- Plant $\dot{\bar{x}} = f(\bar{x}, \bar{u})$
- Constraints:

	Control	State
	$\bar{g}(\bar{x}, \bar{u}) = 0$	$\bar{c}(\bar{x}) = 0$
	$\vec{h}(\bar{x}, \bar{u}) \leq 0$	$\vec{d}(\bar{x}) \leq 0$

- Boundary conditions $\psi[\bar{x}(t_0), \bar{x}(t_f)] = 0$

Cost Function and Plant

- $J = -h(tf)$ (final altitude)
- States $\bar{x} = [h\ x\ y\ V\ \gamma\ \psi]^T$
  - x,y Cartesian system, x east, y north, origin at point runway centerline extended
  - ψ 0 for easterly flight, increases CCW
- Controls $\bar{u} = [C_L\ \sigma]^T$
  - σ negative for right bank
- Equations of motion: flat earth, non-thrusting, aerospace vehicle

Control/State Constraints

- Bank angle can be constrained (40 deg. nominal)

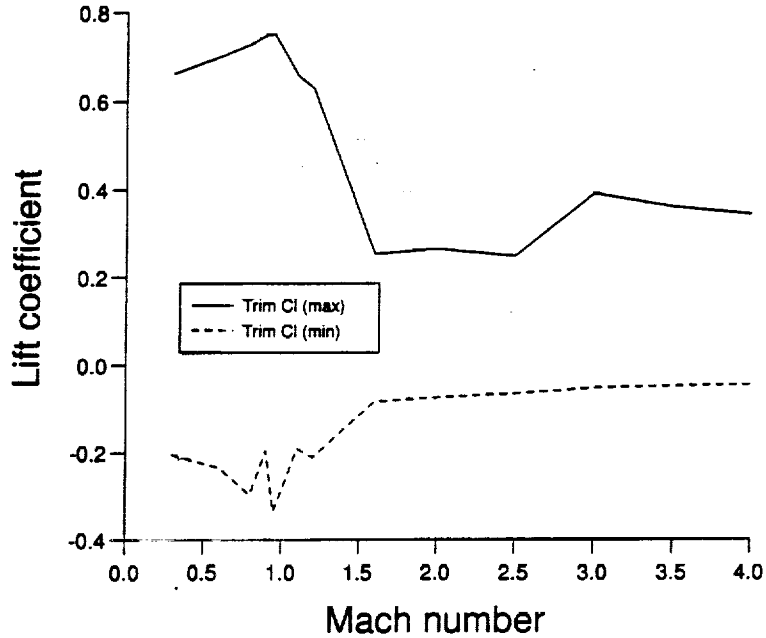
$$-\sigma_{max} \leq \sigma \leq \sigma_{max}$$

- Lift coefficient is constrained between upper and lower trim limits (function of Mach)

$$C_{L_{min}}(M) \leq C_L \leq C_{L_{max}}(M)$$

- Normal and axial load factor constraints (3 g units nominal)

Lift Coefficient Constraint



Initial Conditions

- Initial conditions for abort at T seconds are conditions at T along ascent trajectory followed by primary solid rocket motor (srm) burn, followed by sustainer srm burn

- Example: Initial conditions for abort at T=30

h(t0) = 32882 ft	V(t0) = 1565 ft/sec
x(t0) = - 7409 ft	γ(t0) = 79.7 deg
y(t0) = 45357 ft	ψ(t0) = - 2.0 deg

Final Conditions

For all cases:

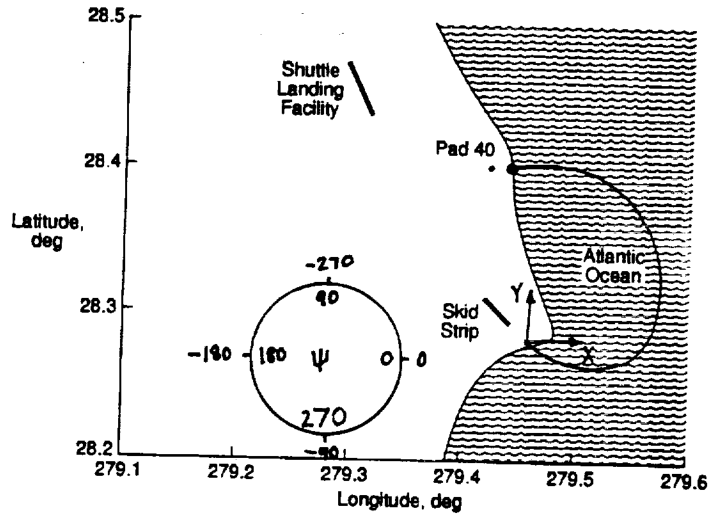
$x(tf) = 0.0$ ft

$y(tf) = 0.0$ ft

$V(tf) = 521.0$ ft/sec

$\gamma(t_f) = -19.0$ deg

$\psi(t_f) = -220.7$ deg



Solution Method

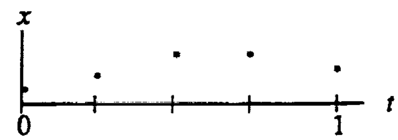
Trajectory Optimization by Differential Inclusion (TODI)

- eliminates controls from problem by constraining state rates
- leads to nonlinear programming problem where parameters are state values at user defined nodes (NPSOL)

The Differential Inclusion Approach Explained on a Simple Example

$$\begin{array}{l} \min \; -x(1) \\ \dot{x} = u, \quad 0 \le u \le 1 \\ x(0) = 0 \end{array}$$

pick states at equidistantly chosen node points



neighboring states have to satisfy either

(differential equation approach)

$$\frac{x_{i+1} - x_i}{\Delta t} = u_i, \quad 0 \le u_i \le 1$$

or

(differential inclusion approach)

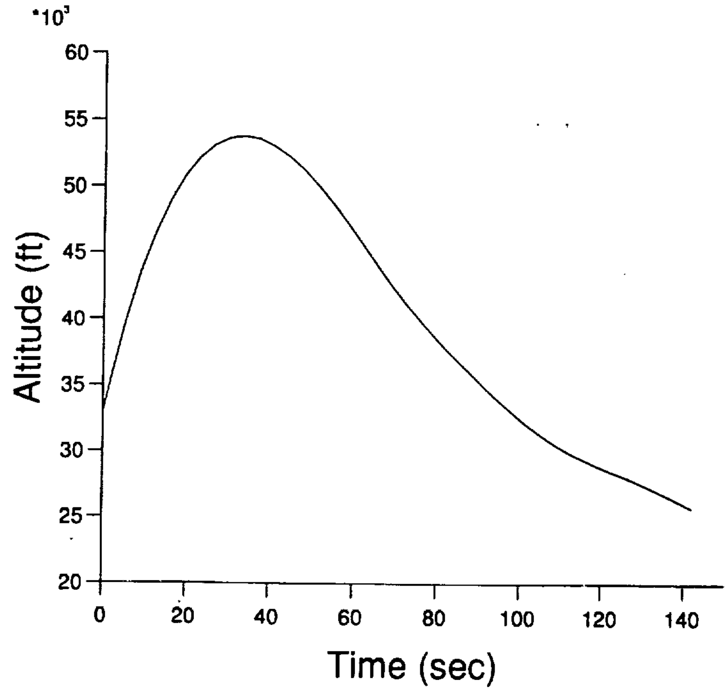
$$\frac{x_{i+1} - x_i}{\Delta t} \ge 0 \quad and \quad \frac{x_{i+1} - x_i}{\Delta t} \le 1$$

General Discretization Scheme

Optimal control problem	Finite dimensional discretization
$\min\limits_{u \in (PWC[0,1])^m} \Phi(x(0), x(1))$	$\min\limits_{[x_0 \dots x_N] \in R^{n \times N}} \Phi(x_0, x_N)$
$\Psi(x(0), x(1)) = 0$	$\Psi(x_0, x_N) = 0$
$\dot{x} = f(x(t), u(t))$ $g(x(t), u(t)) = 0$ $h(x(t), u(t)) \le 0$	for $i = 0, \dots, N-1$: $p\left(\frac{x_{i+1} - x_i}{N}, x_i\right) = 0$ $q\left(\frac{x_{i+1} - x_i}{N}, x_i\right) \le 0$
$c(x(t)) = 0$ $d(x(t)) \le 0$	for $i = 0, \dots, N$: $c(x_i) = 0$ $d(x_i) \le 0$

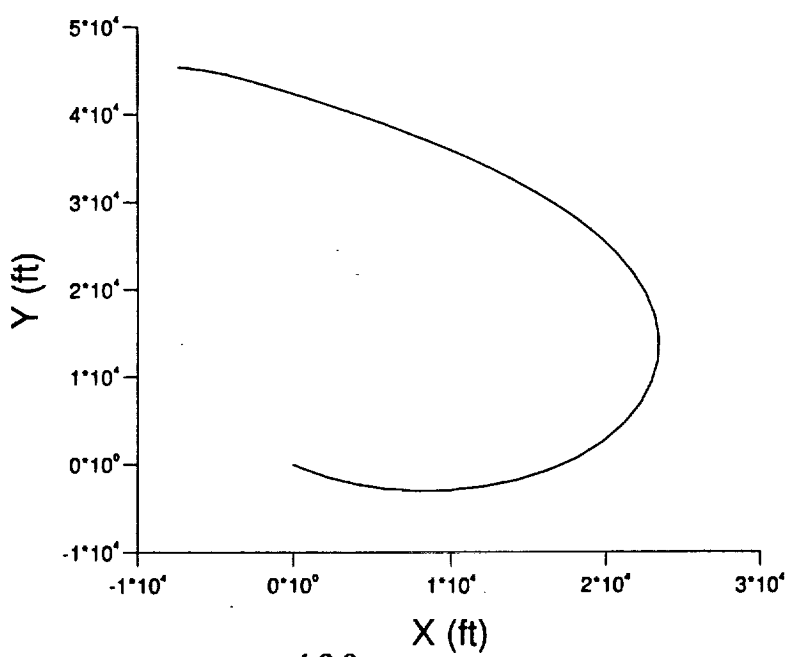
Solution for 30 Second Abort Case

Altitude vs. Time



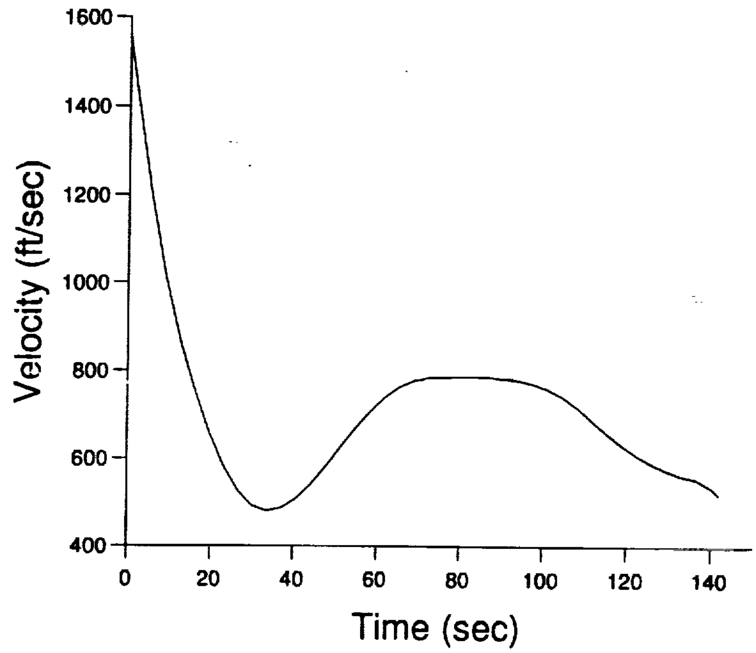
Solution for 30 Second Abort Case

Groundtrack



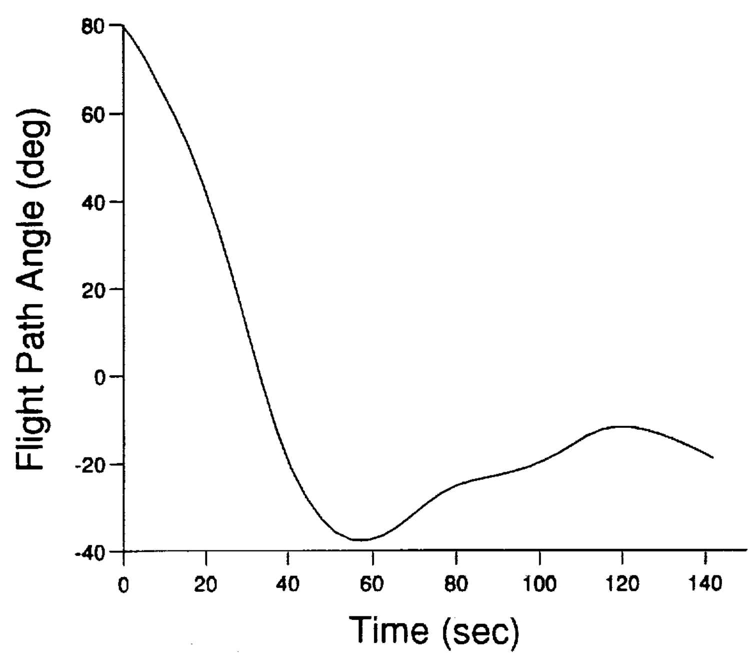
Solution for 30 Second Abort Case

Velocity vs. Time



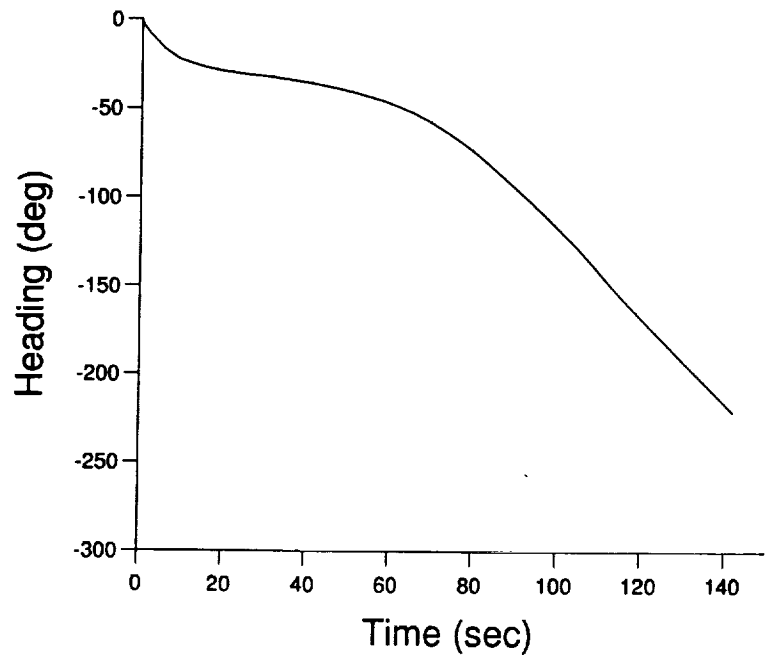
Solution for 30 Second Abort Case

Flight Path Angle vs. Time



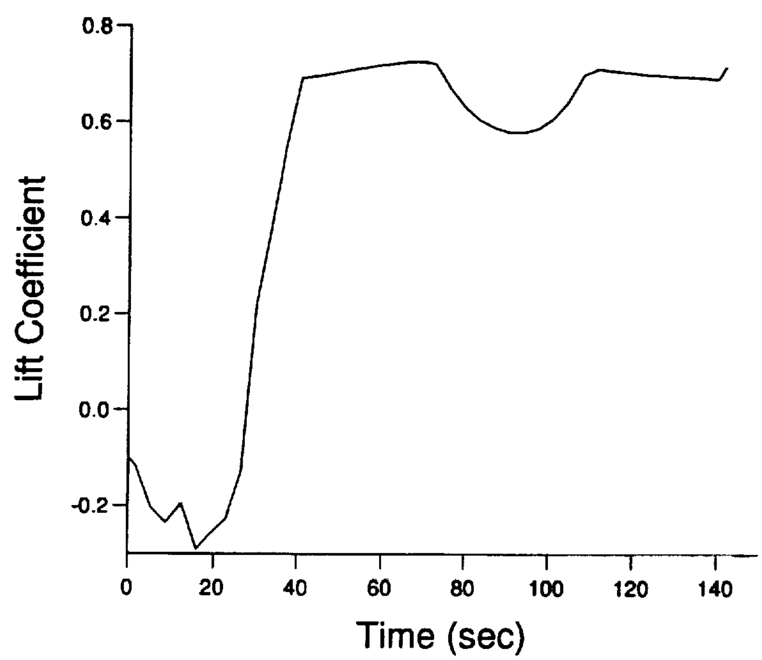
Solution for 30 Second Abort Case

Heading vs. Time



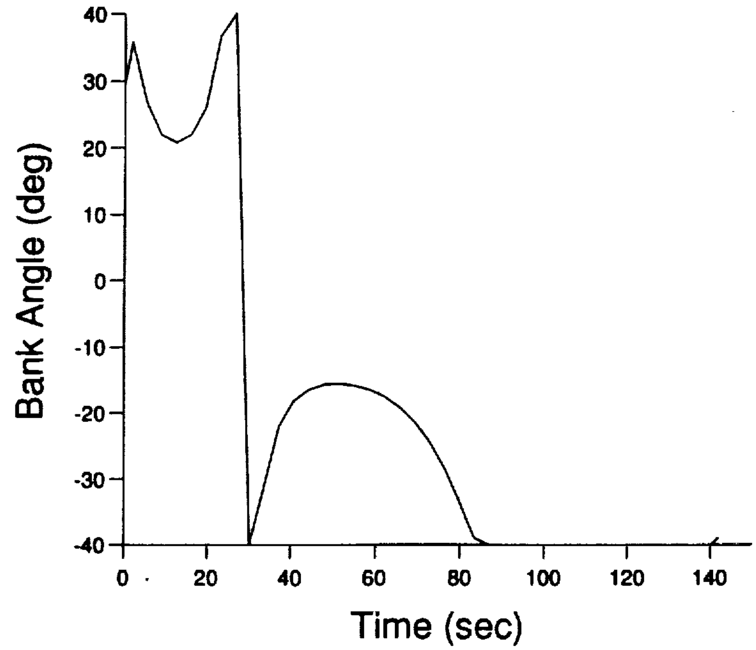
Solution for 30 Second Abort Case

Lift Coefficient vs. Time



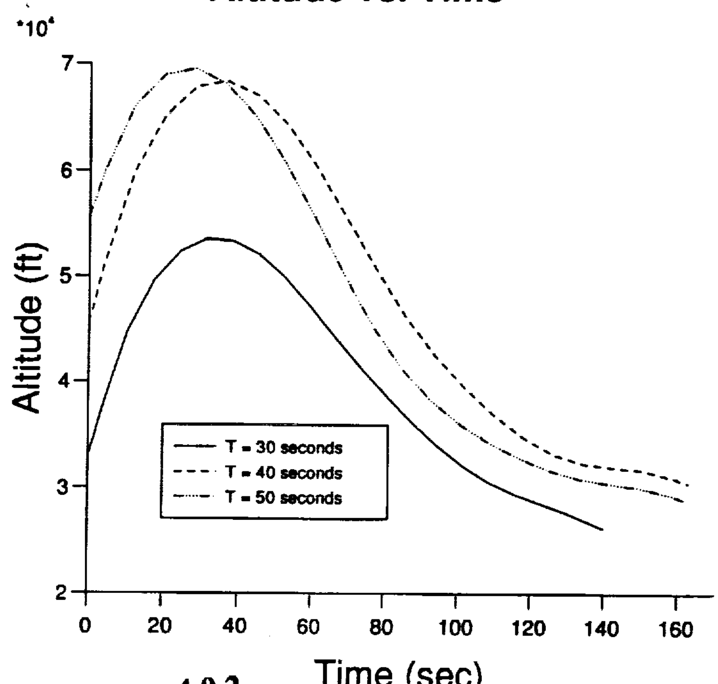
Solution for 30 Second Case

Bank Angle vs. Time



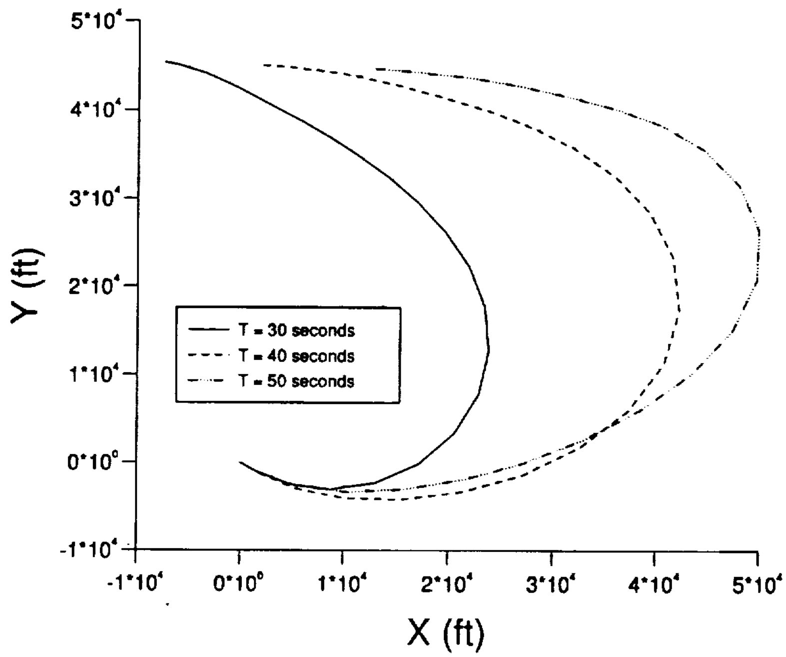
Comparison of T=30,40,50 Sec. Aborts

Altitude vs. Time



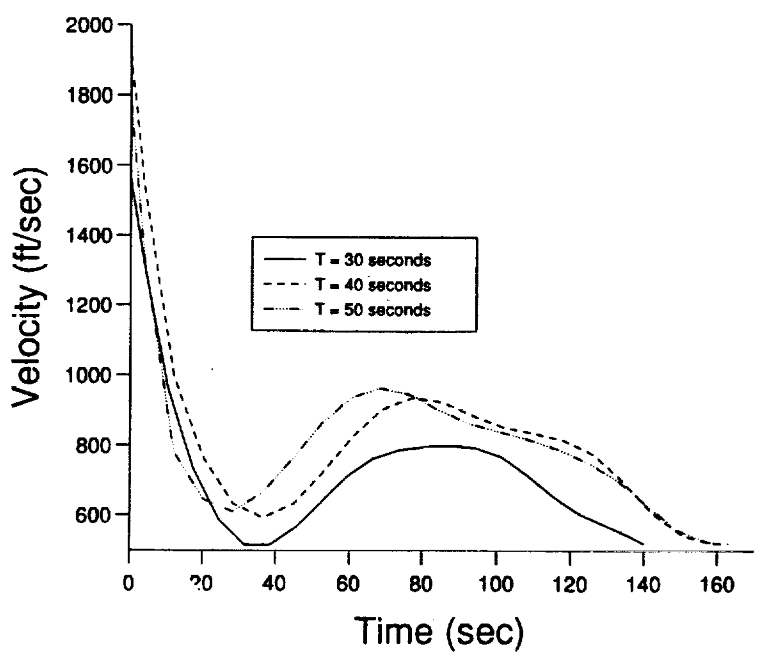
Comparison of T=30,40,50 Sec. Aborts

Groundtrack



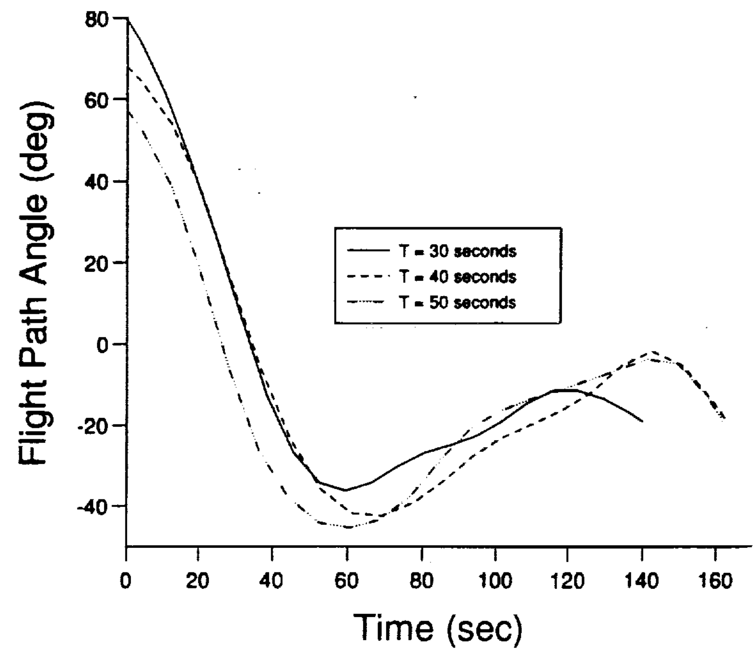
Comparison of T=30,40,50 Sec. Aborts

Velocity vs. Time



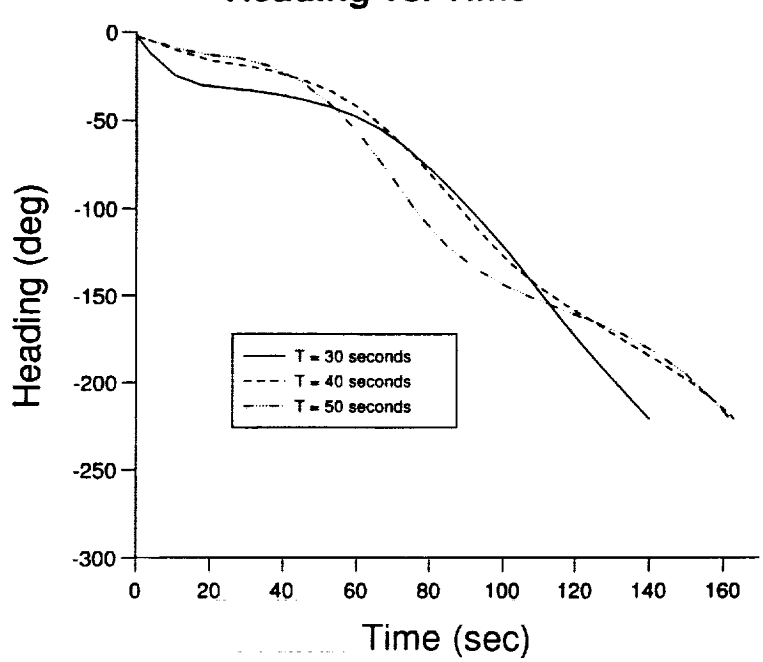
Comparison of T=30,40,50 Sec. Aborts

Flight Path Angle vs. Time



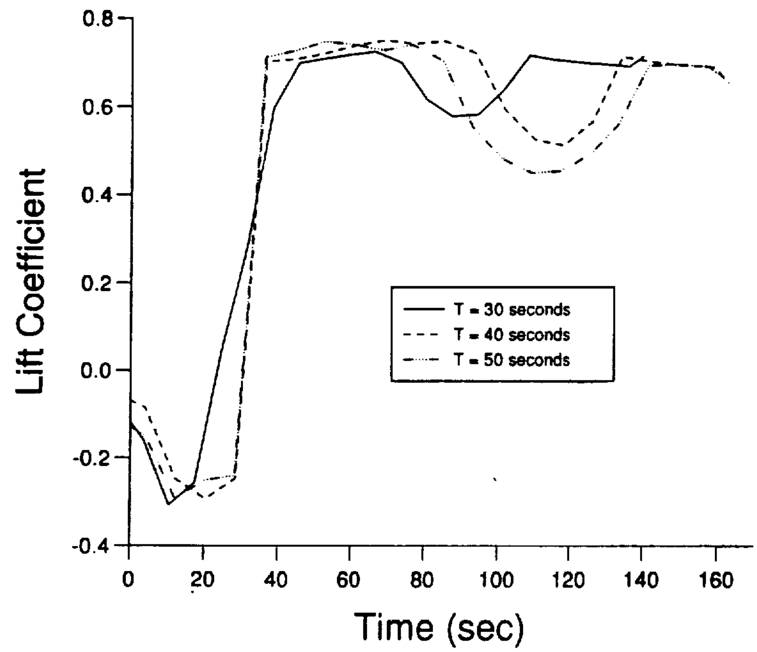
Comparison of T=30,40,50 Sec. Aborts

Heading vs. Time



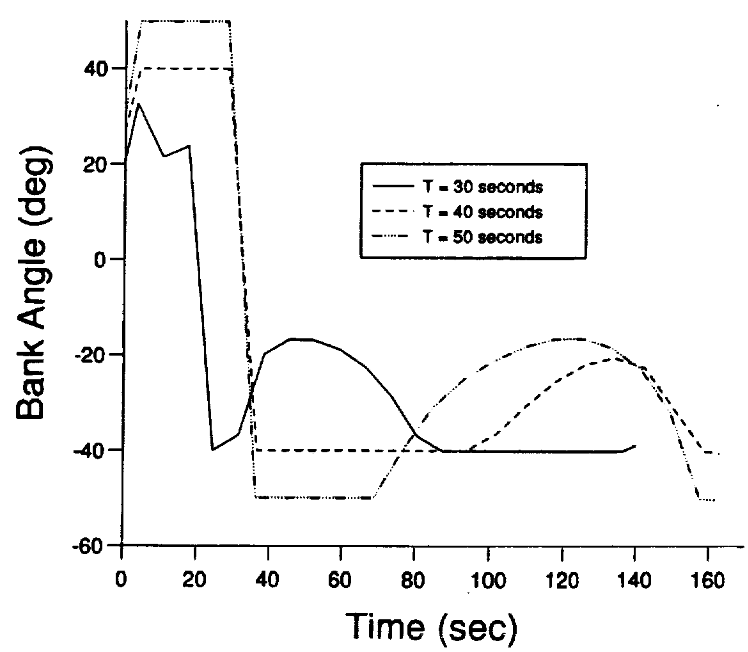
Comparison of T=30,40,50 Sec. Aborts

Lift Coefficient vs. Time



Comparison of T=30,40,50 Sec. Aborts

Bank Angle vs. Time



Concluding Remarks

- When final V is fixed, maximizing final h is nearly same as maximizing final energy ==>calculation of minimum energy trajectories
- Choice of cost function for abort (and reentry) not obvious
- Future work:
  - Single Stage Vehicle (?)
  - experiment to assess "power" of TODI approach compared to traditional shooting

Range Optimal Atmospheric Flight Vehicle Trajectories in Presence of a Dynamic Pressure Limit

BY

Hans Seywald
Analytical Mechanics Associates Inc. (AMA)
Spacecraft Control Branch, NASA LaRC

PURPOSE OF THIS TALK

- Explore nature of range–optimal flight
- Present techniques for identifying temporal structure of optimal control
- Demonstrate in application to an aircraft example

PROBLEM FORMULATION

Cost Function:

$J[u] = -x(t_f)$

State Equations:

$$\dot{E} = (\eta T - D)\frac{v}{W}$$
$$\dot{h} = v \sin\gamma$$
$$\dot{\gamma} = \frac{g}{v}(n - \cos\gamma)$$
$$\dot{x} = v\cos\gamma$$

Initial Conditions:

$E(0) = 38,029.[m]$
$h(0) = 12,119.[m]$
$\gamma(0) = 0.^{\circ}$
$x(0) = 0.[m]$

Final Conditions:

$E(t_f) = 9000.[m]$
$h(t_f) = 942.[m]$
$\gamma(t_f) = -11.5^{\circ}$
$x(t_f)$ be maximized

Final time:

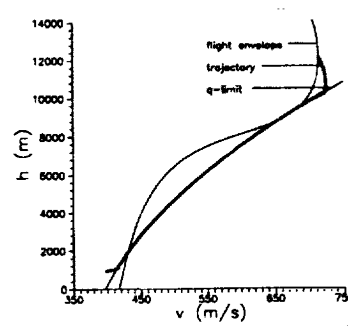
$t_f = 60[sec]$

Control constraint:

$0 \le \eta \le 1$
$|n| \le n_{max}$

State constraint:

$v \le v_{max}(h)$



SIGNIFICANCE FOR PRACTICAL APPLICATION

- Validate optimality of solutions obtained with other methods
- Use optimal solutions to develop guidance laws based on neighboring optimal control
- Decide on choice of discretization (e.g. finite elements)

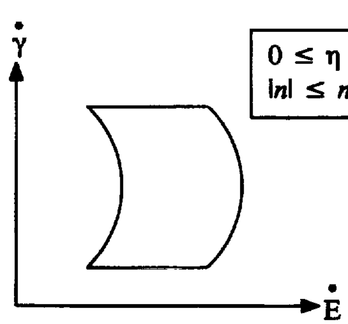
OUTLINE

- Hodograph analysis
- Possible control logics / Optimal switching structures
- Numerical procedures and results
- Summary and Conclusions

HODOGRAPH ANALYSIS

Original formulation:

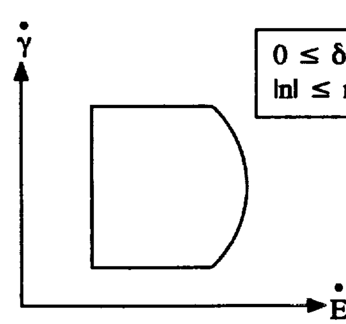
$$\dot{E} = (\eta T - D)\,\frac{V}{W}$$
$$\dot{\gamma} = \frac{g}{V}(n - \cos\gamma)$$



$0 \le \eta \le 1$
$|n| \le n_{max}$

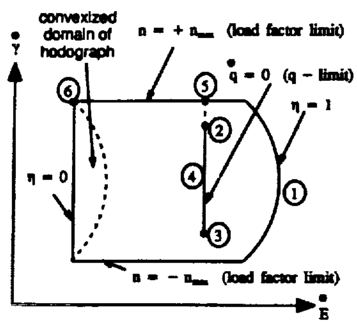
New formulation:

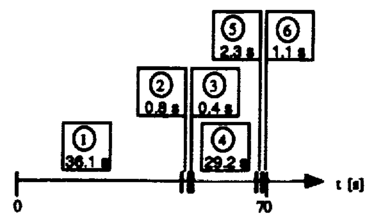
$$\dot{E} = [\delta(T - D + D_{max}) - D_{max}]\,\frac{V}{W}$$
$$\dot{\gamma} = \frac{g}{V}(n - \cos\gamma)$$



$0 \le \delta \le 1$
$|n| \le n_{max}$

POSSIBLE CONTROL LOGICS / OPTIMAL SWITCHING STRUCTURE





1)	$v - v_{max} < 0, \quad \delta = 1,$	$\frac{\partial H}{\partial n} = 0$
2)	$v - v_{max} = 0, \quad \delta = 1,$	$n > 0$ *from* $\frac{d}{dt}(v - v_{max}) = 0$
3)	$v - v_{max} = 0, \quad \delta = 1,$	$n < 0$ *from* $\frac{d}{dt}(v - v_{max}) = 0$
4)	$v - v_{max} = 0, \quad \delta$ *from* $\frac{d}{dt}(v - v_{max}) = 0,$	n *singular*
5)	$v - v_{max} = 0, \quad \delta$ *from* $\frac{d}{dt}(v - v_{max}) = 0,$	$n = n_{max}$
6)	$v - v_{max} < 0,$	$\delta = 0, \quad n = n_{max}$

slides_NASA_workshop

THOROUGH ANALYSIS YIELDS

- 12 different possible control logics are obtained

  6 cases with v_{max}–limit not active
  1 first–order singular case with v_{max}–limit not active
  6 cases with active v_{max}–limit
  1 first–order singular case with v_{max}–limit active
  1 second order singular case with v_{max}–limit active

- To perform higher order optimality tests the Generalized Legendre–Clebsch condition has been extended to the case of singular control in presence of state/control constraints

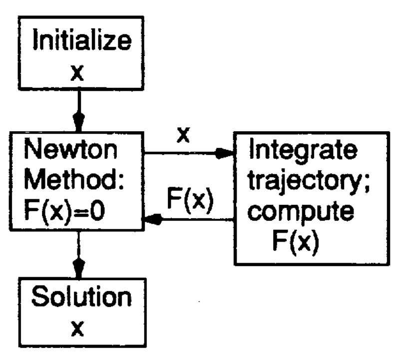
slides_NASA_workshop

REMARKS

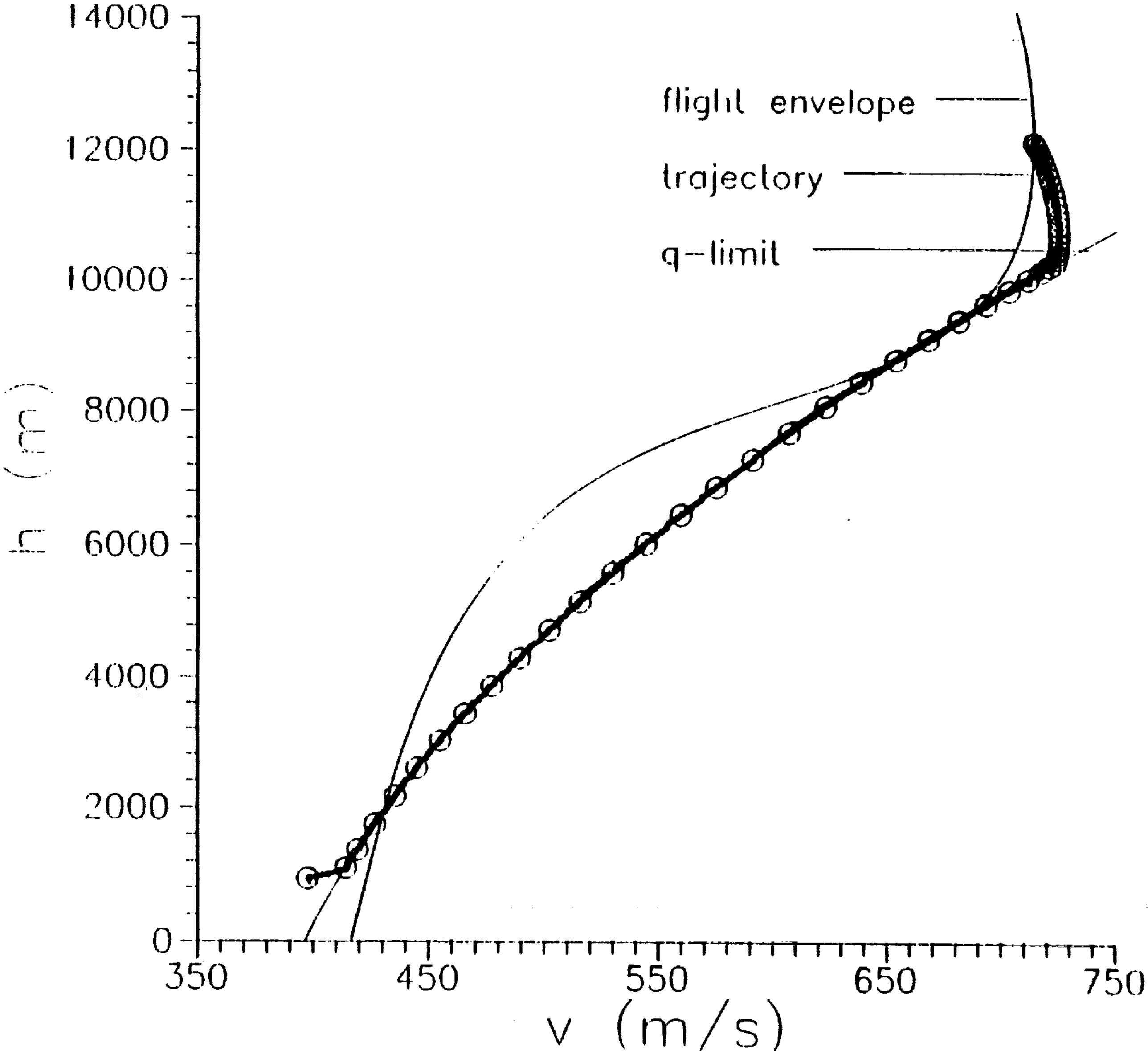
- Switching structure is non–intuitive
- Dynamic pressure constraint makes problem very ill–conditioned

  (i) standard shooting codes fail
  (ii) developed flexible shooting code
  (iii) trick: start integration at the end of singular control

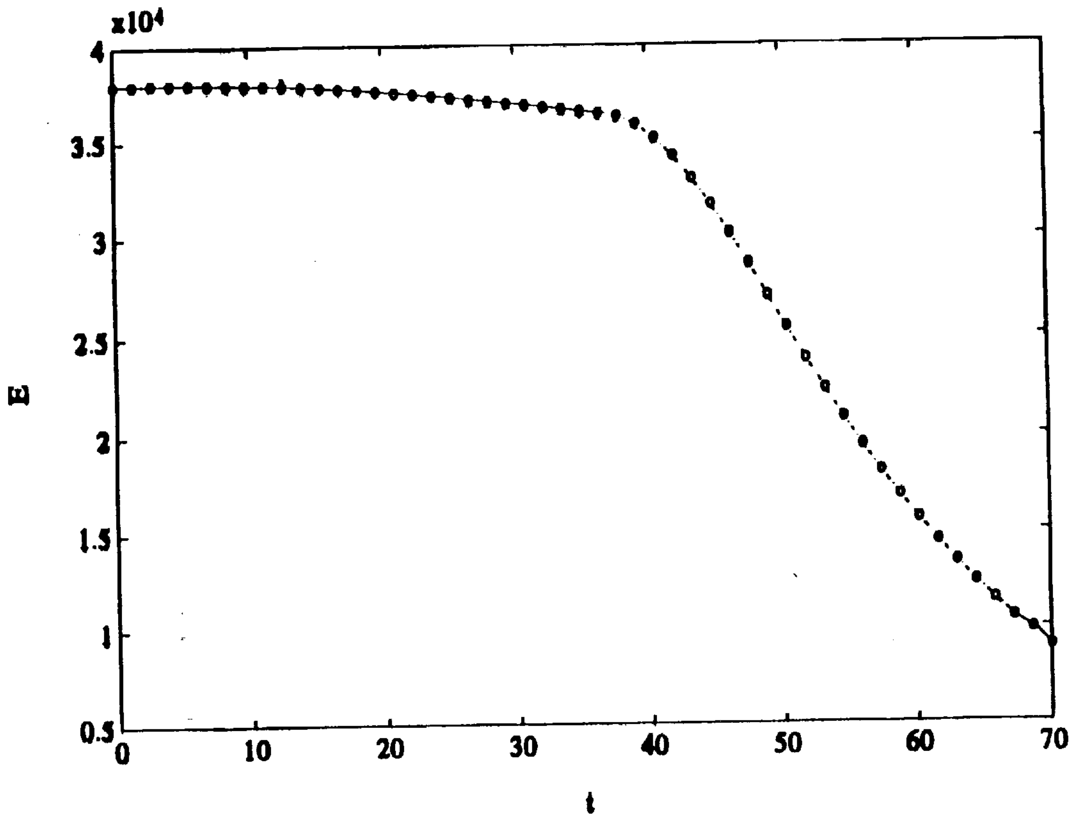
STRUCTURE OF SHOOTING CODE



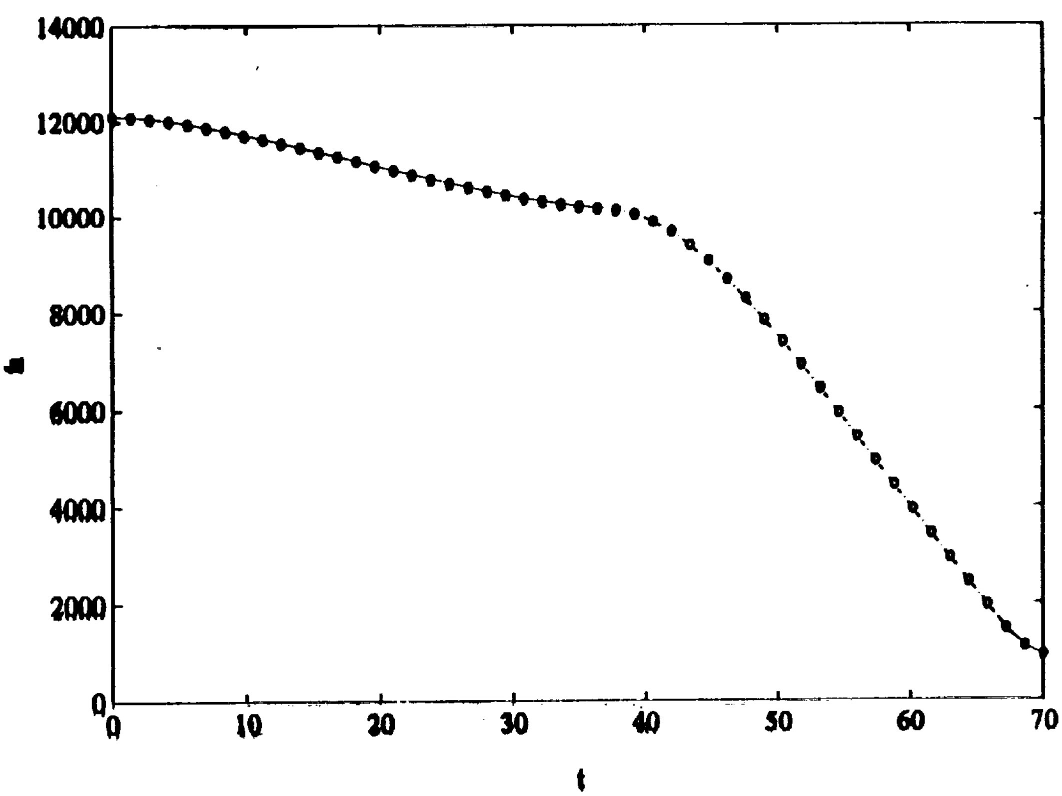
- Boundary value problem: find x such that F(x)=0
- User completely determines function F
- Simple structure allows independent debugging of F(x)



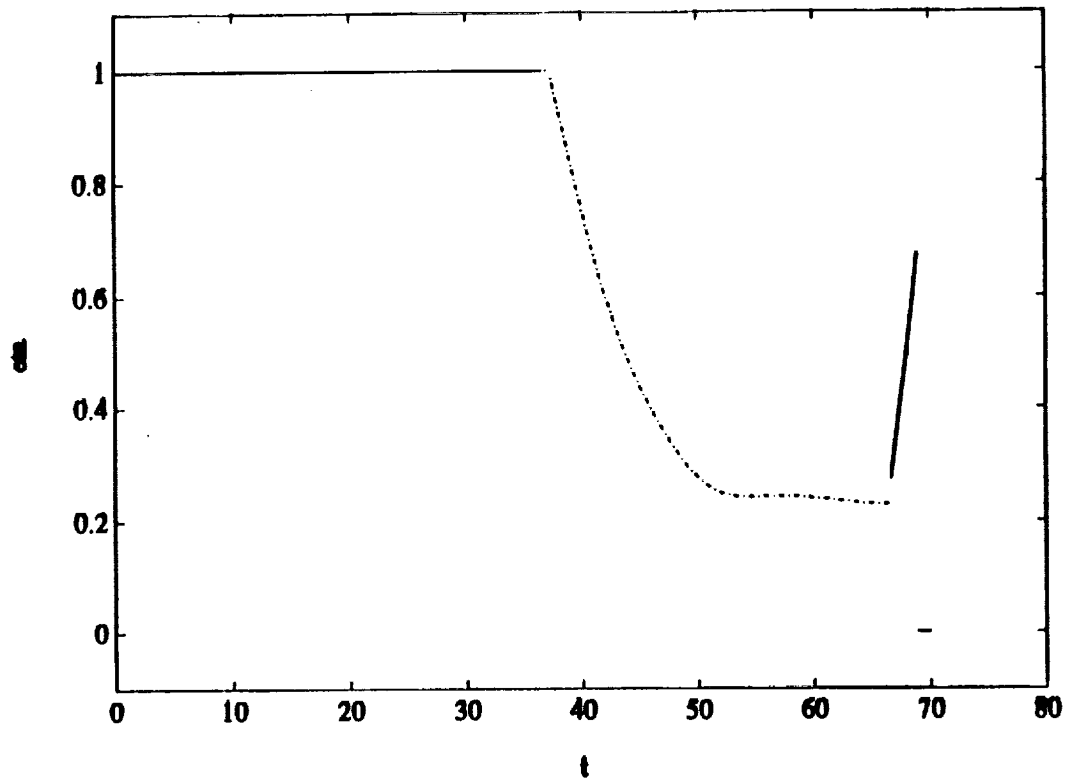
14000
12000
10000
8000
6000
4000
2000
0
h (m)
350
450
550
650
750
v (m/s)
flight envelope
trajectory
q-limit

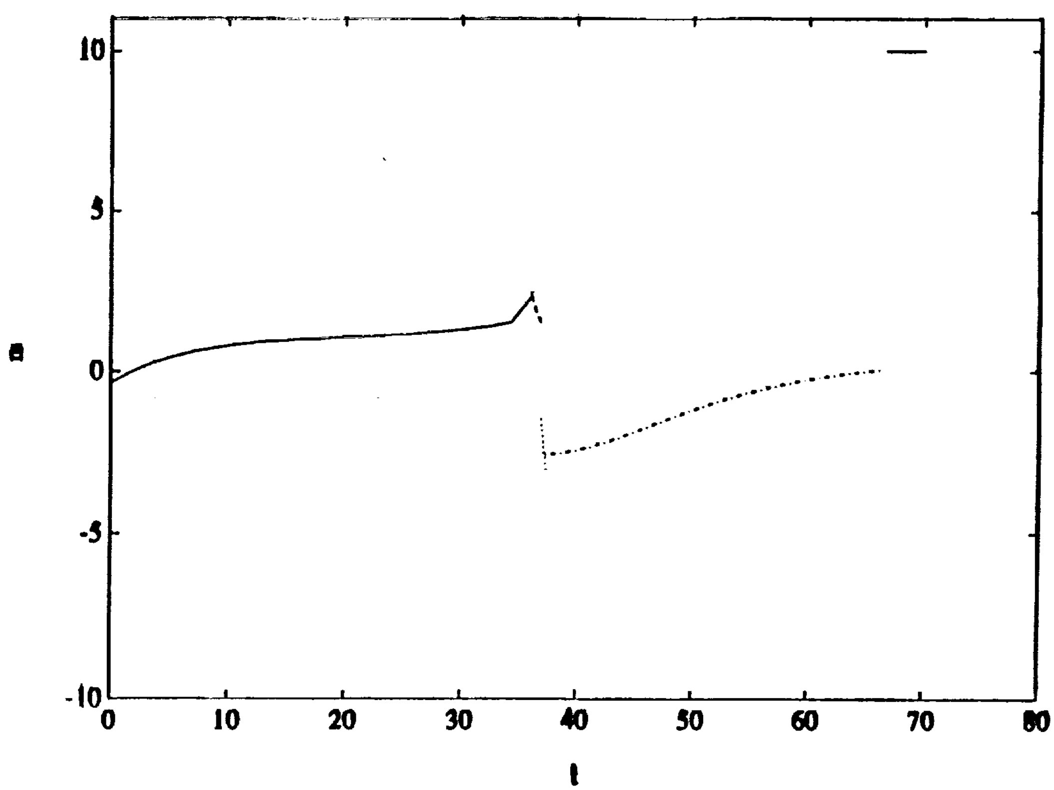


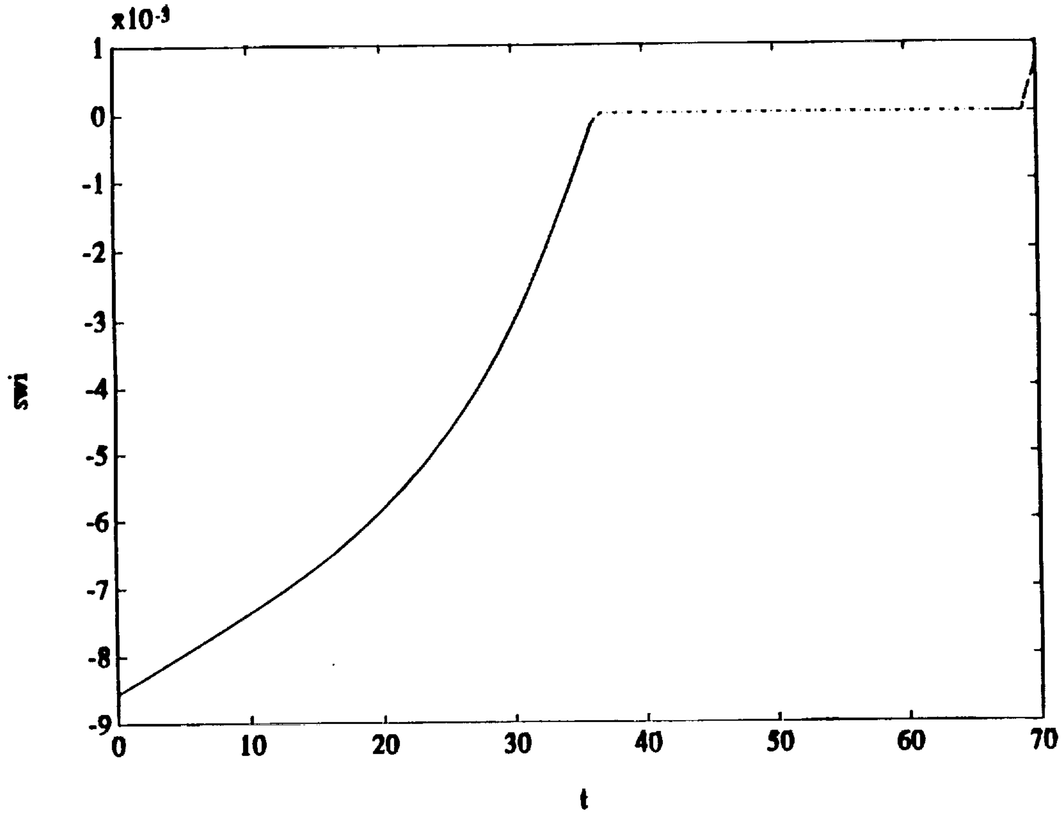
x10^4
4
3.5
3
2.5
2
1.5
1
0.5
E
0
10
20
30
40
50
60
70
t

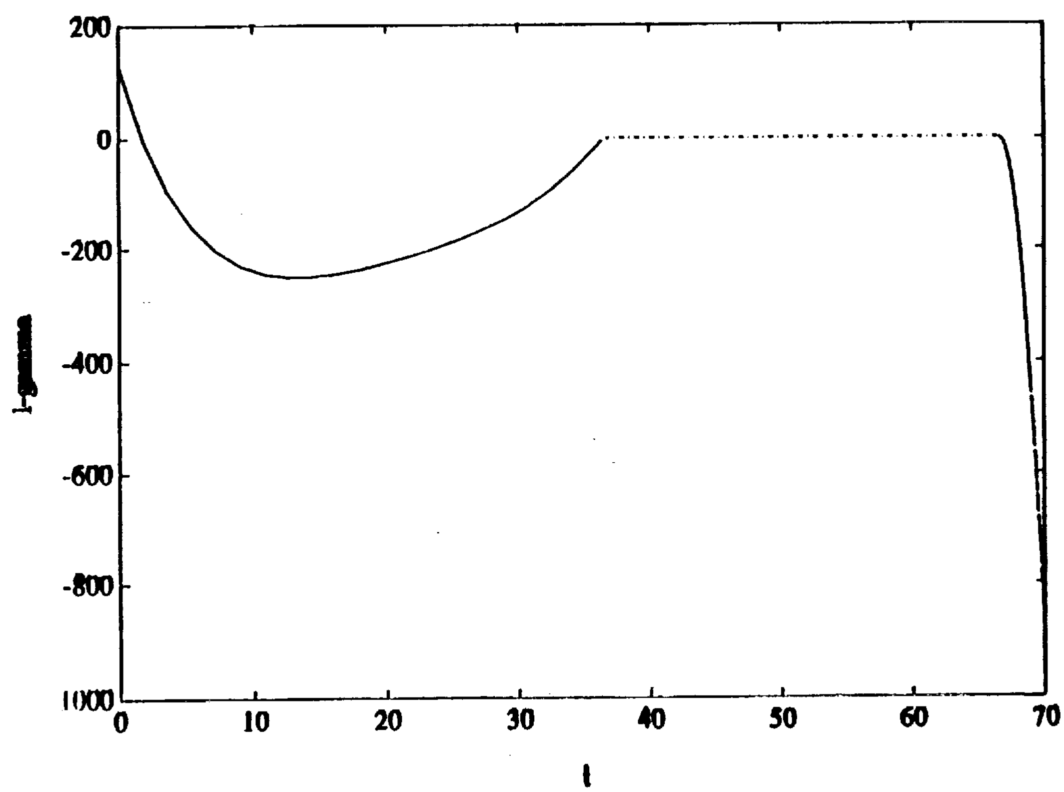


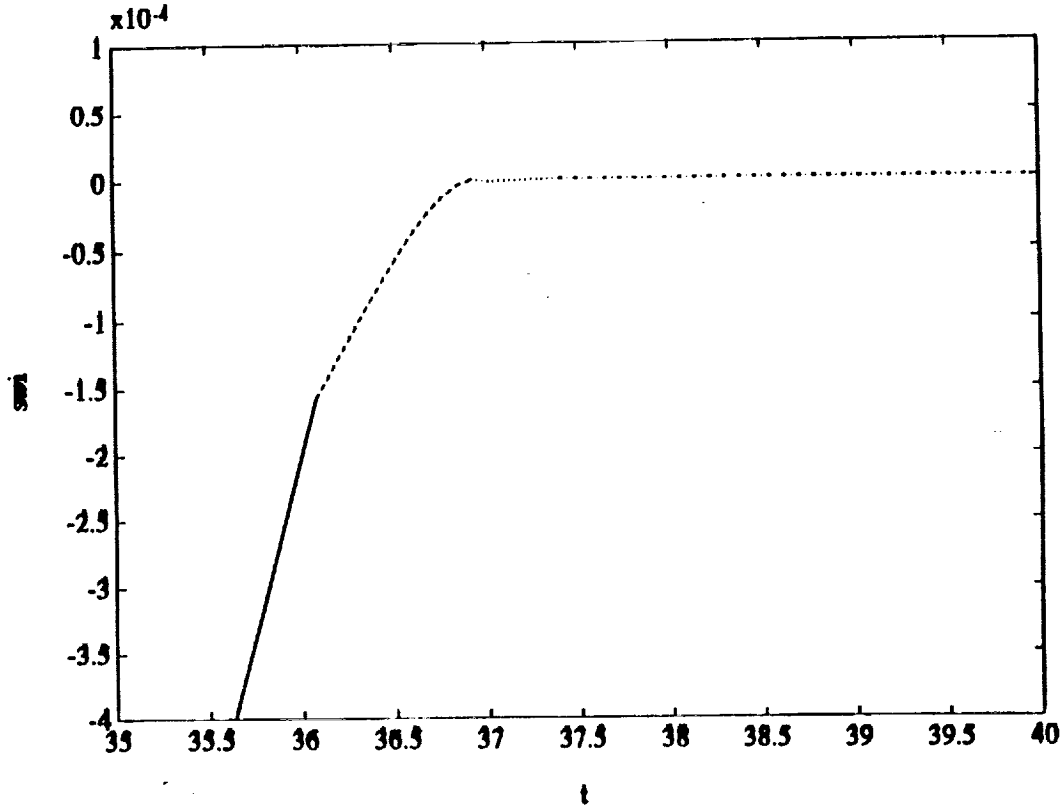
14000
12000
10000
8000
6000
4000
2000
0
0
10
20
30
40
50
60
70
t

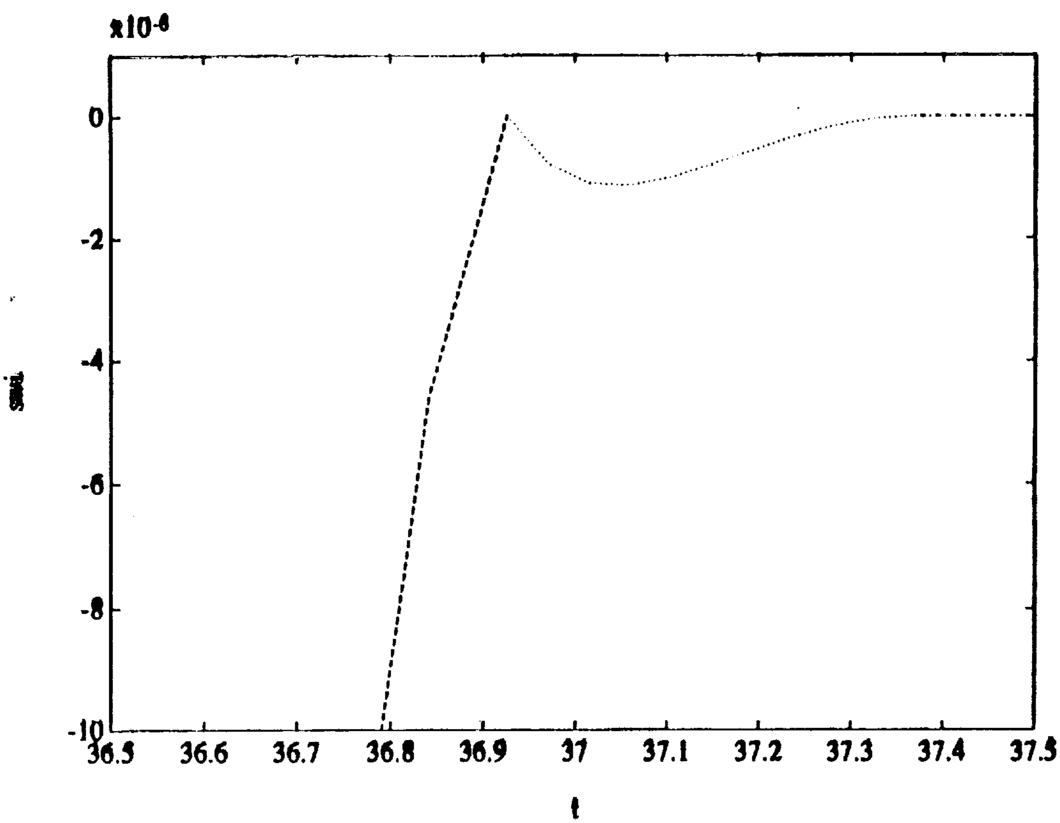

1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
t

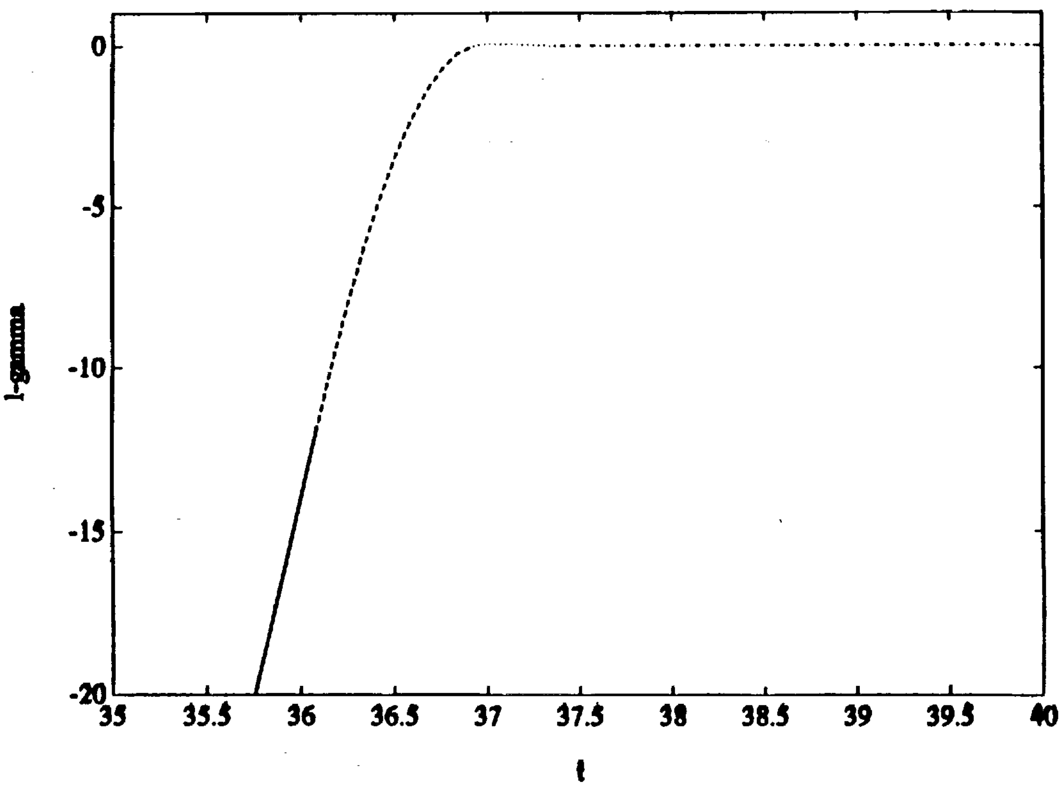

10
5
0
-5
-10
0
10
20
30
40
50
60
70
80
t

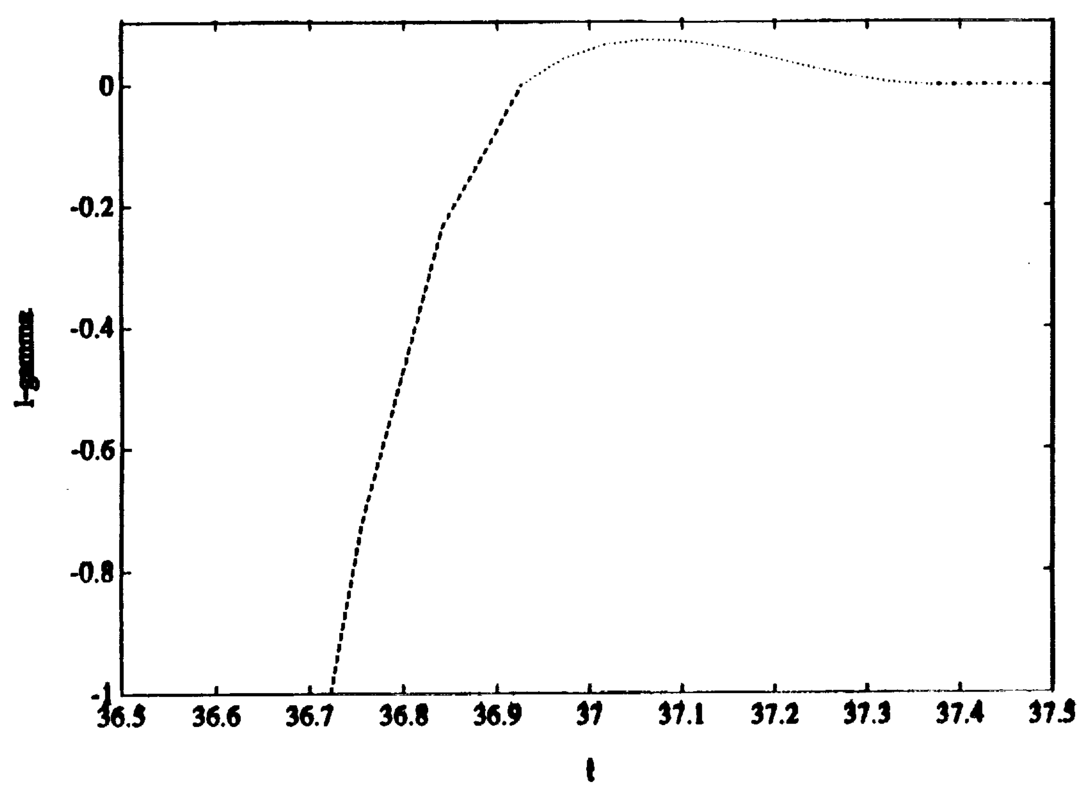

x10-3
swi
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
0
10
20
30
40
50
60
70
t


l-gamma
200
0
-200
-400
-600
-800
1000
0
10
20
30
40
50
60
70
t


x10-4
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
35
35.5
36
36.5
37
37.5
38
38.5
39
39.5
40
t


x10-6
0
-2
-4
-6
-8
-10
36.5
36.6
36.7
36.8
36.9
37
37.1
37.2
37.3
37.4
37.5
t


0
-5
-10
-15
-20
1-gamma
35
35.5
36
36.5
37
37.5
38
38.5
39
39.5
40
t


0
-0.2
-0.4
-0.6
-0.8
-1
1-gamma
36.5
36.6
36.7
36.8
36.9
37
37.1
37.2
37.3
37.4
37.5
t

SUMMARY

- All possible control logics are analyzed

- Optimal switching structures are identified.
  Solutions involve singular control along state constrained arcs

- A flexible multipoint shooting code was developed and applied successfully

- TODI was used to perform sanity check and to guess the optimal switching structure

3-D Air-to-Air Missile Trajectory Shaping Study

by

Renjith Kumar, Hans Seywald
Analytical Mechanics Associates Inc., Hampton, Virginia

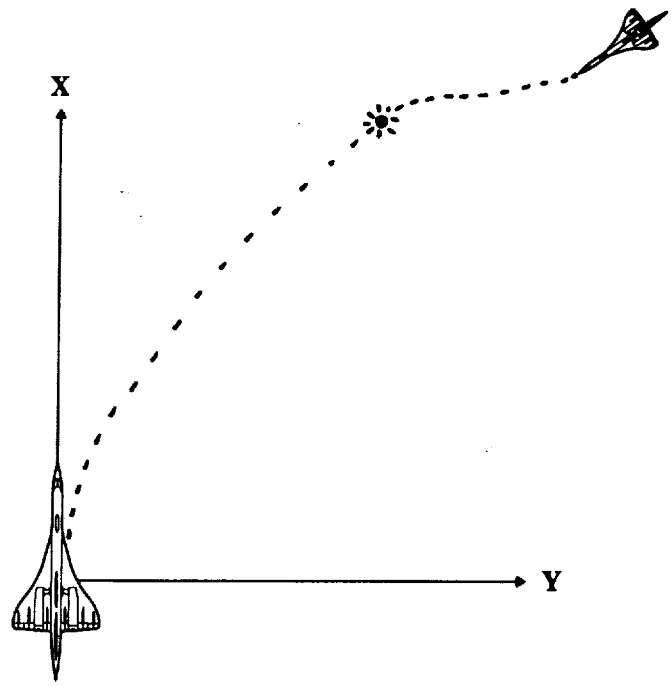
and

Eugene Cliff, Late Henry Kelley
Department of Aerospace Engineering, VPI&SU, Blacksburg, Virginia

HISTORICAL BACKGROUND

- Sir Francis Drake and "Manouevre Board"
- World War II
  - Pure-Pursuit
  - Deviated Pursuit
  - Command to Line-of-sight
  - Collision course
  - Proportional Navigation
- Singular Perturbation (Reduced-Order Modeling)

THE GUIDANCE PROBLEM



PROBLEM FORMULATION

Cost

$$\min \quad -x(t_f)$$

Differential Constraints

$$\dot{x} = V\cos\gamma\cos\chi$$

$$\dot{y} = V\cos\gamma\sin\chi$$

$$\dot{h} = V\sin\gamma$$

$$\dot{E} = \frac{V}{W(t)}\,(T(t) - D(h,M,n))$$

$$\dot{\gamma} = \frac{g}{V}\,(n_v - \cos\gamma)$$

$$\dot{\chi} = \frac{g}{V}\,\frac{n_h}{\cos\gamma}$$

Initial and Final Conditions

$x(0) = 0$	$x(t_f)$ *to be optimized*
$y(0) = 0$	$y(t_f) = y_f$
$h(0) = h_0$	$h(t_f) = h_0$
$E(0) = E_0$	$E(t_f) \geq E_f$
$\gamma(0) = \gamma_0$	$\gamma(t_f)$ *free*
$\chi(0) = \chi_0$	$\chi(t_f)$ *free*

Controls

$n_v\ ,\ n_h$

Control Constraints

$$\sqrt{n^2_V + n^2_h} \leq 30$$

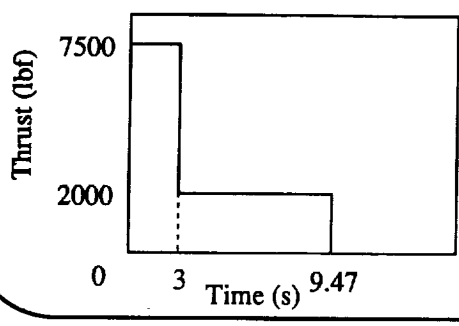
$$\sqrt{n^2_V + n^2_h} \leq \frac{qS}{W} C_{Lmax}(M)$$

DRAG, THRUST & WEIGHT MODEL

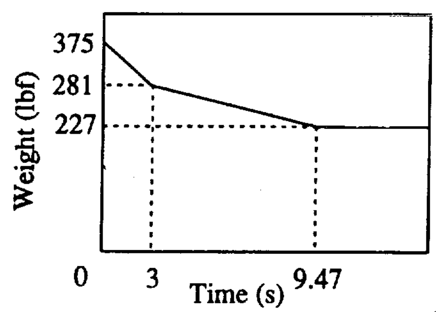
Drag Model

$$D = qS\,[C_{D_0}(M,h) + C_{Di}\,(M)\,(\frac{W}{qS})^{1.8}\,n^{1.8}] \quad \text{where} \quad n = \sqrt{n^2_v + n^2_h}$$

Thrust Model



Weight Model



INDIRECT METHOD

Optimal control problem

$$\min \; \Phi(x(t_f),t_f)$$

$$\dot{x} = f(x,u,t)$$

$$x\,(t_0) = x_0$$

$$\psi\,(x(t_f),t_f) = 0$$

if solution does exist then it satisfies ⇨

Boundary value problem

$$\dot{x} = f(x,u,t)$$

$$\dot{\lambda} = -\frac{\partial H}{\partial x} \quad \text{where} \quad H \doteq \lambda^T f$$

$$\min_{u \in \Omega} \; H(x,\lambda,u,t)$$

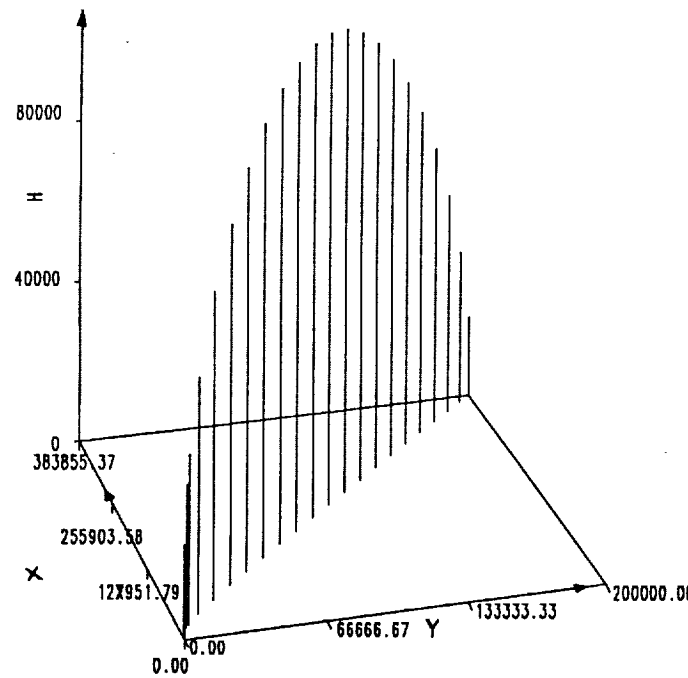
$$x\,(t_0) = x_0$$

$$\psi\,(x(t_f),t_f) = 0$$

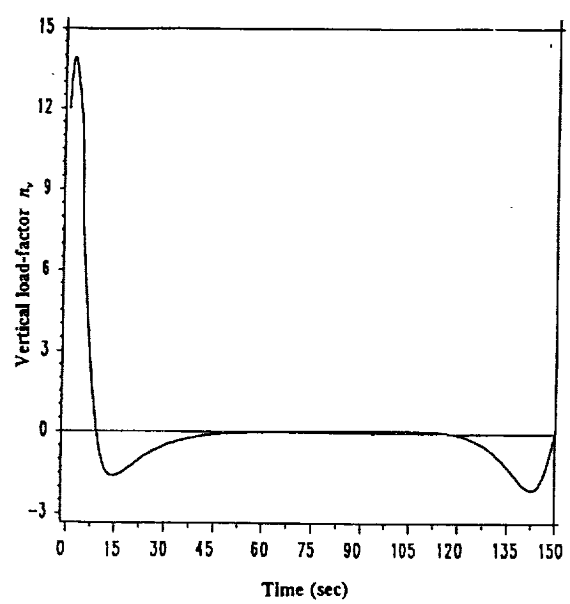
$$\lambda(t_f) = \frac{\partial \Phi}{\partial x(t_f)} + \nu^T \frac{\partial \Psi}{\partial x(t_f)}$$

$$H(t_f) = \frac{\partial \Phi}{\partial t_f} + \nu^T \frac{\partial \Psi}{\partial t_f}$$

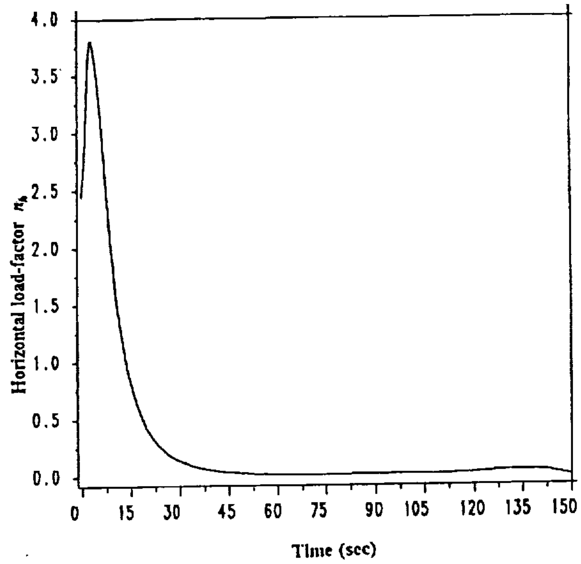
OPTIMAL TRAJECTORY IN 3-D



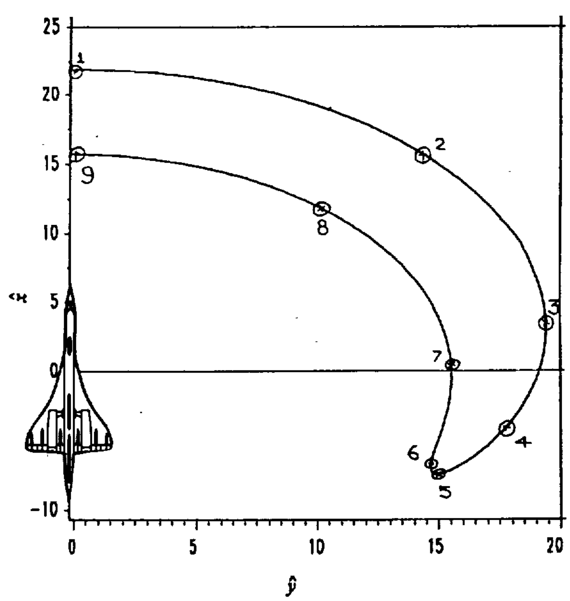
VERTICAL LOAD-FACTOR TIME HISTORY



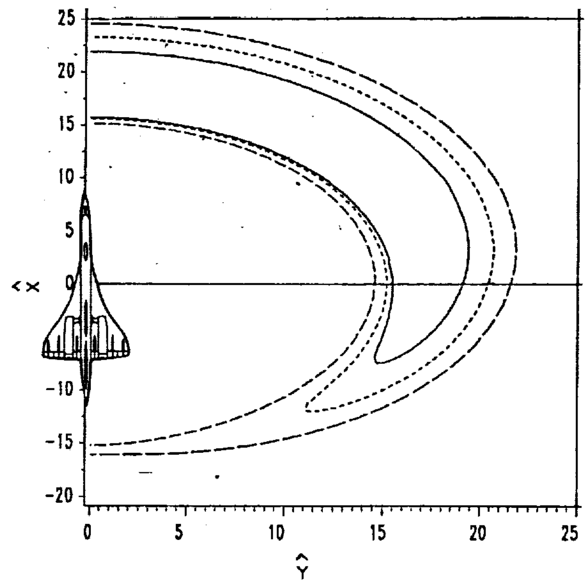
HORIZONTAL LOAD-FACTOR TIME HISTORY



ATTAINABILITY SET FOR FINAL TIME 150s



ATTAINABILITY SET FOR FINAL TIME 150s, 160s, 170s



ACCESSORY MINIMUM PROBLEM

$$\min \ \tfrac{1}{2}x(t_f)^T S_f x(t_f) + \tfrac{1}{2}\int_{t_0}^{t_f} [x^T, u^T] \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} dt$$

$\dot{x} = Fx + Gu$

$x\,(t_0) = x_0$ $\quad$ t_0 *fixed* $\quad$ $A^T = A$ $\quad$ $A_{22} > 0$

$Bx(t_f) - b = 0$ $\quad$ t_f *fixed* $\quad$ $S_f^T = S_f$ $\quad$ $S_f \geq 0$

ACCESSORY MINIMUM PROBLEM (contd.)

$$u = A_{22}^{-1}\,[(-A_{21} - G^T(S - RQ^{-1}R^T))x - G^T RQ^{-1}b]$$

$$\dot{S} = D_{21} + D_{22}S - SD_{11} - SD_{12}S \quad ; \quad S(t_f) = S_f$$

$$\dot{R} = D_{22}R - SD_{12}R \quad ; \quad R(t_f) = B^T$$

$$\dot{Q} = -R^T D_{12} R \quad ; \quad Q(t_f) = 0$$

$\Downarrow$

!! new !!

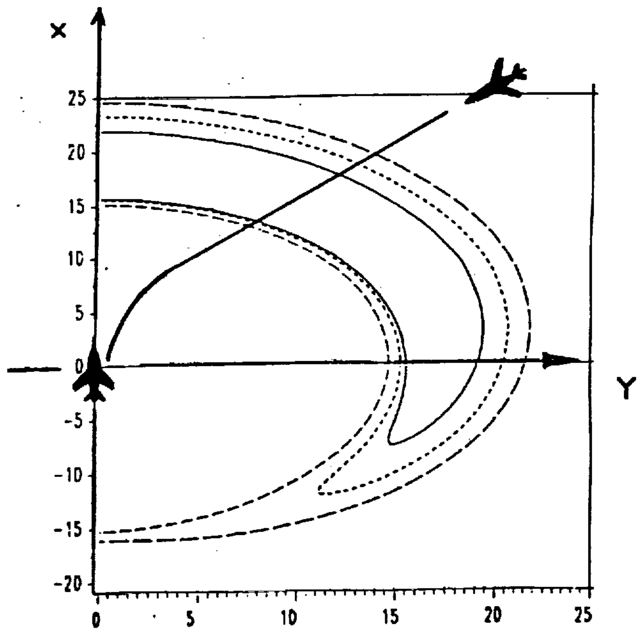
$$H \doteq S - RQ^{-1}R^T$$

$$\dot{H} = D_{21} + D_{22}H - HD_{11} - HD_{12}H$$

THREE PHASE GUIDANCE

- BOOST PHASE GUIDANCE
- MIDCOURSE GUIDANCE
- TERMINAL GUIDANCE

IDENTIFY REFERENCE SOLUTION



MIDCOURSE GUIDANCE (NEIGHBORING SOLUTION)

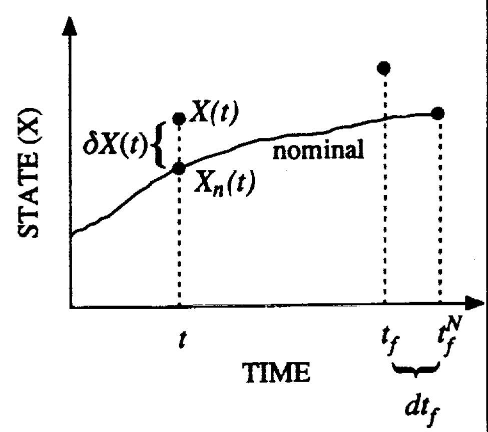
Closed-loop Control

$$u_{CL}(t) = u_{ref}(t) + \delta u(t)$$

$$\delta u(t) = G_1(t)\, \delta X(t) + G_2(t)\, d\psi(t)$$

Change in final time (cost)

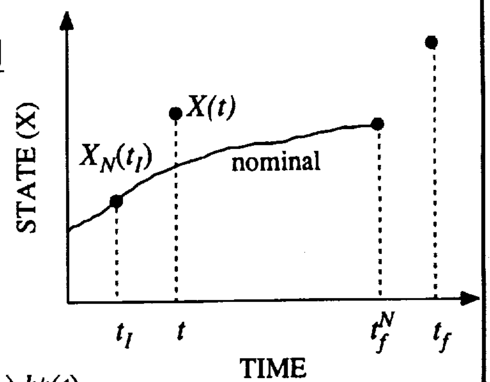
$$dt_f = K_1(t)\, \delta X(t) + K_2(t)\, d\psi(t)$$



MIDCOURSE GUIDANCE (TRANSVERSAL COMPARISON)

$$t_f' = t_f - t = t_f^N - t_I$$

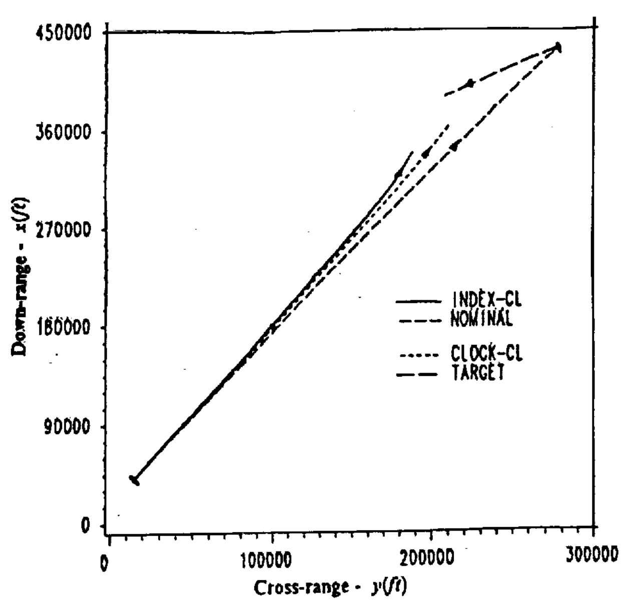
$$t - t_I = \frac{K_1(t_I)[X(t) - X_N(t_I)] + K_2(t_I)[d\psi(t)]}{[1 + K_1(t_I)\dot{X}^N(t_I)]}$$



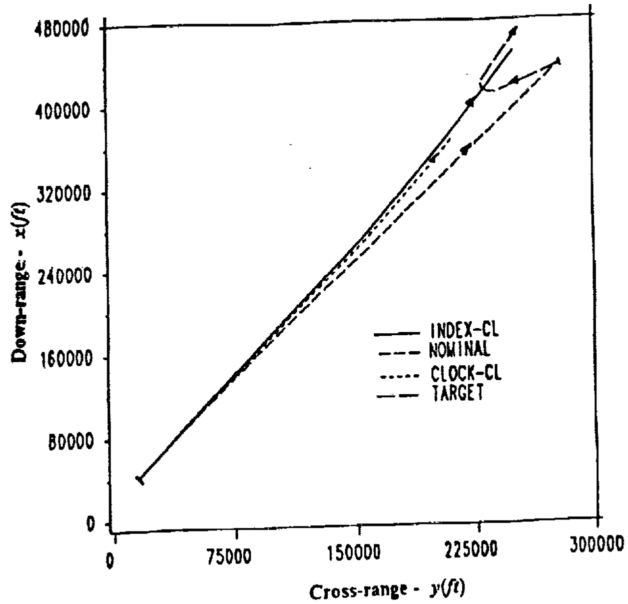
Closed-loop Control

$$u(t) = u^N(t_I) + G_1(t_I)[X(t) - X^N(t_I)] + [G_1(t_I)\dot{X}^N(t_I) + \dot{u}^N(t_I)][t - t_I] + G_2(t_I)d\psi(t)$$

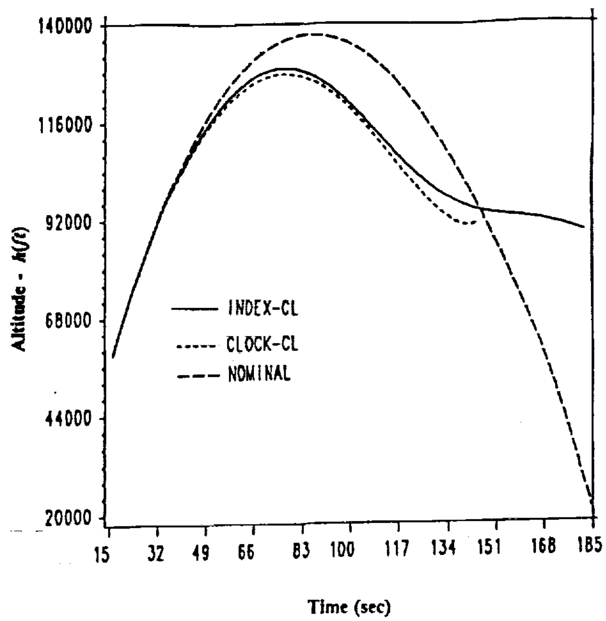
MIDCOURSE GUIDANCE (HORIZONTAL PROJECTION) AGGRESSIVE TARGET



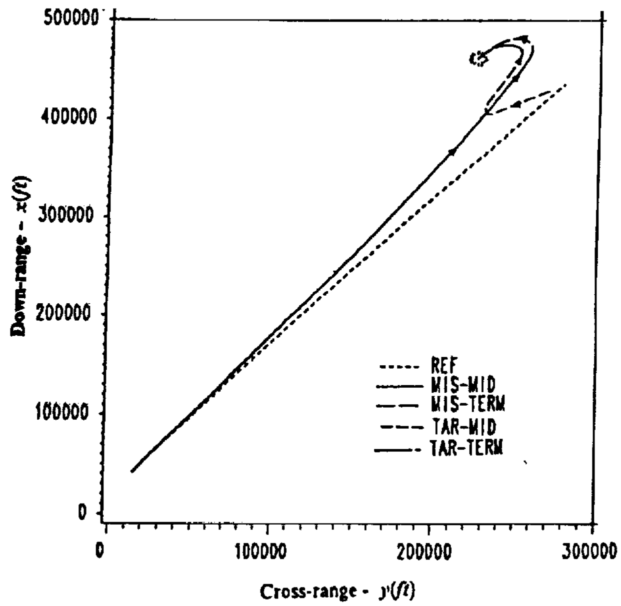
MIDCOURSE GUIDANCE (HORIZONTAL PROJECTION)
RUN-AWAY TARGET



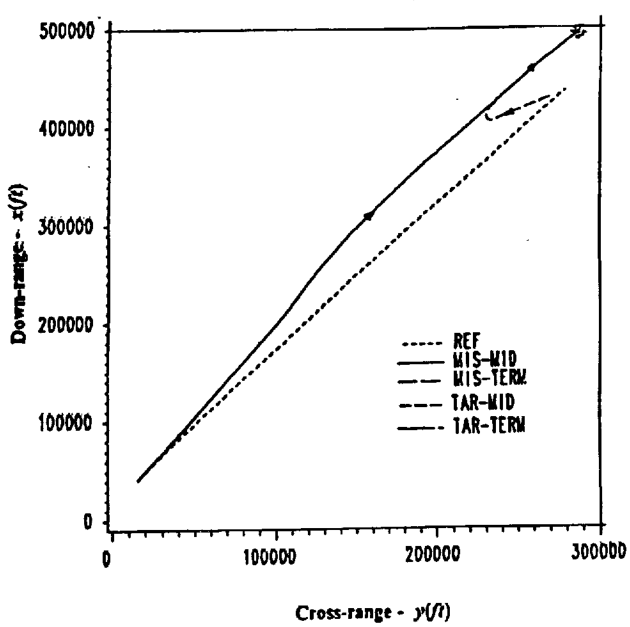
MIDCOURSE GUIDANCE (ALTITUDE)
RUN-AWAY TARGET



NEAR-OPTIMAL GUIDANCE (HORIZONTAL PROJECTION)
SHINAR'S TARGET



HALF-PN GUIDANCE (HORIZONTAL PROJECTION)
SHINAR'S TARGET



SALIENT CONTRIBUTIONS

- Identified attainable sets via intricate homotopy procedures.
- Checked sufficiency conditions for weak local optimality.
  - Derived a new matrix differential equation for conjugate point testing.
- Developed an efficient method of optimal gain evaluation.
- Developed a composite midcourse guidance strategy (half-pn) which saves on-board storage.

Constrained Minimization of Smooth Functions Using A Genetic Algorithm

Lynda J. Foernsler, SCB
Dr. Daniel D. Moerder, SCB
Dr. Bandu N. Pamadi, Vigyan

LaRC Workshop
March 18-19

Purpose

- Discuss the use of a simple genetic algorithm for constrained minimization of differentiable functions with differentiable constraints.
- Assess the performance of this approach

Outline

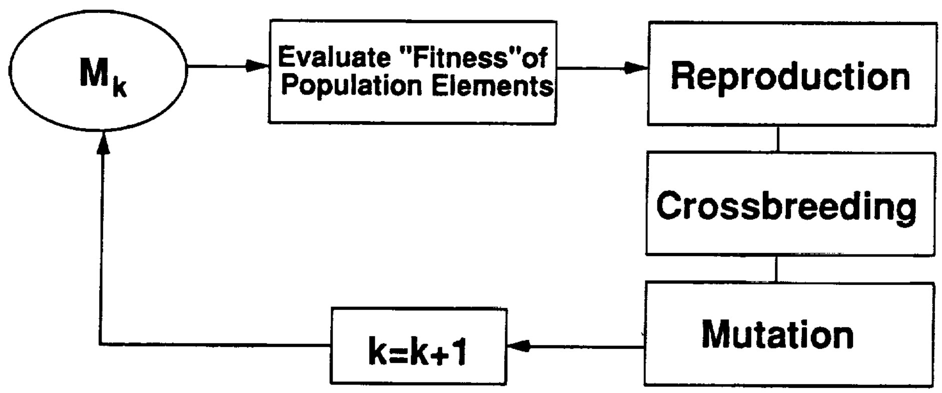
- Genetic Algorithms (GA)
- Problem Formulation for GA
- Numerical Experiment
- Comparison to Penalty Function Approach
- Conclusions
- Future Work

Genetic Algorithms

- Nonderivative, nondescent, random search procedures for unconstrained functional minimization
- Algorithmic structure is based on notions from biology with "survival of the fittest" search heuristic
- Operations performed on successive generations of a population represented by binary coded strings (DNA-analog)

GA Operations

M = population



- Initial population, M_0, is randomly generated

Constrained Function Minimization

$$x^* = \min_{x \in \Re^n} c(x)$$

subject to

$$f_i(x^*) = 0 \quad i \in E$$

$$f_j(x^*) \geq 0 \quad j \in I$$

Kuhn-Tucker (KT) Conditions

$$
\begin{aligned}
\left.\frac{\partial \mathcal{L}(x,\lambda)}{\partial x}\right|_{x^*,\lambda^*} &= 0 & & \\
\lambda_j^* &\geq 0 & & j \in I \\
\lambda_k^* f_k(x^*) &= 0 & & k \in E \cup I \\
f_k &= 0 & & k \in E \\
f_k &\geq 0 & & k \in I
\end{aligned}
$$

where

$$\mathcal{L}(x,\lambda) = c(x) - \sum_{k \in E \cup I} \lambda_k^* f_k(x^*)$$

Problem Formulation For GA

- **Convert the solution of the necessary conditions for a constrained minimum into an unconstrained function minimization**
- **Solve the resulting unconstrained minimization problem**

$$x^* = \arg \min_{x \in X} g(x)$$

where X is the user-specified bounded volume over which the GA takes place.

Unconstrained Minimization Problem Formulation

$$g(x, \lambda^*) = \sum_{i=1}^{n} |\mathcal{L}_{x_i}(x, \lambda^*)| + \sum_{j \in E} |f_j(x)| + \sum_{k \in I} |\min\{0, f_k(x)\}|$$

- estimate λ^* by setting $\mathcal{L}_x(x, \lambda) = 0$

$$\nu(x^*) = (f_x^T(x^*))^+ c_x(x^*)$$

$$\nu_i(x) = \begin{cases} \nu_i(x) & f_i = 0 \quad i \in E \\ |\nu_i(x)| & f_i < 0 \quad i \in I \\ 0 & f_i > 0 \quad i \in I \end{cases}$$

- KT conditions are satisfied by solving the nonsmooth equation

$$g(x, \nu(x)) = 0$$

Genetic Algorithm Function Minimization

$$x^* = \arg\min_{x \in \mathcal{X}} g(x, \nu(x))$$

where $\mathcal{X}$ is the user-specified bounded volume over which the genetic search takes place:

$$\mathcal{X} = \{x : (x_i)_{min} \leq x_i \leq (x_i)_{max};\ i = 1, \ldots, n\}$$

GA Function Minimization

$$x^* = \arg\min_{x \in \mathcal{X}} g(x, \nu(x))$$

where $\mathcal{X}$ is the user-specified bounded volume over which the genetic search takes place:

$$\mathcal{X} = \{x : (x_i)_{min} \leq x_i \leq (x_i)_{max};\ i = 1, \ldots, n\}$$

Numerical Experiment (1)

- Mission: Determine control settings for an energy-state approximation of minimum-fuel ascent to orbit for the Langley Accelerator
- Control variables:

$$\bar{x} = \begin{cases} \alpha, & \text{angle of attack (deg)} \\ h, & \text{altitude (ft)} \\ \delta_E, & \text{elevon deflection (deg)} \\ \delta_T, & \text{thrust vector angle (deg)} \\ \eta, & \text{fuel equivalence ratio} \end{cases}$$

- Cost:

$$c(x) = -\frac{dE}{dm}$$

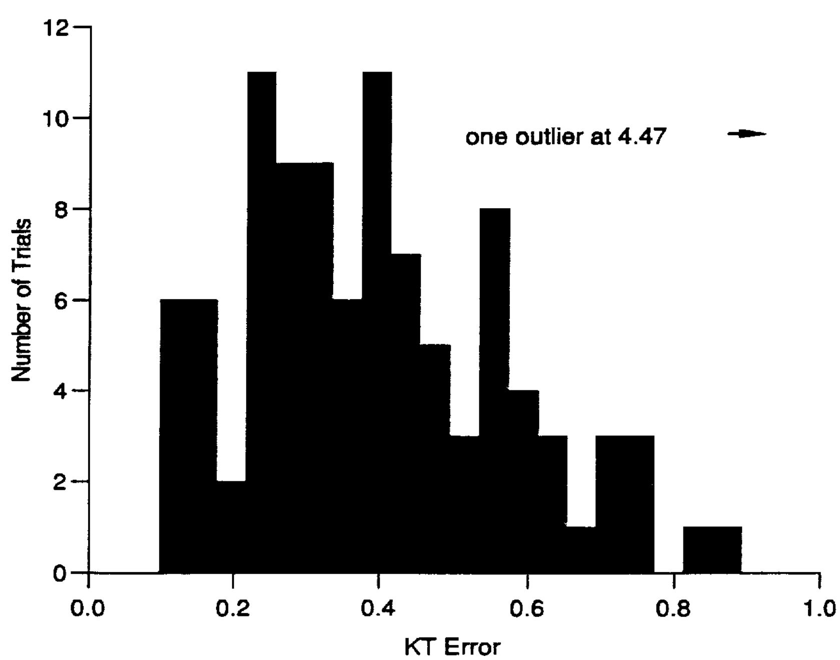
  subject to

  - vertical acceleration balance equality constraint
  - pitch moment balance equality constraint
  - dynamic pressure inequality constraint

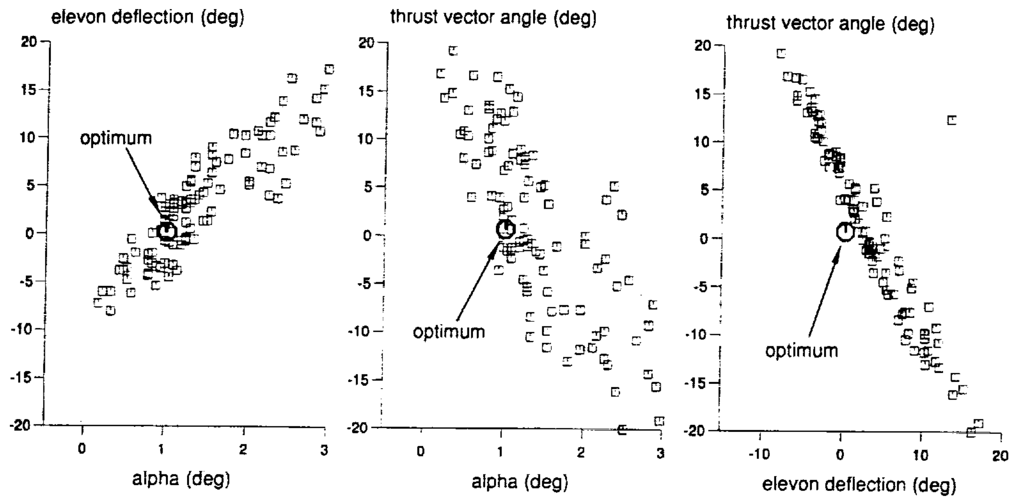
- Monte Carlo Experiment
  - 100 GA runs
  - 600 generations/run

- Used final generation $\bar{x}$ values from GA runs as initial guesses for Newton-Raphson (NR) method

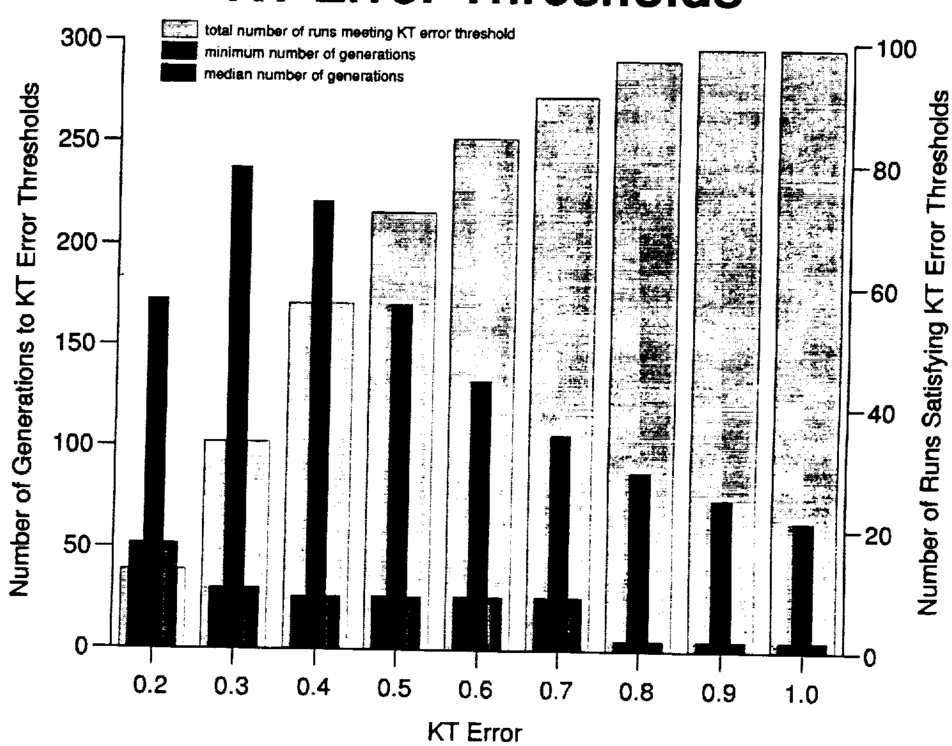
Distribution of KT Error for Aerospace Plane Model



Distribution of Control Settings for KT Approach



KT Error Thresholds



KT Formulation Results

- 82 of the NR runs converged
- 99/100 runs converged within a KT error threshold of .9
- Fewer number of generations/run would have sufficed

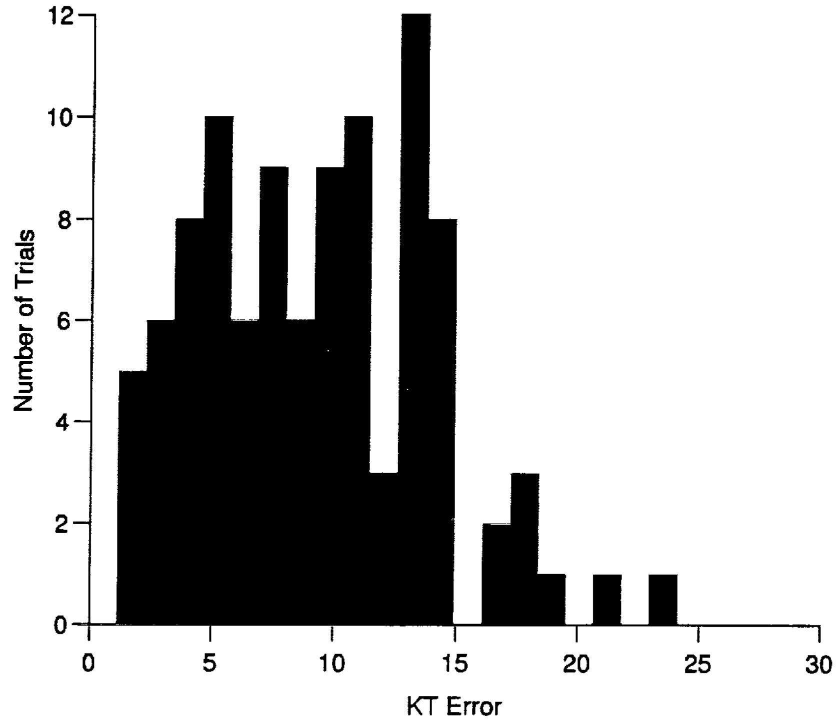
Comparison To Penalty Approach

- Penalty function form:

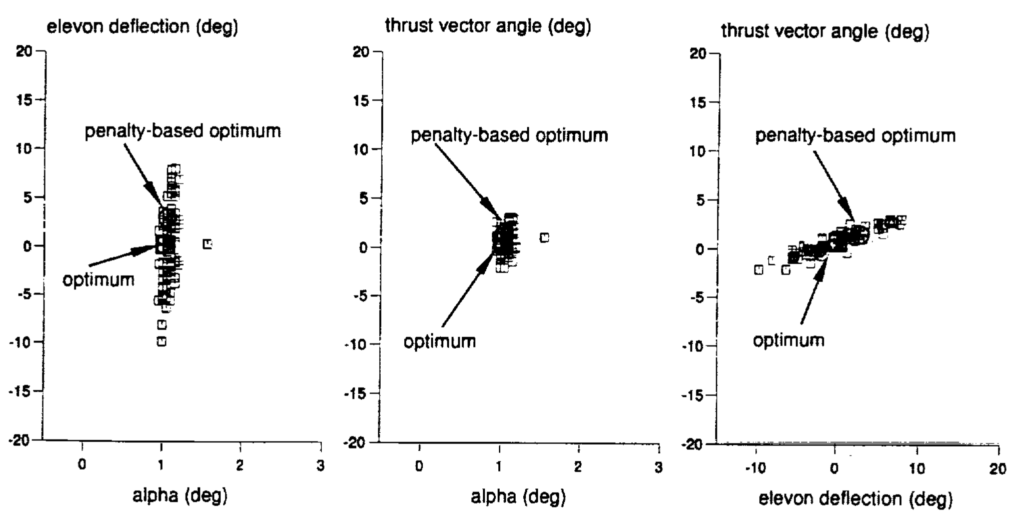
$$x^*_{pen} = \arg\min_{x \in \mathcal{X}} \left\{ c(x) + \sum_{k \in \mathcal{E} \cup \mathcal{I}} p(x, f_k(x)) \right\}$$

- Monte Carlo Experiments
  - 100 GA runs
  - 600 generations/run
  - various penalty-weighting combinations
- Initial Guesses for NR method

Best Penalty Function Histogram



Best Penalty Function Scatter Plots



Penalty Function Results

- 74 of the NR runs converged for the best case
- fine tuning of penalty-weighting combinations is problem specific

Conclusions (1)

- Discussed search characteristics and algorithmic operations of a simple genetic algorithm.
- Discussed method of adapting the KT conditions for a constrained minimization problem to formulate an unconstrained minimization function to be used by a genetic algorithm.
- Demonstrated KT method formulation numerically on an aerospace plane model of the Langley Accelerator

Conclusions (2)

- For this study, KT approach provides reliable initial guesses for Newton-Raphson method
- Unlike the penalty approach, the KT approach
  - minimizes a function whose optimum value is known a priori
  - provides a measure of the constrained stationarity of the solution

Future Work

- Exploit stopping criterion of KT approach
- Extend GA algorithm to include non-smooth cost function and non-smooth constraints