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Analytical Skin Friction and Heat Transfer Formula for Compressible Internal Flows

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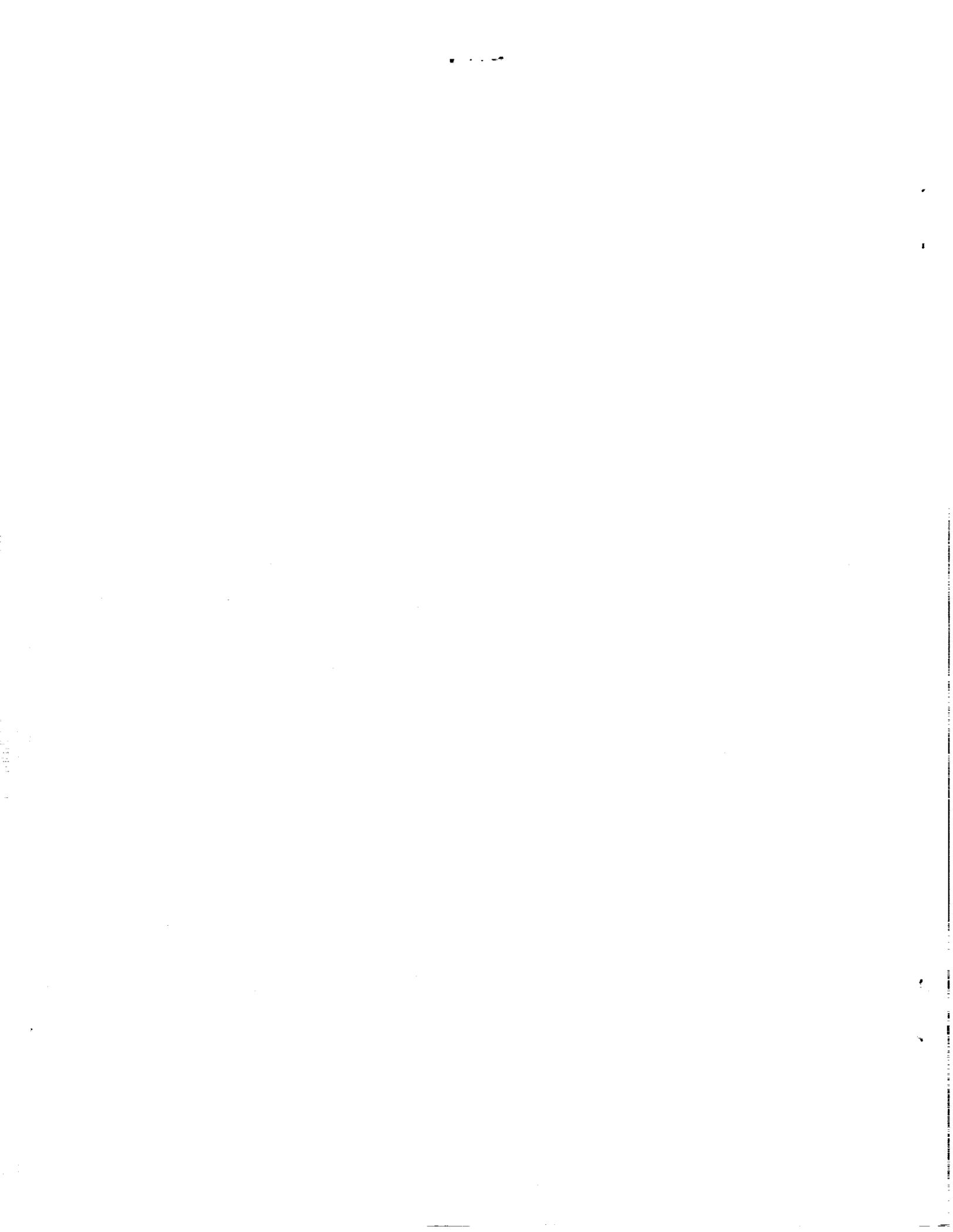
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ANALYTICAL SKIN FRICTION AND HEAT TRANSFER FORMULA
FOR COMPRESSIBLE INTERNAL FLOWS

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SUMMARY

An analytic, closed-form friction formula for turbulent, internal, compressible, fully developed flow has been derived by extending the incompressible law-of-the-wall relation to compressible cases. The model is capable of analyzing heat transfer as a function of constant surface temperatures and surface roughness as well as analyzing adiabatic conditions. The formula reduces to Prandtl's law of friction for adiabatic, smooth, axisymmetric flow. In addition, the formula reduces to the Colebrook equation for incompressible, adiabatic, axisymmetric flow with various roughnesses. Comparisons with available experiments show that the model averages roughly 12.5-percent error for adiabatic flow and 18.5-percent error for flow involving heat transfer.

SYMBOLS

A', B'	"locally" defined variables (no formal association, used for convenience)
B	law-of-wall variable, 5.5
C	locally defined constant
C_f	coefficient of friction, $2\tau_w/\rho u^2$
c_p	specific heat (constant pressure)
c_v	specific heat (constant velocity)
c_1, c_2	integration constants
D	pipe diameter
E	locally defined constant
H	total enthalpy, $h + (1/2)u^2$
h	enthalpy
k^+	roughness height; inner law variable
M	Mach number

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Pr	Prandtl number
p	pressure
R	universal gas constant; radius
Re	Reynolds number
r	recovery factor (Crocco-Busemann relation law), $Pr^{1/3}$
T	temperature
\bar{u}	averaged velocity
u	flow velocity
u^+	velocity; inner law variable
v^*	friction velocity, $(\tau_w/\rho_w)^{1/2}$
y	physical wall variable
y^+	wall variable; inner law variable
γ	ratio of specific heats, c_p/c_v
κ	Von Karman constant, 0.4
λ	Darcy friction factor
κ/D	relative roughness
μ	absolute or dynamic viscosity
ν	kinematic viscosity
ρ	density
τ	shear stress
Subscripts:	
av	average value
w	wall value

INTRODUCTION

A general, closed-form analytical solution describing a particular flow might be considered the "ultimate" mathematical fluid dynamics model because of its inherent flexibility and simplicity of use. Unfortunately, retaining adequate physical information in such a model is usually in conflict with our ability to reduce the solution to closed form. Typically, we are forced to discretize the problem and transform it to a large system of equations that are amenable to numerical solution techniques. The rapid growth in the availability of high-performance computing platforms has permitted the numerical solution of these large systems of governing differential equations by computational fluid dynamics methodologies. However, semiempirical or integral models are still capable of providing closed-form engineering estimates of flow physics in a highly cost-effective manner. This efficiency is particularly desirable for preliminary design analyses, where a large design space may need to be characterized. The goal of the present study was to develop a closed-form friction formula for turbulent, internal, compressible axisymmetric or two-dimensional duct flow. These flows are of interest both

in their own right and as a simple limiting approximation for more complex internal inlet and nozzle flows (ref. 1).

Applications include modeling of acoustic liners for high-speed civil transport (HSCT) nozzles and national aerospace plane (NASP) propulsion system flow-path modeling. Further, this type of analysis may be used to provide first-principle estimates of engine component efficiencies (losses) for cycle analysis tools, such as the NNEP (the NASA/NAVY Engine Program, ref. 2). In addition, because heat transfer analysis is available, this model may provide significant information for use in structural analysis problems.

As indicated, the friction model assumes fully developed flow and can model surface roughness as well as constant-wall-temperature heat transfer. The model is relatively simple because it essentially extends the incompressible law-of-the-wall relation to include compressibility effects. The Crocco-Busemann relation, an approximate solution to the energy equation, is used to model the heat transfer. Consequently, the Prandtl number is assumed to be approximately 1 for this analysis. By requiring the governing equations to reduce to the incompressible limit, necessary constants of integration can be evaluated, thus permitting a closed-form friction coefficient relationship to be determined. In order to gain insight into the model's reliability, the results of the analysis are compared with available experimental data.

ANALYSIS

The basic methodology followed in our development involves two steps. First, a compressible extension to the Prandtl mixing-length hypothesis is integrated to yield an effective velocity relation. Temperature/density dependence is modeled by an approximate energy integral, the Crocco-Busemann relation, and the ideal-gas relation. Second, this effective velocity is averaged and applied to the fully developed flow relation, yielding an implicit equation for the fully developed skin friction coefficient.

To begin our analysis, we consider the shear stress closure by using Prandtl's mixing-length hypothesis

$$\tau \approx \rho_w \left(\frac{\rho}{\rho_w} \right) \kappa^2 y^2 \left(\frac{du}{dy} \right)^2 \quad (1)$$

The energy equation is approximately integrated by using the Crocco-Busemann relation (ref. 3)

$$H = c_1 u + c_2 \quad (2)$$

assuming that $Pr \approx 1$ and the pressure gradient is small. Therefore, with the definition of total enthalpy

$$H = h + \frac{u^2}{2} \quad (3)$$

$$h + \frac{u^2}{2} = c_1 u + c_2$$

and assuming that

$$h \approx c_p T \quad (4)$$

equation (3) then becomes

$$c_p T + \frac{u^2}{2} = c_1 u + c_2 \quad (5)$$

The "no slip" boundary condition dictates that $u(0) = 0$. Evaluating equation (5) at the wall and assuming that the velocity u approaches an average as the temperature T goes to its average, the constants are computed as

$$c_2 = c_p T_w$$

$$c_1 = \frac{c_p(T_{av} - T_w) + u_{av}^2/2}{u_{av}} \quad (6)$$

Substituting into equation (5) gives

$$c_p T + \frac{u^2}{2} = \left[c_p(T_{av} - T_w) + \frac{u_{av}^2}{2} \right] \frac{u}{u_{av}} + c_p T_w \quad (7)$$

And solving for T yields

$$T = \left(T_{av} - T_w + \frac{u_{av}^2}{2c_p} \right) \frac{u}{u_{av}} + T_w - \frac{u^2}{2} \frac{r}{c_p} \quad (8)$$

Note that to partially ameliorate the Prandtl number ($Pr = 1$) assumption, a recovery factor r was added to give somewhat better accuracy. Its value depends on the local Prandtl number and for air in turbulent flow is about $r = 0.88$ (ref. 2).

With the calorically perfect gas assumption

$$c_p = \frac{\gamma R}{\gamma - 1} \quad (9)$$

and

$$M_{av}^2 = \frac{u_{av}^2}{RT\gamma} \quad (10)$$

it can be shown that

$$\frac{T}{T_{av}} = \left[1 - \frac{T_w}{T_{av}} + \frac{1}{2}(\gamma - 1)M_{av}^2 \right] \frac{u}{u_{av}} + \frac{T_w}{T_{av}} - \frac{1}{2}(\gamma - 1)M_{av}^2 \left(\frac{u}{u_{av}} \right)^2 r \quad (11)$$

Multiplying equation (11) by T_{av}/T_w yields

$$\frac{T}{T_w} = \left[\left(\frac{T_{av}}{T_w} - 1 \right) + \frac{1}{2}(\gamma - 1) \frac{T_{av}}{T_w} M_{av}^2 \right] \frac{u}{u_{av}} - \frac{1}{2}(\gamma - 1) M_{av}^2 \frac{T_{av}}{T_w} \left(\frac{u}{u_{av}} \right)^2 r + 1 \quad (12)$$

Applying the equation of state (ideal gas) and relevant boundary layer assumptions (dp/dy small)

$$\frac{dp}{dy} = 0, \quad p_{av} = p_w, \quad p = \rho RT, \quad \frac{\rho_w}{\rho} = \frac{T}{T_w} \quad (13)$$

combined with

$$\frac{T}{T_w} = 1 + A' \left(\frac{u}{u_{av}} \right) - B'^2 \left(\frac{u}{u_{av}} \right)^2$$

where

$$A' \doteq \left(\frac{T_{av}}{T_w} - 1 \right) + \frac{1}{2}(\gamma - 1) M_{av}^2 \frac{T_{av}}{T_w}$$

$$B'^2 \doteq \frac{1}{2}(\gamma - 1) \frac{T_{av}}{T_w} M_{av}^2 r \quad (14)$$

yields

$$\frac{\rho}{\rho_w} = \frac{1}{1 + A' \frac{u}{u_{av}} - B'^2 \left(\frac{u}{u_{av}} \right)^2} \quad (15)$$

This expression is substituted into the original mixing-length formula

$$\frac{\rho}{\rho_w} \left(\frac{du}{dy} \right)^2 = \frac{\tau_w}{\rho_w \kappa^2 y^2} \quad (16)$$

which yields

$$\frac{1}{1 + A' \frac{u}{u_{av}} - B'^2 \left(\frac{u}{u_{av}} \right)^2} \left[\frac{d \left(\frac{u}{u_{av}} \right)}{dy} \right]^2 = \frac{\tau_w}{\rho_w \kappa^2 y^2} \frac{1}{u_{av}^2} \quad (17)$$

Separating and integrating

$$\int \frac{1}{\left[1 + A' \frac{u}{u_{av}} - B'^2 \left(\frac{u}{u_{av}}\right)^2\right]^{1/2}} \frac{d\left(\frac{u}{u_{av}}\right)}{dy} = \left(\frac{\tau_w}{\rho_w}\right)^{1/2} \ln \frac{y}{u_{av}K} + \text{const} \quad (18)$$

yields

$$\frac{v^*}{k} \ln y + \text{const} = \frac{u_{av}}{B'} \arcsin \left[\frac{-2B'^2 \frac{u}{u_{av}} + A'}{(4B'^2 + A'^2)^{1/2}} \right] \quad (19)$$

where friction velocity is defined as

$$v^* = \left(\frac{\tau_w}{\rho_w}\right)^{1/2} \quad (20)$$

To evaluate the integration constant, the constraint applied is that the preceding equation *must* reduce to the incompressible case

$$u^+ = \frac{1}{K} \ln y^+ + B \quad (21)$$

for low Mach number flow ($M_{av} = 0$). The integration constant may be split into two pieces for clarity:

$$\text{const} = \text{const}_1 + \text{const}_2 \quad (22)$$

Rewriting the left-hand-side length scale (which is justifiable because it involves merely adding a constant) gives

$$y \rightarrow y^+ = \frac{yv^*}{v_w} \quad (23)$$

and from the incompressible law of the wall, const_1 is defined as

$$\text{const}_1 \equiv -\left(\frac{1}{K} \ln \frac{v^*}{v_w}\right) = B \quad (24)$$

The second constant const_2 is defined by considering the limit B' defined previously as approaching zero. The term

$$\lim_{B' \rightarrow 0} \frac{u_{av}}{B'} \arcsin \frac{-2B'^2 \frac{u}{u_{av}}}{(4B'^2 + A'^2)^{1/2}} \quad (25)$$

is bounded, whereas

$$\lim_{B' \rightarrow 0} \frac{1}{B'} \arcsin \frac{A'}{(4B'^2 + A'^2)^{1/2}} \quad (26)$$

is *not*. Since physically the limit should be finite, it is justifiable (because, again, it is merely a constant) to define the second integration constant as

$$\text{const}_2 \equiv \frac{1}{B'} \arcsin \frac{A'}{(4B'^2 + A'^2)^{1/2}} \quad (27)$$

This rather subtle but physically reasonable line of argument defines the integration constants.

Further, a classical modification of the incompressible law of the wall may be used to develop an expression that takes the duct roughness into account. To do so, White (ref. 3) suggests the following modification to the incompressible law of the wall:

$$u^+ = \frac{1}{\kappa} \ln \frac{yv^+}{v_w} + B$$

$$B = 5.5 - \frac{1}{\kappa} \ln (1 + 0.3k^+) \quad (28)$$

where

$$k^+ = \frac{kv^+}{v_w}$$

Combining the previous relations yields the Van Driest effective velocity (ref. 2)

$$\frac{v^+}{u_{av}} \frac{1}{k} \ln y^+ + 5.5 - \frac{1}{k} \ln (1 + 0.3k^+) = \frac{1}{B'} \left[\arcsin \frac{A' - 2B'^2 \frac{u}{u_{av}}}{(4B'^2 + A'^2)^{1/2}} - \arcsin \frac{A'}{(4B'^2 + A'^2)^{1/2}} \right] \quad (29)$$

Because all flow properties must be related to either wall values or flow average quantities, the average of the preceding relation must be obtained. By averaging equation (29) over the area, it can be demonstrated that for a pipe the left-hand side becomes

$$\bar{u}^+ = \frac{1}{\kappa} \ln \frac{Rv^*}{v_w} + 1.75 - 2.5 \ln (1 + 0.3k^+) \quad (30)$$

The last term in this equation is the roughness term. Note that for smooth walls ($k^+ = 0$) this roughness term vanishes.

Unfortunately, no such "elementary" integration will be available for the right-hand side (RHS). However, a simple approximation is available that calls for defining this average by replacing u by u_{av} , yielding

$$\text{RHS}_{av} \equiv \frac{1}{B'} \left[\arcsin \frac{A' - 2B'^2}{(4B'^2 + A'^2)^{1/2}} + -\arcsin \frac{A'}{(4B'^2 + A'^2)^{1/2}} \right] \quad (31)$$

Separately, skin friction is defined as

$$\tau_w = \frac{1}{2} C_f \rho_{av} u_{av}^2 \quad (32)$$

where C_f is the desired coefficient of friction. Thus, the friction velocity can be expressed from equations (32) and (19) as

$$\left(\frac{\tau_w}{\rho_w} \right)^{1/2} = v^* = \frac{(2)^{1/2}}{2} C_f^{1/2} \left(\frac{\rho_{av}}{\rho_w} \right)^{1/2} \bar{u} \quad (33)$$

and from equation (13)

$$\frac{\rho_{av}}{\rho_w} = \frac{T_w}{T_{av}} \quad (34)$$

Then we obtain

$$\frac{v^*}{u_{av}} = \frac{(2)^{1/2}}{2} C_f^{1/2} \left(\frac{T_w}{T_{av}} \right)^{1/2} \quad (35)$$

permitting us to write the Reynolds number relation

$$\frac{Rv^*}{v_{av}} = \frac{(2)^{1/2}}{4} C_f^{1/2} \text{Re} \left(\frac{T_w}{T_{av}} \right)^{1/2} \quad (36)$$

where R here is the characteristic duct radius or hydraulic radius. Evaluating viscosity by using Sutherland's relation

$$\frac{\mu_{av}}{\mu_w} = \frac{T_{av}}{T_w} \frac{1.505}{1 + 0.505 \frac{T_w}{T_{av}}} \quad (37)$$

where

$$\frac{v_{av}}{v_w} = \frac{\mu_{av} \rho_w}{\mu_w \rho_{av}}$$

and combining with equation (36) give

$$\frac{Rv^*}{v_w} = \frac{(2)^{1/2}}{4} C_f^{1/2} \text{Re} \frac{T_{av}}{T_w} \left(\frac{1.505}{1 + 0.505 \frac{T_w}{T_{av}}} \right) \quad (38)$$

Recalling that

$$k^+ = \text{Re} \frac{k}{D} \frac{v^*}{u_{av}} \frac{v_{av}}{v_w}$$

where

$$\frac{v^*}{u_{av}} = \frac{(2)^{1/2}}{2} C_f^{1/2} \left(\frac{T_w}{T_{av}} \right)^{1/2}$$

with

$$\frac{v_{av}}{v_w} = \left(\frac{T_{av}}{T_w} \right)^{3/2} \left[\frac{1.505}{1 + 0.505 \frac{T_w}{T_{av}}} \right] \quad (39)$$

then it follows that

$$k^+ = \text{Re} \frac{k}{D} \frac{(2)^{1/2}}{2} C_f^{1/2} \frac{T_{av}}{T_w} \left(\frac{1.505}{1 + 0.505 \frac{T_w}{T_{av}}} \right) \quad (40)$$

Combining the previous relations yields

$$-\frac{u_{av}}{B'v^*} \left(\sin^{-1} \frac{2B'^2 - A'}{(4B'^2 + A'^2)^{1/2}} - \sin^{-1} \frac{A'}{(4B'^2 + A'^2)^{1/2}} \right) = \left[\frac{1}{K} \ln R^+ + 1.75 - 2.5 \ln (1 + 0.3k^+) \right] \quad (41)$$

By substituting equations (38) and (40) into equation (41) and defining the following constants (for both axisymmetric and two-dimensional flows)

$$\begin{aligned} A' &\doteq \left(\frac{T_{av}}{T_w} - 1 \right) + \frac{1}{2}(\gamma - 1)M_{av}^2 \frac{T_{av}}{T_w} \\ B' &\doteq \left[\frac{1}{2}(\gamma - 1)M_{av}^2 \frac{T_{av}}{T_w} r \right]^{1/2} \\ C &\doteq \sin^{-1} \left[\frac{2B'^2 - A'}{(4B'^2 + A'^2)^{1/2}} \right] + \sin^{-1} \left[\frac{A'}{(4B'^2 + A'^2)^{1/2}} \right] \end{aligned} \quad (42)$$

$$E \doteq \frac{T_{av}}{T_w} \left[\frac{1.505}{1 + 0.505 \frac{T_w}{T_{av}}} \right]$$

the final form of the equation is obtained:

$$\frac{C}{C_f^{1/2} \left[\frac{1}{2}(\gamma - 1)M_{av}^2 r \right]^{1/2}} = \text{const} + 1.77 \ln (C_f^{1/2} \text{Re}) + 1.77 \ln E - 1.77 \ln \left(1 + 0.2121 \text{Re} \frac{k}{D} C_f^{1/2} E \right) \quad (43)$$

where const = -0.6005 for axisymmetric flow or 1.5086 for two-dimensional duct flow.

RESULTS AND DISCUSSION

Equation (38) represents the final closed-form solution for the skin friction coefficient. It is immediately obvious that the equation simplifies the solution for isothermal systems where $T_w = T_{av}$. For smooth pipes or ducts the roughness term disappears. An initial comparison with Prandtl's law of friction for smooth pipes

$$\frac{1}{(\lambda)^{1/2}} = 2.0 \log \left[\text{Re}(\lambda)^{1/2} \right] - 0.8$$

shows that the model is consistent to within 2 percent over a range from $\text{Re} = 10^3$ to $\text{Re} = 10^7$ (see table I, from ref. 41). This consistency is really no surprise in that the model was developed to recover the incompressible

limit as described by Prandtl's law. However, Prandtl's law does not consider roughness, heat transfer, or compressibility effects.

In adiabatic systems where

$$\frac{T_w}{T_{av}} = 1 + \frac{1}{2}(\gamma - 1)M^2$$

it is more difficult to see how the formula reduces. A much simpler approach would be to numerically compare the closed solution with experimental data points as a function of T_w/T_{av} .

Experimental data points have thus also been compared directly with the model. The more diverse these experimental flow conditions are, the more fully the model can be tested. Therefore, the experiments of interest range from two-dimensional duct flow to axisymmetric flow, both subsonic and supersonic flows as well as adiabatic and nonadiabatic flows. Discretion must be exercised when comparing actual data points with this model. First, extracting data indirectly off graphs and charts may often lead to inaccurate data readings. Second, experimental data points obtained with the use of Preston tubes are generally regarded as having an uncertainty of 10 to 20 percent (ref. 5). Laser interferometer skin friction (LISF) techniques are generally regarded as having better accuracy (see ref. 5).

Table II shows skin friction estimates predicted by our model and measurements made in a square duct (ref. 6). This comparison is made for adiabatic, smooth, two-dimensional duct flow. In addition, the data point represents an average experimental friction coefficient across the width of the duct. The comparison is encouraging, although if roughness were included, the results would probably be somewhat better.

To show this sensitivity to roughness, table III tabulates skin friction coefficients as a function of roughness for an adiabatic flow. Nevertheless, with roughness effects neglected, the model generally predicts skin friction within errors of less than 20 percent.

In figure 1 the data points are much more scattered. This scattering may be related to the facts that only the inlet conditions were given for this experiment. Local conditions were estimated by using an adiabatic, viscous loss analysis (ref. 1). The results were then fed into the friction model. Therefore, the data points are only estimates of the relevant properties.

Figure 2 compares the model with friction measurements made in a U-tube (ref. 7) in a fully developed region. Less error is associated with this experiment, possibly because data were obtained by using laser interferometry rather than traditional Preston tubes. Because of this, the model agrees within an assumed 10-percent margin of uncertainty. This comparison shows that the model reduces accordingly for low Mach numbers to predict the skin friction for incompressible flows (as expected). Further experiments run on a square duct reveal similar results (ref. 8).

Supersonic, adiabatic pipe flow data (ref. 9) were also compared with the present model. Table IV shows data for four Mach numbers and the corresponding predicted values. Agreement is rather good for all cases.

To fully test the model, not only friction data are needed. Wall and flow temperatures must be known to compare the model's heat transfer capability. Fortunately, several experiments have been done in this area. The first is shown in figure 3 (from ref. 10). Note that the T_{av}/T_w ratio of 1 indicates not an adiabatic system but an isothermal system for that particular location. Table V illustrates the model's capability of predicting flows with roughness.

The recovery factor ($r = 0.88$) has also been used in the Crocco-Busemann relation. The value of the roughness term was chosen rather arbitrarily because no roughness figures were explicitly given. It was estimated to be $k/D = 0.000095$. However, White (ref. 3) suggests that the value chosen represents the point where roughness begins to play a significant role in heat transfer and may be considered a minimum value. However, other values of roughness may be more appropriate. Finally, the average Mach number was sonic ($M = 1$).

With the ability to model surface roughness, the model may be further compared. By solving the friction formula for adiabatic, incompressible flows for various roughnesses, the model should reduce to the Colebrook equation (Moody's chart). Figure 4 compares four roughness plots of the model with the Colebrook equation. As expected, the model recovered the Colebrook equation for small Mach numbers. In spite of this relatively good comparison, it is worth noting that the Moody chart is accurate to ± 15 percent for design calculations (ref. 11). Figure 5 is included to show how the model extends the Moody chart to compressible flow for various Mach numbers.

It is worth noting that figure 5 clearly presents the general trend that for fixed Reynolds numbers *increasing* Mach number tends to *decrease* skin friction coefficient values. Compressibility effects on the internal skin friction coefficient are directly presented in figure 6. Although it is not the intent of this report to discuss the detailed physics of this trend in any great detail, it is desirable to summarize the basic phenomenon. Schlichting (ref. 4) notes that for adiabatic, compressible flow the density must decrease strongly near the wall (by constant pressure and state), thus causing considerable boundary layer thickening. This reduction in the local gradient apparently reduces the skin friction coefficient. Unfortunately, our understanding of turbulent boundary layers is still rather phenomenological and can offer only limited insight into these problems.

Solving for C_f in equation (39) may easily be performed by using various iteration (fixed point) techniques. A short routine using Newton's method was used in this study.

Overall comparison with experimental data yields no observable general trends of the model that may be identified as sources of error. It is worth noting though that because the model is based on Prandtl's turbulence model, the present analysis cannot model the thin laminar sublayer (in fact, it is completely neglected). There is evidence that at low Reynolds numbers this effect becomes significant (ref. 3). White has demonstrated that for incompressible flow at Reynolds numbers of approximately 4000 the effect of the laminar sublayer becomes significant.

SUMMARY OF RESULTS

An analytic skin friction model for compressible, turbulent, internal, fully developed flow involving heat transfer has been developed by extending the incompressible law-of-the-wall relation to compressible cases. The formula recovers Prandtl's incompressible law of friction for pipes (within 2 percent) for incompressible flow. In addition, the model shows good correlation with the Moody chart for similarly low Mach numbers. The model also shows good correlation with available data relating heat transfer and skin friction. Assuming the conditions under which the Reynolds analogy is valid, the skin friction can be directly related to the heat transfer.

RECOMMENDATIONS

In closing, it is worth noting that for these high-speed experiments the local properties (fluid density, Reynolds number, air speed) are difficult to measure. Often, they can only be estimated. As a result, agreement with available experiments is encouraging, but the experiments themselves may introduce some uncertainty.

Further, in contrast to the experimental work done in the area of flat plates, far fewer internal flow experiments have been performed to fully test this model. It is hoped that additional experimentation as well as agreement with numerically implemented turbulence models will prove this model to be accurate for rapid engineering estimation purposes despite its simplicity.

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TABLE I.—MODEL PREDICTION VERSUS PRANDTL'S LAW
(INCOMPRESSIBLE)

Reynolds number, Re	Skin friction coefficient, C_f		Relative error, $\frac{\text{Model} - \text{Prandtl's law}}{\text{Prandtl's law}}$, percent
	Model, $\lambda = 4C_f$	Prandtl's law, $\lambda = 4C_f$	
5.0×10^4	0.0208	0.0209	0.48
1.0×10^6	.0115	.0116	.86
2.0×10^6	.0102	.0104	1.92
1.0×10^7	.0080	.0081	1.23

TABLE II.—MODEL PREDICTION VERSUS EXPERIMENTALLY MEASURED (ref. 6) SKIN FRICTION COEFFICIENT FOR SMOOTH, ADIABATIC, TWO-DIMENSIONAL DUCT FLOW

Reynolds number, Re	Skin friction coefficient, C_f		Relative error, percent
	Model	Experiment	
9.00×10^5	1.50×10^{-3}	1.80×10^{-3}	16
1.00×10^6	1.30×10^{-3}	1.30×10^{-3}	0
2.00×10^7	8.00×10^{-4}	1.00×10^{-3}	20

TABLE III.—MODEL PREDICTION FOR VARIOUS ROUGHNESSES IN TWO-DIMENSIONAL DUCT

[Reynolds number, 1.0×10^6 ;
Mach number, 3.0.]

Skin friction coefficient, C_f	Relative roughness, κ/D
0.00190	0.001
.00160	.0001
.00158	.00005
.00156	.00001

TABLE IV.—MODEL PREDICTION VERSUS EXPERIMENTALLY
 MEASURED (ref. 9) SKIN FRICTION COEFFICIENT
 FOR ADIABATIC PIPE FLOW

Reynolds number, Re	Mach number, M	Skin friction coefficient, C_f		Relative error, percent
		Model	Experiment	
8.00×10^5	2.06	2.75×10^{-3}	2.80×10^{-3}	1.8
3.80×10^4	2.84	4.20×10^{-3}	3.95×10^{-3}	6.3
4.50×10^5	3.14	2.00×10^{-3}	2.45×10^{-3}	18.4
2.25×10^5	3.87	2.30×10^{-3}	2.60×10^{-3}	11.5

TABLE V.—MODEL PREDICTION VERSUS EXPERIMENTALLY MEASURED (ref. 10) SKIN FRICTION FOR ISOTHERMAL PIPE FLOW WITH ROUGHNESS

Temperature ratio, T_{av}/T_w	Reynolds number, Re	Skin friction coefficient ratio, $C_{f,pred}/C_{f,meas}$
0.909	7.0×10^5	1.227
1.00	1.0×10^5	.960
.476	9.0×10^4	.875
.400	1.0×10^4	1.030

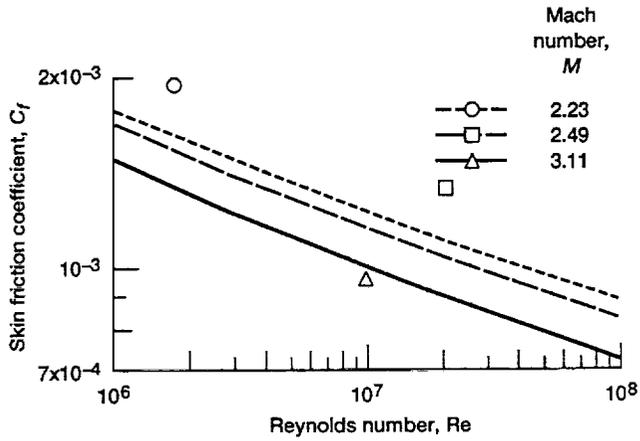


Figure 1.—Skin friction coefficient versus Reynolds number for various Mach numbers.

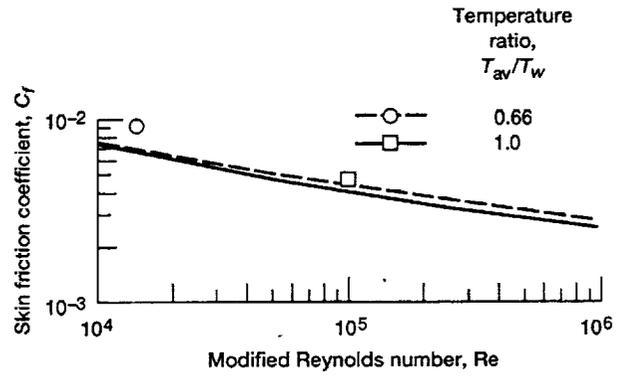


Figure 3.—Skin friction coefficient versus Reynolds number for smooth pipe flow with heat transfer. Mach number, 0.6. (From reference 10.)

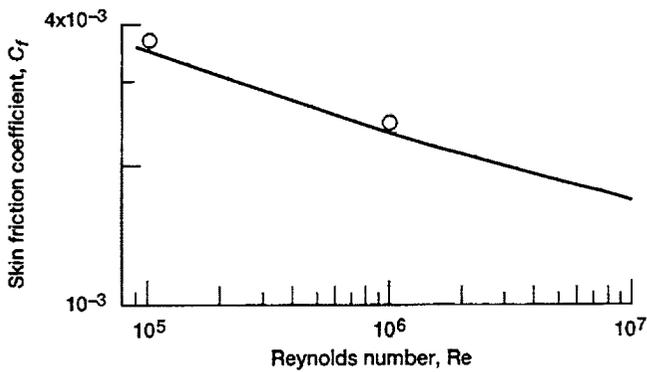


Figure 2.—Skin friction coefficient versus Reynolds number for U-tube. Mach number, 0.1.

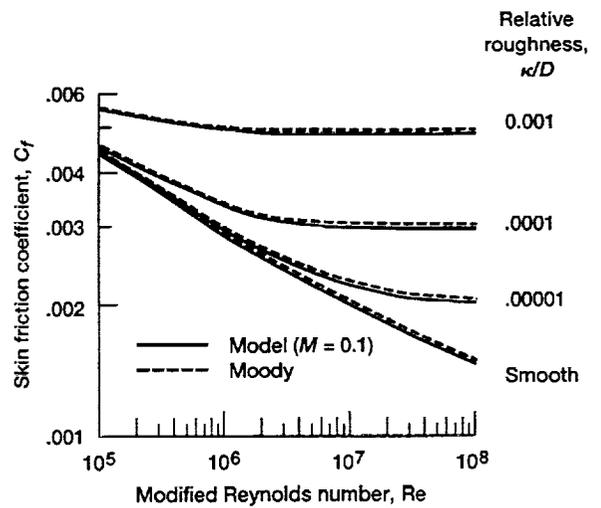


Figure 4.—Comparison with Moody chart.

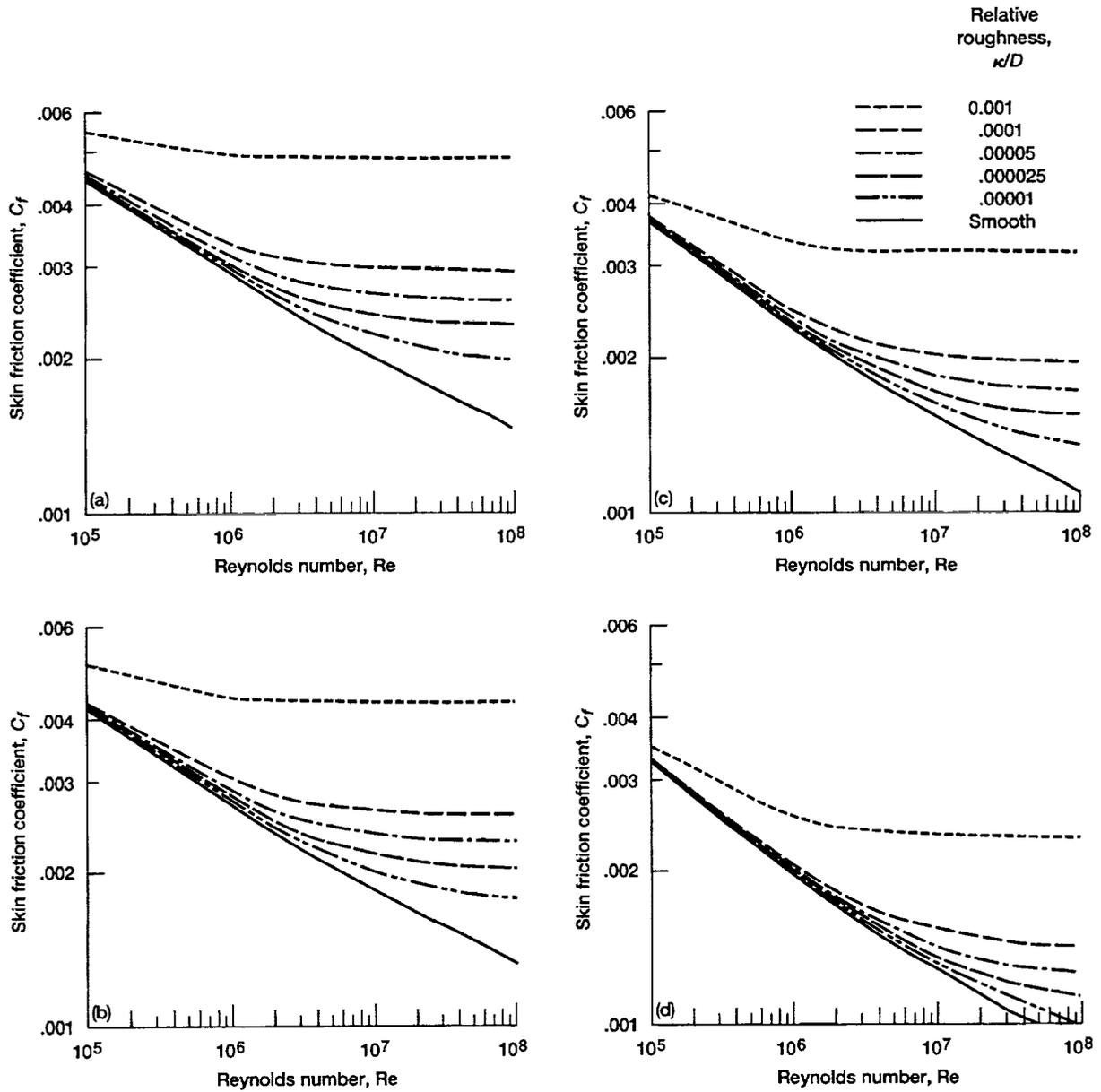


Figure 5.—Skin friction coefficient versus Reynolds number for various Mach numbers and relative roughnesses. (a) Mach 0.1. (b) Mach 1. (c) Mach 2. (d) Mach 3.

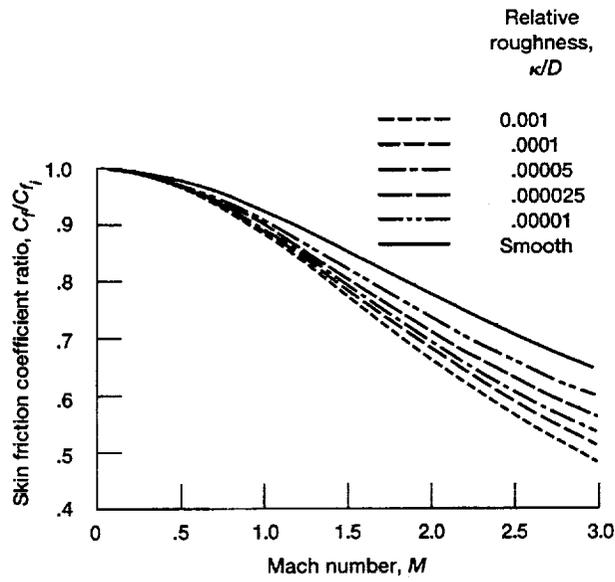


Figure 6.—Compressibility effects on skin friction.



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13. ABSTRACT (Maximum 200 words) An analytic, closed-form friction formula for turbulent, internal, compressible, fully developed flow has been derived by extending the incompressible law-of-the-wall relation to compressible cases. The model is capable of analyzing heat transfer as a function of constant surface temperatures and surface roughness as well as analyzing adiabatic conditions. The formula reduces to Prandtl's law of friction for adiabatic, smooth, axisymmetric flow. In addition, the formula reduces to the Colebrook equation for incompressible, adiabatic, axisymmetric flow with various roughnesses. Comparisons with available experiments show that the model averages roughly 12.5 percent error for adiabatic flow and 18.5 percent error for flow involving heat transfer.			
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