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# MODELING THE SHUTTLE REMOTE MANIPULATOR SYSTEM–ANOTHER FLEXIBLE MODEL

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## ABSTRACT

High fidelity elastic system modeling algorithms are discussed. The particular system studied is the Space Shuttle Remote Manipulator System (RMS) undergoing full articulated motion. The model incorporates flexibility via a methodology the author has been developing. The technique is based in variational principles, so rigorous boundary condition generation and weak formulations for the associated partial differential equations are realized, yet the analyst need not integrate by parts. The methodology is formulated using vector–dyad notation with minimal use of tensor notation, therefore the technique is believed to be affable to practicing engineers. The objectives of this work are to:

1. Determine the efficacy of the modeling method.
2. Determine if the method affords an analyst advantages in the overall modeling and simulation task.

Generated out of necessity were *Mathematica* algorithms that quasi-automate the modeling procedure and simulation development. The project was divided into sections as follows:

1. Model development of a simplified manipulator.
2. Model development for the full-freedom RMS including a flexible movable base on a six degree of freedom orbiter. A rigid-body is attached to the manipulator end-effector.
3. Simulation development for item 2.
4. Comparison to the currently used model of the flexible RMS in the Structures and Mechanics Division of NASA JSC.

At the time of the writing of this report, items 3 and 4 above were not complete.

## INTRODUCTION

Material bodies are inherently of a distributed mass and elasticity nature. Analyst have realized this fact since the early days and developed tools to model these distributed effects [1]. Engineers, challenged with the task of making devices work in a reliable, energy efficient, and inexpensive manner, have been gradually increasing the fidelity of their models by incorporating the distributed properties. The ability to study these high fidelity models grows with the increasing computational capabilities of inexpensive computers.

The literature is teeming with ever-improving ways to model the distributed effects [1]. There are a diverse cross-section of techniques. Some are intuitive to a design engineer [2, 3, 4, 5], while others are mathematically elegant but beyond the training of many practicing engineers [6, 7]. The purpose of this study is to examine the efficacy of the author's attempt at developing a rigorous yet usable method for modeling complicated systems [5].

## METHODOLOGY

### Present Capabilities

Based on discussions,[1] the author understands that the fidelity of the model for the present Shuttle Remote Manipulator System (RMS) simulation is limited to small amplitude vibrations about any "snap shot" configuration of the system. This limitation manifests itself because of the linear finite element (NASTRAN) model used as the progenitor for the modal basis functions. Therefore, RMS slewing maneuver studies are not within the fidelity of the linear model. There exist techniques which allow an analyst to study the slewing maneuvers of systems like the RMS, but these modeling techniques are computationally expensive and/or hard to understand [1], therefore they are not always implemented by practicing engineers. The author believes the technique discussed below gives analysts a familiar yet powerful modeling tool.

### New Capabilities

The main motivating factor for the development of another modeling method was the need to easily derive complete models of complex elastic systems [1, 4, 8, 9, 10, 11, 12]. Although the method discussed herein is still relatively mathematically intense (compared to an equal number of rigid bodies), it has a predisposition for symbolic manipulation. Another impetus for this work is that a simple method may make it possible to bring rigorous flexible system modeling out of the academic domain and into use by product designers. Another catalyst for this effort is that a simple (ultimately an automated) method will make it possible for researchers to rapidly regenerate models based on new continuum assumptions.

The approach is variational in nature. It retains most of the attributes of the analytical approach (i.e. Hamilton's principle), but eliminates most of the pitfalls, such as the need to use Lagrange multipliers for constraints, and excessive algebra. The methodology is vector

[1]Orientation meetings with various engineers from the Structures and Mechanics Division of JSC.

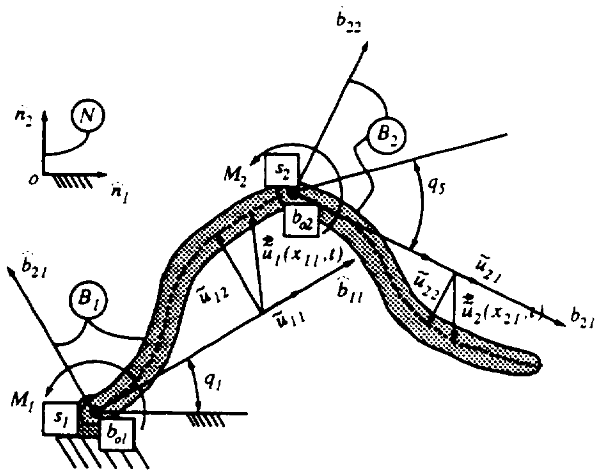

Figure 1: Two Link Flexible Manipulator

based and requires the analyst to perform operations comparable to the operations required for implementing Lagrange's equations. However, it is claimed that the net algebra with the method herein will be less than the net algebra associated to Hamilton's principle or Lagrange's equations. Analysts familiar with Kane's [13] form of d'Alembert's principle will find the technique affable. The complete derivation of the method is shown elsewhere [1, 5].

## SIMPLIFIED MANIPULATOR MODEL

### Preamble

In this section the equations of motion for a flexible two-link planar manipulator will be derived. This section is included for demonstrative purposes. The procedure that is used on the full RMS model is used on a somewhat simpler model so the reader can follow the steps involved. This simpler system was chosen because: a) it is non-trivial due to its distributed elasticity, b) its planar nature allows for heuristic equation verification, and c) Hamilton's principle can be readily applied to it. This example will demonstrate some of the qualities of the new methodology, such as: a) its systematic nature, b) its resulting closed form equations, and c) automatic boundary condition generation.

The system is shown in figure 1. The domain of each beam is one dimensional. The independent coordinates are $x_{11}$ and $x_{21}$ measured from the root of beam $B_1$ and $B_2$, respectively, along the undeformed neutral axis of each beam. The "special" point of beam $B_1$ is labeled $s_1$ (the pivot) and $s_2$ (the joint between the two beams) is for beam $B_2$. The coordinate frames, denoted with $B_1$ and $B_2$, are attached as shown in figure 1. At the root of each beam ($B_1$ and $B_2$) there are massless hubs to which torques $M_1$ and $M_2$ are applied. The angular position of frames $B_1$ and $B_2$ are $q_1$ and $q_5$. Beam deflections are measured with $\tilde{u}_{11}(x_{11}, t)\hat{b}_{11}$ and $\tilde{u}_{21}(x_{21}, t)\hat{b}_{21}$ (elongation), and with $\tilde{u}_{12}(x_{11}, t)\hat{b}_{12}$ and $\tilde{u}_{22}(x_{21}, t)\hat{b}_{22}$ (flexure), as shown in figure 1. The beams have mass per unit length $\rho$, total lengths are

$L_1$ and $L_2$, cross sectional area $A$, area moment of inertia $\mathcal{I}$, and Young's modulus $E$. It is assumed that large deflections and rotational inertia is pertinent, but not shear deformation. Therefore, the beams are modeled with Rayleigh beam theory. The cross-sections of each beam are assumed symmetric about the neutral axis. The intrinsic mass moment of inertia of the cross-section will be taken as $\vec{\vec{I}}_{i_o} = \rho\mathcal{I}\hat{i}_3\hat{i}_3$. $\hat{i}_3$ is normal to the plane of the problem.

## Kinematics

The geometry of the motion for the system is as follows. The position vectors of interest are:

$$
\begin{aligned}
{}^{o}\vec{r}^{\,s_1} &= 0 && (1)\\
{}^{o}\vec{r}^{\,i_{o_1}} &= {}^{s_1}\vec{r}^{\,i_{o_1}} = (x_{11} + \tilde{u}_{11})\,\hat{b}_{11} + \tilde{u}_{12}\hat{b}_{22} && (2)\\
{}^{o}\vec{r}^{\,s_2} &= (L_1 + q_2')\,\hat{b}_{11} + q_3'\hat{b}_{12} && (3)\\
{}^{s_2}\vec{r}^{\,i_{o_2}} &= (x_{21} + \tilde{u}_{21})\,\hat{b}_{21} + \tilde{u}_{22}\hat{b}_{22} && (4)\\
{}^{o}\vec{r}^{\,i_{o_2}} &= (L_1 + q_2')\,\hat{b}_{11} + q_3'\hat{b}_{12} + (x_{21} + \tilde{u}_{21})\,\hat{b}_{21} \\
&\quad + \tilde{u}_{22}\hat{b}_{22} && (5)
\end{aligned}
$$

The angular velocity of frame $B_1$ and $B_2$ and intermediate frames (in the cross-section) $I_1$ and $I_2$ are:

$$
\begin{aligned}
{}^{\mathcal{N}}\vec{\omega}^{\,B_1} &= u_1\hat{b}_{13} && (6)\\
{}^{\mathcal{N}}\vec{\omega}^{\,B_2} &= (u_1 + u_4' + u_5)\,\hat{b}_{13} && (7)\\
{}^{\mathcal{N}}\vec{\omega}^{\,I_1} &= \left(u_1 + \frac{\partial^2\tilde{u}_{12}}{\partial x_{11}\partial t}\right)\hat{b}_{13} && (8)\\
{}^{\mathcal{N}}\vec{\omega}^{\,I_2} &= \left(u_1 + u_4' + u_5 + \frac{\partial^2\tilde{u}_{22}}{\partial x_{21}\partial t}\right)\hat{b}_{23} && (9)
\end{aligned}
$$

The generalized and pseudo-generalized (denoted with a $\prime$) coordinates and velocities are defined as:

$$u_1 = \dot{q}_1, \; u_5 = \dot{q}_5 \qquad (10)$$

$$q_2' = \tilde{u}_{11}(L_1, t), \; q_3' = \tilde{u}_{12}(L_1, t), \; q_4' = \frac{\partial\tilde{u}_{12}(L_1, t)}{\partial x_{11}} \qquad (11)$$

$$
\begin{aligned}
&u_2' = \dot{q}_2' = \frac{\partial\tilde{u}_{11}(L_1, t)}{\partial t}, \; u_3' = \dot{q}_3' = \frac{\partial\tilde{u}_{12}(L_1, t)}{\partial t}, \\
&u_4' = \dot{q}_4' = \frac{\partial^2\tilde{u}_{12}(L_1, t)}{\partial x_{11}\partial t} && (12)
\end{aligned}
$$

The absolute velocity of the point $s_1$ and $s_2$ are also required. Since the system is rotating about $s_1$:

$$^{o}\vec{v}_{\mathcal{N}}^{s_1} = 0 \tag{13}$$

and

$$^{o}\vec{v}_{\mathcal{N}}^{s_2} = \left(u_2' - u_1 q_3'\right)\hat{b}_{11} + \left(u_3' + u_1\left(L_1 + q_2'\right)\right)\hat{b}_{12} \tag{14}$$

where the pseudo-coordinates and pseudo-speeds have been defined above.

The absolute acceleration of the differential beam elements for beam $B_1$ and $B_2$ can be written as:

$$\begin{aligned}
^{o}\vec{a}_{\mathcal{N}}^{i_{o1}} = &\left(\frac{\partial^2\tilde{u}_{11}}{\partial t^2} - \dot{u}_1\tilde{u}_{12} - 2u_1\frac{\partial\tilde{u}_{12}}{\partial t} - \left(x_{11} + \tilde{u}_{11}\right)u_1^2\right)\hat{b}_{11} \\
+ &\left(\frac{\partial^2\tilde{u}_{12}}{\partial t^2} + \left(x_{11} + \tilde{u}_{11}\right)\dot{u}_1 + 2u_1\frac{\partial\tilde{u}_{11}}{\partial t} - \tilde{u}_{12}u_1^2\right)\hat{b}_{12}
\end{aligned} \tag{15}$$

and

$$\begin{aligned}
^{o}\vec{a}_{\mathcal{N}}^{i_{o2}} = &\left(\dot{u}_2' - \dot{u}_1 q_3' - 2u_1 u_3' - \left(L_1 + q_2'\right)u_1^2\right)\hat{b}_{11} \\
&+ \left(\dot{u}_3' + \left(L_1 + q_2'\right)\dot{u}_1 + 2u_1 u_2' - q_3' u_1^2\right)\hat{b}_{12} \\
&+ \left(\frac{\partial^2\tilde{u}_{21}}{\partial t^2} - \left(\dot{u}_1 + \dot{u}_4' + \dot{u}_5\right)\tilde{u}_{22} - \right. \\
&\left. 2\left(u_1 + u_4' + u_5\right)\frac{\partial\tilde{u}_{22}}{\partial t} - \left(x_{21} + \tilde{u}_{21}\right)\left(u_1 + u_4' + u_5\right)^2\right)\hat{b}_{21} \\
&+ \left(\frac{\partial^2\tilde{u}_{22}}{\partial t^2} + \left(x_{21} + \tilde{u}_{21}\right)\left(\dot{u}_1 + \dot{u}_4' + \dot{u}_5\right)\right. \\
&\left. + 2\left(u_1 + u_4' + u_5\right)\frac{\partial\tilde{u}_{21}}{\partial t} - \tilde{u}_{22}\left(u_1 + u_4' + u_5\right)^2\right)\hat{b}_{22}
\end{aligned} \tag{16}$$

respectively. For this problem $i_{oe}$ for each beam is at the centroid of the cross-section.

The methodology also requires the calculations of the "preferred directions" for the variations (pseudo and ordinary), namely the partial speeds. They are determined by inspection of the velocity equations and given in table 1. The partial velocities for the field equations are given as:

$$\begin{aligned}
\frac{\partial^{B_1}\vec{\omega}^{I_1}}{\partial\tilde{u}_{12,1t}} &= \hat{b}_{13}, & \frac{\partial^{B_2}\vec{\omega}^{I_2}}{\partial\tilde{u}_{22,1t}} &= \hat{b}_{23} \\
\frac{\partial^{s_1}\vec{v}_{B_1}^{i_1}}{\partial\tilde{u}_{11,t}} &= \hat{b}_{11}, & \frac{\partial^{s_1}\vec{v}_{B_1}^{i_1}}{\partial\tilde{u}_{12,t}} &= \hat{b}_{12} \\
\frac{\partial^{s_2}\vec{v}_{B_2}^{i_2}}{\partial\tilde{u}_{21,t}} &= \hat{b}_{21}, & \frac{\partial^{s_2}\vec{v}_{B_2}^{i_2}}{\partial\tilde{u}_{22,t}} &= \hat{b}_{22}
\end{aligned} \tag{17}$$

| | ${}^{o}\vec{v}_{\mathcal{N}}^{s_1}$ | ${}^{o}\vec{v}_{\mathcal{N}}^{s_2}$ | ${}^{\mathcal{N}}\vec{\omega}^{B_1}$ | ${}^{\mathcal{N}}\vec{\omega}^{B_2}$ |
|---|---|---|---|---|
| $\frac{\partial}{\partial u_1}$ | 0 | $-q_3'\hat{b}_{11} + (L_1 + q_2')\,\hat{b}_{12} + q_6\hat{b}_{23}$ | $\hat{b}_{13}$ | $\hat{b}_{23}$ |
| $\frac{\partial}{\partial u_2'}$ | 0 | $\hat{b}_{11}$ | 0 | 0 |
| $\frac{\partial}{\partial u_3'}$ | 0 | $\hat{b}_{12}$ | 0 | 0 |
| $\frac{\partial}{\partial u_4'}$ | 0 | 0 | 0 | $\hat{b}_{23}$ |
| $\frac{\partial}{\partial u_5}$ | 0 | 0 | 0 | $\hat{b}_{23}$ |

Table 1: Partial Velocities for Pseudo and Regular Coordinates

The strain energy density functions for the beams $B_i$ ($i = 1, 2$) are (assuming large deflections):

$$\bar{V}_i = \frac{1}{2}EA\left(\frac{\partial \tilde{u}_{i1}}{\partial x_{i1}} + \frac{1}{2}\left(\frac{\partial \tilde{u}_{i2}}{\partial x_{i1}}\right)^2\right)^2 + \frac{1}{2}E\mathcal{I}\left(\frac{\partial^2 \tilde{u}_{i2}}{\partial x_{i1}^2}\right)^2 \tag{18}$$

The torques applied to the massless hubs are:

$$\vec{T}_1 = M_1\hat{b}_{13} \quad \text{and} \quad \vec{T}_2 = M_2\hat{b}_{23} \tag{19}$$

on $B_1$ and $B_2$, respectively. The equations of motion can now be written down.

**Ordinary Differential Equations**

The ordinary differential equations, governing the angular positions are:

$$\begin{aligned} 0 = {} & \frac{\partial\, {}^{o}\vec{v}_{\mathcal{N}}^{s_1}}{\partial u_1} \cdot \left[\vec{F}_{B_1} - \vec{I}_{B_1}\right] + \frac{\partial\, {}^{\mathcal{N}}\vec{\omega}^{B_1}}{\partial u_1} \cdot \left[\vec{T}_{B_1} - \vec{J}_{B_1}\right] \\ & + \frac{\partial\, {}^{o}\vec{v}_{\mathcal{N}}^{s_2}}{\partial u_1} \cdot \left[\vec{F}_{B_2} - \vec{I}_{B_2}\right] + \frac{\partial\, {}^{\mathcal{N}}\vec{\omega}^{B_2}}{\partial u_1} \cdot \left[\vec{T}_{B_2} - \vec{J}_{B_2}\right] \end{aligned} \tag{20}$$

for $u_1$, and

$$\begin{aligned} 0 = {} & \frac{\partial\, {}^{o}\vec{v}_{\mathcal{N}}^{s_1}}{\partial u_5} \cdot \left[\vec{F}_{B_1} - \vec{I}_{B_1}\right] + \frac{\partial\, {}^{\mathcal{N}}\vec{\omega}^{B_1}}{\partial u_5} \cdot \left[\vec{T}_{B_1} - \vec{J}_{B_1}\right] \\ & + \frac{\partial\, {}^{o}\vec{v}_{\mathcal{N}}^{s_2}}{\partial u_5} \cdot \left[\vec{F}_{B_2} - \vec{I}_{B_2}\right] + \frac{\partial\, {}^{\mathcal{N}}\vec{\omega}^{B_2}}{\partial u_5} \cdot \left[\vec{T}_{B_2} - \vec{J}_{B_2}\right] \end{aligned} \tag{21}$$

for $u_5$. The partial velocities are defined in table 1. The forces and torques (applied and inertia) are defined as:

$$\vec{F}_e = \int_{\Omega_f} \left(\mathcal{H}_e\vec{F}_{he} + \mathcal{D}_e\vec{F}_{de}\right) d\Omega_e$$

$$+\int_{\partial\Omega_e}\left(\mathcal{H}_e\vec{F}_{he}+\mathcal{D}_e\vec{F}_{de}\right)d\sigma_e$$

$$\vec{I}_e=\int_{\Omega_e}\rho_e{}^o\vec{a}_{\mathcal{N}}^{*I_e}\,d\Omega_e$$

$$\vec{T}_e=\int_{\Omega_e}\left[{}^{s_e}\vec{r}^{\,i_{oe}}\times\left(\mathcal{H}_e\vec{F}_{he}+\mathcal{D}_e\vec{F}_{de}\right)\right.$$

$$\left.+\mathcal{H}_e\vec{T}_{he}+\mathcal{D}_e\vec{T}_{de}\right]d\Omega_e$$

$$+\int_{\partial\Omega_e}\left[{}^{s_e}\vec{r}^{\,i_{oe}}\times\left(\mathcal{H}_e\vec{F}_{he}+\mathcal{D}_e\vec{F}_{de}\right)\right.$$

$$\left.+\mathcal{H}_e\vec{T}_{he}+\mathcal{D}_e\vec{T}_{de}\right]d\sigma_e$$

$$\vec{J}_e=\int_{\Omega_e}\left[{}^{s_e}\vec{r}^{\,i_{oe}}\times\rho_e{}^o\vec{a}_{\mathcal{N}}^{*I_e}+\right.$$

$$\left.{}^{i_{oe}}\vec{r}^{\,*I_e}\times\rho_e{}^o\vec{a}_{\mathcal{N}}^{\,i_{oe}}+\vec{\vec{I}}_{i_{oe}}\cdot{}^{\mathcal{N}}\vec{\alpha}^{\,I_e}+{}^{\mathcal{N}}\vec{\omega}^{\,I_e}\times\vec{\vec{I}}_{i_{oe}}\cdot{}^{\mathcal{N}}\vec{\omega}^{\,I_e}\right]d\Omega_e$$

The final form of the differential equation are found by taking the indicated dot products and are not displayed here.

### Partial Differential Equations

The field equations governing elongation and bending for $B_1$, are:

$$\begin{aligned}0 &= \frac{\partial}{\partial x_{11}}\left(\frac{\partial\bar{V}_1}{\partial\tilde{u}_{11,1}}\right)-\rho^o\vec{a}_{\mathcal{N}}^{\,i_{o1}}\cdot\hat{b}_{11}\\ &= \frac{\partial}{\partial x_{11}}\left[EA\left(\frac{\partial\tilde{u}_{11}}{\partial x_{11}}+\frac{1}{2}\left(\frac{\partial\tilde{u}_{12}}{\partial x_{11}}\right)^2\right)\right]\\ &-\rho\left(\frac{\partial^2\tilde{u}_{11}}{\partial t^2}-\dot{u}_1\tilde{u}_{12}-2u_1\frac{\partial\tilde{u}_{12}}{\partial t}-(x_{11}+\tilde{u}_{11})u_1^2\right)\end{aligned}\tag{22}$$

for elongation, and:

$$\begin{aligned}0 &= \frac{\partial}{\partial x_{11}}\left(\frac{\partial\bar{V}_1}{\partial\tilde{u}_{12,1}}\right)-\frac{\partial^2}{\partial x_{11}^2}\left(\frac{\partial\bar{V}_1}{\partial\tilde{u}_{12,11}}\right)-\rho^o\vec{a}_{\mathcal{N}}^{\,i_{o1}}\cdot\hat{b}_{12}\\ &+\frac{\partial}{\partial x_{11}}\left[\hat{b}_{13}\cdot\left(\vec{\vec{I}}_{i_{o1}}\cdot{}^{\mathcal{N}}\vec{\alpha}^{\,I_1}+{}^{\mathcal{N}}\vec{\omega}^{\,I_1}\times\vec{\vec{I}}_{i_{o1}}\cdot{}^{\mathcal{N}}\vec{\omega}^{\,I_1}\right)\right]\\ &= \frac{\partial}{\partial x_{11}}\left[EA\left(\frac{\partial\tilde{u}_{11}}{\partial x_{11}}+\frac{1}{2}\left(\frac{\partial\tilde{u}_{12}}{\partial x_{11}}\right)^2\right)\frac{\partial\tilde{u}_{12}}{\partial x_{11}}\right]\\ &-\frac{\partial^2}{\partial x_{11}^2}\left(EI\frac{\partial^2\tilde{u}_{12}}{\partial x_{11}^2}\right)\end{aligned}$$

$$
\begin{aligned}
&-\rho\left(\frac{\partial^2\tilde{u}_{12}}{\partial t^2}+2u_1\frac{\partial\tilde{u}_{12}}{\partial t}+(x_{11}+\tilde{u}_{11})\,\dot{u}_1-\tilde{u}_{12}u_1^2\right)\\
&+\frac{\partial}{\partial x_{11}}\left[\left(\dot{u}_1+\frac{\partial^3\tilde{u}_{12}}{\partial x_{11}\partial t^2}\right)\rho\mathcal{I}\right.\\
&\left.+\rho\mathcal{I}\left(u_1+\frac{\partial^2\tilde{u}_{12}}{\partial x_{11}\partial t}\right)^2(\hat{b}_{13}\cdot\hat{i}_{13})(\hat{b}_{13}\times\hat{i}_{13})\cdot\hat{b}_{13}\right]
\end{aligned}
\tag{23}
$$

for bending.

At $x_{11}=0$, the boundary conditions for $B_1$ are:

$$
\tilde{u}_{11}=\tilde{u}_{12}=\tilde{u}_{12,1}=0 \tag{24}
$$

The boundary conditions at $x_{11}=L_1$ for $B_1$ are:

$$
\frac{\partial\bar{V}_1}{\partial\tilde{u}_{11,1}}=EA\left(\frac{\partial\tilde{u}_{11}}{\partial x_{11}}+\frac{1}{2}\left(\frac{\partial\tilde{u}_{12}}{\partial x_{11}}\right)^2\right)=g'_{11} \tag{25}
$$

for elongation, with:

$$
\begin{aligned}
g'_{12} &= \hat{b}_{13}\cdot\left(\vec{\vec{I}}_{i_{o_1}}\cdot{}^{\mathcal{N}}\vec{\alpha}^{\,l_1}+{}^{\mathcal{N}}\vec{\omega}^{\,l_1}\times\vec{\vec{I}}_{i_{o_1}}\cdot{}^{\mathcal{N}}\vec{\omega}^{\,l_1}\right)\\
&\quad+\frac{\partial\bar{V}_1}{\partial\tilde{u}_{12,1}}-\frac{\partial}{\partial x_{11}}\left(\frac{\partial\bar{V}_1}{\partial\tilde{u}_{12,11}}\right)\\
&= \left(\dot{u}_1+\frac{\partial^3\tilde{u}_{12}}{\partial x_{11}\partial t^2}\right)\rho\mathcal{I}\\
&\quad+\rho\mathcal{I}\left(u_1+\frac{\partial^2\tilde{u}_{12}}{\partial x_{11}\partial t}\right)^2(\hat{b}_{13}\cdot\hat{i}_{13})(\hat{b}_{13}\times\hat{i}_{13})\cdot\hat{b}_{13}\\
&\quad+EA\left(\frac{\partial\tilde{u}_{11}}{\partial x_{11}}+\frac{1}{2}\left(\frac{\partial\tilde{u}_{12}}{\partial x_{11}}\right)^2\right)\frac{\partial\tilde{u}_{12}}{\partial x_{11}}\\
&\quad-\frac{\partial}{\partial x_{11}}\left(E\mathcal{I}\frac{\partial^2\tilde{u}_{12}}{\partial x_{11}^2}\right)
\end{aligned}
\tag{26}
$$

for shear, and

$$
\frac{\partial\bar{V}_1}{\partial\tilde{u}_{12,11}}=E\mathcal{I}\frac{\partial^2\tilde{u}_{12}}{\partial x_{11}^2}=k'_{12} \tag{27}
$$

for bending moment. The intrinsic forcing terms of the boundary conditions at $x_{11}=L_1$ are defined as:

$$
g'_{11} = \frac{\partial^{o}\vec{v}_{\mathcal{N}}^{\,S_1}}{\partial u'_2}\cdot\left[\vec{F}_{B_1}-\vec{I}_{B_1}\right]+\frac{\partial^{\mathcal{N}}\vec{\omega}^{\,B_1}}{\partial u'_2}\cdot\left[\vec{T}_{B_1}-\vec{J}_{B_1}\right]
$$

$$
+\frac{\partial^o \vec{v}_{\mathcal{N}}^{s_2}}{\partial u_2'} \cdot \left[\vec{F}_{B_2} - \vec{I}_{B_2}\right] + \frac{\partial^{\mathcal{N}} \vec{\omega}^{\,B_2}}{\partial u_2'} \cdot \left[\vec{T}_{B_2} - \vec{J}_{B_2}\right] \tag{28}
$$

and

$$
\begin{aligned}
g_{12}' \;=\;& \frac{\partial^o \vec{v}_{\mathcal{N}}^{s_1}}{\partial u_3'} \cdot \left[\vec{F}_{B_1} - \vec{I}_{B_1}\right] + \frac{\partial^{\mathcal{N}} \vec{\omega}^{\,B_1}}{\partial u_3'} \cdot \left[\vec{T}_{B_1} - \vec{J}_{B_1}\right] \\
&+\frac{\partial^o \vec{v}_{\mathcal{N}}^{s_2}}{\partial u_3'} \cdot \left[\vec{F}_{B_2} - \vec{I}_{B_2}\right] + \frac{\partial^{\mathcal{N}} \vec{\omega}^{\,B_2}}{\partial u_3'} \cdot \left[\vec{T}_{B_2} - \vec{J}_{B_2}\right]
\end{aligned} \tag{29}
$$

and

$$
\begin{aligned}
k_{12}' \;=\;& \frac{\partial^o \vec{v}_{\mathcal{N}}^{s_1}}{\partial u_4'} \cdot \left[\vec{F}_{B_1} - \vec{I}_{B_1}\right] + \frac{\partial^{\mathcal{N}} \vec{\omega}^{\,B_1}}{\partial u_4'} \cdot \left[\vec{T}_{B_1} - \vec{J}_{B_1}\right] \\
&+\frac{\partial^o \vec{v}_{\mathcal{N}}^{s_2}}{\partial u_4'} \cdot \left[\vec{F}_{B_2} - \vec{I}_{B_2}\right] + \frac{\partial^{\mathcal{N}} \vec{\omega}^{\,B_2}}{\partial u_4'} \cdot \left[\vec{T}_{B_2} - \vec{J}_{B_2}\right]
\end{aligned} \tag{30}
$$

The field equations for the second member ($B_2$) are:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial x_{21}} \left(\frac{\partial \bar{V}_2}{\partial \tilde{u}_{21,1}}\right) - \rho^o \vec{a}_{\mathcal{N}}^{\,i_{o_2}} \cdot \hat{b}_{21} \\
&= \frac{\partial}{\partial x_{21}} \left[EA\left(\frac{\partial \tilde{u}_{21}}{\partial x_{21}} + \frac{1}{2}\left(\frac{\partial \tilde{u}_{22}}{\partial x_{21}}\right)^2\right)\right] \\
&-\rho\left[\dot{u}_2' - \dot{u}_1 q_3' - 2u_1 u_3' - u_1^2\left(L_1 + q_2'\right)\right]\left(\hat{b}_{11} \cdot \hat{b}_{21}\right) \\
&-\rho\left[\dot{u}_3' + \dot{u}_1\left(L_1 + q_2'\right) + 2u_1 u_2' - u_1^2 q_3'\right]\left(\hat{b}_{12} \cdot \hat{b}_{21}\right) \\
&-\rho\left[\frac{\partial^2 \tilde{u}_{21}}{\partial t^2} - \left(\dot{u}_1 + \dot{u}_4' + \dot{u}_5\right)\tilde{u}_{22}\right. \\
&\left.-2\left(u_1 + u_4' + u_5\right)\frac{\partial \tilde{u}_{22}}{\partial t} - \left(u_1 + u_4' + u_5\right)^2\left(x_{21} + \tilde{u}_{21}\right)\right]
\end{aligned} \tag{31}
$$

for elongation, and:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial x_{21}} \left(\frac{\partial \bar{V}_2}{\partial \tilde{u}_{22,1}}\right) - \frac{\partial^2}{\partial x_{21}^2}\left(\frac{\partial \bar{V}_2}{\partial \tilde{u}_{22,11}}\right) - \rho^o \vec{a}_{\mathcal{N}}^{\,i_{o_2}} \cdot \hat{b}_{22} \\
&+\frac{\partial}{\partial x_{21}}\left[\hat{b}_{23} \cdot \left(\bar{\bar{I}}_{i_{o_2}} \cdot {}^{\mathcal{N}}\vec{\alpha}^{\,I_2} + {}^{\mathcal{N}}\vec{\omega}^{\,I_2} \times \bar{\bar{I}}_{i_{o_2}} \cdot {}^{\mathcal{N}}\vec{\omega}^{\,I_2}\right)\right] \\
&= \frac{\partial}{\partial x_{21}}\left[EA\left(\frac{\partial \tilde{u}_{21}}{\partial x_{21}} + \frac{1}{2}\left(\frac{\partial \tilde{u}_{22}}{\partial x_{21}}\right)^2\right)\frac{\partial \tilde{u}_{22}}{\partial x_{21}}\right]
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{\partial^2}{\partial x_{21}^2}\left(E\mathcal{I}\frac{\partial^2 \tilde{u}_{22}}{\partial x_{21}^2}\right) \\
&-\rho\left[\dot{u}_2' - \dot{u}_1 q_3' - 2u_1 u_3' - u_1^2 (L_1 + q_2')\right]\left(\hat{b}_{11}\cdot\hat{b}_{22}\right) \\
&-\rho\left[\dot{u}_3' + \dot{u}_1 (L_1 + q_2') + 2u_1 u_2' - u_1^2 q_3'\right]\left(\hat{b}_{12}\cdot\hat{b}_{22}\right) \\
&-\rho\left[\frac{\partial^2 \tilde{u}_{22}}{\partial t^2} + (\dot{u}_1 + \dot{u}_4' + \dot{u}_5)(x_{21} + \tilde{u}_{21})\right. \\
&\left.+2(u_1 + u_4' + u_5)\frac{\partial \tilde{u}_{21}}{\partial t} - (u_1 + u_4' + u_5)^2 \tilde{u}_{22}\right] \\
&+\frac{\partial}{\partial x_{21}}\left[\left(\dot{u}_1 + \dot{u}_4' + \dot{u}_5 + \frac{\partial^3 \tilde{u}_{22}}{\partial x_{21}\partial t^2}\right)\rho\mathcal{I}\right. \\
&\left.+\rho\mathcal{I}\left(u_1 + u_4' + u_5 + \frac{\partial^2 \tilde{u}_{22}}{\partial x_{21}\partial t}\right)^2 (\hat{b}_{23}\cdot\hat{\imath}_{23})(\hat{b}_{23}\times\hat{\imath}_{23})\cdot\hat{b}_{23}\right]
\end{aligned}
\tag{32}
$$

for bending.

The boundary conditions for $B_2$ at $x_{21} = 0$ are:

$$
\tilde{u}_{21} = \tilde{u}_{22} = \tilde{u}_{22,1} = 0 \tag{33}
$$

The boundary conditions at $x_{21} = L_2$ are:

$$
0 = EA\left(\frac{\partial \tilde{u}_{21}}{\partial x_{21}} + \frac{1}{2}\left(\frac{\partial \tilde{u}_{22}}{\partial x_{21}}\right)^2\right) \tag{34}
$$

(35)

for elongation, and:

$$
\begin{aligned}
0 = &\left(\dot{u}_1 + \dot{u}_4' + \dot{u}_5 + \frac{\partial^3 \tilde{u}_{22}}{\partial x_{21}\partial t^2}\right)\rho\mathcal{I} \\
&+\rho\mathcal{I}\left(u_1 + u_4' + u_5 + \frac{\partial^2 \tilde{u}_{22}}{\partial x_{21}\partial t}\right)^2 (\hat{b}_{23}\cdot\hat{\imath}_{23})(\hat{b}_{23}\times\hat{\imath}_{23})\cdot\hat{b}_{23} \\
&+EA\left(\frac{\partial \tilde{u}_{21}}{\partial x_{21}} + \frac{1}{2}\left(\frac{\partial \tilde{u}_{22}}{\partial x_{21}}\right)^2\right)\frac{\partial \tilde{u}_{22}}{\partial x_{21}} - \frac{\partial}{\partial x_{21}}\left(E\mathcal{I}\frac{\partial^2 \tilde{u}_{22}}{\partial x_{21}^2}\right)
\end{aligned}
\tag{36}
$$

for shear, and:

$$
0 = E\mathcal{I}\frac{\partial^2 \tilde{u}_{22}}{\partial x_{21}^2} \tag{37}
$$

for moment.

Initial conditions for $q_1$, $q_5$, $u_1$, $u_5$, $\tilde{u}_{11}$, $\tilde{u}_{12}$, $\tilde{u}_{21}$, $\tilde{u}_{22}$, $\frac{\partial \tilde{u}_{11}}{\partial t}$, $\frac{\partial \tilde{u}_{12}}{\partial t}$, $\frac{\partial \tilde{u}_{21}}{\partial t}$, and $\frac{\partial \tilde{u}_{22}}{\partial t}$ must also be specified. The kinematic differential equations for $q_1$ and $q_5$ are given in equation 10.

## Discussion

As can be seen by the presentation above, the methodology facilitates the process of writing equations of motion for complex systems. One can see that the rigorous natural boundary conditions that are generated via variational principles are present but the analyst does not have to integrate by parts. For comparison of the technique with Hamilton's principle see [14] relative to the problem above. For more complex systems, such as those exhibiting nonholonomic constraints, see [1, 15]. For systems undergoing contact/impact, in a hybrid parameter fashion, see [1, 16, 17].

Apparently some of the lingering questions are whether or not the method is attractive to practicing engineers and can the symbolic form of the equations be put in a form suitable for simulation. The author claims yes to the later question and only time will tell on the former question. Massaging the equations into a simulation are the topic of the next section.

With regards to getting numbers from the equations, it is possible to put the equations in a weak form so that the complicated boundary conditions are absorbed into an integrated form of the partial differential equations. Then one has only to choose an appropriate function for the test function used to cast the problem in its weak form. For problems like the manipulator above, Rayleigh-Ritz discretization is probably sufficient. For more complicated continuum bodies, finite element discretization is probably appropriate provided the analyst work with the problem in its weak form so the appropriate boundary conditions are included.

# RMS MODEL

## Model Description

The main subject of this report is the application of the method described above to the Space Shuttle RMS. The RMS is modeled as a system of rigid and continuously elastic bodies, a hybrid parameter mechanical system. The system is broken down as follows (see figure 2).

The orbiter is taken as a six degree of freedom rigid body. The RMS base (assumed to be rigid and labeled $B$) is attached to the orbiter via small displacement small angle springs which approximate the elastic nature of the orbiter. The RMS shoulder yaw motor and housing are assumed to be a rigid body (body $Sy$). Connected to $Sy$ via a nonlinear spring and motor control action is the RMS shoulder pitch body $Sp$ with its actuator. Attached to $Sp$ is the first elastic boom. This boom, as with all booms in this model, is assumed to be a continuum in which $y$ and $z$ deflections along with axial rotation are modeled. The beam model is a Rayleigh beam (intrinsic cross-sectional inertia incorporated) with small deflections. The next body in the chain is the elbow pitch motor and housing. It is attached through a nonlinear actuator to rigid body $EL$. Attached to $EL$ is the next flexible boom, modeled like the first boom. Attached to the second boom is the wrist pitch motor and housing. It is attached to body $P$. Boom three is anchored by body $P$. Boom three is elastic and modeled as described before. At the tip of boom 3 is the motor and housing for the wrist yaw action. The wrist yaw body is rigid and labeled $Y$. Attached to the yaw body through a nonlinear actuator is the wrist roll motor and housing. This is followed by the

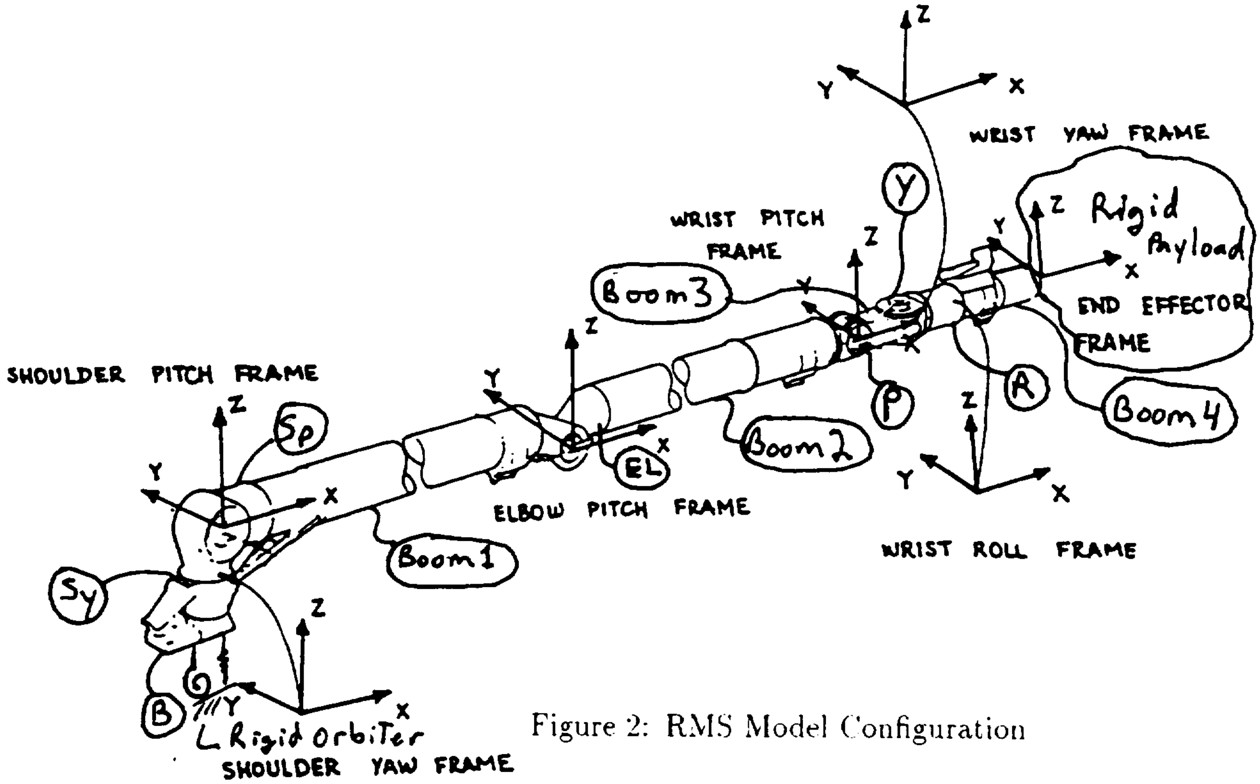


Figure 2: RMS Model Configuration

fourth elastic boom (the end effector). Attached to the end effector is a rigid payload.

## Closed Form Model

In order to attempt writing a closed form model for a system as complicated as the RMS described above, a symbolic manipulating assistant is desired. The author has access to *Mathematica* so this is the tool that was utilized. Before getting into the description of the algorithms developed, justification for the effort will be outlined.

Why should an analyst develop closed form models when there exist other tools that seem to adequately model these systems? The author believes that using tools that are traditionally from the structural analysis realm such as NASTRAN models unnecessarily limit the model to the linear motion about some configuration. It is felt that if the approach of writing complete models first (then reducing to linear if desired) is feasible, in a timely manner, then engineers will utilize these more exact models. In order to facilitate the clock, computer aided modeling is desired; *Mathematica* is an excellent tool for this process. Another advantage to working directly with the closed form model is that the "zero times zero" multiplications that arise in straight out matrix models are avoided. Also repetitive multiplications and additions are readily recognized and can be assigned to a memory location for instant recall. This tight code will make running these complicated models more feasible.

## *Mathematica* Algorithms

*Mathematica* algorithms were developed to mimic the procedure outlined in the previously discussed simplified model of the RMS. The standard notation for *Mathematica* was adjusted so as to mimic engineering vector notation. Then algorithms were developed that recognize the vector dot and cross products, the triple products, and other identities. Differentiation of vectors in multiple coordinate frames was defined. Standard order for the symbols was defined so symbolic cancelation was facilated. Function that aid in the gathering of terms, the distribution of terms, and general manipulation were developed. At this point these algorithms are used via a *Mathematica* notebook running on a NeXT computer. They are not limited to this computer system because the notebooks are portable across multiple computer systems. An example of how one enters symbols for manipulation is shown in the appendix.

### RMS Model Status

Presently the modeling procedure is not complete. All stages of the development are complete up to the point where the actual differential equations suitable for output to FORTRAN format are formed. All the appropriate d'Alembert forces and torques have been calculated along with the appropriate partial velocities, and the weak formulation. Unfortunately the approach taken thus far is very memory hungry so the workstation is using a lot of virtual memory which is time consuming. Refinements to the procedure and algorithms are made in real time and the memory and time consumption problems are being reduced.

### Model Shake Down

Comparisons of the aforementioned model with existing models will be made upon completion of the modeling procedure. It is intended that the efficacy of the technique and model will be tested via metrics such as accuracy, calculation speed, and generality.

## SUMMARY

The rudimental aspects of a procedure to rigorously model complicated systems in a timely manner have been developed. The modeling technique is based on a variational principle based approach for writing the equations of motion, augmented with computer aided modeling algorithms written in *Mathematica* code. The tools developed are being applied to a complicated RMS model in order to establish the efficacy of the modeling technique. The technique shows promise because of its rigor, but the details of the computer aided algorithms need refinement. Numerical studies have yet to be performed.

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This is the symbol manipulations for the robotic arm flex model.

Angular Velocity and Acceleration. omega (W__) are the generalized speed.

Newtonian Frame (N)

Orbiter Frame (O)

```
o[1]:=unitVector[O,o,1]
o[2]:=unitVector[O,o,2]
o[3]:=unitVector[O,o,3]
NwO=omega[N,O]=Wo1 o[1] + Wo2 o[2] + Wo3 o[3]
```

$$Wo1\, \hat{o}_1 + Wo2\, \hat{o}_2 + Wo3\, \hat{o}_3$$

```
NaO=DvDt[N,NwO]//Simplify
```

$$\dot{Wo1}\, \hat{o}_1 + \dot{Wo2}\, \hat{o}_2 + \dot{Wo3}\, \hat{o}_3$$

Manipulator Base Frame (B)

```
b[1]:=unitVector[B,b,1]
b[2]:=unitVector[B,b,2]
b[3]:=unitVector[B,b,3]
OwB=omega[O,B]=Wb1 b[1] + Wb2 b[2] + Wb3 b[3]
```

$$Wb1\ \hat{b}_1 + Wb2\ \hat{b}_2 + Wb3\ \hat{b}_3$$

```
NwB=omega[N,B]=NwO + OwB
```

$$Wb1\ \hat{b}_1 + Wb2\ \hat{b}_2 + Wb3\ \hat{b}_3 + Wo1\ \hat{o}_1 + Wo2\ \hat{o}_2 + Wo3\ \hat{o}_3$$

```
NaB=NaO + DvDt[B,OwB] + NwB >< OwB
```

$$Wb1\ Wo1\ \hat{o}_1 >< \hat{b}_1 + Wb2\ Wo1\ \hat{o}_1 >< \hat{b}_2 + Wb3\ Wo1\ \hat{o}_1 >< \hat{b}_3 +$$

$$Wb1\ Wo2\ \hat{o}_2 >< \hat{b}_1 + Wb2\ Wo2\ \hat{o}_2 >< \hat{b}_2 + Wb3\ Wo2\ \hat{o}_2 >< \hat{b}_3 +$$

$$Wb1\ Wo3\ \hat{o}_3 >< \hat{b}_1 + Wb2\ Wo3\ \hat{o}_3 >< \hat{b}_2 + Wb3\ Wo3\ \hat{o}_3 >< \hat{b}_3 +$$

$$\dot{Wb1}\ \hat{b}_1 + \dot{Wb2}\ \hat{b}_2 + \dot{Wb3}\ \hat{b}_3 + \dot{Wo1}\ \hat{o}_1 + \dot{Wo2}\ \hat{o}_2 + \dot{Wo3}\ \hat{o}_3$$

```
z1=Coefficient[NaB,o[1]><b[1]]
z2=Coefficient[NaB,o[1]><b[2]]
z3=Coefficient[NaB,o[1]><b[3]]
z4=Coefficient[NaB,o[2]><b[1]]
z5=Coefficient[NaB,o[2]><b[2]]
z6=Coefficient[NaB,o[2]><b[3]]
z7=Coefficient[NaB,o[3]><b[1]]
z8=Coefficient[NaB,o[3]><b[2]]
z9=Coefficient[NaB,o[3]><b[3]]
```

Wb1 Wo1

Wb2 Wo1

Wb3 Wo1

Wb1 Wo2

Wb2 Wo2

Wb3 Wo2