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NEAR SURFACE ANALYSIS

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Abstract

This is a study to assist in the understanding of earth near surface structure. Higher order moments are used to detect the density distribution as well as to seek patterns found in geological structures. It is shown how higher order moments at points outside a mass structure are determined as well as how to recover the mass distribution from the higher order moments. It is interesting to note that the first moment at a point P outside the mass structure, $V_0(P)$, is the entire mass and the second moment, $V_1(P)$, is the potential at P due to the mass structure. Usually only the mass and the potential function are used to determine the density distribution in a body. In this study an infinite function sequence $\{V_n(P)\}_{n=0}^{\infty}$ is required to uniquely determine the density distribution.

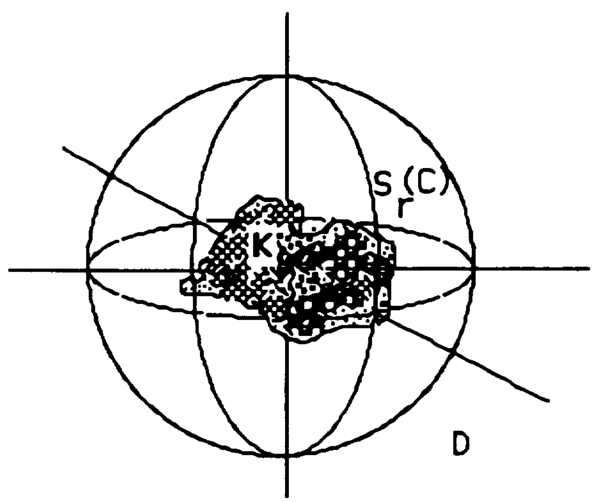
Density Determination

Suppose that A is a material body, i.e., not a point mass, that is modeled by assuming that there is a non negative continuous function g defined on a closed and bounded subset K of euclidean three space E, where K is a geometric approximation of A and g is an approximation to the density function of A. Note that K need not be connected, i.e., K need not be "in one piece".
We shall denote the real number line by R, and the euclidean inner product on by $< \cdot , \cdot >$ which induces a norm $\| \cdot \|$. If r is a positive number and P is a point in E, then

$$S_r(P) = \{ Q \text{ is in } E: \| P - Q \| \leq r \}$$

and $$\mathrm{Bdry}S_r(P) = \{ Q \text{ is in } E: \| P - Q \| = r \}.$$

Since K is bounded there is a number r>0 and a point C, the geometric center, of K such that K is contained in $S_r(C)$. Let D denote the complement of $S_r(C)$ in E.

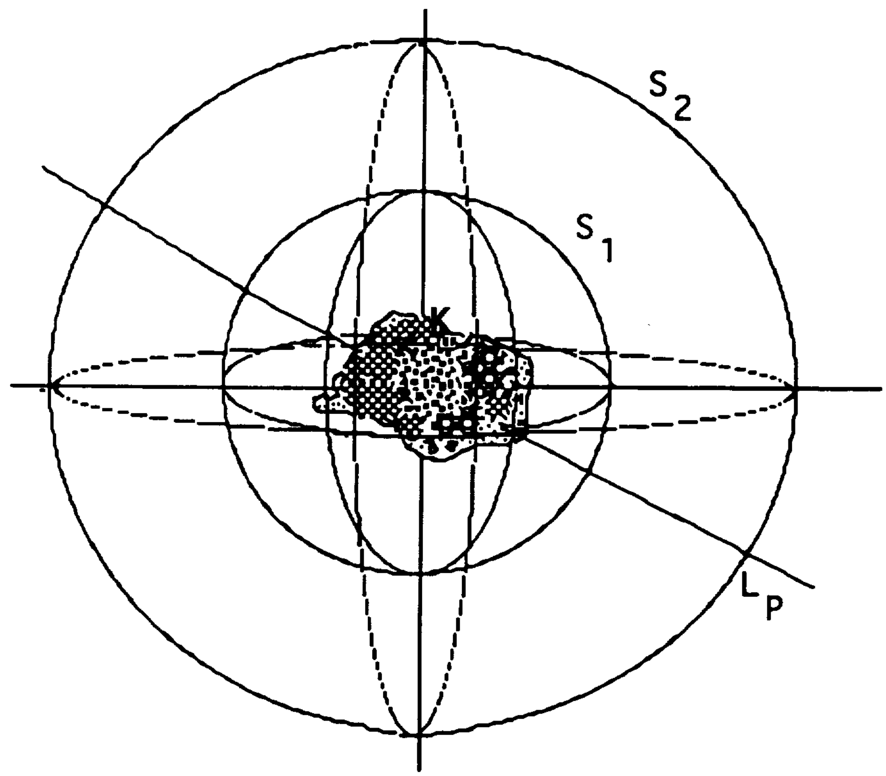


For each non negative integer n, V_n is the real valued function defined on D by

$$V_n(P) = \int_{S_r(C)} \frac{g(Q)}{\| P - Q \|^n} \, dQ \qquad \text{for P in D.}$$

The above integral is a triple integral. Notice that $V_0(P)$ is the mass of K, and $V_1(P)$ is the Newtonian potential at P due to the mass K.

Suppose g is a non negative continuous function defined on E, S_1 is a ball containing K centered at the geometric center of K, S_2 is a ball properly containing S_1 that is concentric with S_1 and also g is zero on the complement of S_2.

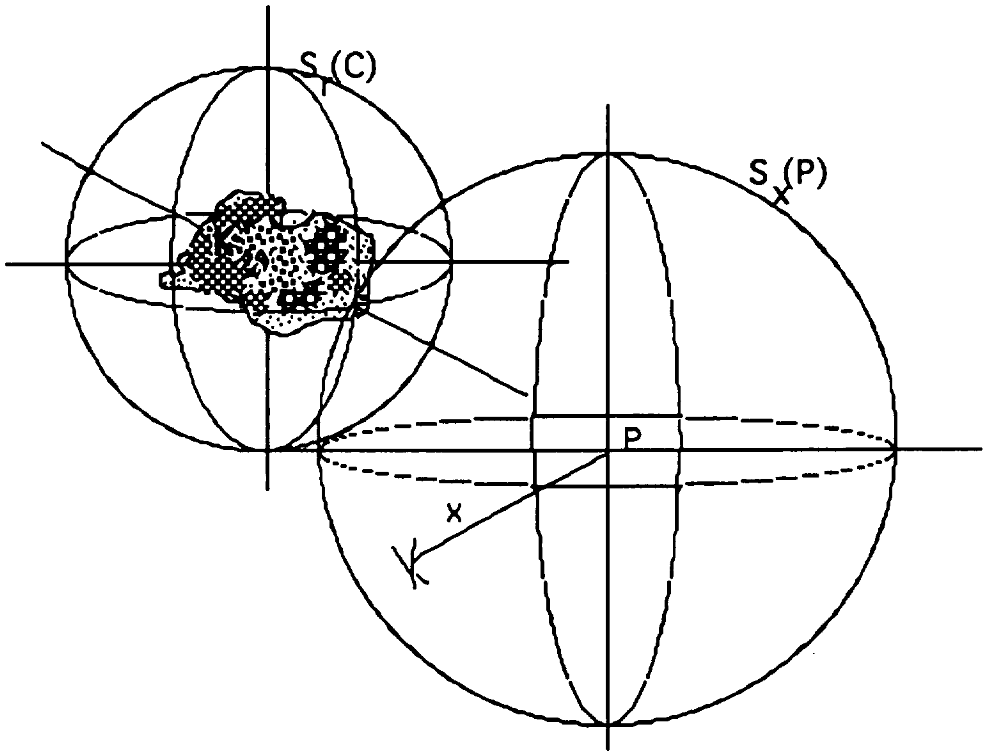


There is a one-to-one correspondence between the set
$V = \{ \{ V_n(P) \}_{n=0}, $ P is in the complement of sphere $S_2 \}$
and g, moreover g can be constructed from the set V.

Suppose P is in D. Let m_P be the mass function defined on the real line R by

$$m_P(x) = 0 \text{ if } x \le 0$$

and $$m_P(x) = \int_{S_x(P)} g(Q)\, dQ \text{ if } x > 0.$$



Observe that for each point P in D, m_P is a real-valued, non decreasing continuous function on R, which has a continuous derivative and hence is of bounded variation on R. (Recall that m_P is constant except on a finite interval of the line.)

$m'_P(x) = 0$ if $x \leq 0$,

$m'_P(x) = \int_{S_r(C) \cap \mathrm{Bdry}S_x(P)} g(x,z)\, dz$ if $S_r(C)$ intersects $\mathrm{Bdry}S_x(P)$, and

$m'_P(x) = 0$ if $x > 0$ and $S_r(C)$ does not intersect $\mathrm{Bdry}S_x(P)$.

If P is a point, then

$$u_P = \inf \{ \| P - Q \| : Q \text{ is in } S_r(C) \} = \| P - C \| - r$$
$$v_P = \sup \{ \| P - Q \| : Q \text{ is in } S_r(C) \} = \| P - C \| + r.$$

Observe that $V_n(P) = \int_{u_P}^{v_P} (1/t^n)\, dm_P(t)$ for $n = 0,1,2,\ldots$

Making a change of variables $s = 1/t$ for $u_p \le t \le v_p$, we have

$$V_n(P) = \int_{u_p^{-1}}^{v_p^{-1}} s^n \, dm_p(1/s) \quad \text{for } n = 0,1,2,\ldots$$

Define for each point P in D

$$A_P = v_p^{-1}, \qquad B_P = u_p^{-1}$$

and

$$M_P(x) = M - m_p(1/x) \qquad A_P \le x \le B_P.$$

Note that $M_p(A_P) = 0$, and $M_p(B_P) = M$,
hence extend M_p to $(0, B_P)$ by defining $M_p(x) = 0$ for $0 \le x \le A_P$.
Finally
$M'_p(A_P) = m'_P(1/A_P)(1/A_P^2) = 0$ and hence M_p has a continuous derivative on $(0, B_P)$.
We now have

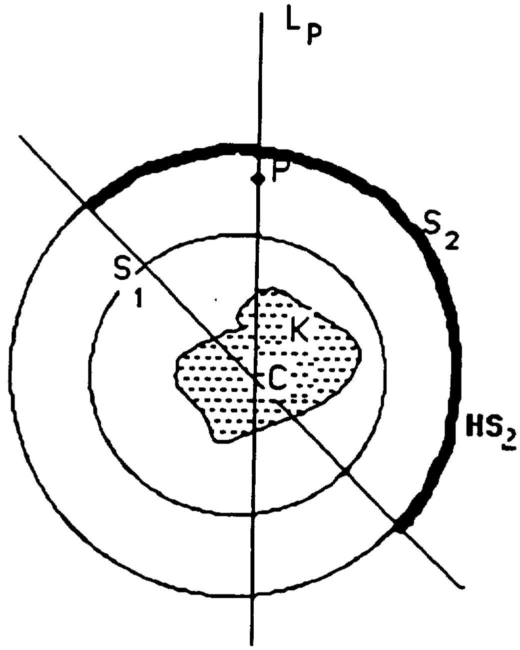
$$V_n(P) = \int_0^{B_P} x^n \, dM_p(x) \quad \text{for } n = 0,1,2,3\ldots,$$

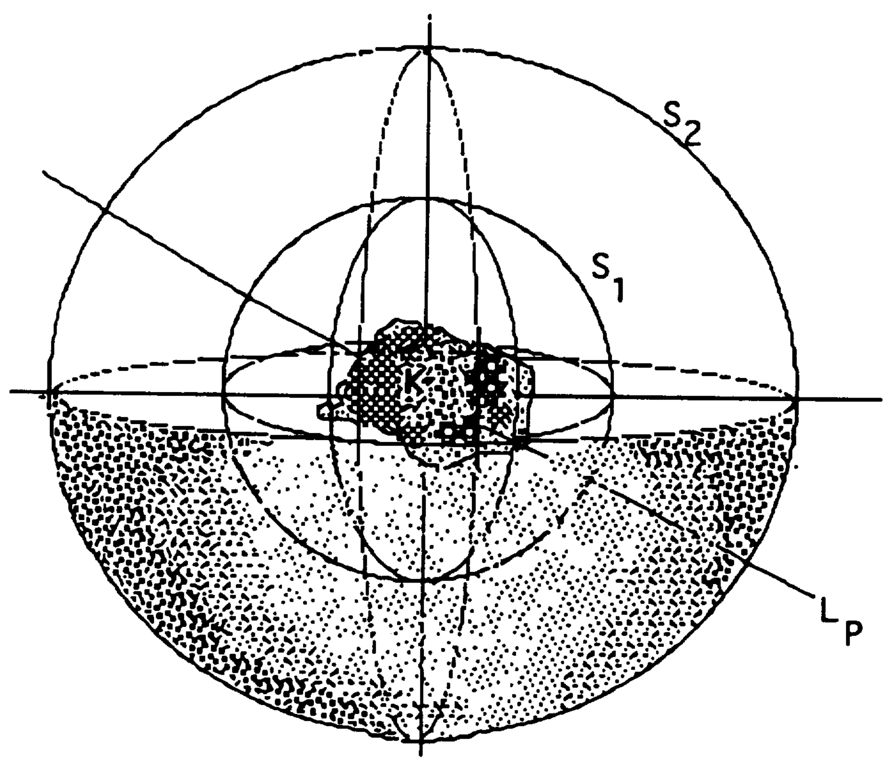
$M'_p(B_P) = m'_p(1/B_P)(1/B_P^2) = 0$,
and the sequence $\{V_n(P)\}_{n=0}$ satisfies the Hausdorf conditions (See appendix 1).

Suppose for each point P on the surface of S_2, where S_2 is a sphere properly containing and concentric with the sphere S_1 which contains K, with center C, (the geometric center of K), the sequence

$$\{ V_n(P) \}_n^{\infty} \quad \text{is known.}$$

For each such point P consider the line L_p containing P and C the center of S_1.





From the sequence $\{V_n(P)\}_n^{\infty}$ we can recover (see A1) the function m_P. Hence for each point P on S_2 we have m_P, then using the method of bilinear forms we may recover (see appendix 2) the function g.

APPENDIX A1

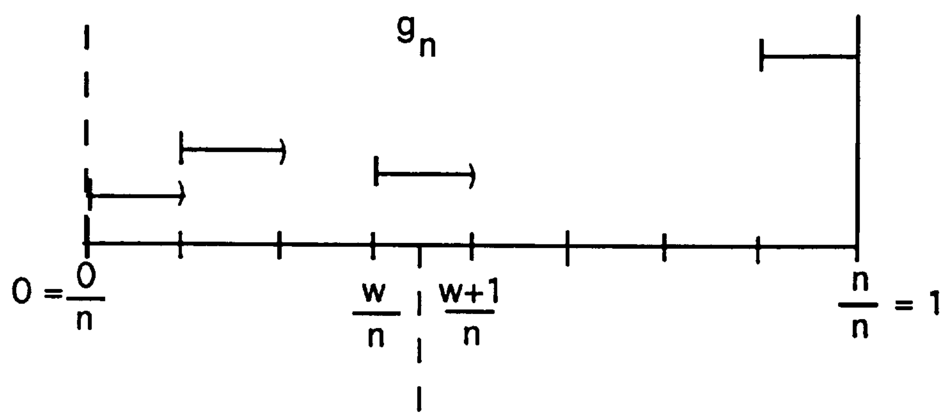
Suppose that the number sequence $\{C_n\}_{n=0}^{\infty}$ is known and satisfies the Hausdorf Condition i.e., there is a positive number H such that

$$\sum_{t=0}^{n} B(n,t) \left| \sum_{s=0}^{n-t} B(n-t,s)\,(-1)^s\, C_{t+s} \right| < H \qquad \text{for } n = 0,1,2,\ldots$$

where B(n,t) is the binomial coefficient n! /(t! (n-t)!).

For each non negative in n and each number x in the number interval [0,1] define

$$g_n(x) = \sum_{t=0}^{w} B(n,t) \sum_{s=0}^{n-t} B(n-t,s)\,(-1)^s\, C_{t+s} \quad \text{if } w/n \le x < ((w+1)/n).$$



The function sequence g_n then converges pointwise to a function g on [0,1]. The function g is of bounded variation on [0,1] with total variation not exceeding H and has the property that

$$\int_0^1 x^n \, dg(x) = C_n \qquad \text{for } n = 0,1,2,\ldots .$$

APPENDIX A2

If for each point P on the surface of a hemisphere we know $M_P(x)$, then we can determine g on K in the following manner.

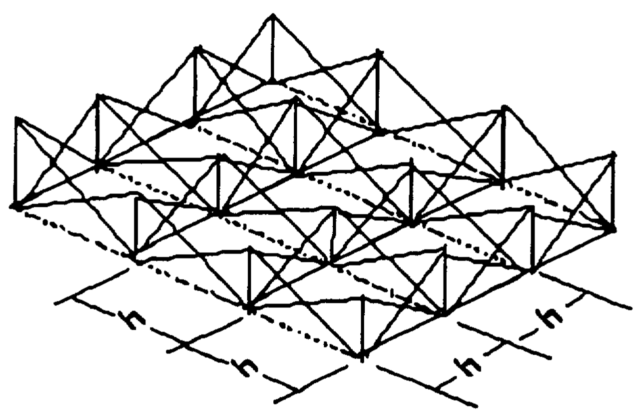
A sequence of functions is created that will converge pointwise to g, on a dense subset of S_2 (**g** is the restriction of g to the dense subset). Since g is continuous, we can extend **g** to g on S_2 .

We define for each positive integer n a function f_n on the three dimensional normalized grid G_n.

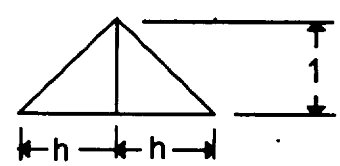
$$f_n(x, y, z) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \Phi_i(x)\, \Phi_j(y)\, \Phi_k(z)\, w_{ijk} \quad \text{for the point } (x,y,z)$$

in the grid G_n where n +1 is the number of grid points on a side and h is 1/n. The problem is to determine w_{ijk} for $1 \le i,j,k \le n$.

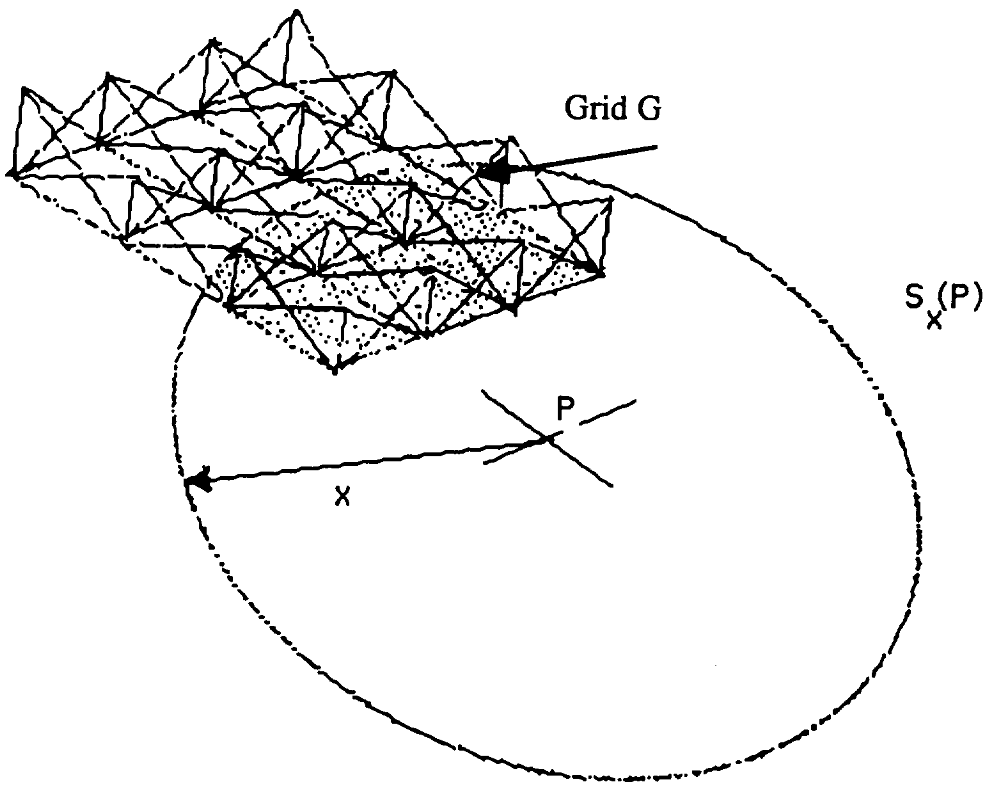
Below is a two dimensional schematic to suggest the three dimensional case.



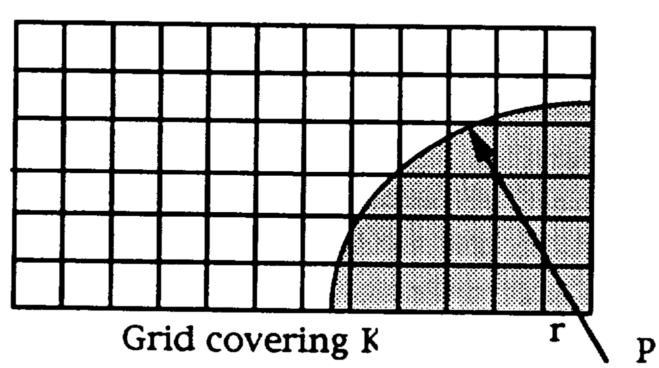
Φ function on grid

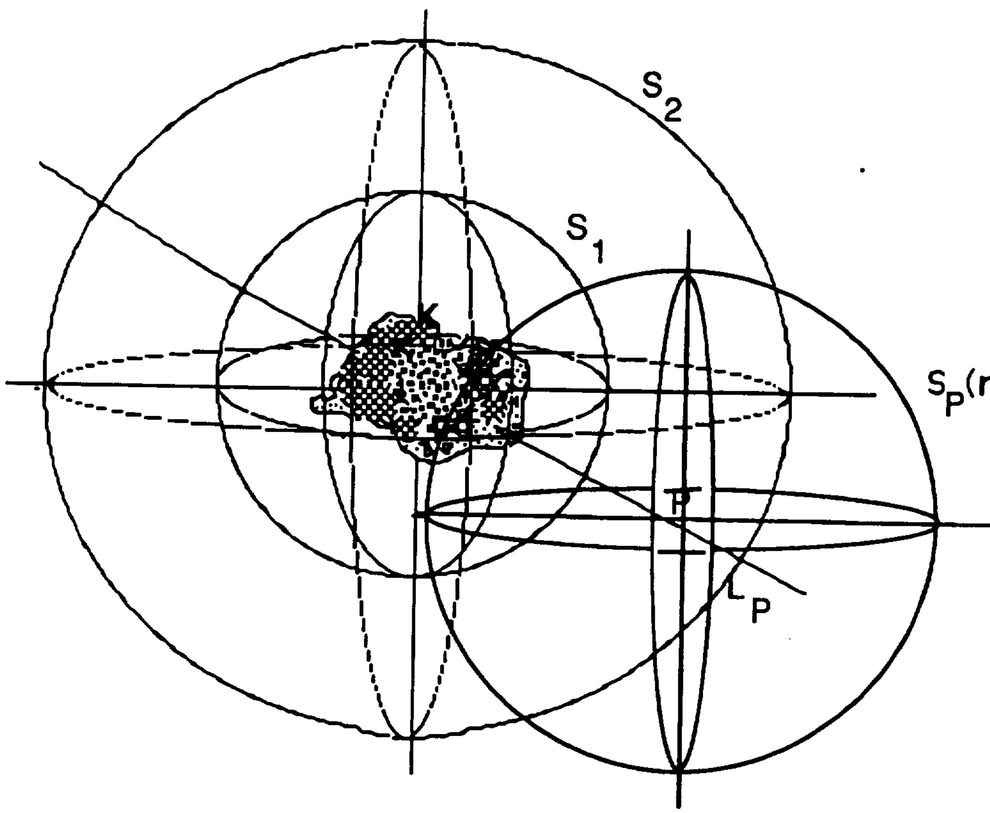


Typical Φ function



The function f_n is integrated over the same portion of the grid as is intersected by the sphere centered at a point P of radius r (see schematic figure below) and set equal to $m_P(r)$, which was constructed from the sequence $\{V_n(P)\}_n$. By selecting various points P and various radii we have a system of linear equations in w_{ijk} that we can solve and thus determine the function f_n. As stated before, the sequence $\{f_n\}_n$ converges pointwise on dense subset to **g**, which is then extended to the function g.





S_2
S_1
$S_P(r)$
P
L_P