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# **Modelling Early Failures on Space Station *Freedom***

Final Report

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**ABSTRACT**

A major problem encountered in planning for Space Station *Freedom* is the amount of maintenance that will be required. To predict the failure rates of components and systems aboard Space Station *Freedom,* the logical approach is to use data obtained from previously flown spacecraft. In order to determine the mechanisms that are driving the failures, models can be proposed, and then checked to see if they adequately fit the observed failure data obtained from a large variety of satellites. For this particular study, failure data and truncation times were available for satellites launched between 1976 and 1984; no data past 1984 was available. The study was limited to electrical subsystems and assemblies, which were studied to determine if they followed a model resulting from a mixture of exponential distributions.

## INTRODUCTION

In order to accurately estimate and reduce the amount of maintenance that will be required on Space Station *Freedom*, it is necessary to understand the mechanisms that cause the failures of its components and systems. Ideally formal life tests would be conducted, where a sample of each type of component would be put on test under environmental and operational conditions identical to those under which it is to be used, and the time to failure for each would be observed. Due to time constraints, it is not always possible to observe all of the items until they fail; in this case some type of censoring mechanism is employed.

There are two basic types of censoring which have been extensively studied. Type I censoring occurs when n items are placed on test and observed for a fixed period of time t. Only the lifetimes of those which fail before time t are known; the others are said to be time censored or *truncated*. In this case the length of the test, t, is fixed, but the number of failures observed, r, is random. In Type II censoring, n items are placed on test and the test is terminated after the $r^{th}$ item fails. In this case, the number of failures, r, is fixed in advance, but the length of time of the test, t, is a random variable. Both of these types of censoring have been treated extensively in the literature. See, for example, Barlow and Proschan (1975), Bain (1978), and Lawless (1982). For information on additional types of censoring, see McCool (1982), and Mann and Singpurwalla (1983).

Unfortunately, the situation encountered in building spacecraft is far from ideal. It is impossible to conduct meaningful life tests on earth because it is not possible to reproduce the microgravity environment. Thus to obtain data on failure of components in microgravity, it is necessary to turn to field data obtained from previously flown spacecraft. The situation is further complicated by the fact that the censoring taking place in this type of situation is neither Type I nor Type II. It is not determined in advance how many components will fail. Furthermore, the truncation times are not known in advance. The lifetime of one component may be truncated by the failure of another--for example, if the attitude control malfunctions, the satellite may fall out of orbit while all of its other components are still functioning; but none of these lifetimes are observable. Also, many components may still be operating at the last time they are observed.

The case where both the number of failures and truncation times are both random variables has not been studied nearly as extensively. In fact, there is no consensus on what it is called. Mann and Singpurwalla (1983) and Lawless (1982) refer to this general case as random censoring, while the entry under "Random Censoring" in the *Encyclopedia of Statistical Sciences, Vol. 1* defines random censoring as a completely different situation. Nevertheless, the particular models given by Mann and Singpurwalla (1983) and Lawless (1982) are not applicable to the satellite data because they assume that failure times are identically distributed.

The approach taken in this study to determine the mechanisms that produce the failures is to propose a mechanism, and then see how well the resulting model fits what is actually observed in the data. If the theoretical survival function differs significantly from the empirical survival function, then the proposed mechanism is not what is actually producing the failures.

## THE SATELLITE DATA

The failure data for over 300 satellites was compiled by Planning Research Corporation and originally published in Bloomquist, et. al. (1978), with an update in 1984. This data is currently being compiled into a data base by Loral Space Information Systems. The data base includes each component of each satellite, classified by subsystem and assembly. The times for all failures are included, and truncation times for those which did not fail.

The way the times were recorded was not consistent for all satellites. Often assemblies are turned off and on during the life of the satellite, so that they are not operating for the entire life of the satellite. Some elements, such as backup systems which are never needed, never get turned on at all. For some of the satellites, the times given were actual operating times for that assembly; for others, they were merely the time since launch, or "survival time." For the purposes of this study, operating times and survival times were treated separately.

Since the data base was still being compiled at the time of this study, it was necessary to limit the number of satellites used in order to obtain some data. Since the more recent satellites are more likely to utilize the same type of technology as Space Station *Freedom*, only satellites launched since 1976 were considered. This was a total of 28 satellites. Of these, only four had recorded operating times; the rest had recorded survival times. The data was further limited by considering only electronic assemblies, because the model considered was proposed for failure of electronic parts.

## THE PROPOSED MODEL

It is generally accepted that the lifetime of electronic components has an exponential distribution, that is

$$f(t|\lambda) = \lambda e^{-\lambda t}, \ t \geq 0,$$

where $\lambda$ is a parameter determined by the failure rate. The exponential distribution possesses a unique property--it has a constant hazard rate. However, when considering many different types of electrical assemblies on different satellites, the odds are that the failure rates will not be the same for all of them. Thus the failure times do not represent a sample from a single exponential distribution, but rather from a mixture of exponentials with varying parameters. While each individual hazard rate is constant, it turns out that the hazard rate of the mixture is actually decreasing (see Barlow and Proschan (1975), p. 102).

Hecht and Hecht (1985) analyzed the original PRC data and concluded that that the failures did indeed possess a decreasing hazard rate. Heydorn, et. al. (1991) proposed a model derived from a mixture of ordinary exponentials and demonstrated that its predictions were close to the results given in Hecht and Hecht (1985). However, they did not have the original data, so their results depended on those given in the Hecht and Hecht report. This study uses the model proposed by Heydorn, et. al. and fits it to the actual satellite data.

The model is obtained by assuming that the electronic components do possess an exponential distribution with parameter $\lambda$. Thus for a fixed $\lambda$ the failures are generated by a Poisson process with parameter $\lambda$. However, $\lambda$ can be considered a random variable since the failure rates are not the same for all components. Taking the Bayesian approach and assuming that the prior distribution of $\lambda$ is uniform on $(0, \infty)$, the posterior distribution of $\lambda$ is a gamma distribution, and the resulting reliability function t is

$$R(t) = \frac{1}{\left(1 + \frac{t}{T}\right)^{\alpha+1}}, \tag{1}$$

where $\alpha$ and T are parameters. The purpose of this study is to find the values of the parameters $\alpha$ and T which best fit the empirical survival function.

## EMPIRICAL SURVIVAL FUNCTION

The next step is to estimate the empirical survival function. If all of the failure times are known, then the estimated survival function is

$$S(t) = \frac{\text{Number of observations} \geq t}{n}, \quad t \geq 0.$$

If, however, some of the survival times are unknown, then this must be modified. Kaplan and Meier (1958) introduced an estimate called the product-limit (PL) estimate which handles the case of random censoring. The estimate is

$$S_{PL}(t) = \prod_{j=1}^{k} \left(1 - \frac{\phi_j}{N-j+1}\right),$$

where k is the total number of events (including both failures and truncations) up to time t, N is the total number of events in the test, and $\phi_j$ is an indicator variable defined by

$$\phi_j = \begin{cases} 0 & \text{if the event is a truncation} \\ 1 & \text{if the event is a failure} \end{cases}.$$

The estimate is a product, where each term can be thought of as the conditional probability of surviving past time $t_j$, given survival to just prior to $t_j$, where $t_j$ is the time of event j. It is a step function, which steps down at each $t_j$. For more information on the properties of the PL estimate, see Lawless (1982) and Peterson (1983).

The problem encountered with using the PL estimate of the reliability function is that when many of the largest times are truncations, none of the terms in the product approach zero, and hence the estimate yields only a small portion of the reliability function. For example, in the data set for satellite operating times, there were a total of 165 observations, of which 14 were failures; the rest were truncations. Furthermore, the 82 largest times were truncations. This meant that the smallest term in product was 82/83. The PL estimate of the survival function for the operating time data is:

| $t$ | $S_{PL}(t)$ |
|---|---|
| 50 | 0.9939 |
| 70 | 0.9878 |
| 76 | 0.9817 |
| 264 | 0.9756 |
| 1440 | 0.9633 |
| 3890 | 0.9572 |
| 4300 | 0.9447 |
| 10500 | 0.9367 |
| 14980 | 0.9285 |
| 20747 | 0.9203 |
| 29361 | 0.9120 |
| 34526 | 0.9009 |

The data set for satellite survival times contained 907 points, of which the last 154 were truncations. The PL estimate of the survival function for this data ranged from 0.9989 to 0.8695.

## ESTIMATING THE PARAMETERS

To find the estimates of $\alpha$ and T in Equation (1) which give the best fit to the PL estimate of the survival function, the first approach was to use the least squares criterion. Since (1) is a nonlinear function, the nonlinear least squares routine in S-plus was used. Unfortunately, the Jacobian matrix was nearly rank deficient, for a multitude of initial values, including the estimates derived in the other approach described below. Thus another approach had to be used to estimate the parameters.

While the survival function given in (1) is nonlinear, it turns out that the inverse of the hazard function associated with it is a linear function. For a survival function S(t), the hazard function is

$$h(t) = -\frac{d\ln S(t)}{dt}.$$

The hazard function associated with the survival function given in (1) is

$$h(t) = \frac{\alpha + 1}{T + t},$$

and its inverse can be expressed as the linear function

$$\frac{1}{h(t)} = \frac{T}{\alpha + 1} + \left(\frac{1}{\alpha + 1}\right) t. \tag{2}$$

Thus the parameters $\alpha$ and T can be estimated by finding the empirical hazard function, taking its inverse, and using least squares to estimate the parameters of the resulting linear equation. These coefficients can then be used to solve for $\alpha$ and T in (2).

The empirical hazard function was estimated by taking the log of the PL estimate of the survival function, then taking successive differences in the resulting values and dividing by the width of the corresponding time interval to approximate the derivative. A linear model was then fit to the inverse of the empirical hazard function, and the parameters $\alpha$ and T were determined.

## RESULTS

To begin the analysis, recall that if the mixed exponential model is correct, the failures should have a decreasing hazard function. Plots of the empirical hazard function for the operating time data and survival time data are given in Figures 1 and 2, respectively. Both plots are extremely noisy, and neither one definitely indicates a decreasing hazard function. No conclusions can be drawn from these plots.

Plots of the empirical survival function and estimated theoretical survival function for the operating time and survival time data are given in Figures 3 and 4, respectively. It can be seen that neither of these demonstrates a very good fit. In particular, the empirical survival function initially drops at a much faster rate than the theoretical one for both sets of data. The data appears to display early failures, or "infant mortality" that is not adequately explained by the model. Based on these plots, the mixed exponential model does not seem to adequately explain the failures. A different model must be sought.

In conclusion, this research indicates that a mixture of exponential distributions does not adequately explain electronic failures seen in previously flown satellites. In particular, it does not model the early failures very well. A different model will be needed to explain the mechanism generating the failures.

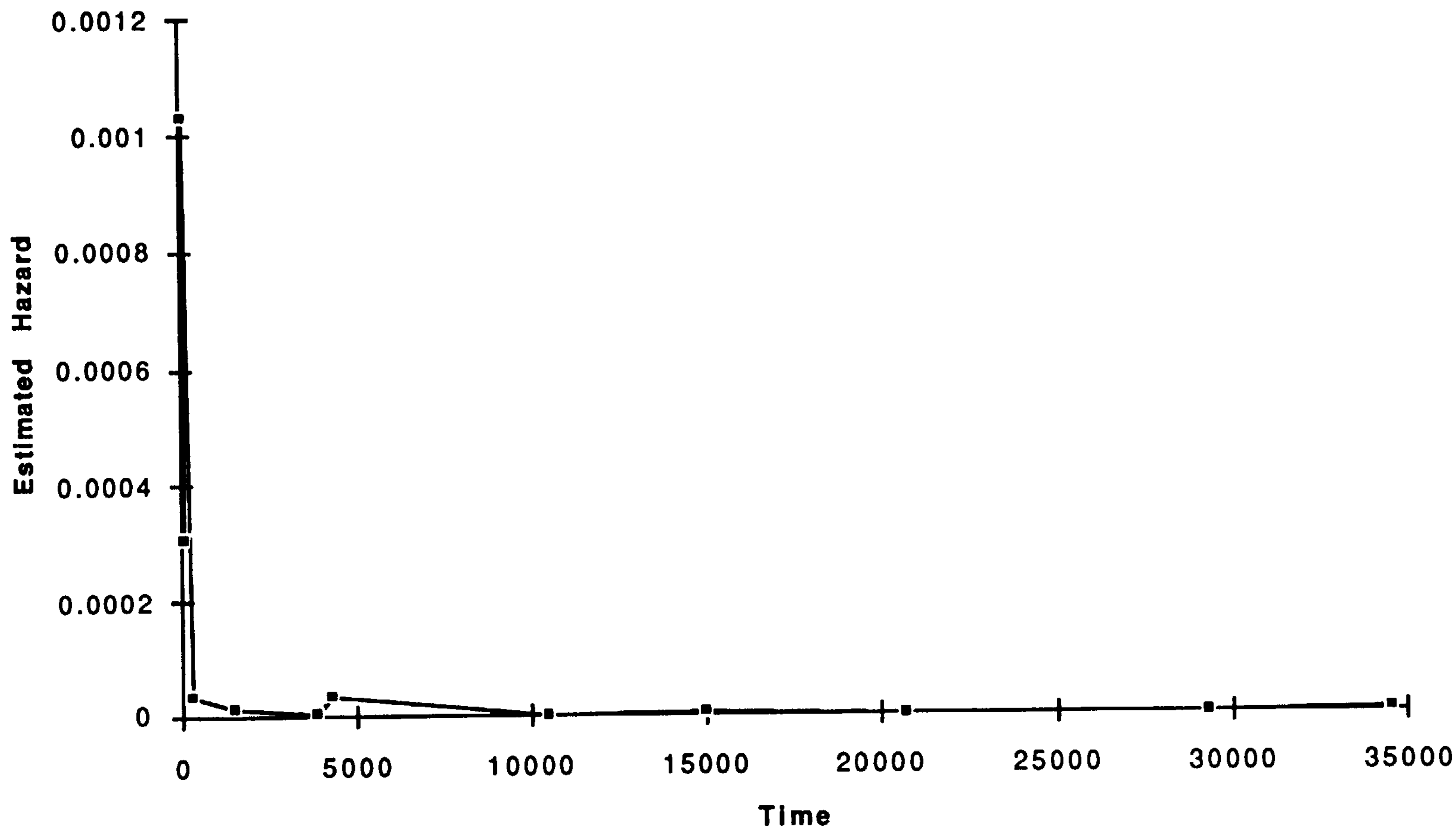


Figure 1.- Empirical hazard function of operating time data

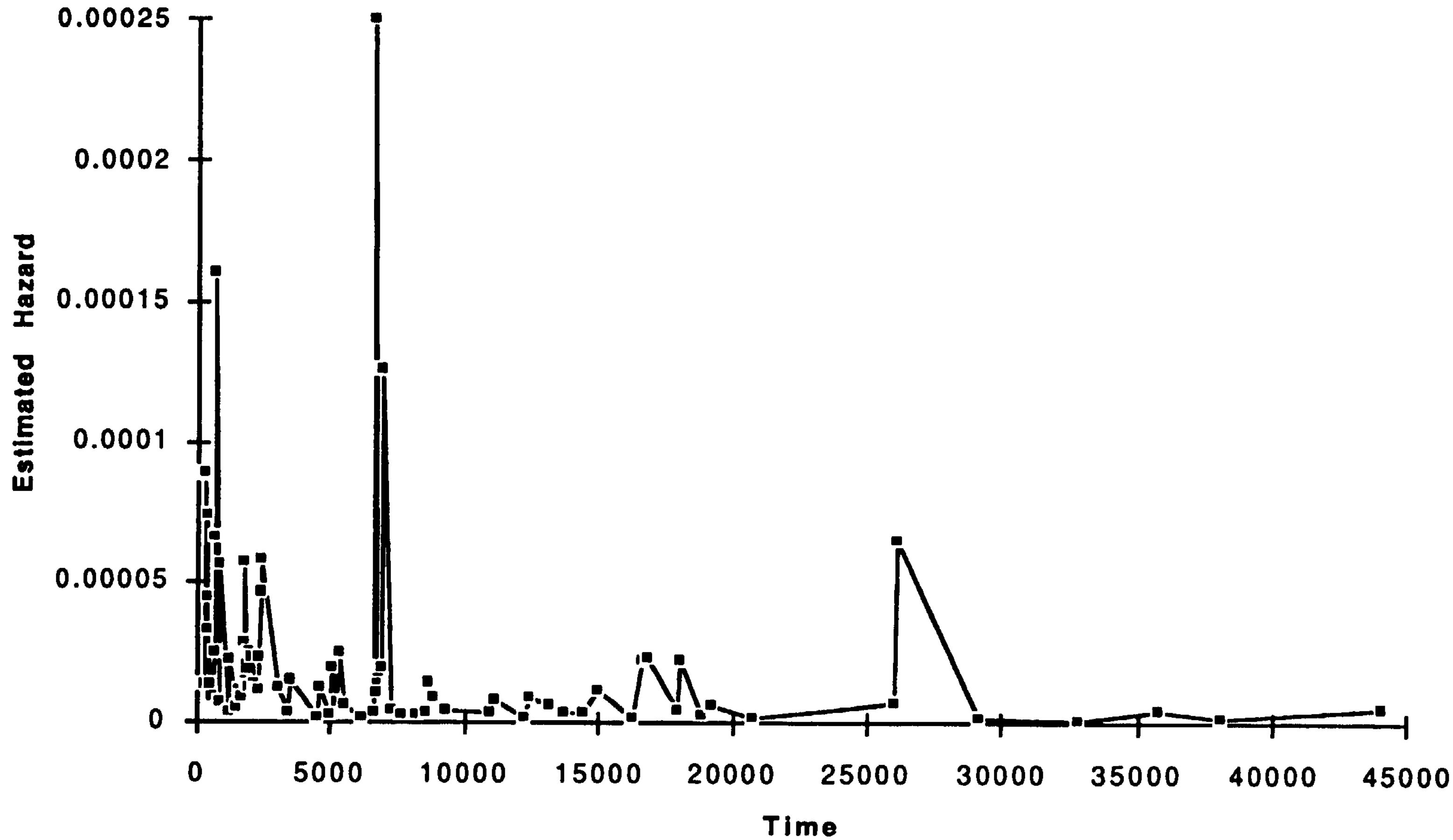


Figure 2.- Empirical hazard function of survival time data

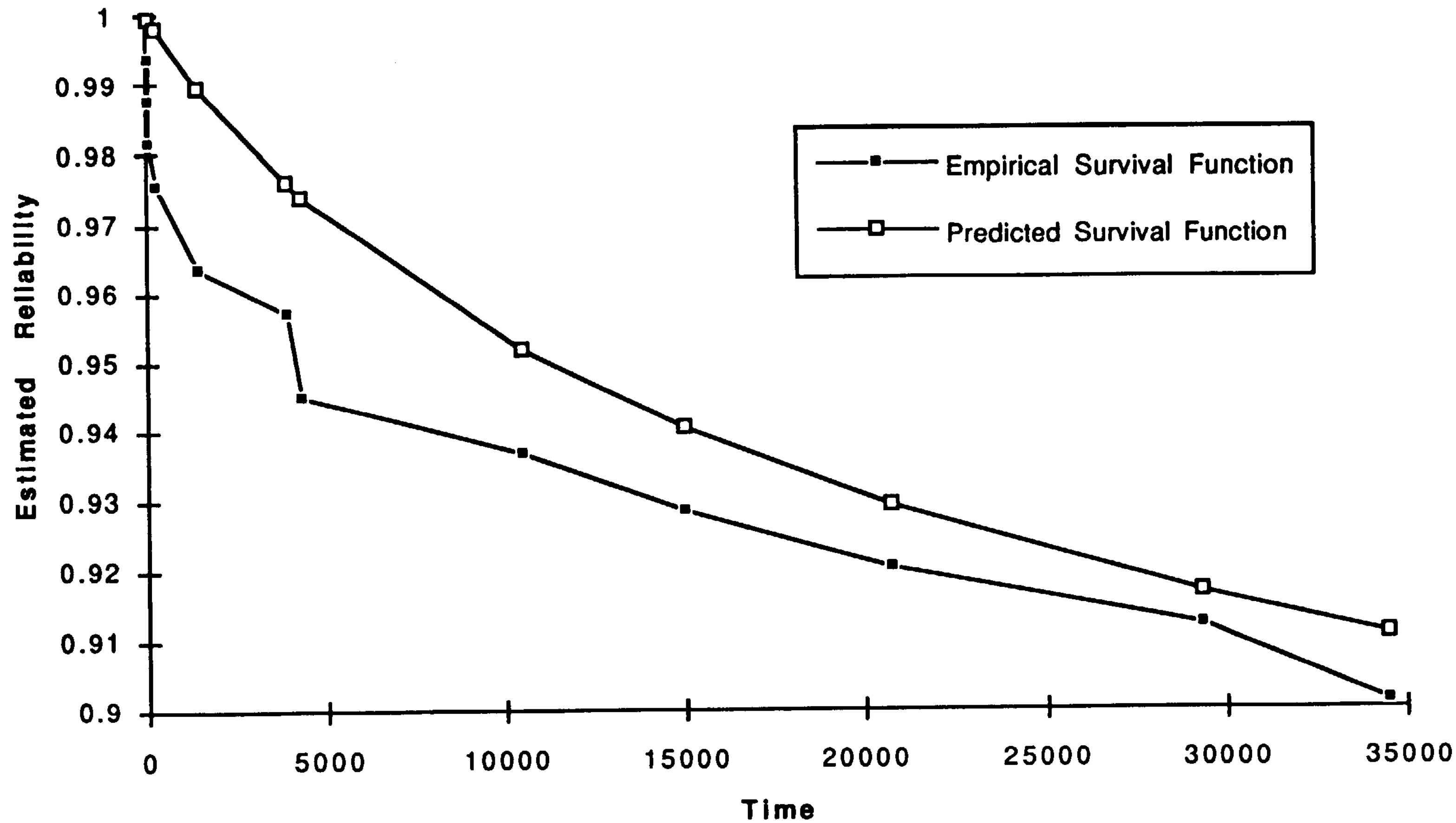


Figure 3.- Estimated survival functions of operating time data

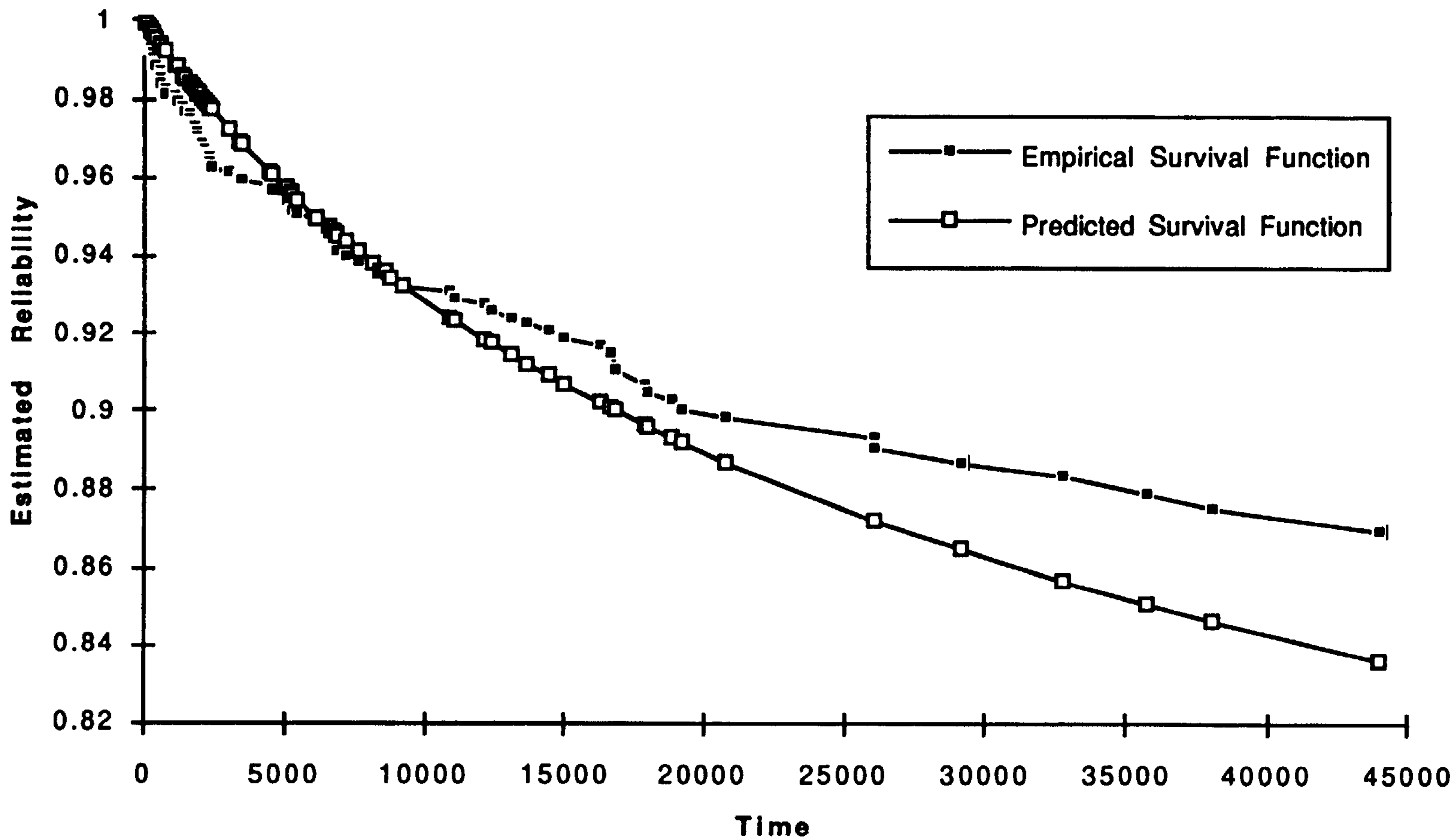


Figure 4.- Estimated survival function of survival time data

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