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Propagation of Experimental Uncertainties from the Tunnel to the Body Coordinate System in 3-D LDV Flow Field Studies

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Summary

This report provides an analysis of experimental laser Doppler velocimetry (LDV) data uncertainties that propagate from measurements in the tunnel coordinate system to results in the model system. Calculations of uncertainties as functions of the variables that comprise the final result requires assessment of the contribution each variable makes. Such an analysis enables and necessitates the experimentalists to identify and address the contributing error sources in the experimental measurement system. This provides an opportunity to improve the quality of data derived from experimental systems. This is especially important in experiments where small changes in test conditions are expected to produce small, detectable changes in results. In addition, the need for high-quality experimental data for CFD method validation demands a thorough assessment of experimental uncertainty.

Transforming from one Cartesian coordinate system to another by three sequential rotations, equations were developed to transform the variables initially obtained in the original coordinates into variables in the final coordinate system. Based on the transformation equations, propagation equations for errors in the experimentally-derived flow quantities were derived for a model at angle of attack. Experimental uncertainties were propagated from the tunnel coordinate system into the model system. Comparisons between results for the two systems revealed a variety of increases and decreases in bias and precision errors.

Introduction

Changes in experimental test conditions, with expected changes in test results, require high repeatability of experimental data and estimates of experimental uncertainties. For example, the validation of computational fluid dynamics (CFD) methods requires experimental data of high quality with the best estimates of experimental uncertainties. Laser Doppler velocimetry (LDV) is frequently used in wind and water tunnels to make measurements in off-body flow fields. This technique provides accurate measurements of flow mean and fluctuating velocities from which various turbulence relationships, e.g., Reynolds normal and shear stresses, can be calculated.

Many publications of experimental LDV results include an analysis of expected errors or uncertainty in some form. Several documents are dedicated specifically to error or uncertainty analysis. References 1 - 3 are examples of such analyses. Experimental results are often transformed into coordinate systems other than those in which they were measured. For example, if one or more of the axes in a multi-axis LDV system are not congruent with the tunnel axes, the measured quantities must be resolved into the tunnel system. In addition, if the model being tested is at an angle relative to the tunnel coordinate system, the measured quantities must also be transformed into the model coordinate system.

The propagation of experimental uncertainties through the various coordinate transformations is required to provide accurate estimates of uncertainties in the final, transformed experimental results. Systematic procedures for propagating uncertainty have been described by several authors. One of the most current references in this area is by Coleman and Steele (reference 4).

This report addresses the propagation of experimental uncertainties in LDV flow field measurements from the tunnel coordinate system into the model coordinate system. The purpose is to provide a systematic way to transform contributing errors of various calculated flow quantities into final results.

Symbols and Abbreviations

a, A	transformation matrix elements
ANSI	American National Standards Institute
ASME	American Society of Mechanical Engineers
b, B	transformation matrix elements
B	bias limit
c, C	transformation matrix elements
d, D	transformation matrix elements
$\hat{e}$	unit vector
LDV	laser doppler velocimetry
N	number of samples
P	precision limit, $P = t_{95}S$
r	result in an uncertainty analysis
RSS	root-sum-square
S	precision index
t_{95}	statistical t-distribution value giving 95% confidence level
U	uncertainty or velocity, in/sec
u,v,w	total instantaneous velocities, in/sec
$\overline{u}$, $\overline{v}$, $\overline{w}$	mean velocity components, in/sec
u',v',w'	instantaneous velocity fluctuations, in/sec
$\sqrt{\overline{u'^2}}$, $\sqrt{\overline{v'^2}}$, $\sqrt{\overline{w'^2}}$	rms velocity fluctuations, in/sec
$\overline{u'u'}$, $\overline{v'v'}$, $\overline{w'w'}$	Reynolds normal stresses, in^2/sec^2
$\overline{u'v'}$, $\overline{v'w'}$, $\overline{w'u'}$	Reynolds shear stresses, in^2/sec^2

x	variable in uncertainty analysis
x, y, z	cartesian coordinates, in
α	angle of attack, deg
β	sideslip angle, deg
Δ	implies "uncertainty" when preceeding quantities in uncertainty analysis
δ_{ij}	Kronecker delta
ϕ	roll angle, deg
ρ	correlation coefficient between variables in uncertainty analysis

subscripts:

B_{ij}	correlation coefficient for bias errors, eq (2)
i or j	elemental component
ij	matrix element indices
j	jet
LDV	LDV coordinate system
P_{ij}	correlation coefficient for precision errors, equation (2)
r	result
rms	root mean square
RSS	root-sum-square
t	tunnel
x	variable
x, y, z	unit vector axes
∞	free stream condition
1, 2, 3	intermediate axis systems

superscripts:

'	fluctuating quantity
‾	mean quantity
$\rightarrow$	vector
^	unit vector

Analysis of Experimental Uncertainties

The concepts of bias and precision errors are the fixed and random errors, respectively, that occur in experimentation. The bias errors are fixed, systematic or constant errors that induce an offset from the true value of the quantity being measured. The precision errors are random variation or repeatability errors of the quantity being measured. The total uncertainty is then formed by a combination of a bias limit and a precision error estimate. The bias limit, B, is an estimated limit "of a confidence interval on the true value of the bias" (reference 4). The combination technique used in the present study is the root-sum-square (RSS) method, where the total uncertainty is given by,

$$U_{RSS} = (B^2 + P_x^2)^{1/2},$$

and where B is the bias limit estimate and P_X is the precision limit. The precision limit is the product of the precision index (precision error estimate) and t_{95}, the value of t from the statistical t distribution that gives a 95% confidence level to the estimate of the precision (random) error. For experimental sample sizes greater than 30, the standard in reference 5 recommends using a value for t_{95} of 2 for 95% confidence estimates. The equation for the total uncertainty becomes,

$$U_{RSS} = [\ B^2 + (2S)^2\ ]^{1/2}, \tag{1}$$

where S is the precision error estimate or precision index.

Propagation of Uncertainty

Coleman and Steele (reference 4) describe the propagation of uncertainty in a general uncertainty analysis. If a result, r, is a function of many variables (x_1, x_2, x_3, . . . , x_N), the uncertainty, U_r, in a calculated result is

$$U_r = \left[\left(\frac{\partial r}{\partial x_1}U_{x_1}\right)^2 + \left(\frac{\partial r}{\partial x_2}U_{x_2}\right)^2 + \cdots + \left(\frac{\partial r}{\partial x_N}U_{x_N}\right)^2\right]^{\frac{1}{2}}.$$

The U_x's are the uncertainties in the variables, x_i. In a detailed uncertainty analysis, the bias and precision limit estimates must be propagated separately. Then the estimates are combined into the total uncertainty (using the RSS method in this case). If the bias limits (or precision limits) of the different variables are not independent of each other, there are cross terms in the expressions for the bias and precision limits as follows.

$$B_r = \left\{\sum_{i=1}^{N}\left[\left(\frac{\partial r}{\partial x_i}B_{x_i}\right)^2 + \sum_{j=1}^{N}\frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j}B_{x_i}B_{x_j}\rho_{B_{ij}}\left(1-\delta_{ij}\right)\right]\right\}^{\frac{1}{2}}, \text{ and}$$

$$P_r = \left\{ \sum_{i=1}^{N} \left[\left(\frac{\partial r}{\partial x_i} P_{x_i} \right)^2 + \sum_{j=1}^{N} \frac{\partial r}{\partial x_i} \frac{\partial r}{\partial x_j} P_{x_i} P_{x_j} \rho_{P_{ij}} \left(1 - \delta_{ij}\right) \right] \right\}^{\frac{1}{2}}, \qquad (2)$$

where the ρ's are correlation coefficients associated with each type of error and δ_{ij} is one if $i = j$ and zero if $i \neq j$. If the error terms are independent (i.e., the errors in the measurements of the variables are independent), the cross terms dissapear ($\rho = 0$) in equations (2).

The bias limit for r is

$$B_r = \left[\left(\frac{\partial r}{\partial x_1} B_{x_1} \right)^2 + \left(\frac{\partial r}{\partial x_2} B_{x_2} \right)^2 + \cdots + \left(\frac{\partial r}{\partial x_N} B_{x_N} \right)^2 \right]^{\frac{1}{2}},$$

and the precision limit for r is

$$P_r = \left[\left(\frac{\partial r}{\partial x_1} P_{x_1} \right)^2 + \left(\frac{\partial r}{\partial x_2} P_{x_2} \right)^2 + \cdots + \left(\frac{\partial r}{\partial x_N} P_{x_N} \right)^2 \right]^{\frac{1}{2}},$$

where the B_X's and the P_X's are the bias and precision limits for the measured variables, x_i. The equation for the propagated precision error estimate (precision index) is

$$S_r = \left[\left(\frac{\partial r}{\partial x_1} S_{x_1} \right)^2 + \left(\frac{\partial r}{\partial x_2} S_{x_2} \right)^2 + \cdots + \left(\frac{\partial r}{\partial x_N} S_{x_N} \right)^2 \right]^{\frac{1}{2}},$$

where the S_X's are the estimated precision errors in the measured results.

In a data acquisition program, the results for the measurement instrument may be modified by at least one coordinate transformation to yield final results. For example, there may be a transformation from the LDV coordinate system to the tunnel coordinate system, if one or more of the LDV axes are not aligned with the tunnel axes. Then, if the model is at an angle or angles relative to the tunnel, there is a transformation to the model coordinate system. Therefore, the uncertainty analysis is required to account for this transformation. The propagation of the uncertainties into the calculated results, including coordinate transformation, was carried out in a manner similar to that used by Neuhart, et al (reference 6), where the transformation from the LDV coordinate system to the tunnel system was developed.

The description of the development of the coordinate transformation equations, from the tunnel to the model coordinate system, are described in the following section. Next, the method for estimating uncertainties in the final, transformed results will be shown. Finally, an example calculation is given to demonstrate the process. Uncertainties in the calculated quantities from reference 6, in the tunnel coordinate system for a forebody/strake model at angle of attack, are listed in table 1. The results of propagation of the uncertainties into the model coordinate system are listed in table 2. The tabulated results show the propagated precision error estimates (precision indicies) instead of the precision limit. The total uncertainty, however, does contain the precision limit, $P = t_{95}S$. All quantities calculated were normalized by appropriate terms, as shown in the tables. Final estimated uncertainties in table 2 are presented with two significant figure accuracy since more significant figures would be inappropriate for such estimates.

Coordinate Transformation

The derivation of the transformation equations follows the development of Neuhart, et al., (reference 6), which was based on the work of Morrison, et al, in reference 7. In figure 1, the x-y-z axes shown form the orthogonal, model coordinate system. The total velocity vector defined in this system would be

$$\bar{U} = u\hat{e}_x + v\hat{e}_y + w\hat{e}_z,$$

where the $\hat{e}_x, \hat{e}_y, \hat{e}_z$ vector quantities represent unit vectors in the x, y, and z directions, respectively. This represents the velocity defined in the model coordinate system. The velocity defined in the tunnel coordinate system (x_1, y_1, z_1 axes in figure 1) would be represented generally as

$$\bar{U}_t = u_t\hat{e}_{x_1} + v_t\hat{e}_{y_1} + w_t\hat{e}_{z_1},$$

where the $\hat{e}_{x_1}, \hat{e}_{x_1}, \hat{e}_{x_1}$ quantities are the unit vectors in the tunnel system coordinate directions. In general, the two coordinate systems have the same origin, but the axes are not coincident. By successive rotations (similar to Euler rotations), the transformation equations between the tunnel and the model at sideslip, angle of attack, and/or roll can be obtained.

Beginning with the x_1, y_1, z_1 axes in figure 1, a rotation is made about the z_1 axis (sideslip, β). As shown in figure 2, the relationship between the respective unit vectors can be represented by

$$\hat{e}_{x_1} = \hat{e}_{x_2}\cos\beta - \hat{e}_{y_2}\sin\beta$$
$$\hat{e}_{y_1} = \hat{e}_{x_2}\sin\beta + \hat{e}_{y_2}\cos\beta,$$
$$\hat{e}_{z_1} = \hat{e}_{z_2}$$

or in matrix form,

$$\begin{bmatrix}\hat{e}_{x_1}\\ \hat{e}_{y_1}\\ \hat{e}_{z_1}\end{bmatrix} = \begin{bmatrix}\cos\beta & -\sin\beta & 0\\ \sin\beta & \cos\beta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\hat{e}_{x_2}\\ \hat{e}_{y_2}\\ \hat{e}_{z_2}\end{bmatrix}.$$

A subsequent rotation (figure 3) about the y_2 axis (angle of attack, α) yields,

$$\hat{e}_{x_2} = \hat{e}_{x_3}\cos\alpha + \hat{e}_{z_3}\sin\alpha$$
$$\hat{e}_{y_2} = \hat{e}_{y_3},$$
$$\hat{e}_{z_2} = -\hat{e}_{x_3}\sin\alpha + \hat{e}_{z_3}\cos\alpha$$

or,

$$\begin{bmatrix}\hat{e}_{x_2}\\ \hat{e}_{y_2}\\ \hat{e}_{z_2}\end{bmatrix} = \begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ 0 & 1 & 0\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}\begin{bmatrix}\hat{e}_{x_3}\\ \hat{e}_{y_3}\\ \hat{e}_{z_3}\end{bmatrix}.$$

A final rotation about the x_3 axis (roll, ϕ, figure 4) results in

$$\hat{e}_{x_3} = \hat{e}_{x}$$
$$\hat{e}_{y_3} = \hat{e}_{y}\cos\phi - \hat{e}_{z}\sin\phi,$$
$$\hat{e}_{z_3} = \hat{e}_{y}\sin\phi + \hat{e}_{z}\cos\phi$$

or,

$$\begin{bmatrix}\hat{e}_{x_3}\\ \hat{e}_{y_3}\\ \hat{e}_{z_3}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi & \cos\phi\end{bmatrix}\begin{bmatrix}\hat{e}_{x}\\ \hat{e}_{y}\\ \hat{e}_{z}\end{bmatrix}.$$

The relationship between the unit vectors in the tunnel coordinate system (x_1, y_1, z_1) and those in the model system (x,y,z) is, by substitution for intermediate unit vectors, the product of the three matricies.

$$\begin{bmatrix} \hat{e}_{x_1} \\ \hat{e}_{y_1} \\ \hat{e}_{z_1} \end{bmatrix} = \begin{bmatrix} \hat{e}_{x_t} \\ \hat{e}_{y_t} \\ \hat{e}_{z_t} \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \hat{e}_x \\ \hat{e}_y \\ \hat{e}_z \end{bmatrix}$$

Therefore, the relationship between the velocity vectors is

$$\begin{bmatrix} u_t \\ v_t \\ w_t \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix},$$

or in shorthand form,

$$\begin{bmatrix} u_t \\ v_t \\ w_t \end{bmatrix} = [A_{ij}][B_{ij}][C_{ij}] \begin{bmatrix} u \\ v \\ w \end{bmatrix} = [D_{ij}] \begin{bmatrix} u \\ v \\ w \end{bmatrix}.$$

Since the desired result will be in the model coordinate system, the inverse process will be used. This process is defined by

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = [C_{ij}]^{-1}[B_{ij}]^{-1}[A_{ij}]^{-1} \begin{bmatrix} u_t \\ v_t \\ w_t \end{bmatrix} = [D_{ij}]^{-1} \begin{bmatrix} u_t \\ v_t \\ w_t \end{bmatrix}. \qquad (3)$$

Utilizing the rule that the product of a matrix and it's inverse equals the identity matrix, the terms in each rotation matrix were determined. The first equation, for sideslip, is shown below.

$$\begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{32} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Multiplying the terms to form nine equations, the a_{ij} terms were determined. The solution matrix for sideslip, a_{ij}, is

$$\begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The above inverse matrix, a_{ij}, was verified by multiplying it by the original, A_{ij}, matrix to yield the identity matrix.

The transformation matrix for angle of attack was derived using the following equation.

$$\begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{32} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The solution matrix for angle of attack, b_{ij}, is

$$\begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}.$$

Finally, the transformation matrix for model roll was obtained using

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{32} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The transformation matrix for roll, c_{ij}, is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}.$$

The resulting full transformation from the tunnel to body coordinate system is given by equation (3) as

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_t \\ v_t \\ w_t \end{bmatrix}.$$

The three separate matricies can be combined to form a single matrix for the three body rotations.

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ \sin\alpha\cos\beta\sin\phi - \sin\beta\cos\phi & \sin\alpha\sin\beta\sin\phi + \cos\beta\cos\phi & \cos\alpha\sin\phi \\ \sin\alpha\cos\beta\cos\phi + \sin\beta\sin\phi & \sin\alpha\sin\beta\cos\phi - \cos\beta\sin\phi & \cos\alpha\cos\phi \end{bmatrix} \begin{bmatrix} u_t \\ v_t \\ w_t \end{bmatrix}$$

or in short form,

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \left[D_{ij}\right]^{-1} \begin{bmatrix} u_t \\ v_t \\ w_t \end{bmatrix} = \left[d_{ij}\right] \begin{bmatrix} u_t \\ v_t \\ w_t \end{bmatrix}. \qquad (4)$$

Since the total instantaneous velocities are the sum of a mean and an instantaneous fluctuating velocity,

$$u = \overline{u} + u'$$
$$v = \overline{v} + v' \quad ,$$
$$w = \overline{w} + w'$$

then the following transformations exist.

$$\begin{bmatrix} \overline{u} \\ \overline{v} \\ \overline{w} \end{bmatrix} = \left[d_{ij}\right] \begin{bmatrix} \overline{u}_t \\ \overline{v}_t \\ \overline{w}_t \end{bmatrix}, \qquad (5)$$

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \left[d_{ij}\right] \begin{bmatrix} u_t' \\ v_t' \\ w_t' \end{bmatrix}.$$

From the definition of the Reynolds normal and shear stresses,

$$\overline{u'u'} = \frac{\sum_{i=1}^{N} u_i u_i}{N} - \overline{u}\,\overline{u} \quad , \qquad (6)$$

and

$$\overline{u'v'} = \frac{\sum_{i=1}^{N} u_i v_i}{N} - \overline{u}\,\overline{v} \quad , \qquad (7)$$

where u_i and v_i are total instantaneous velocity values. Substituting the matrix equations (4) and (5) for the u_i and $\overline{u}$ values into equation (6) yields the following transformation equation for the $\overline{u'u'}$ Reynolds normal stress.

$$\overline{u'u'} = \frac{\sum_{i=1}^{N}\begin{pmatrix} d_{11}d_{11}u_t u_t + d_{11}d_{12}u_t v_t + d_{11}d_{13}u_t w_t \\ +d_{12}d_{11}v_t u_t + d_{12}d_{12}v_t v_t + d_{12}d_{13}v_t w_t \\ +d_{13}d_{11}w_t u_t + d_{13}d_{12}w_t v_t + d_{13}d_{13}w_t w_t \end{pmatrix}}{N}$$

$$-\begin{pmatrix} d_{11}d_{11}\overline{u_t}\,\overline{u_t} + d_{11}d_{12}\overline{u_t}\,\overline{v_t} + d_{11}d_{13}\overline{u_t}\,\overline{w_t} \\ +d_{12}d_{11}\overline{v_t}\,\overline{u_t} + d_{12}d_{12}\overline{v_t}\,\overline{v_t} + d_{12}d_{13}\overline{v_t}\,\overline{w_t} \\ +d_{13}d_{11}\overline{w_t}\,\overline{u_t} + d_{13}d_{12}\overline{w_t}\,\overline{v_t} + d_{13}d_{13}\overline{w_t}\,\overline{w_t} \end{pmatrix}.$$

Substituting for the u_i, v_i, $\overline{u}$, and $\overline{v}$ quantities in equation (7) from equations (4) and (5) gives the transformation equation for the $\overline{u'v'}$ Reynolds shear stress.

$$\overline{u'v'} = \frac{\sum_{i=1}^{N}\begin{pmatrix} d_{11}d_{21}u_t u_t + d_{11}d_{22}u_t v_t + d_{11}d_{23}u_t w_t \\ +d_{12}d_{21}v_t u_t + d_{12}d_{22}v_t v_t + d_{12}d_{23}v_t w_t \\ +d_{13}d_{21}w_t u_t + d_{13}d_{22}w_t v_t + d_{13}d_{23}w_t w_t \end{pmatrix}}{N}$$

$$-\begin{pmatrix} d_{11}d_{21}\overline{u_t}\,\overline{u_t} + d_{11}d_{22}\overline{u_t}\,\overline{v_t} + d_{11}d_{23}\overline{u_t}\,\overline{w_t} \\ +d_{12}d_{21}\overline{v_t}\,\overline{u_t} + d_{12}d_{22}\overline{v_t}\,\overline{v_t} + d_{12}d_{23}\overline{v_t}\,\overline{w_t} \\ +d_{13}d_{21}\overline{w_t}\,\overline{u_t} + d_{13}d_{22}\overline{w_t}\,\overline{v_t} + d_{13}d_{23}\overline{w_t}\,\overline{w_t} \end{pmatrix}$$

Similar types of derivations were done for the other two normal stresses and two shear stresses, and are not shown here. Using the rule that the summation of a sum of terms equals the sum of the summations of the individual terms $\left(\sum(a+b+c) = \sum a + \sum b + \sum c\right)$ and rearranging terms yields transformation equations that are a function of normal and shear stresses in the tunnel coordinate system. The result is a set of transformation equations that are expressed as the following matrix equation.

$$
\begin{bmatrix} \overline{u'u'} \\ \overline{v'v'} \\ \overline{w'w'} \\ \overline{u'v'} \\ \overline{w'u'} \\ \overline{v'w'} \end{bmatrix} = \begin{bmatrix} d_{11}d_{11} & d_{12}d_{12} & d_{13}d_{13} & 2d_{11}d_{12} & 2d_{11}d_{13} & 2d_{12}d_{13} \\ d_{21}d_{21} & d_{22}d_{22} & d_{23}d_{23} & 2d_{21}d_{22} & 2d_{21}d_{23} & 2d_{22}d_{23} \\ d_{31}d_{31} & d_{32}d_{32} & d_{33}d_{33} & 2d_{31}d_{32} & 2d_{31}d_{33} & 2d_{32}d_{33} \\ d_{11}d_{21} & d_{12}d_{22} & d_{13}d_{23} & d_{11}d_{22}+d_{12}d_{21} & d_{11}d_{23}+d_{13}d_{21} & d_{12}d_{23}+d_{13}d_{22} \\ d_{31}d_{11} & d_{32}d_{12} & d_{33}d_{13} & d_{31}d_{12}+d_{32}d_{11} & d_{31}d_{13}+d_{33}d_{11} & d_{32}d_{13}+d_{33}d_{12} \\ d_{21}d_{31} & d_{22}d_{32} & d_{23}d_{33} & d_{21}d_{32}+d_{22}d_{31} & d_{21}d_{33}+d_{23}d_{31} & d_{22}d_{33}+d_{23}d_{32} \end{bmatrix} \begin{bmatrix} \overline{u_t'u_t'} \\ \overline{v_t'v_t'} \\ \overline{w_t'w_t'} \\ \overline{u_t'v_t'} \\ \overline{w_t'u_t'} \\ \overline{v_t'w_t'} \end{bmatrix}
$$

In an uncertainty analysis, the contribution of the bias and precision limits of each variable determines the total uncertainty of the result. Clearly, each resultant Reynolds stress error term can be can be a function of up to nine variables. These are the six tunnel system stresses plus the three rotation angles in the d_{ij} matricies. For simplicity, we will consider the most common case where $\alpha \neq 0$ and $\beta = \phi = 0$. The resulting transformation matrix for the mean velocities is

$$
\left[d_{ij}\right] = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix},
$$

and for the Reynolds normal and shear stresses,

$$
\begin{bmatrix} \overline{u'u'} \\ \overline{v'v'} \\ \overline{w'w'} \\ \overline{u'v'} \\ \overline{w'u'} \\ \overline{v'w'} \end{bmatrix} = \begin{bmatrix} \cos^2\alpha & 0 & \sin^2\alpha & 0 & -2\cos\alpha\sin\alpha & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sin^2\alpha & 0 & \cos^2\alpha & 0 & 2\cos\alpha\sin\alpha & 0 \\ 0 & 0 & 0 & \cos\alpha & 0 & -\sin\alpha \\ \cos\alpha\sin\alpha & 0 & -\cos\alpha\sin\alpha & 0 & \cos^2\alpha-\sin^2\alpha & 0 \\ 0 & 0 & 0 & \sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} \overline{u_t'u_t'} \\ \overline{v_t'v_t'} \\ \overline{w_t'w_t'} \\ \overline{u_t'v_t'} \\ \overline{w_t'u_t'} \\ \overline{v_t'w_t'} \end{bmatrix}.
$$

The resulting equations for the transformed mean and instantaneous fluctuating velocities are

$$\begin{aligned}\overline{u} &= \overline{u_t}\cos\alpha - \overline{w_t}\sin\alpha \\ \overline{v} &= \overline{v_t} \\ \overline{w} &= \overline{u_t}\sin\alpha + \overline{w_t}\cos\alpha\end{aligned}\quad , \tag{8}$$

and

$$\begin{aligned}u' &= u_t'\cos\alpha - w_t'\sin\alpha \\ v' &= v_t' \\ w' &= u_t'\sin\alpha + w_t'\cos\alpha\end{aligned}\quad .$$

The transformation equations for the Reynolds stresses are

$$\left.\begin{aligned}\overline{u'u'} &= \overline{u_t'u_t'}\cos^2\alpha + \overline{w_t'w_t'}\sin^2\alpha - 2\overline{w_t'u_t'}\cos\alpha\sin\alpha \\ \overline{v'v'} &= \overline{v_t'v_t'} \\ \overline{w'w'} &= \overline{u_t'u_t'}\sin^2\alpha + \overline{w_t'w_t'}\cos^2\alpha + 2\overline{w_t'u_t'}\cos\alpha\sin\alpha\end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned}\overline{u'v'} &= \overline{u_t'v_t'}\cos\alpha - \overline{v_t'w_t'}\sin\alpha \\ \overline{w'u'} &= \overline{u_t'u_t'}\cos\alpha\sin\alpha - \overline{w_t'w_t'}\cos\alpha\sin\alpha + \overline{w_t'u_t'}(\cos^2\alpha - \sin^2\alpha) \\ \overline{v'w'} &= \overline{u_t'v_t'}\sin\alpha + \overline{v_t'w_t'}\cos\alpha\end{aligned}\right\}. \tag{10}$$

The transformed rms fluctuating velocities u_{rms}, v_{rms}, and w_{rms} are calculated as the square root of the transformed $\overline{u'u'}$, $\overline{v'v'}$, and $\overline{w'w'}$, respectively, after transformation. This is done since, for example,

$$u_{rms} = \sqrt{\overline{u'^2}} = \sqrt{\overline{(u_i - \overline{u})^2}} = \left[\frac{\sum_{i=1}^{N}(u_i - \overline{u})^2}{N}\right]^{1/2} = \sqrt{\overline{u'u'}}. \tag{11}$$

Also, therefore,

$$v_{rms} = \sqrt{\overline{v'v'}},$$
$$w_{rms} = \sqrt{\overline{w'w'}}.$$

The transformation equations for the rms velocity fluctuations involve the square root of transformation equations (9) for $\overline{u'u'}$, $\overline{v'v'}$, and $\overline{w'w'}$. These equations contain Reynolds normal and shear stresses in the tunnel coordinate system (see equations (9)). However, according to equation (11), $\overline{u_t'u_t'} = {u_{t_{rms}}}^2$, and also $\overline{v_t'v_t'} = {v_{t_{rms}}}^2$ and $\overline{w_t'w_t'} = {w_{t_{rms}}}^2$. Therefore, using equations (9) and (11),

$$\left.\begin{aligned} u_{rms} &= \sqrt{\overline{u'u'}} = \left({u_{t_{rms}}}^2 \cos^2\alpha + {w_{t_{rms}}}^2 \sin^2\alpha - 2\overline{w_t'u_t'}\cos\alpha\sin\alpha\right)^{1/2} \\ v_{rms} &= \sqrt{\overline{v'v'}} = \sqrt{\overline{v_t'v_t'}} \\ w_{rms} &= \sqrt{\overline{w'w'}} = \left({u_{t_{rms}}}^2 \sin^2\alpha + {w_{t_{rms}}}^2 \cos^2\alpha + 2\overline{w_t'u_t'}\cos\alpha\sin\alpha\right)^{1/2} \end{aligned}\right\}. \quad (12)$$

Propagation of Uncertainty into the Model Coordinate System

Refering to the transformation equations (8) for the mean velocities, the equations for the propagation of the bias limits and precision indices (errors) can be derived using equations (2). Since

$$\overline{v} = \overline{v_t},$$

and

$$\frac{\partial \overline{v}}{\partial \overline{v_t}} = 1$$

and, for example,

$$B_{\bar{v}} = \left[\left(\frac{\partial \bar{v}}{\partial \bar{v}_t} B_{\bar{v}_t}\right)^2\right]^{\frac{1}{2}},$$

the propagated bias limits and precision errors for the v mean velocities are equal to those for the measured quantities.

That is,

$$B_{\bar{v}} = B_{\bar{v}_t}, \text{ and } S_{\bar{v}} = S_{\bar{v}_t}.$$

For the $\bar{u}$ and $\bar{w}$ velocity components, the bias limit equations become (from equations (2))

$$B_{\bar{u}}^{\,2} = \left(\frac{\partial \bar{u}}{\partial \bar{u}_t} B_{\bar{u}_t}\right)^2 + \left(\frac{\partial \bar{u}}{\partial \bar{w}_t} B_{\bar{w}_t}\right)^2 + \left(\frac{\partial \bar{u}}{\partial \bar{\alpha}} B_{\bar{\alpha}}\right)^2 + \frac{\partial \bar{u}}{\partial \bar{u}_t}\frac{\partial \bar{u}}{\partial \bar{w}_t}\rho_{u_t w_t} B_{\bar{u}_t} B_{\bar{w}_t} + \quad \ldots,$$

and

$$B_{\bar{w}}^{\,2} = \left(\frac{\partial \bar{w}}{\partial \bar{u}_t} B_{\bar{u}_t}\right)^2 + \left(\frac{\partial \bar{w}}{\partial \bar{w}_t} B_{\bar{w}_t}\right)^2 + \left(\frac{\partial \bar{w}}{\partial \bar{\alpha}} B_{\bar{\alpha}}\right)^2 + \frac{\partial \bar{w}}{\partial \bar{u}_t}\frac{\partial \bar{w}}{\partial \bar{w}_t}\rho_{u_t w_t} B_{\bar{u}_t} B_{\bar{w}_t} + \quad \ldots.$$

The bias limit in α was ±0.226° (±0.004 rad). The assumption is made that the errors in the measurements of the variables u_t, w_t, and α are independent. The cross terms drop out ($\rho_{u_t w_t} = \rho_{u_t \alpha} = \rho_{w_t \alpha} = 0$), resulting in (from equations (8))

$$B_{\bar{u}}^{\,2} = \left(\cos\alpha\, B_{\bar{u}_t}\right)^2 + \left(-\sin\alpha\, B_{\bar{w}_t}\right)^2 + \left(-\bar{u}_t \sin\alpha - \bar{w}_t \cos\alpha\right)^2 B_{\bar{\alpha}}^{\,2} \text{ and,}$$

$$B_{\bar{w}}^{\,2} = \left(\sin\alpha\, B_{\bar{u}_t}\right)^2 + \left(\cos\alpha\, B_{\bar{w}_t}\right)^2 + \left(\bar{u}_t \cos\alpha - \bar{w}_t \sin\alpha\right)^2 B_{\bar{\alpha}}^{\,2}.$$

Similarly, for the precision errors,

$$S_{\overline{u}}^{2} = \left(\cos\alpha\, S_{\overline{u_t}}\right)^2 + \left(-\sin\alpha\, S_{\overline{w_t}}\right)^2 \text{. and}$$
$$S_{\overline{w}}^{2} = \left(\sin\alpha\, S_{\overline{u_t}}\right)^2 + \left(\cos\alpha\, S_{\overline{w_t}}\right)^2 .$$

The uncertainty in the angle α is limited to the bias limit only.

As before, the total uncertainty is then calculated as

$$U_{\overline{u}} = \left(B_{\overline{u}}^2 + \left(2S_{\overline{u}}\right)^2\right)^{\frac{1}{2}},$$
$$U_{\overline{v}} = \left(B_{\overline{v}}^2 + \left(2S_{\overline{v}}\right)^2\right)^{\frac{1}{2}},$$
$$U_{\overline{w}} = \left(B_{\overline{w}}^2 + \left(2S_{\overline{w}}\right)^2\right)^{\frac{1}{2}}.$$

The transformed rms fluctuating velocities were calculated as the square root of the transformed Reynolds normal stresses (equations (12)). Therefore, similar to the mean velocities, the bias and precision errors in v_{rms} quantities were

$$B_{v_{rms}} = B_{v_{t_{rms}}} \text{ , } S_{v_{rms}} = S_{v_{t_{rms}}} .$$

The u and w rms fluctuating velocities were given by the transformation equations (12). Cross terms are provided for the $u_{t_{rms}}$ and $\overline{w_t{'}u_t{'}}$, and $w_{t_{rms}}$ and $\overline{w_t{'}u_t{'}}$ pairs since they are related in the $\overline{w_t{'}u_t{'}}$ covariances. Assuming errors in measurements of

$u_{t_{rms}}$, $w_{t_{rms}}$, and α, were independent (eliminating those cross terms), the equations for the bias limits were derived from

$$B_{u_{rms}}{}^2 = \left(\frac{\partial u_{rms}}{\partial u_{t_{rms}}} B_{u_{t_{rms}}}\right)^2 + \left(\frac{\partial u_{rms}}{\partial w_{t_{rms}}} B_{w_{t_{rms}}}\right)^2 + \left(\frac{\partial u_{rms}}{\partial \overline{w_t{}'u_t{}'}} B_{\overline{w_t{}'u_t{}'}}\right)^2 + \left(\frac{\partial u_{rms}}{\partial \alpha} B_{\alpha}\right)^2$$
$$+\left(\frac{\partial u_{rms}}{\partial u_{t_{rms}}}\right)\left(\frac{\partial u_{rms}}{\partial \overline{w_t{}'u_t{}'}}\right) B_{u_{t_{rms}}} B_{\overline{w_t{}'u_t{}'}} + \left(\frac{\partial u_{rms}}{\partial w_{t_{rms}}}\right)\left(\frac{\partial u_{rms}}{\partial \overline{w_t{}'u_t{}'}}\right) B_{w_{t_{rms}}} B_{\overline{w_t{}'u_t{}'}}$$

and

$$B_{w_{rms}}{}^2 = \left(\frac{\partial w_{rms}}{\partial u_{t_{rms}}} B_{u_{t_{rms}}}\right)^2 + \left(\frac{\partial w_{rms}}{\partial w_{t_{rms}}} B_{w_{t_{rms}}}\right)^2 + \left(\frac{\partial w_{rms}}{\partial \overline{w_t{}'u_t{}'}} B_{\overline{w_t{}'u_t{}'}}\right)^2 + \left(\frac{\partial w_{rms}}{\partial \alpha} B_{\alpha}\right)^2$$
$$+\left(\frac{\partial w_{rms}}{\partial u_{t_{rms}}}\right)\left(\frac{\partial w_{rms}}{\partial \overline{w_t{}'u_t{}'}}\right) B_{u_{t_{rms}}} B_{\overline{w_t{}'u_t{}'}} + \left(\frac{\partial w_{rms}}{\partial w_{t_{rms}}}\right)\left(\frac{\partial w_{rms}}{\partial \overline{w_t{}'u_t{}'}}\right) B_{w_{t_{rms}}} B_{\overline{w_t{}'u_t{}'}}$$

The assumption is made that the elements in the cross terms are perfectly correlated, giving correlation coeffcients of one. This assumption, yielding conservative uncertainty estimates, will be adopted in all further developments.

Inserting the appropriate terms in the above equations and performing some algebraic manipulation yields

$$B_{u_{rms}}{}^2 = \left(\frac{u_{t_{rms}} \cos^2\alpha}{u_{rms}} B_{u_{t_{rms}}}\right)^2 + \left(\frac{w_{t_{rms}} \sin^2\alpha}{u_{rms}} B_{w_{t_{rms}}}\right)^2 + \left(\frac{-\cos\alpha \sin\alpha}{u_{rms}} B_{\overline{w_t{}'u_t{}'}}\right)^2$$
$$+\left[\cos\alpha\sin\alpha\left(w_{t_{rms}}{}^2 - u_{t_{rms}}{}^2\right) - \overline{w_t{}'u_t{}'}\left(1-2\sin^2\alpha\right)\right]^2 \frac{B_\alpha{}^2}{u_{rms}{}^2},$$
$$+\left(\frac{u_{t_{rms}} \cos^2\alpha}{u_{rms}}\right)\left(\frac{-\cos\alpha\sin\alpha}{u_{rms}}\right) B_{u_{t_{rms}}} B_{\overline{w_t{}'u_t{}'}} + \left(\frac{w_{t_{rms}} \sin^2\alpha}{u_{rms}}\right)\left(\frac{-\cos\alpha\sin\alpha}{u_{rms}}\right) B_{w_{t_{rms}}} B_{\overline{w_t{}'u_t{}'}}$$

and

$$B_{w_{rms}}{}^2 = \left(\frac{u_{t_{rms}} \sin^2\alpha}{w_{rms}} B_{u_{t_{rms}}}\right)^2 + \left(\frac{w_{t_{rms}} \cos^2\alpha}{w_{rms}} B_{w_{t_{rms}}}\right)^2 + \left(\frac{\cos\alpha\sin\alpha}{w_{rms}} B_{\overline{w_t'u_t'}}\right)^2$$
$$+\left[\cos\alpha\sin\alpha\left(u_{t_{rms}}{}^2 - w_{t_{rms}}{}^2\right) + \overline{w_t'u_t'}\left(1-2\sin^2\alpha\right)\right]^2 \frac{B_\alpha{}^2}{w_{rms}{}^2}$$
$$+\left(\frac{u_{t_{rms}} \sin^2\alpha}{w_{rms}}\right)\left(\frac{\cos\alpha\sin\alpha}{w_{rms}}\right) B_{u_{t_{rms}}} B_{\overline{w_t'u_t'}} + \left(\frac{w_{t_{rms}} \cos^2\alpha}{w_{rms}}\right)\left(\frac{\cos\alpha\sin\alpha}{w_{rms}}\right) B_{w_{t_{rms}}} B_{\overline{w_t'u_t'}}$$

The precision errors were defined by

$$S_{u_{rms}}{}^2 = \left(\frac{\partial u_{rms}}{\partial u_{t_{rms}}} S_{u_{t_{rms}}}\right)^2 + \left(\frac{\partial u_{rms}}{\partial w_{t_{rms}}} S_{w_{t_{rms}}}\right)^2 + \left(\frac{\partial u_{rms}}{\partial \overline{w_t'u_t'}} S_{\overline{w_t'u_t'}}\right)^2$$
$$+\left(\frac{\partial u_{rms}}{\partial u_{t_{rms}}}\right)\left(\frac{\partial u_{rms}}{\partial \overline{w_t'u_t'}}\right) S_{u_{t_{rms}}} S_{\overline{w_t'u_t'}} + \left(\frac{\partial u_{rms}}{\partial w_{t_{rms}}}\right)\left(\frac{\partial u_{rms}}{\partial \overline{w_t'u_t'}}\right) S_{w_{t_{rms}}} S_{\overline{w_t'u_t'}},$$

and

$$S_{w_{rms}}{}^2 = \left(\frac{\partial w_{rms}}{\partial u_{t_{rms}}} S_{u_{t_{rms}}}\right)^2 + \left(\frac{\partial w_{rms}}{\partial w_{t_{rms}}} S_{w_{t_{rms}}}\right)^2 + \left(\frac{\partial w_{rms}}{\partial \overline{w_t'u_t'}} S_{\overline{w_t'u_t'}}\right)^2$$
$$+\left(\frac{\partial w_{rms}}{\partial u_{t_{rms}}}\right)\left(\frac{\partial w_{rms}}{\partial \overline{w_t'u_t'}}\right) S_{u_{t_{rms}}} S_{\overline{w_t'u_t'}} + \left(\frac{\partial w_{rms}}{\partial w_{t_{rms}}}\right)\left(\frac{\partial w_{rms}}{\partial \overline{w_t'u_t'}}\right) S_{w_{t_{rms}}} S_{\overline{w_t'u_t'}}.$$

The resulting equations for the precision errors in the u_{rms} and w_{rms} quantities are

$$S_{u_{rms}}{}^2 = \left(\frac{u_{t_{rms}} \cos^2\alpha}{u_{rms}} S_{u_{t_{rms}}}\right)^2 + \left(\frac{w_{t_{rms}} \sin^2\alpha}{u_{rms}} S_{w_{t_{rms}}}\right)^2 + \left(\frac{-\cos\alpha\sin\alpha}{u_{rms}} S_{\overline{w_t'u_t'}}\right)^2$$
$$+\left(\frac{u_{t_{rms}} \cos^2\alpha}{u_{rms}}\right)\left(\frac{-\cos\alpha\sin\alpha}{u_{rms}}\right) S_{u_{t_{rms}}} S_{\overline{w_t'u_t'}} + \left(\frac{w_{t_{rms}} \sin^2\alpha}{u_{rms}}\right)\left(\frac{-\cos\alpha\sin\alpha}{u_{rms}}\right) S_{w_{t_{rms}}} S_{\overline{w_t'u_t'}},$$

and

$$S_{w_{rms}}^{\ 2} = \left(\frac{u_{t_{rms}} \sin^2\alpha}{w_{rms}} S_{u_{t_{rms}}}\right)^2 + \left(\frac{w_{t_{rms}} \cos^2\alpha}{w_{rms}} S_{w_{t_{rms}}}\right)^2 + \left(\frac{\cos\alpha \sin\alpha}{w_{rms}} S_{\overline{w_t'u_t'}}\right)^2$$
$$+\left(\frac{u_{t_{rms}} \sin^2\alpha}{w_{rms}}\right)\left(\frac{\cos\alpha \sin\alpha}{w_{rms}}\right) S_{u_{t_{rms}}} S_{\overline{w_t'u_t'}} + \left(\frac{w_{t_{rms}} \cos^2\alpha}{w_{rms}}\right)\left(\frac{\cos\alpha \sin\alpha}{w_{rms}}\right) S_{w_{t_{rms}}} S_{\overline{w_t'u_t'}}$$

The bias limits and precision errors for the Reynolds normal stresses were calculated using the transformation equations (9). Again, since

$$\overline{v'v'} = \overline{v_t'v_t'}$$

the bias limits and precision error estimates for $\overline{v'v'}$ were derived as

$$B_{\overline{v'v'}} = B_{\overline{v_t'v_t'}}, \text{ and } S_{\overline{v'v'}} = S_{\overline{v_t'v_t'}}.$$

Assuming that the errors in the measurements of $\overline{u_t'u_t'}$, $\overline{w_t'w_t'}$, and α are independent for this quantity (eliminating those cross terms), the equations for the bias limits in $\overline{u'u'}$ can be derived from

$$B_{\overline{u'u'}}^{\ 2} = \left(\frac{\partial \overline{u'u'}}{\partial \overline{u_t'u_t'}} B_{\overline{u_t'u_t'}}\right)^2 + \left(\frac{\partial \overline{u'u'}}{\partial \overline{w_t'w_t'}} B_{\overline{w_t'w_t'}}\right)^2 + \left(\frac{\partial \overline{u'u'}}{\partial \overline{w_t'u_t'}} B_{\overline{w_t'u_t'}}\right)^2 + \left(\frac{\partial \overline{u'u'}}{\partial \alpha} B_\alpha\right)^2$$
$$+\left(\frac{\partial \overline{u'u'}}{\partial \overline{u_t'u_t'}}\right)\left(\frac{\partial \overline{u'u'}}{\partial \overline{w_t'u_t'}}\right) B_{\overline{u_t'u_t'}} B_{\overline{w_t'u_t'}} + \left(\frac{\partial \overline{u'u'}}{\partial \overline{w_t'w_t'}}\right)\left(\frac{\partial \overline{u'u'}}{\partial \overline{w_t'u_t'}}\right) B_{\overline{w_t'w_t'}} B_{\overline{w_t'u_t'}} .$$

Based on the transformation equation (9) for the $\overline{u'u'}$ Reynolds normal stress, the bias limit is

$$B_{\overline{u'u'}}^{\ 2} = \left(\cos^2\alpha\, B_{\overline{u_t'u_t'}}\right)^2 + \left(\sin^2\alpha\, B_{\overline{w_t'w_t'}}\right)^2 + \left(-2\cos\alpha\sin\alpha\, B_{\overline{w_t'u_t'}}\right)^2$$
$$+\left[2\cos\alpha\sin\alpha\left(\overline{w_t'w_t'} - \overline{u_t'u_t'}\right) - 2\overline{w_t'u_t'}\left(1-2\sin^2\alpha\right)\right]^2 B_\alpha^{\ 2}$$
$$-2\cos\alpha\sin\alpha\left(\cos^2\alpha\, B_{\overline{u_t'u_t'}} B_{\overline{w_t'u_t'}} + \sin^2\alpha\, B_{\overline{w_t'w_t'}} B_{\overline{w_t'u_t'}}\right)$$

The bias limit in $\overline{w'w'}$ can be formulated from

$$B_{\overline{w'w'}}^{\ 2} = \left(\frac{\partial\overline{w'w'}}{\partial\overline{u_t'u_t'}} B_{\overline{u_t'u_t'}}\right)^2 + \left(\frac{\partial\overline{w'w'}}{\partial\overline{w_t'w_t'}} B_{\overline{w_t'w_t'}}\right)^2 + \left(\frac{\partial\overline{w'w'}}{\partial\overline{w_t'u_t'}} B_{\overline{w_t'u_t'}}\right)^2 + \left(\frac{\partial\overline{w'w'}}{\partial\alpha} B_\alpha\right)^2$$
$$+\left(\frac{\partial\overline{w'w'}}{\partial\overline{u_t'u_t'}}\right)\left(\frac{\partial\overline{w'w'}}{\partial\overline{w_t'u_t'}}\right) B_{\overline{u_t'u_t'}} B_{\overline{w_t'u_t'}} + \left(\frac{\partial\overline{w'w'}}{\partial\overline{w_t'w_t'}}\right)\left(\frac{\partial\overline{w'w'}}{\partial\overline{w_t'u_t'}}\right) B_{\overline{w_t'w_t'}} B_{\overline{w_t'u_t'}}$$

Using the transformation equation for $\overline{w'w'}$ from equation (9), the bias limit is expressed as

$$B_{\overline{w'w'}}^{\ 2} = \left(\sin^2\alpha\, B_{\overline{u_t'u_t'}}\right)^2 + \left(\cos^2\alpha\, B_{\overline{w_t'w_t'}}\right)^2 + \left(2\cos\alpha\sin\alpha\, B_{\overline{w_t'u_t'}}\right)^2$$
$$+\left[2\cos\alpha\sin\alpha\left(\overline{u_t'u_t'} - \overline{w_t'w_t'}\right) + 2\overline{w_t'u_t'}\left(1-2\sin^2\alpha\right)\right]^2 B_\alpha^{\ 2}$$
$$+2\cos\alpha\sin\alpha\left(\sin^2\alpha\, B_{\overline{u_t'u_t'}} B_{\overline{w_t'u_t'}} + \cos^2\alpha\, B_{\overline{w_t'w_t'}} B_{\overline{w_t'u_t'}}\right)$$

The precision indicies had a similar form, without the α terms, given as,

$$S_{\overline{u'u'}}^{\ 2} = \left(\cos^2\alpha\, S_{\overline{u_t'u_t'}}\right)^2 + \left(\sin^2\alpha\, S_{\overline{w_t'w_t'}}\right)^2 + \left(-2\cos\alpha\sin\alpha\, S_{\overline{w_t'u_t'}}\right)^2$$
$$-2\cos\alpha\sin\alpha\left(\cos^2\alpha\, S_{\overline{u_t'u_t'}} S_{\overline{w_t'u_t'}} + \sin^2\alpha\, S_{\overline{w_t'w_t'}} S_{\overline{w_t'u_t'}}\right)$$

and,

$$S_{\overline{w'w'}}^{\;2} = \left(\sin^2\alpha\, S_{\overline{u_t'u_t'}}\right)^2 + \left(\cos^2\alpha\, S_{\overline{w_t'w_t'}}\right)^2 + \left(2\cos\alpha\sin\alpha\, S_{\overline{w_t'u_t'}}\right)^2$$
$$+2\cos\alpha\sin\alpha\left(\sin^2\alpha\, S_{\overline{u_t'u_t'}} S_{\overline{w_t'u_t'}} + \cos^2\alpha\, S_{\overline{w_t'w_t'}} S_{\overline{w_t'u_t'}}\right).$$

The uncertainty analysis for the Reynolds shear stresses was based on the transformation equations (10). In this case, the LDV velocity measurements were required to be made from the same seeding particle for correlation of velocity fluctuations. Therefore, the $\overline{u'v'}$, and $\overline{v'w'}$ stresses each had a third, cross term in their expressions for bias limits and precision errors. The $\overline{w'u'}$ stresses had two cross terms. The equations for determination of the bias limits for the three shear stresses can be derived from

$$B_{\overline{u'v'}}^{\;2} = \left(\frac{\partial\overline{u'v'}}{\partial\overline{u_t'v_t'}} B_{\overline{u_t'v_t'}}\right)^2 + \left(\frac{\partial\overline{u'v'}}{\partial\overline{v_t'w_t'}} B_{\overline{v_t'w_t'}}\right)^2 + \left(\frac{\partial\overline{u'v'}}{\partial\alpha} B_\alpha\right)^2$$
$$+\left(\frac{\partial\overline{u'v'}}{\partial\overline{u_t'v_t'}}\right)\left(\frac{\partial\overline{u'v'}}{\partial\overline{v_t'w_t'}}\right) B_{\overline{u_t'v_t'}} B_{\overline{v_t'w_t'}},$$

$$B_{\overline{v'w'}}^{\;2} = \left(\frac{\partial\overline{v'w'}}{\partial\overline{u_t'v_t'}} B_{\overline{u_t'v_t'}}\right)^2 + \left(\frac{\partial\overline{v'w'}}{\partial\overline{v_t'w_t'}} B_{\overline{v_t'w_t'}}\right)^2 + \left(\frac{\partial\overline{v'w'}}{\partial\alpha} B_\alpha\right)^2$$
$$+\left(\frac{\partial\overline{v'w'}}{\partial\overline{u_t'v_t'}}\right)\left(\frac{\partial\overline{v'w'}}{\partial\overline{v_t'w_t'}}\right) B_{\overline{u_t'v_t'}} B_{\overline{v_t'w_t'}},$$

and

$$B_{\overline{w'u'}}^{\;2} = \left(\frac{\partial\overline{w'u'}}{\partial\overline{u_t'u_t'}} B_{\overline{u_t'u_t'}}\right)^2 + \left(\frac{\partial\overline{w'u'}}{\partial\overline{w_t'w_t'}} B_{\overline{w_t'w_t'}}\right)^2 + \left(\frac{\partial\overline{w'u'}}{\partial\overline{w_t'u_t'}} B_{\overline{w_t'u_t'}}\right)^2$$
$$+\left(\frac{\partial\overline{w'u'}}{\partial\overline{u_t'u_t'}}\right)\left(\frac{\partial\overline{w'u'}}{\partial\overline{w_t'u_t'}}\right) B_{\overline{u_t'u_t'}} B_{\overline{w_t'u_t'}} + \left(\frac{\partial\overline{w'u'}}{\partial\overline{w_t'w_t'}}\right)\left(\frac{\partial\overline{w'u'}}{\partial\overline{w_t'u_t'}}\right) B_{\overline{w_t'w_t'}} B_{\overline{w_t'u_t'}}.$$
$$+\left(\frac{\partial\overline{v'w'}}{\partial\alpha} B_\alpha\right)^2$$

The equations for the bias limits are as follows.

$$B_{\overline{u'v'}}^2 = \left(\cos\alpha\, B_{\overline{u_t'v_t'}}\right)^2 + \left(-\sin\alpha\, B_{\overline{v_t'w_t'}}\right)^2 + \left(-\overline{u_t'v_t'}\sin\alpha - \overline{v_t'w_t'}\cos\alpha\right)^2 B_\alpha^2, - \cos\alpha\sin\alpha\, B_{\overline{u_t'v_t'}} B_{\overline{v_t'w_t'}}$$

$$B_{\overline{v'w'}}^2 = \left(\sin\alpha\, B_{\overline{u_t'v_t'}}\right)^2 + \left(\cos\alpha\, B_{\overline{v_t'w_t'}}\right)^2 + \left(\overline{u_t'v_t'}\cos\alpha - \overline{v_t'w_t'}\sin\alpha\right)^2 B_\alpha^2 + \cos\alpha\sin\alpha\, B_{\overline{u_t'v_t'}} B_{\overline{v_t'w_t'}}$$

and

$$B_{\overline{w'u'}}^2 = \left(\frac{\sin 2\alpha}{2} B_{\overline{u_t'u_t'}}\right)^2 + \left(\frac{-\sin 2\alpha}{2} B_{\overline{w_t'w_t'}}\right)^2 + \left[\left(1-2\sin^2\alpha\right) B_{\overline{w_t'u_t'}}\right]^2 + \frac{\sin 4\alpha}{4} B_{\overline{u_t'u_t'}} B_{\overline{w_t'u_t'}} - \frac{\sin 4\alpha}{4} B_{\overline{w_t'w_t'}} B_{\overline{w_t'u_t'}} + \left[\left(1-2\sin^2\alpha\right)\left(\overline{u_t'u_t'} - \overline{w_t'w_t'}\right) - \overline{w_t'u_t'}\left(2\sin 2\alpha\right)\right]^2 B_\alpha^2$$

The precision error equations were determined from the following relationships.

$$S_{\overline{u'v'}}^2 = \left(\frac{\partial \overline{u'v'}}{\partial \overline{u_t'v_t'}} S_{\overline{u_t'v_t'}}\right)^2 + \left(\frac{\partial \overline{u'v'}}{\partial \overline{v_t'w_t'}} S_{\overline{v_t'w_t'}}\right)^2 + \left(\frac{\partial \overline{u'v'}}{\partial \overline{u_t'v_t'}}\right)\left(\frac{\partial \overline{u'v'}}{\partial \overline{v_t'w_t'}}\right) S_{\overline{u_t'v_t'}} S_{\overline{v_t'w_t'}}$$

$$S_{\overline{v'w'}}^2 = \left(\frac{\partial \overline{v'w'}}{\partial \overline{u_t'v_t'}} S_{\overline{u_t'v_t'}}\right)^2 + \left(\frac{\partial \overline{v'w'}}{\partial \overline{v_t'w_t'}} S_{\overline{v_t'w_t'}}\right)^2 + \left(\frac{\partial \overline{v'w'}}{\partial \overline{u_t'v_t'}}\right)\left(\frac{\partial \overline{v'w'}}{\partial \overline{v_t'w_t'}}\right) S_{\overline{u_t'v_t'}} S_{\overline{v_t'w_t'}}$$

$$S_{\overline{w'u'}}^2 = \left(\frac{\partial \overline{w'u'}}{\partial \overline{u_t'u_t'}} S_{\overline{u_t'u_t'}}\right)^2 + \left(\frac{\partial \overline{w'u'}}{\partial \overline{w_t'w_t'}} S_{\overline{w_t'w_t'}}\right)^2 + \left(\frac{\partial \overline{w'u'}}{\partial \overline{w_t'u_t'}} S_{\overline{w_t'u_t'}}\right)^2$$
$$+ \left(\frac{\partial \overline{w'u'}}{\partial \overline{u_t'u_t'}}\right)\left(\frac{\partial \overline{w'u'}}{\partial \overline{w_t'u_t'}}\right) S_{\overline{u_t'u_t'}} S_{\overline{w_t'u_t'}} + \left(\frac{\partial \overline{w'u'}}{\partial \overline{w_t'w_t'}}\right)\left(\frac{\partial \overline{w'u'}}{\partial \overline{w_t'u_t'}}\right) S_{\overline{w_t'w_t'}} S_{\overline{w_t'u_t'}}$$

The resulting equations for the precision error estmates are,

$$S_{\overline{u'v'}}^2 = \left(\cos\alpha\, S_{\overline{u_t'v_t'}}\right)^2 + \left(-\sin\alpha\, S_{\overline{v_t'w_t'}}\right)^2$$
$$- \cos\alpha \sin\alpha\, S_{\overline{u_t'v_t'}} S_{\overline{v_t'w_t'}}$$

$$S_{\overline{v'w'}}^2 = \left(\sin\alpha\, S_{\overline{u_t'v_t'}}\right)^2 + \left(\cos\alpha\, S_{\overline{v_t'w_t'}}\right)^2$$
$$+ \cos\alpha \sin\alpha\, S_{\overline{u_t'v_t'}} S_{\overline{v_t'w_t'}}$$

$$S_{\overline{w'u'}}^2 = \left(\frac{\sin 2\alpha}{2} S_{\overline{u_t'u_t'}}\right)^2 + \left(\frac{-\sin 2\alpha}{2} S_{\overline{w_t'w_t'}}\right)^2 + \left[\left(1 - 2\sin^2\alpha\right) S_{\overline{w_t'u_t'}}\right]^2$$
$$+ \frac{\sin 4\alpha}{4} S_{\overline{u_t'u_t'}} S_{\overline{w_t'u_t'}} - \frac{\sin 4\alpha}{4} S_{\overline{w_t'w_t'}} S_{\overline{w_t'u_t'}}$$

Example calculation. The calculation of the bias limit in the $\frac{\overline{v'w'}}{U_\infty^2}$ nondimensional shear stress is presented in this section. The equation for estimating this error is listed below.

$$B_{\overline{v'w'}}^2 = \left(\sin\alpha\, B_{\overline{u_t'v_t'}}\right)^2 + \left(\cos\alpha\, B_{\overline{v_t'w_t'}}\right)^2$$
$$+ \left(\overline{u_t'v_t'}\cos\alpha - \overline{v_t'w_t'}\sin\alpha\right)^2 B_\alpha^2$$
$$+ \cos\alpha \sin\alpha\, B_{\overline{u_t'v_t'}} B_{\overline{v_t'w_t'}}$$

All velocity quantities used in this equation were nondimensionalized by U_∞^2, yielding the bias limit in the nondimensional shear stress. The angle α is

25°. The bias limit in this angle, B_α, is ±0.226° (±0.004 rad). The bias limits in $\frac{\overline{u_t'v_{t'}}}{U_\infty^2}$ and $\frac{\overline{v_t'w_{t'}}}{U_\infty^2}$ were obtained from table 1c and are ±0.0000029 and ±0.0000012, respectively. Average measured values of $\frac{\overline{u_t'v_{t'}}}{U_\infty^2}$ and $\frac{\overline{v_t'w_{t'}}}{U_\infty^2}$ were used and they were 0.0029 and 0.0016, respectively. (If this analysis was being done on a point-by-point basis, the values at the particular point in the tunnel would be used.) The four terms, in order of their appearance in the bias error equation, are 1.50×10^{-12}, 1.18×10^{-12}, 6.10×10^{-11}, and 1.33×10^{-12}. The total bias limit is ±0.0000081. Clearly, the third term dominates this error, which is the error in the angle of attack measurement. This result is typical of most systems, and better accuracy in angle of attack measurement will have the greatest effect in reducing this bias error.

This concludes the discussion of the propagation of uncertainty for the calculated quantities. The tabulated results for all of the transformed, final quantities in the model coordinate system are given in tables 2a, 2b, and 2c. All transformed quantities were normalized by the appropriate terms, as shown in the tables. In all cases where cross terms existed in the error equations, the sign of the correlation coefficient was unknown. It was therefore assumed to be the sign that would maximize the error estimate.

Analysis of Uncertainty Results

The significance of the final uncertainty results can be examined by comparing them to values in the tunnel coordinate system. The uncertainty results relative to the results in the tunnel coordinate system showed changes in all quantities associated with the u and w velocity components.

Since the effect of angle of attack, α, was the only effect examined, and the u and w vectors only are affected by α rotations, all v-only related quantities remained unchanged. All quantities associated with the w-related variables showed the largest differences from the values in the tunnel system.

In general, bias limits and precision errors in the six Reynolds stresses increased, except for the errors in $\frac{\overline{v'v'}}{U_\infty}$, which did not change. There was a slight decrease in uncertainty in the u_{rms} fluctuating velocity due to decreases in bias limit and precision error. The uncertainty in the w_{rms} fluctuating velocity increased primarily due to an increase in bias limit. Bias limit increases offset precision error decreases in the mean u velocity. Both errors increased for the mean w velocity.

To address the effect of model sideslip and/or roll on uncertainties in the mean velocities, the full transformation in equation (4) would be used to formulate the transformation equations. The number of variables in the bias error propagation equations would increase by the number of additional angular rotations applied to the model. To evaluate the effect on rms fluctuating velocities and the Reynolds stresses, sequential application of the angular rotations would reduce complexity. Although not proven here, it is assumed that this would involve developing the propagation equations for each angle separately, setting the other angles equal to zero. The equations for each angular rotation would then be applied separately to each succeeding result.

Conclusions

An analysis of the propagation of estimated experimental LDV data uncertainties from the tunnel coordinate system to the model system has been presented. Transforming from one Cartesian coordinate system to another by three sequential rotations, equations were developed to transform the variables initially obtained in the original coordinates into variables in the final coordinate system. Based on the transformation equations, propagation equations for errors in the experimentally-derived flow quantities were derived for a model at angle of attack. Experimental uncertainties were then propagated from the tunnel coordinate system into the model system.

Comparisons between results for the two systems revealed a variety of increases and decreases in bias and precision estimates. Quantities associated with the w-related variables primarily increased and showed the largest differences from the values in the tunnel system. In some cases, u-related quantities decreased after transformation, or did not change significantly.

Calculations of uncertainties as functions of the variables that comprise the experimental results allows assessment of the contribution each variable makes. Such an analysis enables and necessitates the experimentalists to identify and address the contributing error sources in the experimental measurement system. This provides an opportunity to improve the quality of data derived from experimental systems. This is especially important in experiments where small changes in test conditions are expected to produce small, detectable changes in results. In addition, the need for high-quality experimental data for CFD method validation demands a thorough assessment of experimental uncertainty.

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Table 1a

Uncertainty Estimates for Results in Tunnel Coordinate System-

Mean and Fluctuating (rms) Velocities

(Reference 3 - Strake Test)

(U_∞=3 in/sec)

	$\overline{u_t} / U_\infty$	$\overline{v_t} / U_\infty$	$\overline{w_t} / U_\infty$	$\sqrt{\overline{u_t'^2}} / U_\infty$	$\sqrt{\overline{v_t'^2}} / U_\infty$	$\sqrt{\overline{w_t'^2}} / U_\infty$
<u>Strake Test</u>						
Bias	+.0101/	+.0254/	+.00822/	±.00195	±.0119	±.00441
	-.0112	-.0281	-.00957			
Precision	±.00168	±.00888	±.00110	±.00119	±.0060	±.000775
Total	+.011/	+.031/	+.0085/	±.0031	±.017	±.0047
Uncert.	-.012	-.033	-.0098			

Table 1b

Uncertainty Estimates for Results in Tunnel Coordinate System-

Reynolds Normal Stresses

(Reference 3 - Strake Test)

(U_∞=3 in/sec)

	$\overline{u_t'u_t'}/U_\infty^2$	$\overline{v_t'v_t'}/U_\infty^2$	$\overline{w_t'w_t'}/U_\infty^2$
<u>Strake Test</u>			
Bias	0	±.0000513	0
Precision	±.000056	±.00149	±.000024
Total uncert.	±.000056	±.0030	±.000024

Table 1c

Uncertainty Estimates for Results in Tunnel Coordinate System-

Reynolds Shear Stresses

(Reference 3 - Strake Test)

(U_∞=3 in/sec)

	$\overline{u_t'v_t'}/U_\infty^2$	$\overline{v_t'w_t'}/U_\infty^2$	$\overline{w_t'u_t'}/U_\infty^2$
Strake Test			
Bias	±.0000029	±.0000012	0
Precis.	±.00012	±.000066	±.000020
Total uncert.	±.00024	±.00013	±.000020

Table 2a

Uncertainty Estimates for Results in Model Coordinate System-

Mean and Fluctuating (rms) Velocities

(Reference 3 - Strake Test)

(U_∞=3 in/sec)

	$\bar{u}/U_\infty$	$\bar{v}/U_\infty$	$\bar{w}/U_\infty$	$\sqrt{\overline{u'^2}}/U_\infty$	$\sqrt{\overline{v'^2}}/U_\infty$	$\sqrt{\overline{w'^2}}/U_\infty$
<u>Strake Test</u>						
Bias	+.0110/ -.0120	+.0254/ -.0281	+.00909/ -.0103	±.00181	±.0119	±.00546
Precision	±.00159	±.00888	±.00122	±.00117	±.0060	±.000771
Total Uncert.	+.011/ -.012	+.031/ -.033	+.0094/ -.011	±.0030	±.017	±.0057

Table 2b

Uncertainty Estimates for Results in Model Coordinate System-

Reynolds Normal Stresses

(Reference 3 - Strake Test)

(U_∞=3 in/sec)

	$\overline{u'u'}/U_\infty^2$	$\overline{v'v'}/U_\infty^2$	$\overline{w'w'}/U_\infty^2$
<u>Strake Test</u>			
Bias	±.0000110	±.0000513	±.0000110
Precision	±.000059	±.00149	±.0000322
Total uncert.	±.00012	±.0030	±.000065

Table 2c

Uncertainty Estimates for Results in Model Coordinate System-

Reynolds Shear Stresses

(Reference 3 - Strake Test)

(U_∞=3 in/sec)

	$\overline{u'v'}/U_\infty^2$	$\overline{v'w'}/U_\infty^2$	$\overline{w'u'}/U_\infty^2$
Strake Test			
Bias	±.0000111	±.0000081	±.0000009
Precis.	±.000125	±.0000958	±.0000339
Total uncert.	±.00025	±.00019	±.000068

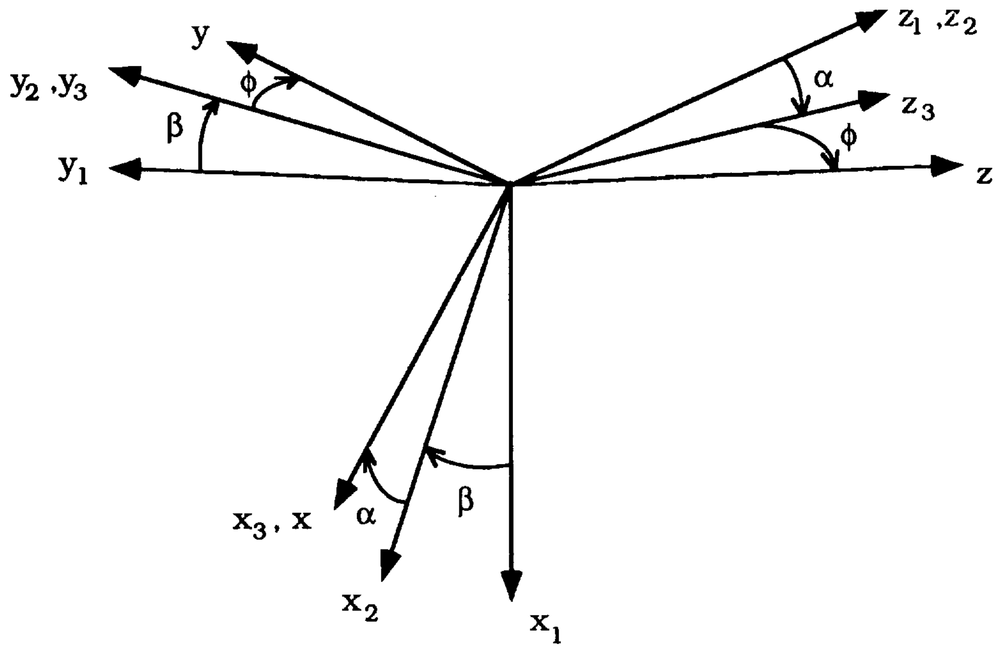

Figure 1. Rotations from the Tunnel (x_1, y_1, z_1) to Model (x, y, z) Coordinate System

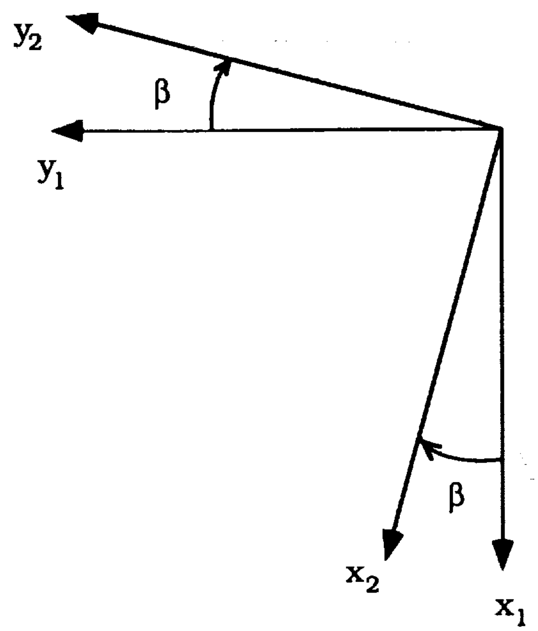

Figure 2. Rotations from the Tunnel (x_1, y_1, z_1) to x_2, y_2, z_2 Coordinate System

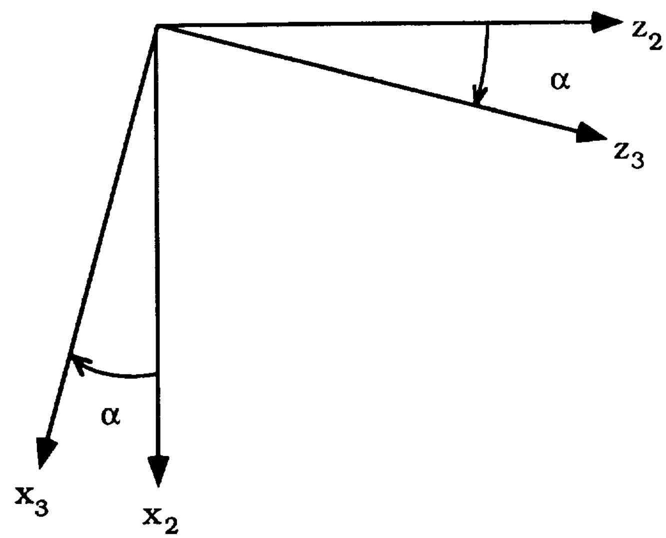

Figure 3. Rotations from the x_2, y_2, z_2 to x_3, y_3, z_3 Coordinate System

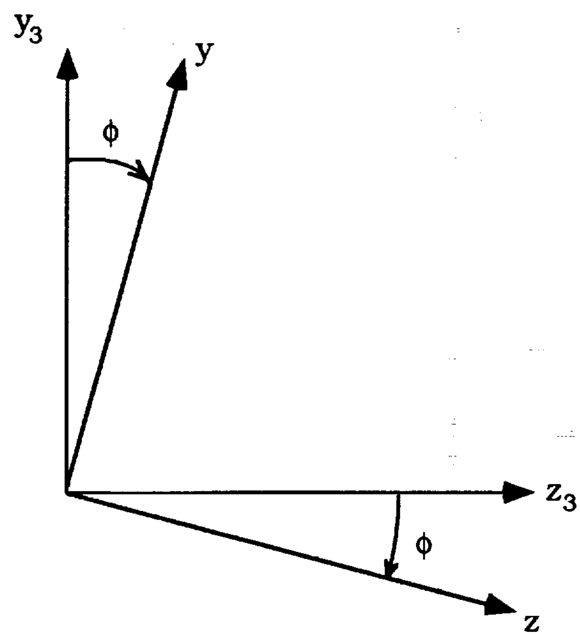

Figure 4. Rotations from the x_3, y_3, z_3 to Model (x , y , z) Coordinate System

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13. ABSTRACT *(Maximum 200 words)*

This report provides an analysis of experimental laser Doppler velocimetry (LDV) data uncertainties that propagate from measurements in the tunnel coordinate system to results in the model system. Calculations of uncertainties as functions of the variables that comprise the final result requires assessment of the contribution each variable makes. Such an analysis enables and necessitates the experimentalists to identify and address the contributing error sources in the experimental measurement system. This provides an opportunity to improve the quality of data derived from experimental systems. This is especially important in experiments where small changes in test conditions are expected to produce small, detectable changes in results. In addition, the need for high-quality experimental data for CFD method validation demands a thorough assessment of experimental uncertainty.

Transforming from one Cartesian coordinate system to another by three sequential rotations, equations were developed to transform the variables initially obtained in the origiinal coordinates into variables in the final coordinate system. Based on the transformation equations, propagation equations for errors in the experimentally-derived flow quantities were derived for a model at angle of attack. Experimental uncertainties were then propagated from the tunnel coordinate system into the model system.

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