REALTIME MITIGATION OF GPS SA ERRORS USING LORAN-C

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Abstract

The hybrid use of Loran-C with the Global Positioning System (GPS) has been shown capable of providing a sole-means of enroute air radionavigation. By allowing pilots to fly direct to their destinations, use of this system is resulting in significant time savings and therefore fuel savings as well. However, a major error source limiting the accuracy of GPS is the intentional degradation of the GPS signal known as Selective Availability (SA). SA-induced position errors are highly correlated and far exceed all other error sources (horizontal position error: 100 meters, 95%). Realtime mitigation of SA errors from the position solution is highly desirable. This paper discusses how that can be achieved. The stability of Loran-C signals is exploited to reduce SA errors. The theory behind this technique will be discussed and results using bench and flight data will be given.

Introduction

The hybrid use of Loran-C with the Global Positioning System (GPS) has been shown to be capable of providing a sole-means of enroute air radionavigation [1]. Standardization committees such as the RTCA are currently working on developing minimum operational performance standards for this system. By allowing pilots to fly direct to their destinations, use of this system will result in significant time savings and therefore fuel savings as well. By not confining all aircraft to a small portion of the airspace (which results when using the Victor airways), the risk of collision undoubtedly will be reduced as well.

However, a major error source limiting the accuracy of GPS is the intentional degradation of the GPS signal known as Selective Availability (SA). SA manifests itself in the form of erroneous orbital data broadcast by the satellites and in dithering of the satellite clock. The result is position determination which, according to the Department of Defense (DoD), will be in error by one hundred meters 95% of the time in the horizontal plane. Previous work performed at Ohio University showed that SA-induced position errors are highly correlated [2]. Since the correlation time is on the order of minutes, it follows that the error falls well within the passband of the

aircraft's dynamic response. The result is that the aircraft will follow the deviations induced by SA.

Realtime mitigation of SA errors from the position solution is highly desirable. This paper discusses how that can be achieved. The stability of Loran-C signals is exploited to reduce SA errors. In the typical hybrid use of Loran-C and GPS, the Loran-C signal stability is not exploited. This stems from the relatively poor absolute accuracy of Loran-C (relative to GPS). However, it is possible to use the stability of Loran-C positioning to reduce SA-induced GPS positioning errors. The theory behind this technique will be discussed and results will be given. First, the phenomenon of SA will be described.

Selective Availability

As mentioned in the introduction, SA is an intentional corruption of the GPS signal by the DoD to limit the accuracy available to the public. The degradation is achieved in two ways. First, false satellite orbit parameters are broadcast to the users. This results in incorrect positioning of the satellites in the navigation solution. Secondly, code and carrier tracking errors are induced through dithering the satellite clock (carrier frequency). The erroneous orbit parameters lead to position errors which vary slowly throughout the satellite pass. Code-phase and carrier-phase errors due to the dithering of the satellite clock are random but also are highly correlated. Correlation times of several minutes are typical. As a result, simple filtering schemes are not effective and aircraft will follow the deviations. Virtually all of the information available to date about SA has been gathered through data collection efforts by civilian organizations. The DoD, however, has stated that SA shall be instituted in such a way as to yield horizontal position errors at a 95% level of 100 meters [3].

Mitigation Methodology

The heart of the mitigation scheme lies in the differences between Loran-C and SA-induced GPS position errors. Loran-C position errors in general are biased and noisy. The level of noise depends upon the receiver architecture

and specifically upon the tracking loop bandwidth. However, noise levels on the order of 5 to 10 meters can be achieved for airborne applications. The Loran-C position bias is primarily composed of unmodeled additional secondary phase factors (ASF). In general the bias does not remain constant over any given flight path but the variation is usually quite slow in comparison to the clock component of GPS SA error. This phenomenon is what makes Loran-based SA mitigation possible. The long-term stability of the Loran-C measurements is exploited to smooth the SA-induced variations in the GPS measurements.

Conceptually, the mitigation scheme works as follows. The Loran-C sensor computes the horizontal position of the aircraft. A vertical input is needed and is supplied by the barometric altimeter (again, a sensor which is biased but stable). The combination provides a three-dimensional position of the aircraft. Range values are computed from the GPS satellites to the Loran-C/altimeter position. These range values are then the reference against which the measured GPS pseudoranges are filtered.

Note that the technique depends upon the assumption that SA error is composed only of high frequency components relative to the Loran-C bias error variations. Strictly speaking, this assumption is not valid since the orbital component of SA error is slowly varying. However, as was shown in [2], the clock component of SA error has periods on the order of five to ten minutes. As such it is a high frequency error source relative to the non-noise component of Loran-C error. Although this has not been rigorously proven, flight data (to be shown later) supports the conclusion. Thus, the technique is able to reduce the clock component (or roughly speaking, the variance) of SA error.

The filtering is accomplished by complementary Kalman filters which are applied to each pseudorange measurement [4,5]. The inputs to each filter are the given GPS pseudorange measurement and the corresponding range computed from the satellite to the Loran-C/altimeter position. At each measurement epoch (current time given by the index k), the complementary Kalman filter is executed as follows:

$$d_k^- = d_{k-1}^+ + (L_k - L_{k-1})$$ (1)

$$p_k^- = p_{k-1}^+ + q$$ (2)

$$k_k = \frac{p_k^-}{p_k^- + r}$$ (3)

$$d_k^+ = d_k^- + k_k(z_k - d_k^-)$$ (4)

$$p_k^+ = (1 - k_k)p_k^-$$ (5)

where the subscript represents the time index. The superscripts '-'and '+'represent predicted and estimated quantities respectively. 'd*' represents the estimated pseudorange with variance q. 'z'represents the measured pseudorange with error variance r. Note that r is due primarily to SA. 'L'represents the range computed from the satellite to the Loran-C/altimeter position. 'p' represents the prediction or estimation error variance. 'k' is the Kalman gain. In equation (1), the current pseudorange prediction is computed by updating the previous pseudorange estimate with the difference between the current and previous Loran-C/altimeter ranges. The prediction error variance is computed in equation (2) and is used to compute the Kalman gain in equation (3). The difference between the measured and predicted pseudoranges is weighted by the Kalman gain in the computation of the current estimate (equation 4). Finally, the current estimation error variance is computed (equation 5).

Position Solution

Given at least four GPS pseudoranges, position may be computed. As will be shown in the next two sections, significant reduction in SA-error may be achieved when using the mitigation technique just described.

For both the simulation and flight test results (to be shown later), the ordinary least-squares (OLS) estimator is used to determine position and clock bias from the pseudoranges. In the absence of measurement errors, the relationship between satellite location, receiver location, clock bias and pseudorange is given by:

$$R_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} + b$$ (6)

where R_i is the pseudorange to the ith satellite, $(x_i, y_i, z_i)$are the coordinates of the satellite, $(x, y, z)$are the coordinates of the receiver and b is the receiver clock bias (converted to units of distance through multiplication by the speed of light). Since the receiver coordinates and clock bias must be solved for simultaneously, at least four measurements are required.

However, instead of attempting simultaneous solution of non-linear equations, the standard technique is to solve iteratively a set of equations which have been linearized about an initial estimated position and clock bias

(x_o, y_o, z_o, b_o). This is achieved by forming a Taylor series expansion and retaining the zeroth and first order terms:

$$R_i = R_{io} + (\delta x)\frac{\partial R_i}{\partial x}\Big|_{x_o, y_o, z_o} + (\delta y)\frac{\partial R_i}{\partial y}\Big|_{x_o, y_o, z_o} \qquad (7)$$
$$+ (\delta z)\frac{\partial R_i}{\partial y}\Big|_{x_o, y_o, z_o} + (\delta b)\frac{\partial R_i}{\partial b}$$

where R_{io} is the range from the satellite to the initial position estimate. δx, δy, δz and δb represent the corrections to the initial estimates. If the initial estimate is close to the truth, no iterations are required. However, if the initial estimate is not close, the corrections are used to update the initial estimate and the process is repeated. Convergence is declared if the magnitudes of the corrections are below a desired threshold.

The partial derivatives are evaluated as follows:

$$\frac{\partial R_i}{\partial x} = \frac{x_o - x}{R_i - b_o} = \alpha_{i1} \qquad (8)$$

$$\frac{\partial R_i}{\partial y} = \frac{y_o - y}{R_i - b_o} = \alpha_{i2} \qquad (9)$$

$$\frac{\partial R_i}{\partial z} = \frac{z_o - z}{R_i - b_o} = \alpha_{i3} \qquad (10)$$

$$\frac{\partial R_i}{\partial b} = 1 = \alpha_{i4} \qquad (11)$$

Substitution of equations (8) through (11) into (7) yields:

$$\delta R_i = (\delta x)\alpha_{i1} + (\delta y)\alpha_{i2} + (\delta z)\alpha_{i3} + (\delta b)\alpha_{i4} \qquad (12)$$

where:

$$\delta R_i = R_i - R_{io} \qquad (13)$$

Four pseudorange measurements allow for the following simultaneous set of equations:

$$\begin{pmatrix} \delta R_1 \\ \delta R_2 \\ \delta R_3 \\ \delta R_4 \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{pmatrix} \begin{pmatrix} \delta x \\ \delta y \\ \delta z \\ \delta b \end{pmatrix} \qquad (14)$$

which may be rewritten more succinctly:

$$y = H\underline{\beta} \qquad (15)$$

The presence of measurement errors may be accounted for by the addition of an error vector:

$$y = H\underline{\beta} + \underline{\epsilon} \qquad (16)$$

The ordinary least-squares solution is then given by:

$$\underline{\beta}_{OLS} = (H^T H)^{-1} H^T y \qquad (17)$$

After one iteration then, the position and clock bias estimate is given by:

$$\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ \hat{b} \end{pmatrix} = \begin{pmatrix} x_o \\ y_o \\ z_o \\ b_o \end{pmatrix} + \begin{pmatrix} \delta x \\ \delta y \\ \delta z \\ \delta b \end{pmatrix} \qquad (18)$$

Simulation

To determine the feasibility of the technique, a simulation was performed. A simple flight-path was modeled with the aircraft traveling to the east for 900 seconds at 100 meters/second, followed by a 2g turn and then returning to the west (figure 1). For the sake of simplicity in the calculations, a static satellite constellation was modeled. In order to focus on the effects of SA, all other GPS error sources were assumed to be zero. The Loran-C/altimeter errors were modeled in the position domain by a constant 200 meter bias on each axis.

The SA model was obtained from collected data using the System Identification procedure described in [2]. In order to model SA rather than the combination of SA and receiver noise, integrated Doppler data (rather than pseudorange data) were used. The System Identification procedure yielded a 16th order autoregressive (AR) filter. When Gaussian white noise (of proper variance) is input to this filter model, the output is statistically equivalent to the collected SA data. An example of the output is given in figure 2.

The positioning errors resulting from the SA corruption are given in figures 3 and 4. Both the east and north components of the position error exhibit similar characteristics to the SA error on the pseudorange measurements. As discussed earlier, the errors are highly correlated and reach up to 100 meters. However, use of the Loran-C/altimeter data in the complementary Kalman filter yields significant reduction of SA error (figures 5 and 6).
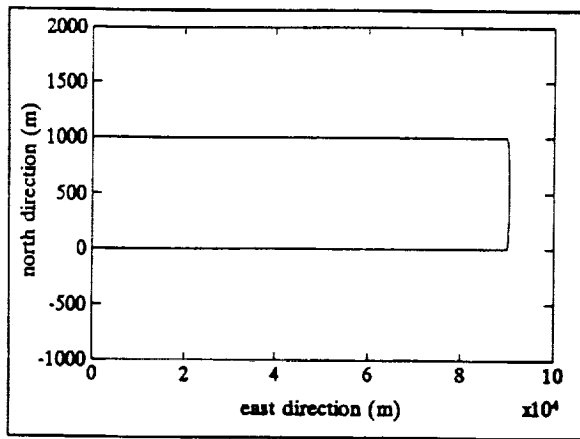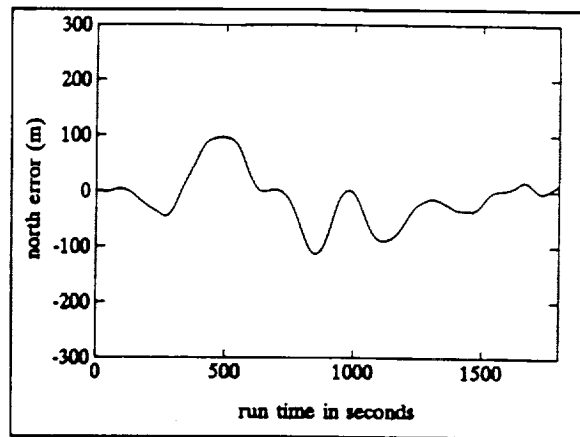
57

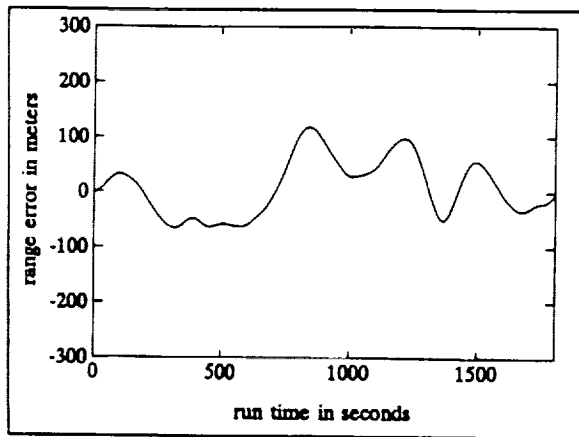Figure 1. Simulated flight path.

Figure 2. Pseudorange error due to SA.

Figure 3. Raw GPS position error due to SA.
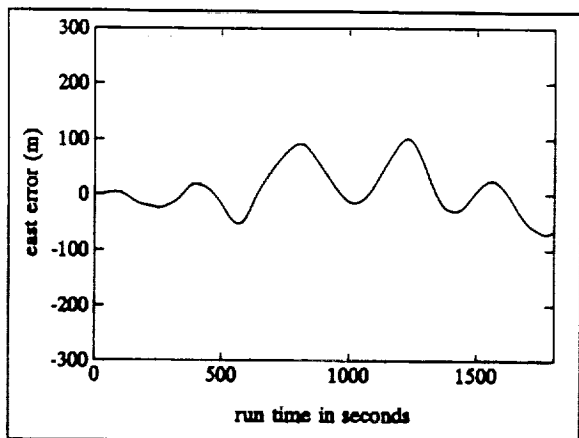

Figure 4. Raw GPS position error due to SA.
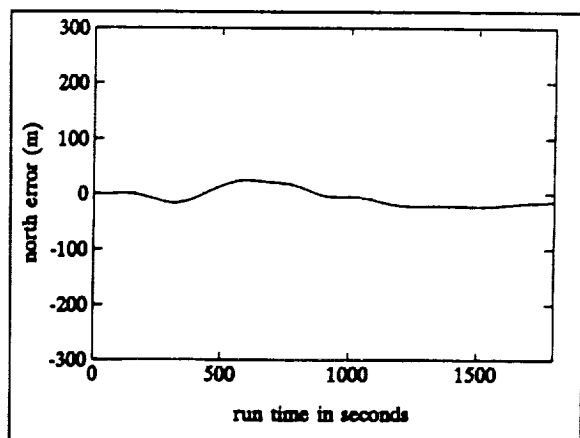

Figure 5. Complementary Kalman filter results.

Figure 6. Complementary Kalman filter results.

Flight Test

Although extremely encouraging, the simulation results were obtained using a simple model for Loran-C position errors. In order to verify the robustness of the technique, actual flight data was used. This is necessary since Loran-C position error bias is spatially dependent.

The flight data employed here were collected during a trip from Cleveland to Athens, Ohio in Fall of 1990 (figure 7). It may be recalled that SA was temporarily turned off at that time because of military use of civilian GPS receivers during Operation Desert Shield [6]. As a result, the GPS horizontal positioning accuracy is on the order of 10-20 meters [1]. For this flight, the GPS-derived position was therefore used as a rough truth reference.

SA was generated by the model described earlier and added to the raw GPS pseudorange measurements (figure 8). As expected, the Loran-C position error is biased but the bias is not constant with position (figures 9 and 10). As was done earlier, altimeter error was modeled as a constant 200 meter bias. Raw SA-induced position errors are as expected with large excursions and high correlation (figures 11 and 12). Again, position errors after smoothing are significantly reduced (figures 13 and 14). It is important to note that even in the face of spatially varying Loran-C position errors, the mitigation scheme continues to perform well.

Conclusions

A technique has been described whereby the stability of Loran-C signals are exploited to reduce SA-induced GPS position errors. The viability of the technique has been confirmed using simulations as well as actual flight data. Future work will consider the possibility of realtime SA model identification.
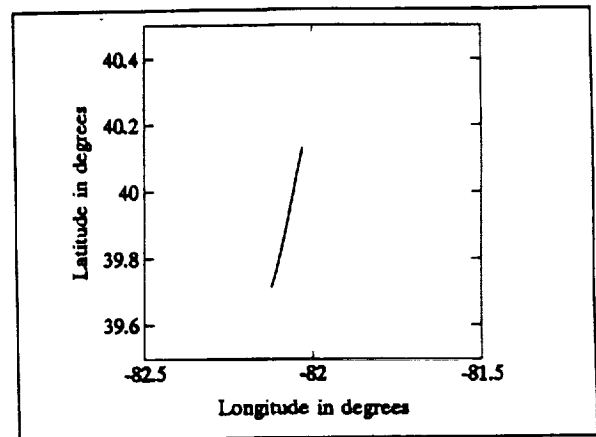
Acknowledgements

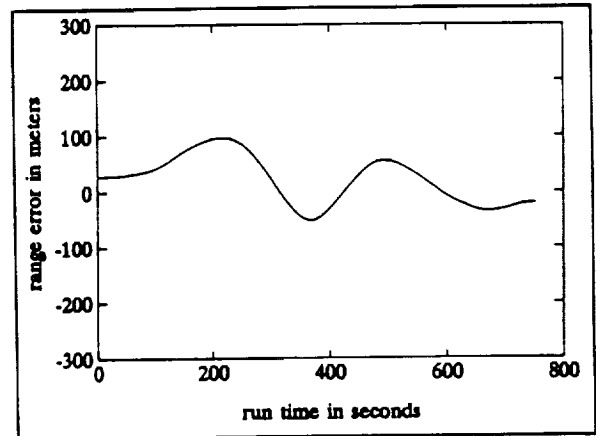**Figure 7.** Ground track, 1990-Nov-01.

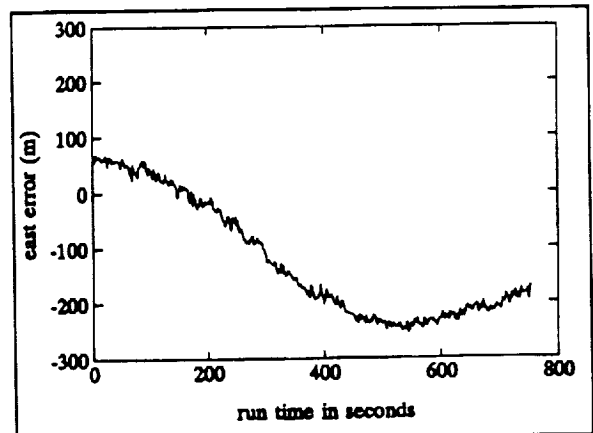**Figure 8.** Pseudorange error due to SA.

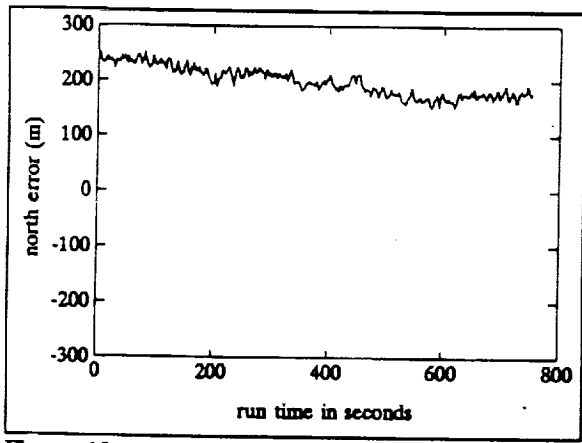**Figure 9.** Loran-C position error.
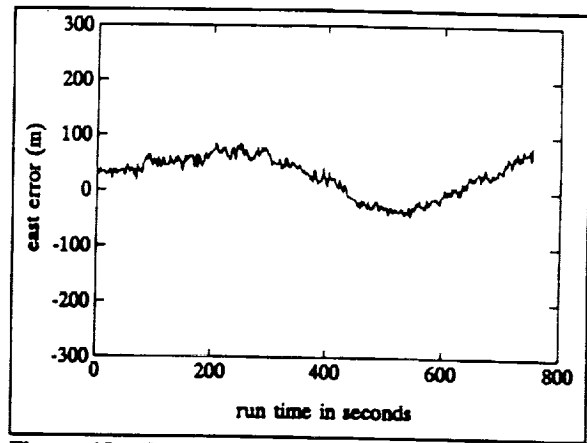
Figure 10. Loran-C position error.
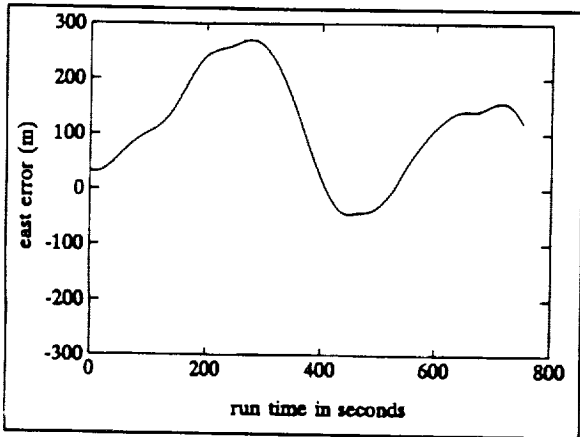
Figure 13. Complementary Kalman filter results.
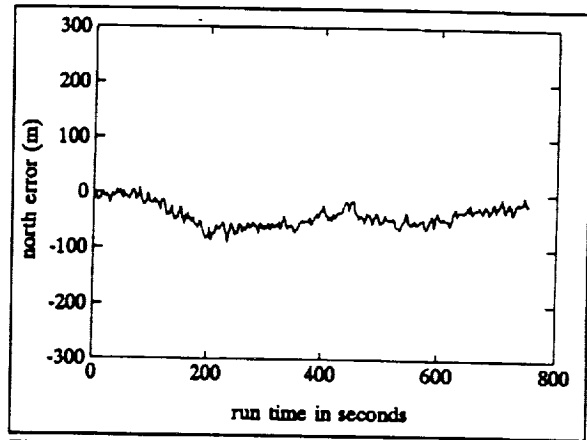
Figure 11. Raw GPS position error due to SA.
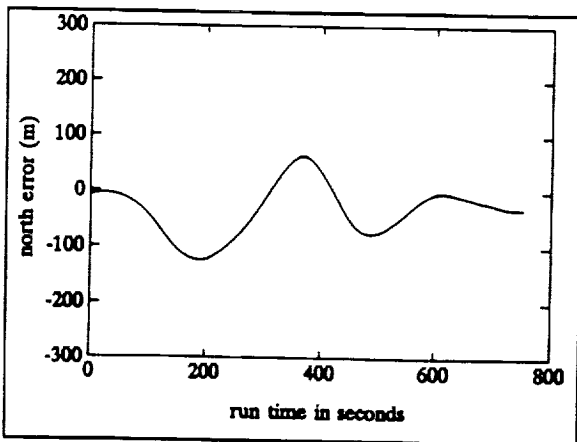
Figure 14. Complementary Kalman filter results.

Figure 12. Raw GPS position error due to SA.

References

1. van Graas, F. "Hybrid GPS/Loran-C: A Next-Generation Of Sole Means Air Navigation," Ph.D. Dissertation, Department of Electrical and Computer Engineering, Ohio University, Athens, Ohio, November 1988.

2. Braasch, M. "A Signal Model for GPS," NAVIGATION: Journal of the Institute of Navigation, Vol. 37, No. 4, Winter 1990-91.

3. Wysocki, J. "GPS and Selective Availability - The Military Perspective," GPS World, July/August 1991.

4. Hwang, P. and R. G. Brown, "GPS Navigation: Combining Pseudorange with Continuous Carrier Phase Using a Kalman Filter," NAVIGATION: Journal of the Institute of Navigation, Vol. 37, No. 2, Summer 1990.

5. van Graas, F. and M. Braasch, "GPS Interferometric Attitude and Heading Determination: Initial Flight Test Results," NAVIGATION: Journal of the Institute of Navigation, Vol. 38, No. 4, Winter 1991-92.

6. Anonymous, "Antispoofing, Selective Availability Moves Surprise Civil Users," Global View, GPS World, November/December 1990.