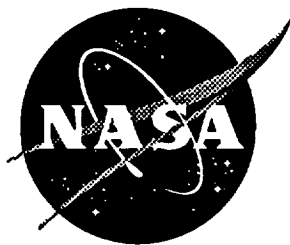NASA Technical Memorandum 109104

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NEW CONSTITUTIVE EQUATION FOR THE VOLUME VISCOSITY IN FLUIDS

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Introduction

The traditional stress tensor for a viscous fluid

$$\sigma_{ij} = -\left[p - \lambda \, \frac{\partial u_m}{\partial x_m}\right]\delta_{ij} + \mu\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] \tag{1}$$

comprises terms representing the thermodynamic pressure, the volume viscosity, and the shear viscosity. Unlike the shear viscosity μ, which can be derived from the Boltzmann Transport Equation and thus lends itself to a microscopic interpretation [1], the traditional volume viscosity λ has no physical basis and no relationship to established dissipative processes in fluids. In other words, neither its value nor even its order of magnitude can be predicted from the fundamental physical properties of a given fluid. In order to circumvent this obstacle to an orderly analysis of viscous flows, Stokes hypothesized that the volume losses in fluids are negligibly small [2] (which is true by definition for incompressible flows). This so-called "Stokes hypothesis"

$$\lambda = -(2/3)\mu \tag{2}$$

has been utilized in a large volume of both experimental and theoretical data in many classes of compressible convective flows.

In nonconvective (periodic or acoustic) flow, on the other hand, the point of view that volume losses are negligibly small has no basis in fact, to which the vast literature on sound absorption in fluids will attest. The underlying absorption processes in gases and some liquids are well understood and known to be attributable to a variety of relaxation processes, which accordingly constitute the physical basis for the volume viscosity. In gases the most prominent of these under ordinary experimental conditions are the relaxations of the molecular degrees of freedom.

The purpose of this memorandum is to close the longstanding gap between acoustics and fluid dynamics with regard to volumetric losses in fluids. We start out by reviewing the fundamental physics of relaxation and its mathematical representation. Then we apply the lossy Navier-Stokes Equation to periodic (acoustic) flow and show that the traditional

expression for volume viscosity leads to a result which contradicts that describing acoustical relaxation. We demonstrate that the addition of a second volume viscosity term resolves the conflict and leads to a direct correspondence between the volume viscosity and the acoustic relaxation parameters. We proceed to discuss the formulation of volume viscosity in the presence of multiple relaxations. Finally, we present an example of the role of volume viscosity in a convective compressible flow. Although our discussion will be confined to gases, our analysis and conclusions will apply equally well to liquids since the mathematical representation of relaxation is the same in both media.

Symbols

c	real part of sound speed
$\mathbf{c}$	complex sound speed
c_o	relaxed sound speed
k	real part of acoustic wave number
L	length scale for linearly accelerating flow
L_{max}	length of contraction section
M	Mach number
$p(x)$	thermodynamic pressure
p_o	thermodynamic pressure at x=0
$p_R(x)$	relaxed pressure
$\Delta p_k(x)$	pressure increment due to kth relaxation
$\bar{p}(x)$	reduced pressure, equation (37)
$\tilde{p}(x)$	amplitude of acoustical pressure variation
R_e	Reynolds number, equation (32d)
t	time
T	time scale for pressure relaxation, equation (32c)
$u(x)$	one-dimensional flow velocity
$\bar{u}(x)$	reduced flow velocity, equation (32a)
u_i	ith component of flow velocity
U_e	exit flow velocity of contraction section
U_o	upstream flow velocity
x	spatial coordinate
x_i	ith component of spatial coordinate
$\bar{x}$	reduced distance, equation (32b)
X	mole fraction of gas constituent
α	imaginary part of acoustic wave number
ϵ	relaxation strength
η_p	pressure relaxation coefficient
η_v	density relaxation coefficient

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κ	reduced density relaxation coefficient, equation (32e)
λ	traditional volume viscosity coefficient
Λ	acoustical wavelength
μ	shear viscosity coefficient
$\rho(x)$	gas density
ρ_0	equilibrium gas density
$\tilde{\rho}(x)$	amplitude of acoustical density variation
$\rho_R(x)$	relaxed density
$\Delta\rho_k(x)$	density increment due to kth relaxation
σ_{ij}	stress component
τ	experimental relaxation time, equation (11b)
τ_{ps}	isentropic relaxation time at constant pressure
τ_{vs}	isentropic relaxation time at constant volume (density)
ω	acoustical angular frequency

Numerical Subscripts

i,j,m	indicates stress component (i,j,m = 1,2,3)
k	indicates relaxation process (k = 1,2,...)

Mathematical Operators

D/Dt	$= \partial/\partial t + u\partial/\partial x$, one-dimensional total time derivative
δ_{ij}	$= 0$ if $i \neq j$, $= 1$ if $i=j$, Kroniker delta
O( )	indicates order of magnitude

Acoustic Relaxation in Gases

The following derivation of the acoustic relaxation equations is based on small periodic (harmonic) variations, one-dimensional propagation, and a single relaxation process. The equations of motion

$$\rho_0 \frac{\partial u}{\partial t} = - \frac{\partial p}{\partial x} \qquad (3)$$

and continuity

$$\frac{\partial u}{\partial x} = - \frac{1}{\rho_0} \frac{\partial \rho}{\partial t} \qquad (4)$$

lead to a quasi-wave equation

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$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2 p}{\partial x^2} \tag{5}$$

which is satisfied by travelling wave solutions of the form

$$\rho = \overset{\sim}{\rho}\, e^{i(\omega t - kx)} \tag{6a}$$

$$p = \tilde{p}\, e^{i(\omega t - kx)} \tag{6b}$$

When we insert (6a) and (6b) into the acoustical equation of state [3]

$$(1 + \tau_{ps}\frac{\partial}{\partial t})\rho = \frac{1}{c_o{}^2}(1 + \tau_{vs}\frac{\partial}{\partial t})p \tag{7}$$

we obtain the ratio $\overset{\sim}{\rho}/\tilde{p}$, which is the the reciprocal complex sound speed squared:

$$\frac{\overset{\sim}{\rho}}{\tilde{p}} = \frac{1}{c^2} = \frac{1}{c_o{}^2}\frac{1+i\omega\tau_{vs}}{1+i\omega\tau_{ps}} + O(\epsilon^2) \tag{8}$$

Upon separating equation (8) into real and imaginary parts, we isolate the terms representing the sound dispersion (real) and absorption (imaginary):

$$\frac{c_o{}^2}{c^2} = 1 - \epsilon\frac{\omega^2\tau_{ps}{}^2}{1+\omega^2\tau_{ps}{}^2} - i\epsilon\frac{\omega\tau_{ps}}{1+\omega^2\tau_{ps}{}^2} + O(\epsilon^2) \tag{9}$$

or, equivalently

$$\frac{c^2}{c_o{}^2} = 1 + \frac{\epsilon}{1-\epsilon}\frac{\omega^2\tau^2}{1+\omega^2\tau^2} + i\frac{\epsilon}{\sqrt{1-\epsilon}}\frac{\omega\tau}{1+\omega^2\tau^2} + O(\epsilon^2) \tag{10}$$

As indicated in equations (9) and (10), a single relaxation process contains two defining parameters, a "relaxation strength" and a "relaxation time." The parameters used in phenomenological theories of relaxation are the isentropic relaxation times at constant pressure τ_{ps} and at constant volume τ_{vs}, but those obtained from acoustical measurements are the relaxation strength ϵ and a third relaxation time τ. The parameter pairs are interrelated by

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$$\epsilon = \frac{\tau_{ps} - \tau_{vs}}{\tau_{ps}} \tag{11a}$$

$$\tau = \sqrt{\tau_{ps}\tau_{vs}} \tag{11b}$$

or conversely,

$$\tau_{ps} = \tau/\sqrt{1-\epsilon} \tag{12a}$$

$$\tau_{vs} = \tau\sqrt{1-\epsilon} \tag{12b}$$

The relaxation strength is a measure of the capacity of the medium
to convert translational energy of the flow into internal energy.
It can be evaluated with great precision from known thermodynamic
properties of the medium. The relaxation time is a measure of
the rapidity with which the medium reacts to changes in
translational energy. Because microscopic theories of the
relaxation time are generally imprecise, quantitative evaluation
depends on experiment. Information on the relaxation processes
occurring in air are given later in this memorandum.

It is important to note that the relaxation strength depends
upon the difference between τ_{ps} and τ_{vs}, a fact not taken into
account in previous treatments of the volume viscosity.

The real part of equation (10) yields the dispersion of the
sound speed squared, shown in the plot of figure 1. The
"dispersion step" has a height of $\epsilon/(1-\epsilon)$, equal to the
difference between the low and high frequency limiting solutions
of the plot, and an inflection point where the condition $\omega\tau=1$ is
fulfilled.

The imaginary part of equation (10) yields the sound
absorption per unit wavelength:

$$\alpha\Lambda \frac{c_o^2}{c^2} = \frac{\pi\epsilon}{\sqrt{1-\epsilon}} \frac{\omega\tau}{1+\omega^2\tau^2} \tag{13a}$$

$$\alpha\Lambda = \frac{\pi\epsilon}{\sqrt{1-\epsilon}} \frac{\omega\tau}{1+\omega^2\tau^2} + O(\epsilon^2) \tag{13b}$$

The plot of equation (13b) in Fig. 2 shows an absorption peak of height $\pi\epsilon/(2\sqrt{1-\epsilon})$ and a location at $\omega\tau=1$.

Both plots reveal that a relaxation process is ineffective at very low frequencies, where the internal degrees of freedom have ample time to equilibrate with translation, and at very high frequencies, where changes in translation occur too rapidly for equilibration to take place. In convective compressible flows, then, the volume viscosity would be expected to be most effective on time scales on the order of the relaxation time τ.

Traditional Volume Viscosity and Application to Acoustics

We observe that the acoustical wave equation (5) does not contain a dissipative term. Dissipation is accounted for in the complex sound speed, which is derived from the acoustical equation of state (7). The introduction of a physically meaningful volume viscosity implies a transfer of the relaxation terms from the equation of state to the Navier-Stokes Equation. Then, the equation of state will serve simply to determine the stationary temperature of the medium.

To investigate the case of damped acoustical propagation, let us insert the traditional stress tensor [equation (1)] into the one-dimensional Navier-Stokes Equation, retaining only the dilatational terms. In place of equation (3) we obtain

$$\rho_0 \frac{\partial u}{\partial t} = \frac{\partial \sigma_{11}}{\partial x} = -\frac{\partial p}{\partial x} + \eta_v \frac{\partial^2 u}{\partial x^2} \tag{14}$$

where for one-dimensional propagation

$$\eta_v = \lambda + 2\mu \tag{15}$$

Equation (14) together with the continuity equation (4) leads to the lossy wave equation:

$$\frac{\partial^2 \rho}{\partial t^2} - \frac{\eta_v}{\rho_0} \frac{\partial^3 \rho}{\partial x^2 \partial t} = \frac{\partial^2 p}{\partial x^2} \tag{16}$$

Solutions of the form (6a) and (6b) yield the complex sound speed:

$$\frac{\tilde{\varrho}}{\tilde{p}} = \frac{1}{c^2} = \frac{k^2}{\omega^2 - i\omega(\eta_v/\rho_o)k^2} \qquad (17)$$

from which

$$\frac{c_o^2}{c^2} = 1 - \frac{\omega^2\eta_v^2/\rho_o^2 c_o^4}{1+\omega^2\eta_v^2/\rho_o^2 c_o^4} + i\,\frac{\omega\eta_v/\rho_o c_o^2}{1+\omega^2\eta_v^2/\rho_o^2 c_o^4} + O(\epsilon^2) \qquad (18)$$

A term-by-term comparison between equations (18) and (9) reveals that the quantity $\eta_v/\rho_o c_o^2$ can be identified with $-\tau_{ps}$, but that the two equations are compatible only if $\epsilon=1$--an absurd conclusion, for Kneser proves that the relaxation strength of a diatomic gas is bounded by $\epsilon<0.082$ for the vibrational relaxation and $\epsilon<0.16$ for the rotational relaxation [4]. Therefore, we conclude that the volume viscosity cannot be represented by the traditional constitutive equation (1), containing only a single term.

New Constitutive Equation for the Volume Viscosity

Taking a cue from the acoustical equation of state (7), we hypothesize that the second volume viscosity term is proportional to the time derivative of the pressure. Then the stress tensor becomes

$$\sigma_{ij} = -\left[p - \eta_p \frac{Dp}{Dt} - \eta_v \frac{\partial u_m}{\partial x_m}\right]\delta_{ij} + \mu\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_m}{\partial x_m}\delta_{ij}\right] \qquad (19)$$

where for one-dimensional flow the factor 2/3 is replaced by 2, and the operator

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x}$$

is used to allow for pressure changes in convective flow. Let us examine the acoustical consequences. When we substitute the nonconvective dilatational terms of (19) into the one-dimensional Navier-Stokes Equation, we now obtain

$$\rho_o \frac{\partial u}{\partial t} = \frac{\partial \sigma_{11}}{\partial x} = -\frac{\partial p}{\partial x} + \eta_v \frac{\partial^2 u}{\partial x^2} + \eta_p \frac{\partial^2 p}{\partial x\partial t} \qquad (20)$$

and the lossy wave equation becomes

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$$\frac{\partial^2 \rho}{\partial t^2} - \frac{\eta_v}{\rho_o} \frac{\partial^3 \rho}{\partial x^2 \partial t} = \frac{\partial^2 p}{\partial x^2} - \eta_p \frac{\partial^3 p}{\partial x^2 \partial t} \qquad (21)$$

As before, solutions of the form (6a) and (6b) lead to the complex sound speed

$$\frac{\tilde{\rho}}{\tilde{p}} = \frac{1}{c^2} = \frac{k^2 - i\omega k^2 \eta_p}{\omega^2 - i\omega(\eta_v/\rho_o)k^2} = \frac{1}{c_o^2} \frac{1 - i\omega\eta_p}{1 - i\omega\eta_v/\rho_o c_o^2} + O(\epsilon^2) \qquad (22)$$

Comparison of the complex sound speed derived from the volume viscosity [equation (22)] with that derived from the acoustical equation of state [equation (8)] yields the volume viscosity parameters in terms of the acoustical relaxation parameters:

$$\eta_p = -\tau_{vs} \qquad (23a)$$

$$\eta_v = -\rho_o c_o^2 \tau_{ps} \qquad (23b)$$

Both η_p and η_v are negative quantities in order to provide the proper phase relationships when equation (21) is applied to acoustic flow.

Volume viscosity and pressure relaxation cannot be modeled adequately via a single constitutive constant. The difficulty is due to the more fundamental character of pressure, in comparison to either volumetric viscous effects or shear stresses. Since pressure is accepted universally as a thermodynamic variable which can be used in the determination of other thermodynamic properties, it is not logical to force pressure to be related directly to the material rate of change of density, through the conservation of mass equation and a single volume viscosity coefficient. Furthermore, it is not logical to assume that pressure and density are determined uniquely for unsteady flows of simple fluids through an equilibrium equation of state, if those fluids possess internal degrees of freedom (which equilibrate at different rates). These relaxation effects are easily visualized in terms of the behavior of air when it is subjected to high frequency acoustic excitations. It is certainly evident that the temporal behavior of the density, produced by these acoustic oscillations, does not correspond to that produced by a reversible, quasi-static, cyclic equilibrium process. In fact, it is known that pressure must respond more rapidly to these propagating disturbances than density because the disturbances consist primarily of collisional

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exchanges of translational momentum, and pressure is a direct measure of translational momentum. Furthermore, since the nitrogen and oxygen molecules in air possess internal degrees of freedom that equilibrate more slowly than their translational counterparts, acoustically excited air is forced to exist in a regime where density and pressure are no longer in phase with each other, and temporal variations in internal molecular temperatures can be substantially different from their translational temperature histories. While it may be possible to ignore volume viscosity contributions completely, via Stokes' hypothesis, it is not possible to ignore these phase shifts between density, temperature(s), and pressure that are manifest by these fundamental departures from thermodynamic equilibrium. If pressure equilibrium is enforced by an idealized equation of state, then the pressure is modified improperly via the traditional volume viscosity. It is certainly true that the pressure relaxation constant, introduced in our constitutive model, can be used along with volume viscosity; but, even further, it is quite possible that the pressure relaxation constant follows only the dynamic viscosity in importance as a constitutive constant.

Multiple Relaxations

The volumetric dissipation in many gases, notably air, is characterized not by one relaxation but by multiple relaxations occurring simultaneously. For the acoustical case Bauer shows that the effects of the individual relaxation processes upon the compressibility, as evidenced by the dispersion and absorption terms in equation (9), are additive [3]. The treatment here for generalized flow, then, is similarly based on the assumption that the effects of the individual relaxation processes are additive. A discussion of coupling among the relaxation processes appears at the end of this section.

We define $p_R(x)$ and $\rho_R(x)$ as the "relaxed" pressure and density, i.e. the solutions obtained without losses. Then we assume that the pressure $p(x)$ and density $\rho(x)$ are composed of the relaxed contribution plus the sum of the contributions from all the relaxation processes. If the gas is a mixture, then each contribution must be multiplied by the mole fraction X_k of the relaxing component of the mixture:

$$p(x) = p_R(x) + \sum_k X_k \Delta p_k(x), \qquad k = 1, 2, \ldots \qquad (24a)$$

$$\rho(x) = \rho_R(x) + \sum_k X_k \Delta \rho_k(x) \qquad (24b)$$

where $\Delta p_k(x)$ and $\Delta \rho_k(x)$ are the changes in pressure and density due to the kth relaxation alone. These changes are not necessarily perturbations and can, in fact, be very large. If $p_k(x)$ and $\rho_k(x)$ are solutions to Navier-Stokes Equation for the kth single relaxation process alone, using equation (19), then the changes are simply

$$\Delta p_k(x) = p_k(x) - p_R(x) \qquad (25a)$$

$$\Delta \rho_k(x) = \rho_k(x) - \rho_R(x) \qquad (25b)$$

The above procedure is based on the condition that the presence of one relaxation process does not influence the effects due to another; in other words, the relaxations are decoupled. In reality this is not the case, for there are two types of coupling among the individual relaxations: gas-kinetic and nonlinear.

Gas-kinetic coupling arises from the fact that the reactants and products of molecular reactions can appear in more than one reaction. The relaxation times τ_{psk} and τ_{vsk} are no longer those of the decoupled kth relaxation alone. Rather they are determined from the solutions to a set of linear simultaneous equations involving all the coupled reactions. Since the coupling is linear, the formulation expressed by equations (24)-(25) remains valid; only the values of τ_{psk} and τ_{vsk} have to be adjusted. For air this subject is treated in reference 5.

The second type of coupling is due to the nonlinear nature of the convective operator $u\partial/\partial x$. When operating on equation (24), it will introduce mixed terms in the Navier-Stokes Equation. Additional terms will be required in the series to account for the mixing. Generally, it may be expected that the volumetric losses and pressure and density changes are sufficiently small to permit equations (24)-(25) to remain very accurate approximations for most gases.

An Example: Linearly Accelerating Flow

In order to examine the behavior of this constitutive model, it is convenient to examine a class of steady, one-dimensional flows. That class of flows includes flows through standing normal shock waves, but we are interested here in flows which are more typically encountered in the subsonic contraction sections of wind tunnels. If the velocity is given by $u = u(x)$, the material or total time derivative is given by

$$\frac{D}{Dt} = u\frac{d}{dx}$$

and conservation of mass requires that

$$\frac{d}{dx}(\rho u) = 0 \tag{26}$$

Thus, if we assume that the density and velocity are prescribed at the coordinate origin--say ρ_o and U_o--conservation of mass is equivalent to the requirement

$$\rho(x)u(x) = \rho_o U_o \tag{27}$$

Conservation of linear momentum can be written

$$\rho u\frac{du}{dx} = \frac{d}{dx}(\rho_o U_o u) = \frac{d\sigma_{11}}{dx} \tag{28}$$

and if we assume that $\sigma_{11}(0) = -p_o$, equation (28) can be integrated to yield

$$\sigma_{11}(x) = -p_o - \rho_o U_o[U_o - u(x)] \tag{29}$$

which is a kinematical relationship not affected directly by the constitutive model.

The constitutive model can be examined via the kinematical equation for normal stress. That is, σ_{11} must satisfy the relationship

$$\sigma_{11}(x) = -p(x) + \eta_p u\frac{dp}{dx} + \eta_v\frac{du}{dx} = -p_o - \rho_o U_o(U_o - u) \tag{30}$$

Before proceeding to the particular flow case, it is useful to distinguish between "lossless" pressure variations (where η_p, η_v, and μ are taken to be zero) and "lossy" effects. That is, we

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define $\Delta\bar{p}(x)$:

$$\Delta\bar{p}(x) = \frac{p(x) - p_o - \rho_o U_o(U_o - u)}{\rho_o U_o^2} \qquad (31)$$

and introduce the reduced and dimensionless variables

$$\bar{u} = u/U_o \qquad (32a)$$

$$\bar{x} = x/L \qquad (32b)$$

$$T = -\eta_p U_o/L = Mc_o \tau_{vs}/L \qquad (32c)$$

$$R_e = \rho_o U_o L/\mu = \rho_o Mc_o L/\mu \qquad (32d)$$

$$\kappa = -\eta_v/\mu = \rho_o c_o^2 \tau_{ps}/\mu \qquad (32e)$$

Then, our constitutive model must satisfy

$$\Delta\bar{p} = -T\bar{u}(d\Delta\bar{p}/d\bar{x}) + (T\bar{u} - \kappa/R_e)(d\bar{u}/d\bar{x}) \qquad (33)$$

For simplicity, we consider linearly accelerating flow systems, where:

$$\bar{u}(\bar{x}) = 1 + \bar{x} \qquad (34)$$

We note that in the case of this "contoured" wind tunnel contraction flow, the characteristic length L is related to the length of the contraction section L_{max} and the exit velocity U_e by:

$$L = \frac{L_{max} U_o}{U_e - U_o} \text{ , with } 0 \leq x \leq \frac{U_e - U_o}{U_e} \qquad (35)$$

Here the lossless or "relaxed" pressure would decrease linearly (from p_o) with $\bar{x}$:

$$\bar{p}_R = \bar{p}_o - \bar{x} \qquad (36)$$

where the overbar indicates a reduced pressure

$$\bar{p} = p/\rho_o U_o^2 \qquad (37)$$

The reduced lossy pressure variation can be gotten by integrating equations (33) and (34), with $\Delta\bar{p}(0) = 0$, to get:

$$\Delta\bar{p} = \frac{T}{T+1}\bar{x} + \left[\frac{\kappa}{R_e} - \frac{T}{T+1}\right]\left[(1+\bar{x})^{-1/T} - 1\right] \qquad (38)$$

which yields the interesting relation that volume viscosity will cause $\Delta\bar{p}$ to be negative, since $(1+\bar{x})^{-1/T} - 1$ is negative for $\bar{x}>0$; whereas the pressure lag contributes an additional effect which is positive rather than negative. Since $\Delta\bar{p} = -\kappa/R_e$, when $T=0$, the pressure lag effect changes the lossy pressure behavior fundamentally, by eliminating the instantaneous and constant $\Delta\bar{p}$ contribution, and introducing a contribution which is initially zero and becomes positive.

The significant relaxing degrees of freedom in air are the vibration of N_2, vibration of O_2, rotation of N_2 and O_2 taken together, and translation of all constituents. For relatively long time scales ($>1\ \mu$s) only the two vibrational relaxations need be considered. Applying equation (38) together with (24a) and designating N_2 and O_2 with subscripts "N" and "X," we find for the reduced pressure:

$$\bar{p}(\bar{x}) = \bar{p}_o + X_N \left\{ \frac{T_N}{T_N+1}\ \bar{x} + \left[\frac{\kappa_N}{R_e} - \frac{T_N}{T_N+1} \right] \left[(1+\bar{x})^{-1/T_N} - 1 \right] \right\}$$

$$+ X_X \left\{ \frac{T_X}{T_X+1}\ \bar{x} + \left[\frac{\kappa_X}{R_e} - \frac{T_X}{T_X+1} \right] \left[(1+\bar{x})^{-1/T_X} - 1 \right] \right\} \qquad (39)$$

Values for the relaxation times and relaxation strengths for N_2 and O_2 are taken from reference 6. Then equations (12a,b) and (23a,b) are used to find τ_{ps}, τ_{vs}, η_p, and η_v. These are listed for a temperature of 20°C and humidity values of 0 and 1 mole percent in table 1. Because of the small relaxation strengths of N_2 and O_2, both τ_{ps} and τ_{vs} are nearly equal to the experimentally determined relaxation time τ for each gas constituent.

Equation (39) is plotted in figures 3 and 4 for a temperature $T = 20$°C, length scale $L = 1$m, and absolute humidities $h = 0$ and 1 mole percent, respectively. The terms $T/(T+1)$ related to pressure relaxation lead to a positive pressure increment, while the terms κ/R_e related to the traditional volume viscosity lead to a negative pressure increment (or decrement). A net pressure decrement is possible only if the volume viscosity terms exceed the pressure relaxation terms, as occurs at subsonic Mach numbers. In fact, at a speed Mach 0.9 the decrement turns into an increment at a reduced distance $\bar{x} \sim 0.4 - 0.5$. At Mach 1 and higher the increment is positive over the whole range of reduced

distance. Unlike the acoustical case, there do not appear to be readily discernible conditions for which the increment reaches a maximum.

Conclusions

The traditional volume viscosity cannot be given the same fundamental status as dynamic viscosity. Not only does the traditional volume viscosity fail to evolve naturally from kinetic theory, but we have presented explanations showing that the volume viscosity coefficient must represent multiple molecular relaxation processes, including volumetric dissipation, simultaneously when the conventional Newtonian-fluid model is used. Furthermore, because of the direct relationship between the divergence of the velocity vector and the particle rate of change of density, volume viscosity in effect forces normal stresses, including pressure, to be modeled in a manner which is inconsistent with the more rigorous requirements resulting from nonequilibrium acoustic equations of state. We have proposed that a pressure relaxation contribution be added directly to the constitutive model for normal stresses, in order to bring the constitutive model into better agreement with accepted nonequilibrium behavior.

Using accepted acoustical theory, we have estimated the values for the pressure relaxation coefficients and volume viscosities in pure nitrogen and oxygen at 20°C. Those estimates indicate that the nitrogen contributions are nearly an order of magnitude larger than the oxygen contributions, but both coefficients are influenced strongly by humidity. Our estimates show that the volume viscosity is several million times larger than the dynamic viscosity for perfectly dry nitrogen, but the ratio of volume to dynamic viscosity decreases by a factor of 32 at moderately high humidity levels (at 20°C). In addition, if pressure fluctuations are on the order of 1 Pa/s, the pressure relaxation contribution to normal stresses is nearly an order of magnitude larger than the volume viscosity contribution, when the modified constitutive model is employed.

The application to a simple problem in one-dimensional convective flow reveals that there are conditions for which the volumetric losses in air are negligible (low Mach number, moderate humidity), in agreement with the Stokes' hypothesis, but other conditions for which they are substantial (high Mach number, low humidity). The latter case suggests caution to the

advocates of the popular notion that computers will replace wind tunnels, for a computational algorithm is effective only to the extent that it incorporates the significant physical interactions. A computation ignoring or misrepresenting volumetric losses may fail to predict effects which would be observed in wind tunnel tests.

The constitutive model proposed here should apply equally well to liquids, since the acoustical equation of state is the same as for gases [7]. The constitutive coefficients can be determined from acoustical relaxation data through equations (23a-b). Then the volumetric losses due to thermal relaxation [8], for example, can be computed for convective flow in liquids.

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Table 1. Volume Viscosity Parameters for N_2 and O_2 at 20°C.

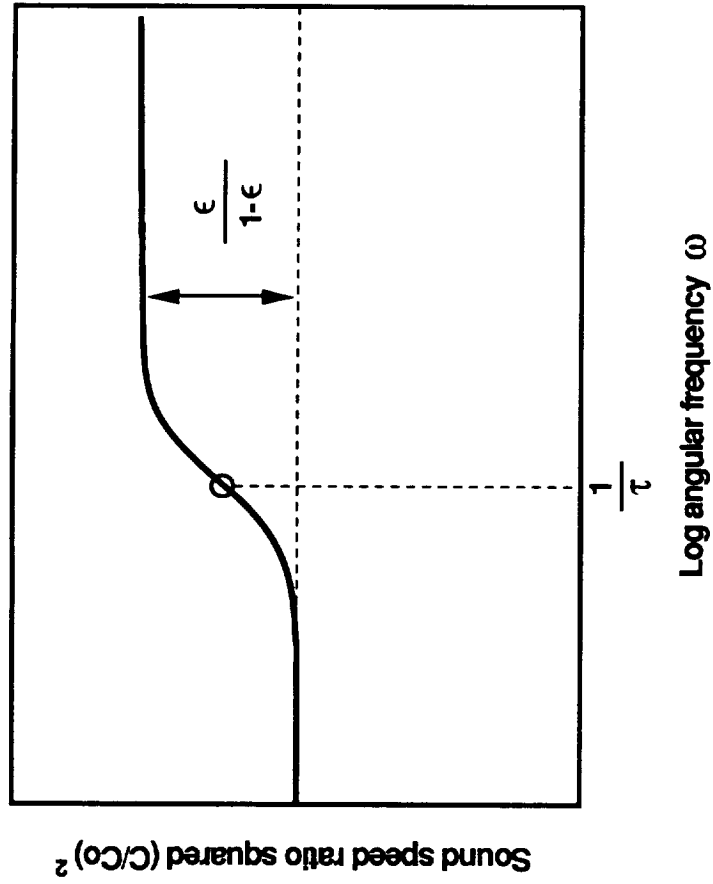| Humidity, mole percent			0		1	
Air constituent			N_2	O_2	N_2	O_2
Parameter		Unit				
Relaxation strength ϵ			0.00016	0.0032	0.00016	0.0032
Relaxation times	τ	μs	17700	6630	551	5.37
	τ_{ps}	μs	17700	6640	551	5.38
	τ_{vs}	μs	17700	6620	551	5.36
Volume viscosity	η_p	μs	−17700	−6620	−551	−5.36
coefficients	η_v	Pa.s	−2512.6	−943.6	−78.24	−0.764
Mole fraction	X		0.79	0.21	0.79	0.21

Figure 1. Sound speed ratio squared $(c/c_0)^2$, showing the dispersion characteristic of a single relaxation process.
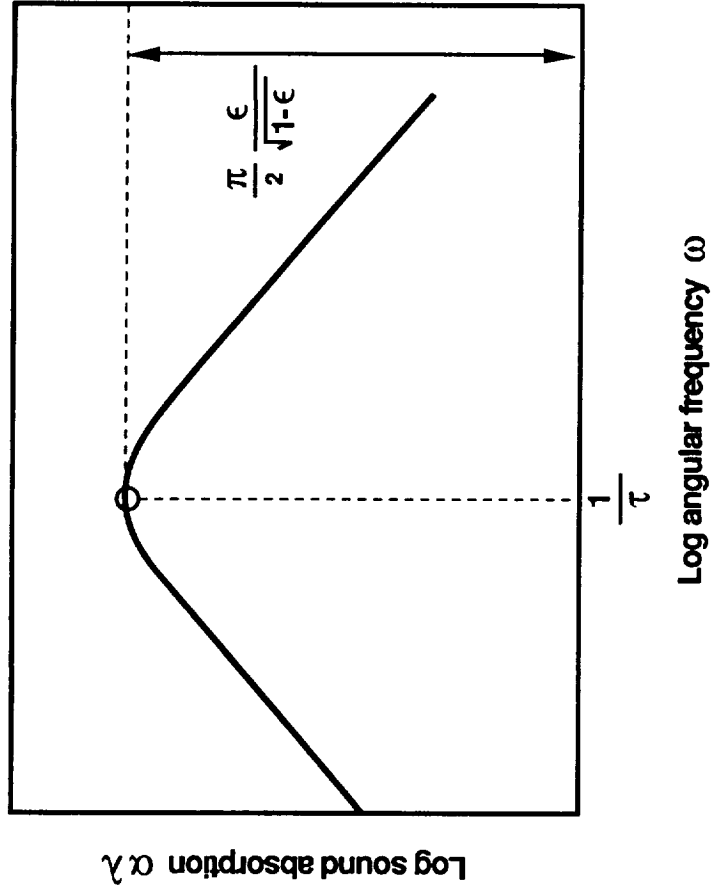

Figure 2. Sound absorption per unit wavelength, showing the relaxation peak characteristic of a single relaxation process.
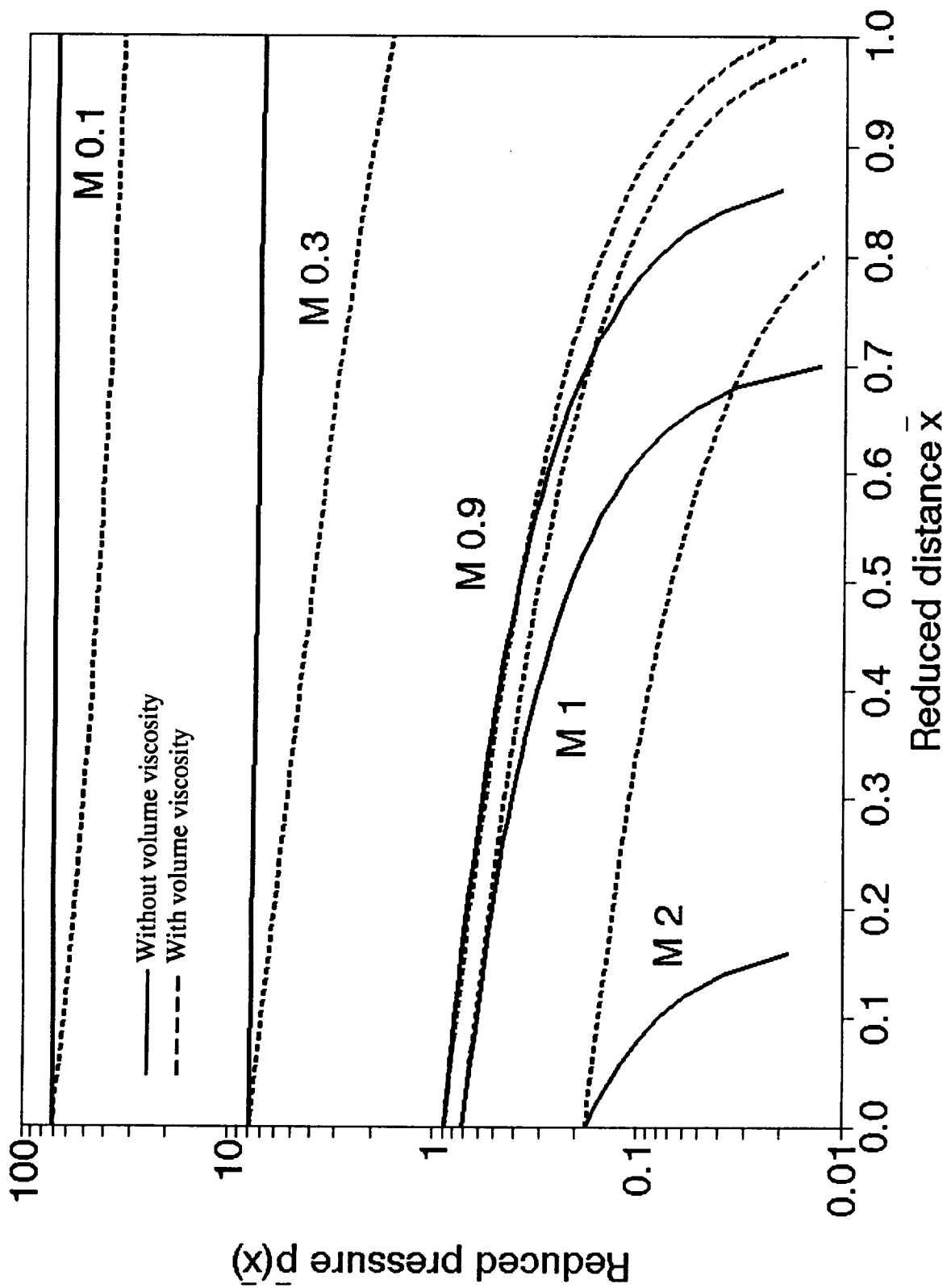
17

Figure 3. Reduced pressure versus reduced distance for a linearly accelerating flow in air at various downstream Mach numbers. The temperature is 20°C, pressure 1 atm, humidity 0 percent mole ratio, and scale length L = 1m.

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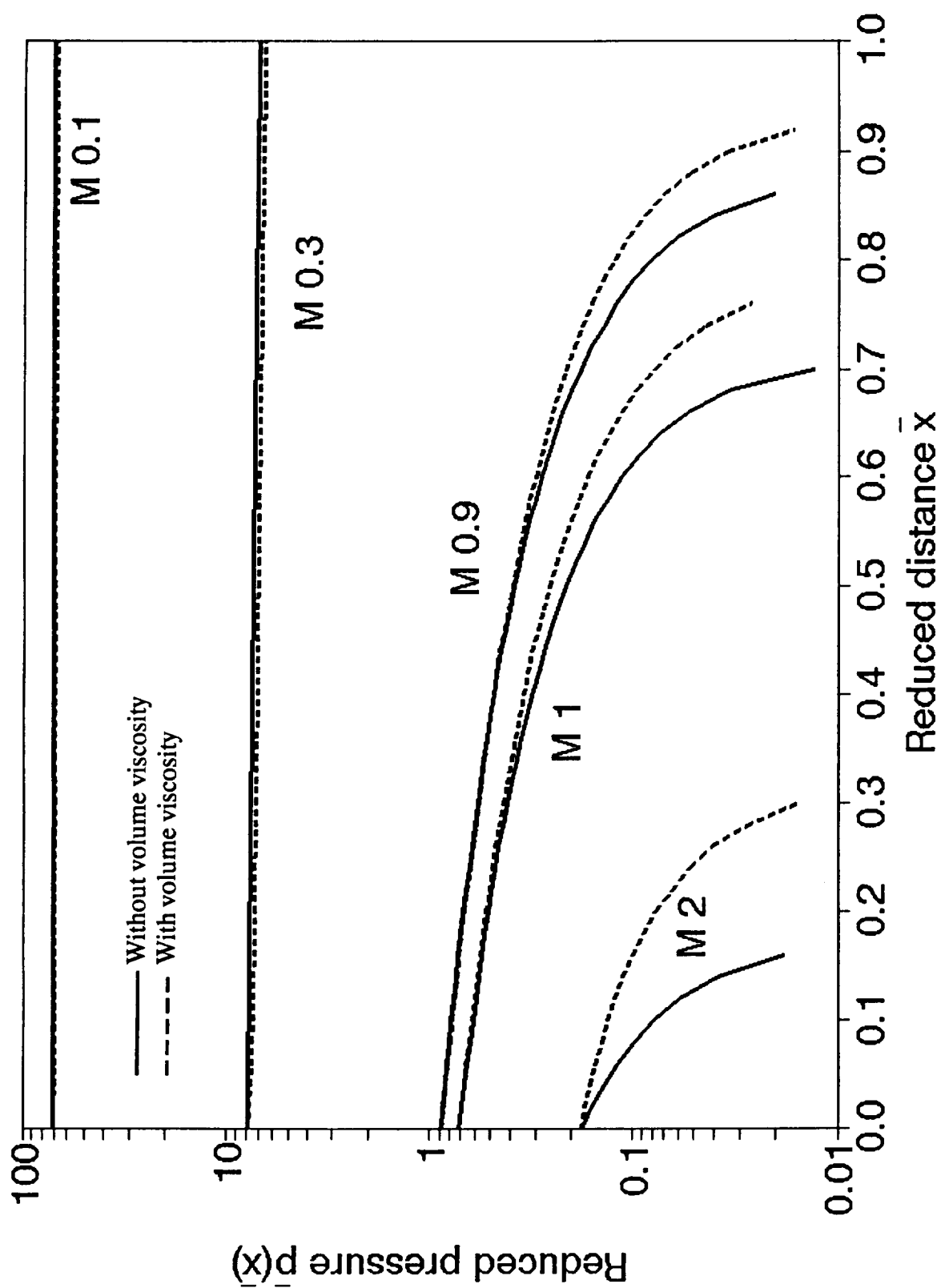Figure 4. Same as figure 3 except for a humidity of 1 percent mole ratio (43.4 percent relative humidity).

REPORT DOCUMENTATION PAGE

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11. SUPPLEMENTARY NOTES

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13. ABSTRACT (Maximum 200 words)

The traditional volume viscosity, Stokes' hypothesis, and acoustical relaxation are reviewed. The lossy Navier-Stokes Equation is applied to periodic (acoustic) flow, and it is shown that the traditional volume viscosity leads to a result which contradicts that describing acoustical relaxation. It is demonstrated that the addition of a second volume viscosity term to the constitutive equation, to account for pressure relaxation, resolves the conflict and leads to a direct correspondence between the volume viscosity parameters and the acoustical relaxation parameters. The representation of volume viscosity is formulated for the case of multiple relaxations, as occur in air. Finally, an application of the new constitutive equation to a simple convective compressible flow, namely a linearly accelerating flow, demonstrates the impact of volume viscosity upon the flow and the physical conditions for which it is important.

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