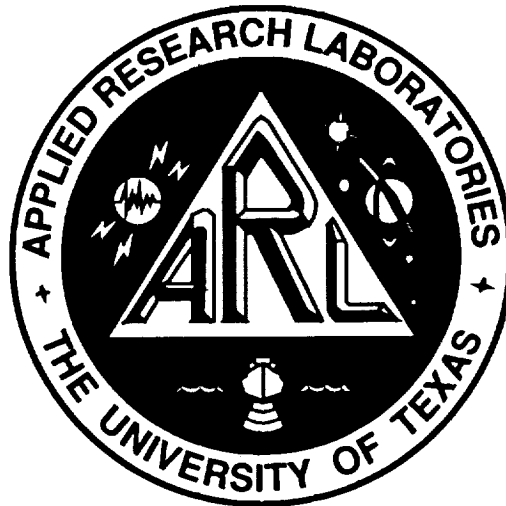# Sonic Boom Propagation through Turbulence; A Ray Theory Approach

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In this work, a ray theory approach is used to examine the propagation of sonic booms through a turbulent ground layer, and to make predictions about the received waveform. The rays are not propagated one at a time, as is typical in ray theory; instead, sufficient rays to represent a continuous wave front are propagated together. New rays are interpolated as needed to maintain the continuity of the wave front. In order to predict the received boom signature, the wave front is searched for eigenrays after it has propagated to the receiver.

OVERVIEW

- **Rays describing a wave front propagate through an instantaneous "snapshot" of the turbulence.**

- **Turbulence produces focusing and defocusing of portions of the wave front, which results in caustic formation, wave front "folding", and multiple eigenray paths to the receiver.**

- **The eigenrays to the receiver are identified.**

- **The respective arrival times and ray tube areas of the eigenrays, along with the identification of caustics, generate the predicted waveform at the receiver.**

- **If repeated many times, this generates a statistical description of the predicted wave form characteristics.**

The Comte-Bellot turbulence model (Ref. 1) is used to generate an instantaneous "snapshot" of the turbulent field. The transient acoustic wave is assumed to be sufficiently short in duration such that the time-dependance of the turbulent field may be neglected.

Turbulence Model
(Comte-Bellot '91)

Instantaneous realization of incompressible, isotropic turbulence is represented by a sum of Fourier modes:

$$w(x) = \sum_{j=1}^{N} a_j \cos(k_j \cdot x + \phi_j)$$

where directions of a_j and k_j are random with the provision that

$$(a_j \cdot k_j) = 0 \quad \text{for each } j$$

For a given mode k_j, the magnitude $|a_j|$ is given by

$$|a_j| \sim \sqrt{E(k)\,\delta k}$$

δk is the separation between modes.
The spectral energy density $E(k)$ is given by the Von Kármán model:

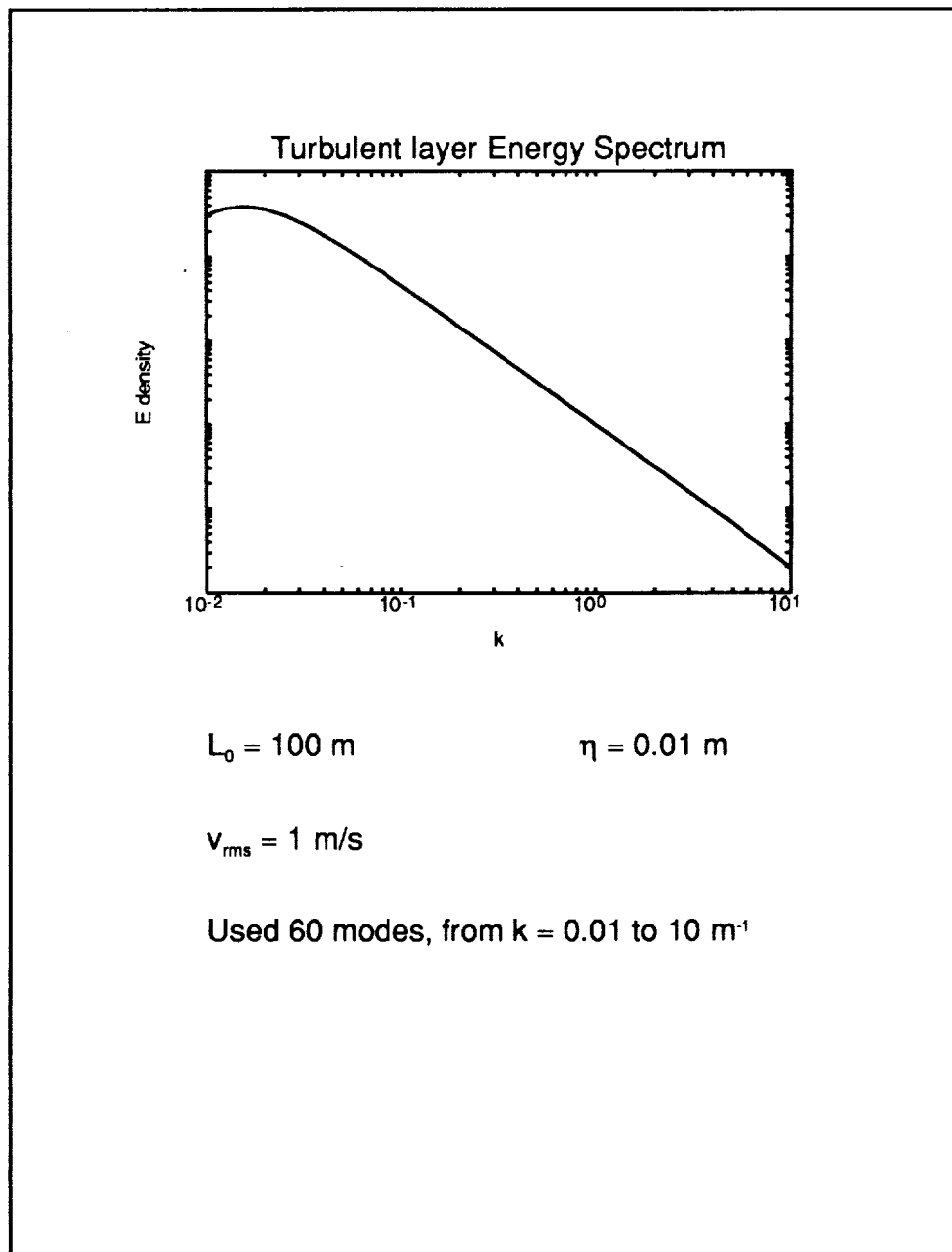$$E(k) \sim \frac{k^4}{\left(k^2 + \frac{1}{L_0^2}\right)^{17/6}} \exp(-2.25\,(\eta k)^{4/3})$$

L_0 = integral length scale
η = Kolmogorov scale

The magnitude of the rms velocity is related by

$$|v_{rms}| = \frac{1}{2} \sum_{j=1}^{N} |a|^2$$

In this model, we use 60 Fourier modes logarithmically distributed between wavenumbers 10^{-2} and 10 (m^{-1}). The integral length scale was chosen to be 100m, as was the thickness of the turbulent layer. The rms wind velocity was chosen to be 1 m/s. This corresponds to a mild turbulent layer, such as might be found in the morning on a clear day. These values are used for all of the remaining figures and discussions.

Turbulent layer Energy Spectrum

E density

k

$L_0 = 100$ m $\eta = 0.01$ m

$V_{rms} = 1$ m/s

Used 60 modes, from k = 0.01 to 10 m^{-1}

A number of rays, with the same starting conditions, are propagated through different realizations or "snapshots" of the turbulent field. Each ray will be displaced by the turbulence away from the undistorted ray path, which in this case would be represented by a horizontal line.
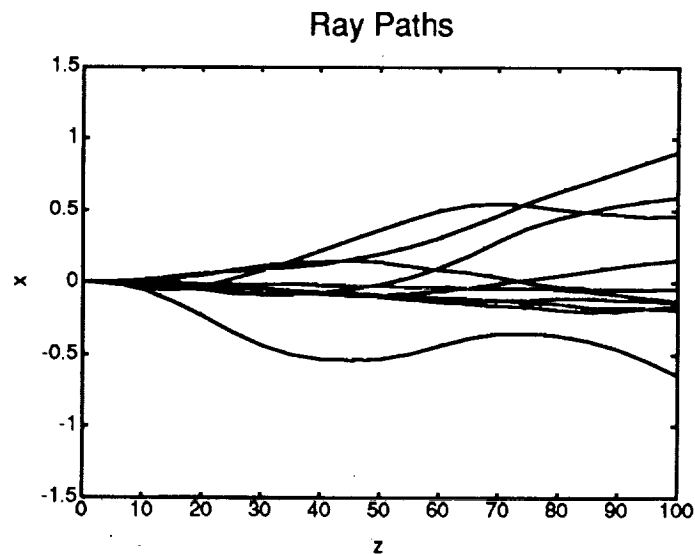
Ray Paths

- Each ray represents a different realization of the turbulent layer.

- The turbulence displaces each ray from the horizontal (undistorted) ray path.

If a large number of rays are propagated through different turbulence realizations, a pattern or distribution of ray displacement may be developed. In our case, 95% of the rays fall within a circle of radius 2 meters around the zero-turbulence ray path. By symmetry arguments, this means that any eigenrays have a 95% probability of starting within 2 meters of the zero-turbulence eigenray. This statistical approach allows us to drastically reduce our eigenray search area to a feasible quantity.

Ray displacement distribution after 100 m propagation

• 95% of the rays fall within a circle of radius 2 m
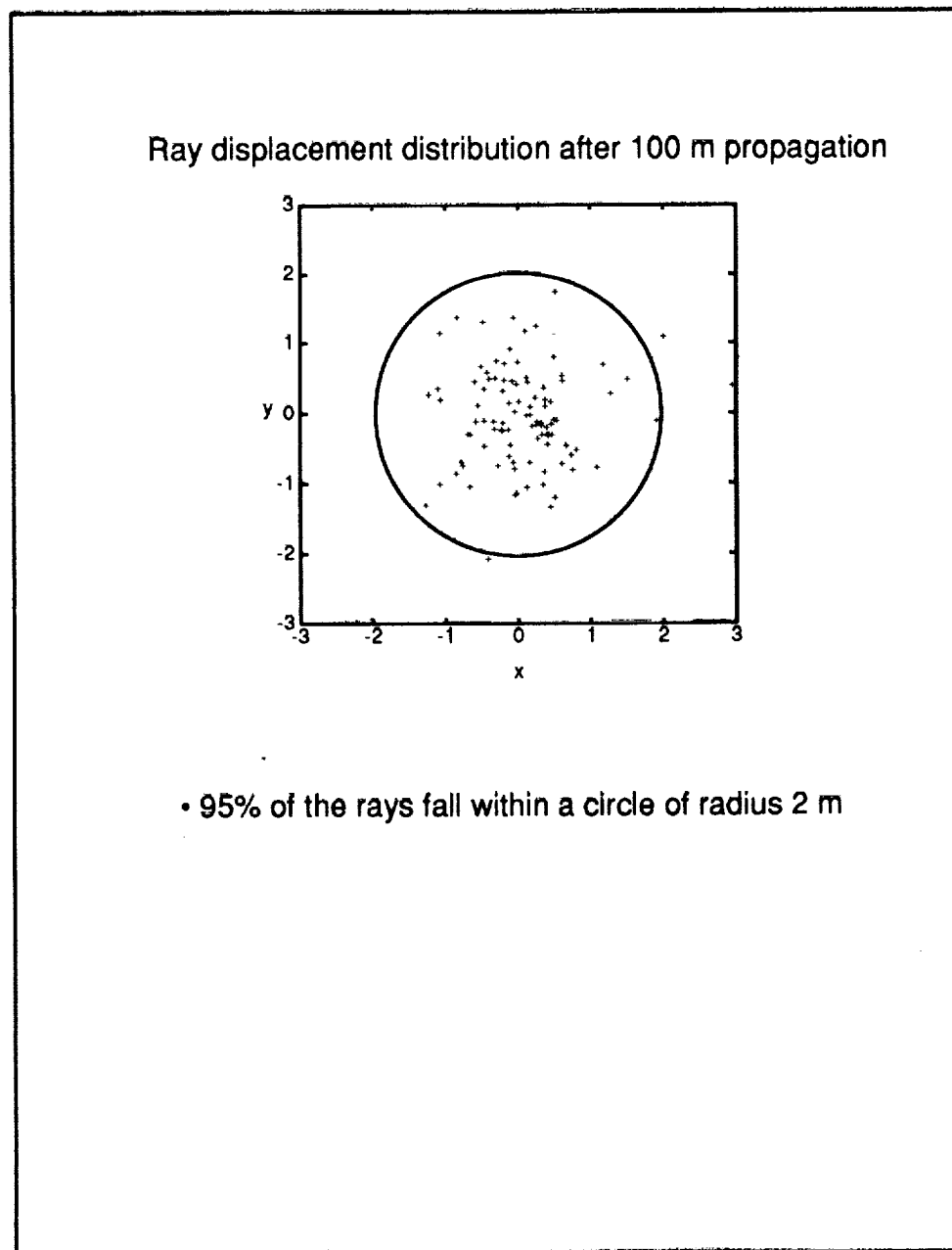
The development of the ray tube area along the ray paths gives an indication of how much the turbulent field is distorting the wave fronts. A ray crossing the horizontal as indicates that the ray has passed through a caustic at that point.

Ray tube areas
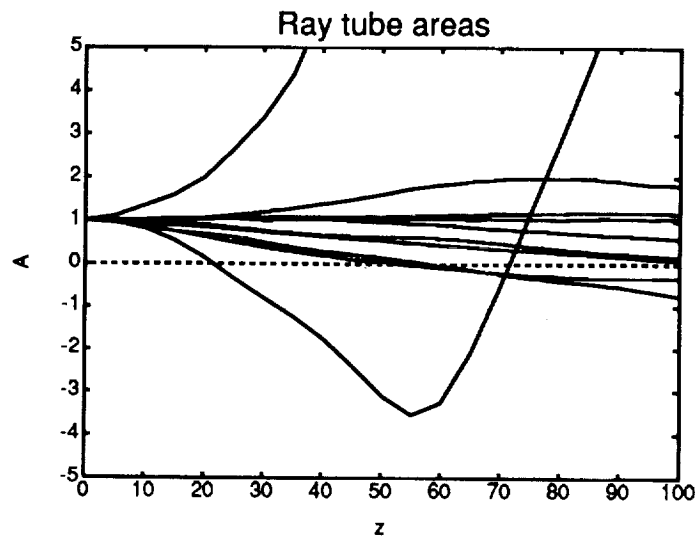
- Each ray represents a different realization of the turbulent layer.

- Crossing the horizontal axis indicates that the ray has encountered a caustic.

In this figure, a linear "slice" of a wavefront is propagated through 100 m of turbulence. In this case, the distortion is slight and no caustics are observed.
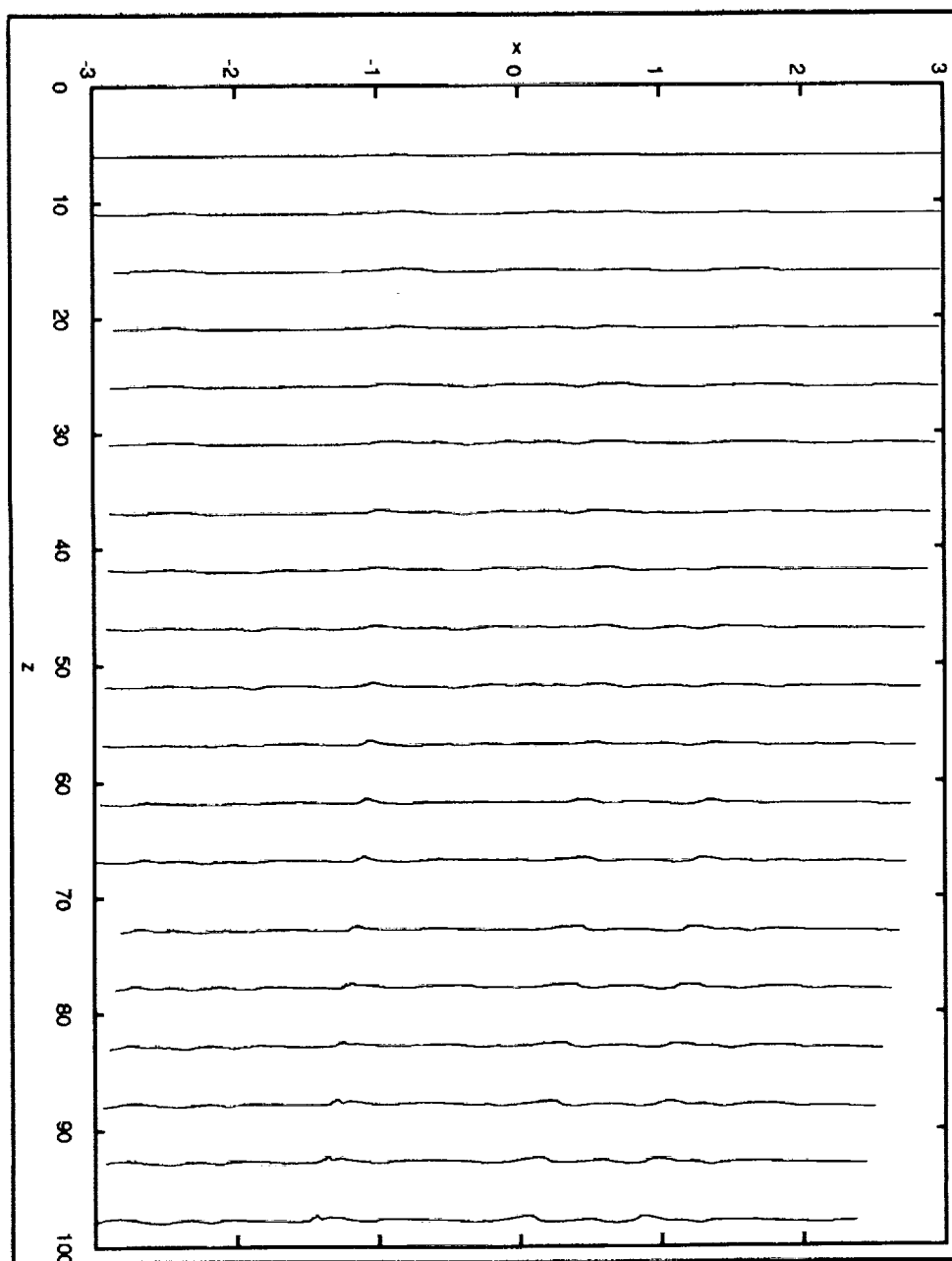
Again, a linear "slice" of the wavefront is propagated through a realization of turbulence. Although the statistical parameters are unchanged, in this case, the turbulence has a marked effect on the wavefronts. After 20 m, caustics begin to form which eventually overlap, producing, in the end, a highly folded wave front, with multiple eigenrays to the receiver.
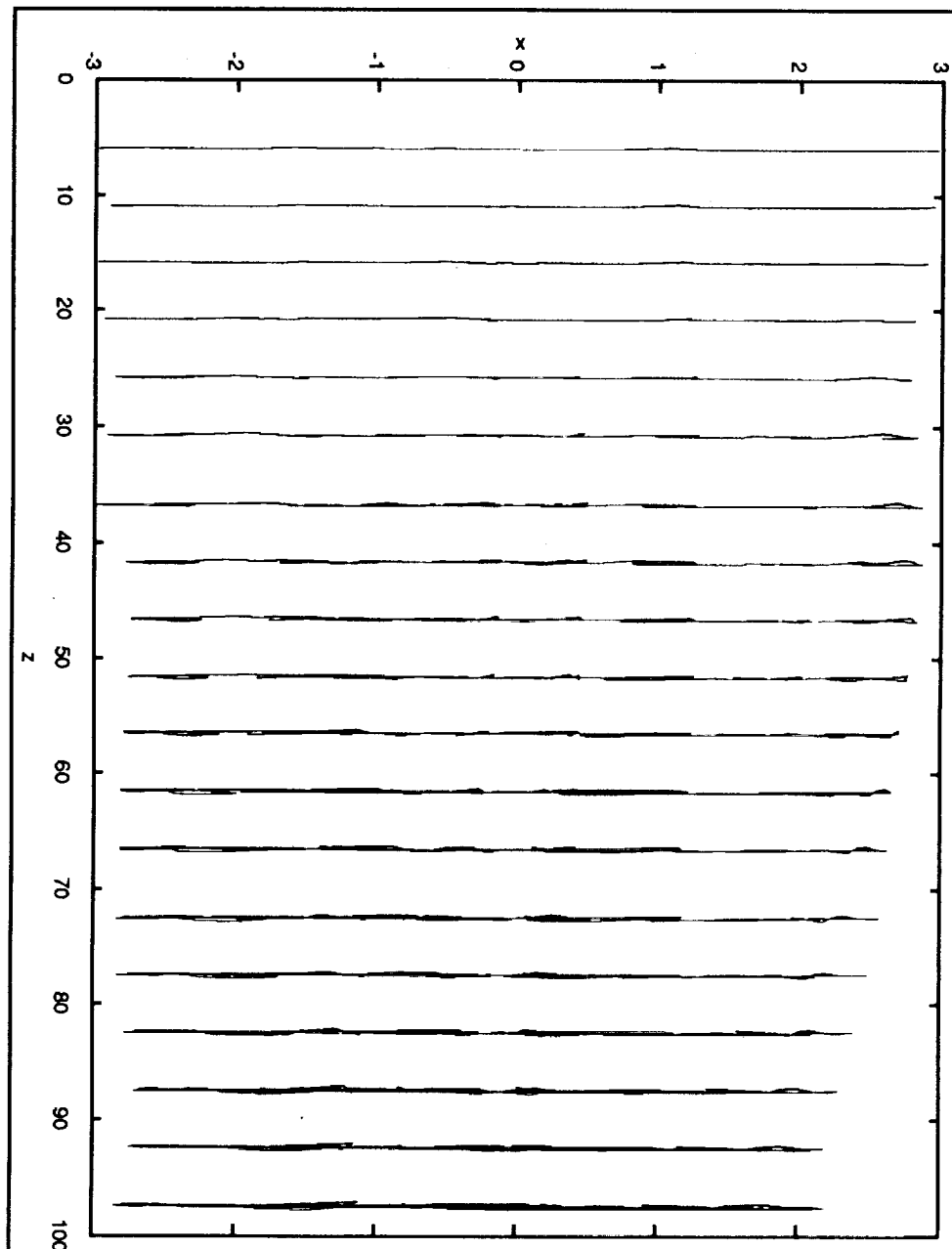
It must be remembered that the wavefront distortion is three-dimensional. The two plots below show the distortion of the original wavefront "slice" in the previous two figures, show in the plane normal to the direction of propagation. Note that the second figure shows considerable distortion due to the presence of numerous caustics.
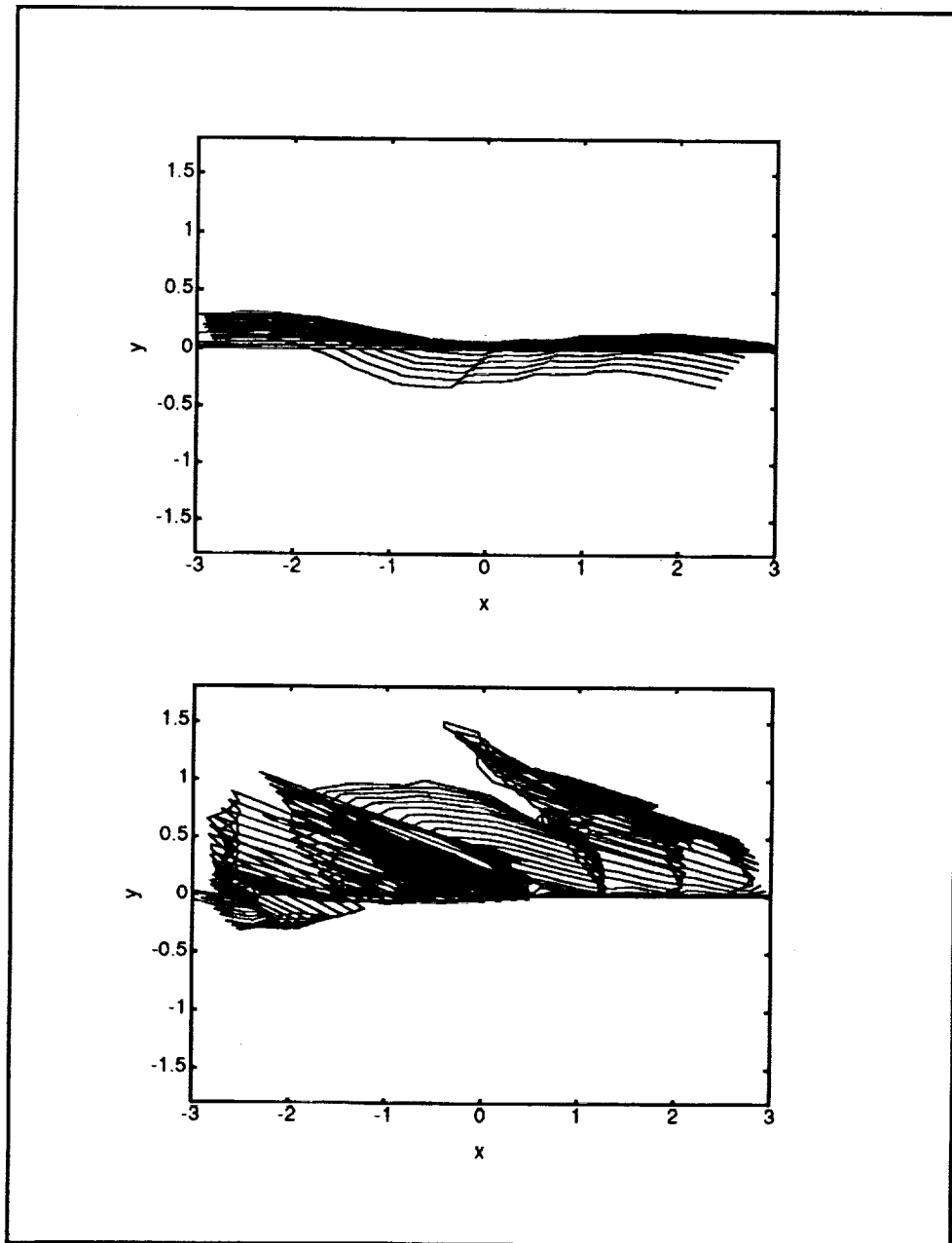
For four realizations of the turbulent field, the wave fronts were propagated, eigenrays were found, and the resultant waveforms were calculated. The initial waveform was generated by the ZEPHYRUS model (Ref. 2); it represents a typical sonic boom waveform in the absence of turbulence. The next two plots demonstrate theresulting waveforms when the wave front is spreading, or defocusing, and when the wave front is focusing, but has not formed a caustic. The last plot displays the U wave resulting from multiple eigenrays, some of which have passed through one or more caustics.
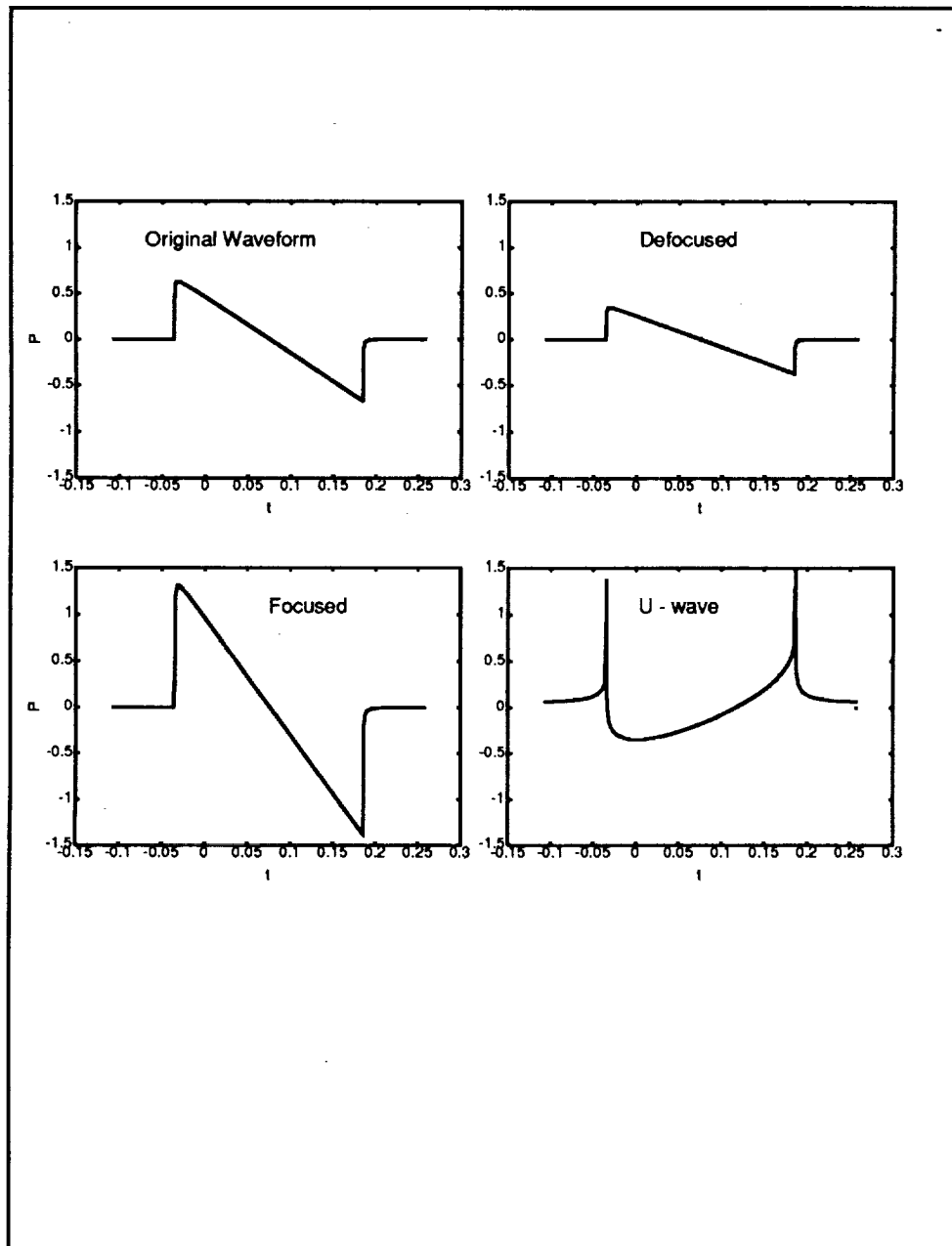
In nonlinear geometric acoustics, the effects of self-refraction may usually be ignored. Although nonlinear effects may displace a ray from the small-signal ray path, the properties of the wave front is usually slowly varying in the plane of the wave front and so, the equivalent nonlinear wave front is virtually identical to the original. As we've seen, however, in the case of propagation through turbulence, we've seen that the wave fronts may become very distorted and so the assumptions that lead to neglecting self-refraction must be examined more closely. This is most easily tested by comparing the same ray with and without the nonlinear correction. We first start with the nonlinear ray path equations given below.

Nonlinear Ray Equations

The ray path equations may be modified to include self-refraction as follows:

$$\frac{d\mathbf{x}}{dt} = \left[(\mathbf{W} + \mathbf{u}') + \frac{\left(c_0 + \frac{P'\alpha}{c_0^2} \right)^2 \mathbf{p}}{(1 - \mathbf{W} \cdot \mathbf{p} - \mathbf{u}' \cdot \mathbf{p})} \right]$$

$$\frac{d\mathbf{p}}{dt} = \left[\left(\sum_i - p_i \nabla'(W_i + u'_i) \right) - \frac{\left(c_0 + \frac{P'\alpha}{c_0^2} \right) \nabla'\left(c_0 + \frac{P'\alpha}{c_0^2} \right) |\mathbf{p}|^2}{(1 - \mathbf{W} \cdot \mathbf{p} - \mathbf{u}' \cdot \mathbf{p})} \right]$$

where

$$\alpha = \frac{(\beta - 1) c_0}{\rho_0}$$

$\mathbf{p}$ is the slowness vector
P' and $\mathbf{u}'$ are the acoustical overpressure and particle velocity

and ∇' is the spatial operator in retarded time coordinates:

$$\nabla' = \nabla + \mathbf{p}\frac{\partial}{\partial t}$$

The simplest wave front property to calculate is the ray tube area. When we compare the results for a number of different turbulence realizations, we see that the ray tubes with and without the nonlinear correction give almost the same result. This indicates that, for these environments, the wave fronts remain sufficiently smooth that we may continue to ignore the effects of self-refraction.
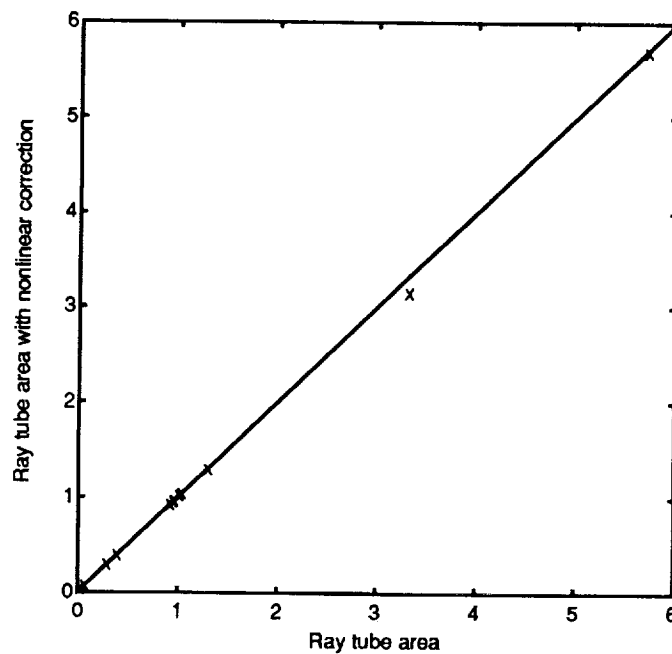
- Initial 200 Pa acoustic overpressure

- The nonlinear correction to the ray paths makes little difference.

The ray theory approach has been demonstrated to be a useful tool for the investigation of propagation through turbulence. The next step will likely be to attempt prediction for more severe turbulence, to see if waveforms of more complex structure that have been observed, such as multiply peaked or rounded, can be simulated by this method.

It is fortunate that the nonlinear distortion of the ray paths may be neglected, as this simplifies the goal of sonic boom prediction.

Conclusions

- **A ray theory approach provides a useful tool for investigating the properties of propagation through turbulence.**

- **Wavefront folding and multiple eigenrays are good candidates for explaining some of the structure commonly observed in sonic booms.**

- **Nonlinear distortion of the ray paths may be safely ignored.**

References

(1)   Karweit, M., Blanc-Benon, P., Juvé, D., and Comte-Bellot, G. (1991). "Simulation of the propagation of an acoustic wave through a turbulent velocity field: A study of phase variance," J. Acoust. Soc. Am. 89, pp. 52-62.

(2)   Robinson, Leick D.: *Sonic Boom Propagation through an Inhomogeneous, Windy Atmosphere*, Ph.D. dissertation, The University of Texas at Austin, December 1991.