19940025111

N94-29614

442582

Combining GPS and VLBI Earth-Rotation Data for Improved Universal Time

A. P. Freedman
Tracking Systems and Applications Section

The Deep Space Network (DSN) routinely measures Earth orientation in support of spacecraft tracking and navigation using very long-baseline interferometry (VLBI) with the deep-space tracking antennas. The variability of the most unpredictable Earth-orientation component, Universal Time 1 (UT1), is a major factor in determining the frequency with which the DSN measurements must be made. The installation of advanced Global Positioning System (GPS) receivers at the DSN sites and elsewhere may soon permit routine measurements of UT1 variation with significantly less dependence on the deep-space tracking antennas than is currently required. GPS and VLBI data from the DSN may be combined to generate a precise UT1 series, while simultaneously reducing the time and effort the DSN must spend on platform-parameter calibrations. This combination is not straightforward, however, and a strategy for the optimal combination of these data is presented and evaluated. It appears that, with the aid of GPS, the frequency of required VLBI measurements of Earth orientation could drop from twice weekly to once per month. More stringent real-time Earth-orientation requirements possible in the future would demand significant improvements in both VLBI and GPS capabilities, however.

I. Introduction

One of the largest error sources for high-precision spacecraft tracking and navigation is Earth-orientation variability.[1,2] The Deep Space Network (DSN) is currently engaged in monitoring platform-parameter variability through the Time and Earth Motion Precision Observations (TEMPO) program of twice-weekly very long-baseline-interferometry (VLBI) measurements. This burden on the DSN radio telescopes is expected to increase as ever higher Earth-orientation accuracy is needed for space missions in the late 1990s and beyond.

Earth-orientation measurements can be made using a Global Positioning System (GPS) receiver network currently being installed at the three DSN sites and at a number of additional sites around the globe [1]. If an automated, rapid-turnaround data collection and reduction system is in place, these GPS data can be used in conjunc-

[1] R. Treuhaft and L. Wood, "Revisions in the Differential VLBI Error Budget and Applications for Navigation in Future Missions," JPL Interoffice Memorandum 335.4-601 (internal document), Jet Propulsion Laboratory, Pasadena, California, December 31, 1986.

[2] S. W. Thurman, "DSN Baseline Coordinate and Station Location Errors Induced by Earth Orientation Errors," JPL Engineering Memorandum 314-488 (internal document), Jet Propulsion Laboratory, Pasadena, California, July 25, 1990.

tion with DSN VLBI data to produce a highly accurate Earth-orientation series in near-real time [2]. Such a system would help to reduce the frequency with which VLBI Earth-orientation measurements must currently be made, thereby allowing more time for direct spacecraft tracking and communications.

The component of Earth orientation that is the most variable and unpredictable is the Earth's rotation rate, as measured by Universal Time 1 (UT1). Its unpredictability is a major motivation for the twice-weekly TEMPO measurements currently being performed. GPS techniques promise to deliver daily estimates of the UT1 rate of change (also known as length of day or LOD) with a precision of better than 0.1 msec/day [2]. Unfortunately, random error in LOD measurements can accumulate to produce estimates of UT1 (i.e., integrated LOD) that are in error by an amount that could exceed mission requirements for UT1 accuracy. (Currently, a 50-nrad tracking accuracy requirement necessitates knowledge of Earth rotation at the ~0.6-msec level; future missions may require 0.1-msec-level real-time UT1 knowledge.[3]) Hence, some sort of synergism is required between the GPS and VLBI techniques such that VLBI data would constrain long-term excursions in the GPS-derived UT1 time series while the GPS data would guarantee accurate monitoring of high-frequency UT1 variability.

This article presents a method for combining DSN VLBI and GPS data. Section II discusses the nature of UT1 and LOD and highlights the need for frequent, accurate, LOD measurements. Section III reviews the capabilities of both VLBI and GPS for measuring UT1 and LOD. Section IV discusses the combination of these two data types and various issues that need to be considered in order to generate the most effective combination.

II. The Time Variability of UT1 and LOD

The angle of rotation of the Earth is represented by a time, UT1 (equal to the rotation angle times a constant), and is usually discussed with respect to a reference rotation angle that is growing at a constant, predefined rate. Two commonly used reference angles, both based on atomic time standards, are Universal Coordinated Time (UTC), which is always maintained within 1 second of UT1, and International Atomic Time (TAI). The LOD is a measure of the rate of change of UT1,[4] and is defined by

$$L \approx -L_0 \frac{\partial U}{\partial t} \tag{1}$$

where L is the excess length of day, U represents UT1–UTC, and L_0 is the nominal length of the day (86,400 sec) [3]. LOD currently has a magnitude of about 2 msec, indicating that the value of UT1–UTC decreases by about 2 msec every 24 hours.

LOD can vary in an unpredictable manner, typically changing by a few hundredths of a millisecond over one day. Figure 1 shows a sample of smoothed LOD over a recent 1-year period; note both the stochastic, high-frequency behavior and the more pronounced regular variations at semi-annual and ~40-day periods. In this and subsequent figures illustrating LOD, tidal effects have been removed according to the model of Yoder et al. [4].

If unmodeled, this variability in LOD can, within a few days, lead to an accumulated UT1 error that exceeds the current accuracy requirement of 0.6 msec. To illustrate this, Fig. 2 shows the growth in error in UT1–UTC as a function of time for three simple cases for the behavior of LOD, according to

$$U_{error} = U_{true} - U_{predicted} = \underbrace{\int_{t_0}^{t} [-L(\tau)]\, d\tau}_{U_{true}} - \underbrace{\int_{t_0}^{t} [-L_r]\, d\tau}_{U_{predicted}} \tag{2}$$

where $\tau \equiv time/L_0$ is dimensionless. In all three cases, the assumed reference value of UT1, $U_{predicted}$, is based on a value of LOD, L_r, that remains constant with time; thus, $U_{predicted}$ varies linearly with time. In case one, LOD itself grows linearly with time, i.e, $L(t) = L_r(t_0) + \alpha(t - t_0)$. In case two, LOD grows linearly with time for 2 days (as in case one), and then remains constant at its new value. In the third case, LOD is assumed to grow in proportion to the square root of time, i.e., $L(t) = L_r(t_0) + \alpha(t - t_0)^{1/2}$. The constant α is assumed to be 0.07 msec/day in the first two cases and 0.07 msec/$\sqrt{\text{day}}$ in the third case. (Note that in these examples, the units of time for the integration are given in days, while LOD and UT1 are measured in milliseconds.) From Fig. 2, it is apparent that the maximum permissible UT1 error, 0.6 msec, is exceeded after 4 to 6 days. Although LOD behavior tends to be irregular, occasional secular variations in LOD (see Fig. 1) can result in unpredictable rapid growth of UT1 error akin to that illustrated by the three cases shown in Fig. 2; at times such as these, frequent measurements of LOD or UT1 are critical.

[3] Treuhaft and Wood, op cit.

[4] LOD as defined in this article is more accurately termed ΔLOD, the excess length of day (the difference between the length of the day and L_0), but the term LOD is retained based on common usage.

A more realistic model for the day-to-day variability of LOD and UT1 may be found from power spectral density (PSD) plots of LOD, such as that shown in Fig. 3. For periods longer than about 10 days, the spectrum is characterized by a $(frequency)^{-2}$ dependence, corresponding to a random-walk stochastic process [5]. LOD can thus be characterized, over subseasonal time scales, as an integrated white-noise (i.e., a random-walk) process, while UT1 can be viewed as an integrated random walk. The white-noise PSD, Q, obtained from Fig. 3 is $Q \approx 0.005$ msec^2/day. After a time $\Delta t = t - t_0$, the expected variance of LOD is $Q\Delta t$, while the corresponding expected error is $\sim 0.07\sqrt{\Delta t}$ msec (when Δt is measured in days). This stochastic model for LOD still produces an unacceptably large UT1 error after 6 days.

Given the typical variability of LOD, accurate UT1 or LOD values must be available at least every 5 days in order to meet the current DSN requirements. In fact, the larger but less frequent LOD variations argue for measurements at least every 4 days. Furthermore, these time intervals do not take into account the required data processing time. The turnaround time for processing TEMPO data is typically two to three days, at which time the UT1 uncertainty has already grown to at least 0.2 msec. Since a new measurement must be available within 3 more days and it takes 2 to 3 days to process the next data point, a new TEMPO measurement must be made within one more day. The net effect, assuming a 2- or 3-day turnaround time, is to require UT1 measurements at least every 3 days.

III. Measuring UT1 and LOD With VLBI and GPS

Very long-baseline interferometry is sensitive to the orientation of the VLBI baseline in a celestial reference frame defined by quasars. Since the angle corresponding to UT1 is one component of the Earth's orientation in inertial space, UT1 is a directly measurable quantity, although multiple baselines are required to unambiguously remove the effects of polar motion. Twice-weekly TEMPO measurements, one per week on each of two different intercontinental baselines, provide regular UT1 and polar motion data to the DSN with typically a 2- or 3-day turnaround time (although 1-day turnaround has been demonstrated). Other VLBI networks, such as the National Geodetic Survey's International Radio Interferometric Surveying (IRIS) network and the U.S. Naval Observatory's VLBI network (NAVNET), also provide regular UT1 measurements, albeit with slower turnaround times that are unsuitable for the needs of the DSN. The formal errors listed for the TEMPO data are currently at the 0.1- to 0.2-msec level [6], although combining and smoothing data from the two baselines can generate improved UT1 estimates with formal errors below 0.1 msec.

Each VLBI network relies on its own reference frames, both celestial and terrestrial, to produce Earth-rotation measurements. These reference frames may have small offsets with respect to one another, resulting in systematic differences between the Earth-rotation time series generated by each network [7]. These UT1 offsets are generally small and slowly varying, but they need to be considered in any precise estimate of UT1, especially if data from multiple techniques are combined.

GPS methods differ fundamentally from VLBI techniques. To determine Earth orientation with GPS, one first needs to define a terrestrial reference frame. This is usually done by fixing the locations of a few select ground receivers. These sites are known as fiducial sites and are tied by local ground surveys to nearby, collocated VLBI antennas whose relative positions are known precisely through VLBI [8]. A precise set of initial values, derived from VLBI and from satellite laser ranging (SLR), is used to orient this Earth-fixed frame with respect to a celestial reference frame, which then permits the GPS satellite orbit epoch states to be constrained in inertial space. The behavior of the satellite network is governed by dynamical models, such that movements of the solid Earth within that framework, i.e., Earth orientation, can be observed. UT1–UTC, however, cannot be determined simply by monitoring the relative motions of the terrestrial network and the GPS satellite orbits, as it is defined with respect to an arbitrary location on the celestial sphere. The signature in the data residuals of an error in UT1 is identical to that due to an error in the satellites ascending node locations. In addition, since the GPS satellites have almost identical orbital radii, inclinations, and eccentricities, the oblateness of the Earth induces precession of the GPS orbits that is nearly indistinguishable from a slow UT1 variation.

Although GPS techniques cannot directly measure the absolute value of UT1 and have difficulty distinguishing long-term UT1 changes from precession of the GPS orbits, they are sensitive to short-term changes in UT1. The large number of GPS satellites and receivers provide robust and redundant networks for measuring the displacements of the terrestrial frame with respect to the GPS orbits that result from rapid UT1 variations. Regular GPS measurements of LOD are, in fact, already being made, although with a less than ideal satellite configuration and station network [9]. These values are weekly averages and have typical errors of about 0.15 msec. The capability of GPS to measure LOD is expected to improve dramatically by the mid-1990s. The full constellation of 24 satellites will be available, along with a worldwide, high-precision GPS tracking network with the potential for near-real-time data processing. LOD measurements are thus expected at the 0.05-msec level or better in the next few years [2].

The terrestrial reference frame used in GPS processing, although tied to a VLBI terrestrial frame through the fiducial technique, is not necessarily identical to that of VLBI, since local site ties between VLBI and GPS antenna phase centers may contain errors. These site ties can also vary over time, due either to changes in antenna positions or to improvements in the local ground surveys. In addition, as VLBI site location and site velocity estimates improve, both VLBI and GPS reference frames and the Earth orientation time series may be affected.

VLBI is a proven, currently operating technique to measure UT1, whereas precise, routine, operational determination of UT1 using GPS techniques lies at least a year or two in the future. Why, then, is there interest in supplementing VLBI with GPS?

As the tracking and navigation duties of the DSN expand and higher precision is needed, more frequent UT1 data will be required. As shown in Section II, the current frequency of TEMPO measurements is barely adequate in providing near-real-time Earth-rotation information; in fact, the TEMPO data must be supplemented with meteorologically derived atmospheric angular-momentum information (which has been shown empirically to be highly correlated with LOD) to ensure reasonable accuracy of current UT1 estimates [10]. The higher-precision future Earth-rotation requirements of the DSN will require more frequent VLBI data if alternative techniques are not available. Dedicating DSN radio telescopes to obtaining Earth rotation at daily or 2-day intervals would heavily tax the already-overburdened DSN tracking schedule. Hence, future high-precision Earth-platform determination is critically dependent on new high-accuracy technologies with the potential for rapid data turnaround, such as GPS.

IV. Combining VLBI and GPS Data

Three main issues need to be addressed in order to accurately and reliably combine GPS and VLBI Earth-rotation data. They are (1) growth of UT1 error with time, (2) reference-frame definition and compatibility, and (3) systematic technique-dependent errors. They will be discussed, in turn, below.

A. Growth of Error in UT1

Any strategy for the synergistic use of GPS and VLBI Earth-rotation data to monitor UT1 must take into account error characteristics of both GPS and VLBI, along with the observed variability of Earth rotation. Since VLBI is able to measure UT1 directly whereas the strength of GPS lies in its ability to monitor LOD (the change from day to day in UT1), VLBI is needed to provide an initial estimate of UT1. Subsequent measurements of LOD by GPS can then permit the calculation of UT1 by daily integration. Since errors in UT1 will accumulate over time, VLBI data are also needed to calibrate the GPS-determined time series. This periodic VLBI calibration should overcome various problems related to long-term monitoring of Earth rotation with GPS, including the effects of orbit precession, data noise, satellite force model errors, etc.

Figure 4 schematically illustrates this plan for GPS and VLBI synergism. At some epoch, a VLBI observation is made, yielding a precise estimate (and formal error) of UT1. Daily measurements of LOD are then obtained by a rapid-turnaround global GPS network, and UT1 and its uncertainty determined by integrating these LOD values. At some point, the accumulating uncertainty in UT1 necessitates a new VLBI observation, which fixes the value of UT1 at the new epoch and constrains its uncertainty. Note that Fig. 4 illustrates only a near-real-time method for estimating UT1; a UT1 time series of uniformly high precision will still be available after a delay of 2 weeks or more.

To determine the frequency with which periodic VLBI measurements should be made, the sources of UT1 error must be evaluated. Formal errors for UT1 derived from TEMPO measurements are currently at the 0.1-msec level, although VLBI system improvements should reduce this uncertainty in the next few years. In this study, a formal VLBI-derived UT1 error of either 0.1 msec (conservative) or 0.05 msec (optimistic) and a 2-day turnaround time for processing these data are assumed. A UT1 measurement is assumed to represent the average value of UT1 over a few hours, obtained from back-to-back, 3-hour TEMPO VLBI sessions on each of two baselines. (Note that this differs from the current observing strategy of one 3-hour session on alternate baselines every few days.)

Based on extensive covariance analyses [2], the LOD error anticipated from GPS when an optimal satellite and station configuration is established is less than 0.05 msec. To be conservative in this analysis, and for ease of presentation, we assume GPS-derived LOD also to be accurate to either 0.1 msec or 0.05 msec. Note that the GPS observable is, in actuality, an estimate of the UT1 rate-of-change over a 12- to 24-hour period, but this is essentially equivalent to the average LOD. GPS data are assumed to be generated either every 24 or 48 hours, and to be available after a 24-hour processing delay.

All error estimates mentioned are the one-sigma formal errors and are assumed to be Gaussian. In addition, no correlation is assumed between VLBI and GPS errors, even though technically there may be some correlation if any VLBI-derived parameters are used to initialize the GPS data processing scheme.

Figure 5 illustrates a few examples of UT1 error behavior. (The Appendix describes the mathematical model used to generate this and the subsequent figure.) Both the VLBI UT1 and daily GPS LOD measurements are assumed to have identical formal errors (either 0.10 or 0.05 msec). At $t = 0$ days, a VLBI measurement of UT1 is made. The reduced data point is available at $t =$ 2 days; by this time, however, the real-time knowledge of UT1 based solely on VLBI will have degraded due to the stochastic nature of LOD. Fortunately, GPS data are being obtained each day, providing a realistic assessment on day 2 of the LOD for day 0 and day 1 (recall that there is a 24-hour delay for GPS data). Subsequently, GPS-derived LOD data with their formal errors control the growth of uncertainty in UT1. At some point (in this case, $t =$ 40 days), a new VLBI measurement is made in response to the accumulating uncertainty in UT1. Again, it takes 2 days for the measurement to become available, and the cycle continues.

In the case where formal errors for VLBI UT1 and GPS LOD are both 0.1 msec, the 0.6-msec DSN threshold for UT1 is reached in 35 days. Thus, DSN VLBI platform-parameter measurements would be needed approximately monthly. If the measurement errors for the two techniques drop to the 0.05-msec level, VLBI measurements of UT1 might be needed only once every 3 or 4 months. In this latter case, the DSN could even collect GPS LOD data every other day and still free up the VLBI system for almost 2 months.

If, however, DSN requirements for near-real-time Earth rotation drop to the 0.2-msec level, the picture is not so pleasant. This requirement could not be met with 0.1-msec quality data. Only with high-quality (~0.05-msec) data from both VLBI and GPS, and GPS LOD measurements obtained daily, could a 0.2-msec requirement be met without VLBI calibrations for any appreciable length of time. In this case, DSN VLBI measurements would be needed about every 2 weeks. This is still less than half as often as TEMPO currently operates, and the alternative, TEMPO-only option would require TEMPO measurements to be taken daily. To meet a 0.1-msec real-time UT1 requirement, even VLBI and GPS data with errors at the 0.05-msec level are inadequate. Measurement errors must be reduced to the 0.02-msec level or better, and DSN VLBI calibration measurements must be made at least weekly to meet this more stringent Earth-rotation requirement.

Figure 6 addresses a subtle issue concerning the near-real-time need for UT1. On any given day, the estimate of UT1 for the preceding day is an empirical value based on combined VLBI and GPS data. To generate an actual real-time estimate of UT1, this value for the preceding day must be adjusted for the behavior expected of LOD over the additional day, and the error estimate corrected accordingly. Two cases are illustrated for this theoretical behavior: a random walk and worst case linear growth. The random-walk model is simply the stochastic behavior of LOD described in Section II, while the worst case scenario corresponds to an unusually large daily change in LOD. These two cases correspond to growth in the uncertainty of LOD over one day of 0.07 msec and 0.10 msec, respectively.

Figure 6 shows the expected error for these two models if GPS data are taken either daily or, for purposes of comparison, every other day. A VLBI UT1 measurement with a 0.05-msec formal error is made on day 0 and becomes available on day 2. GPS LOD measurements with a 0.05-msec formal error are made either daily or on alternate, even-numbered days. After every LOD measurement, the uncertainty in LOD is assumed to grow until the next LOD measurement, according to one of three models: no growth, random-walk (stochastic) growth, or rapid linear ("maximum") growth. The alternate-day LOD data curves oscillate because the uncertainty in UT1 24 hours after a GPS measurement is made is smaller than the UT1 uncertainty 48 hours after the preceding measurement is made. Shown for comparison are the estimated uncertainties in UT1 if no GPS data were available to constrain the variability of LOD (the "VLBI only" curves).

When daily GPS LOD data are used to estimate the current value of UT1, the modeled behavior of the LOD does not significantly affect real-time UT1 estimates on the day after the last data are taken. If GPS data are obtained on alternate days, however, the estimated UT1 uncertainties are significantly affected by the assumed LOD behavior. In this case, rapid LOD variations can produce UT1 errors exceeding 0.2 msec very quickly. Hence, once-a-day estimates of LOD by GPS are still the preferred measurement frequency.

B. Reference-Frame Compatibility

As discussed in Section III, GPS and VLBI may generate estimates of Earth-rotation parameters using slightly different terrestrial reference frames. Each frame should be internally stable over time, because a principal source of reference-frame instability, global tectonic motions, is a slowly varying process. Hence, differences between the two reference frames should manifest themselves in the Earth-rotation data as a more-or-less constant bias and, perhaps, a rate (i.e., a secularly growing offset) between the two time series. These bias and rate terms may be estimated empirically, based on previously measured time series of UT1 and LOD from the two techniques. The terrestrial frames may also shift relative to each other in an unpredictable manner when fiducial sites are changed, antennas are moved, or improved local site ties between the GPS and VLBI antenna phase centers are determined.

As long as the site network and data reduction method for each technique remain the same, the biases and rates should not significantly vary. However, changes in the reference site network, collocated site ties, VLBI radio source catalog, or data processing software may generate UT1 time-series glitches that need to be closely monitored. These time-series offsets or slope changes might be evident only weeks or months after they commence, when a sufficient quantity of data of diverse types has accumulated to distinguish the time-series characteristics of individual techniques. Fortunately, reference-frame adjustments are not expected to occur often.

C. Systematic Errors

Related to reference-frame discrepancies are the effects of systematic errors in each technique, either in data collection or in data processing, that contaminate the resulting UT1 and LOD time series. Unlike reference-frame errors, these need not be of a long period. Systematic GPS-derived LOD errors have yet to be investigated, since the data themselves are not yet available at nearly the required frequency or precision. However, numerous orbit and baseline studies have isolated a number of possible systematic error sources for GPS techniques in general. These include orbit mismodeling, incorrect fiducial site ground ties, and mismodeled atmospheric propagation effects [11].

Isolating these systematic errors will require periodic campaigns of simultaneous GPS and VLBI Earth-rotation measurements.[5] Intercomparing these time series and incorporating the results of other Earth orientation measurement techniques, such as IRIS and NAVNET VLBI, SLR, and lunar laser ranging (LLR), should help to pinpoint systematic errors. This type of analysis is ongoing for currently existing techniques [12].

V. Conclusions

Within the next few years, high-precision GPS techniques will be able to generate Earth-rotation values of great utility to the DSN. By combining these LOD data with VLBI-derived estimates of UT1, a high-quality, real-time UT1 time series could become available for critical DSN spacecraft navigation and tracking applications. By reducing the amount of antenna time the DSN must commit to Earth-platform-parameter estimation, use of GPS data would permit a greater percentage of radio telescope time to be available for spacecraft telemetry and tracking. This would be all the more valuable given the expected increase in demand on the DSN radio antennas from the large number of planned and ongoing interplanetary missions over the next decade.

Although a high-quality GPS receiver network will already be in place at the DSN sites and at numerous other locations around the globe in support of TOPEX/POSEIDON tracking, the DSN must also establish a mechanism for rapid GPS data collection and processing. A prototype system already exists for sub-daily orbit determination,[6] but a fully operational system with 12-hour or less turnaround time and an explicit goal of measuring Earth orientation has yet to be implemented.

Estimates of predicted UT1 uncertainty suggest that, given a small but reasonable GPS measurement error, the frequency of TEMPO VLBI measurements can be reduced from twice weekly to monthly or less (although this once-a-month TEMPO session must include measurements on both baselines). If a future real-time Earth-rotation requirement drops to a third of its present value (i.e., to $\leq$0.2 msec), the only feasible way to fulfill this requirement, short of daily TEMPO measurements, is the synergistic combination of daily GPS LOD measurements and weekly or semimonthly VLBI UT1 measurements. Furthermore, to achieve a 0.1-msec real-time Earth-rotation capability, significant improvements in both VLBI and GPS techniques are also needed. Thus, both VLBI and GPS appear to be techniques essential for meeting DSN platform-parameter requirements in the coming years.

[5] The first of these multiple-technique simultaneous-observing sessions will occur in 1991.

[6] G. Blewitt, personal communication, Jet Propulsion Laboratory, Pasadena, California, 1990.

References

[1] R. E. Neilan, W. G. Melbourne, and G. L. Mader, "The Development of a Global GPS Tracking System in Support of Space and Ground-Based GPS Programs," to be published in *GPS and Other Radio Tracking Systems* (proceedings of IAG Symposium 102, August 3–12, 1989, Edinburgh, Scotland), ed. Y. Bock and N. Leppard, New York: Springer-Verlag, 1989.

[2] A. P. Freedman, "Determination of Earth Orientation Using the Global Positioning System," *TDA Progress Report 42-99*, vol. July–September 1989, Jet Propulsion Laboratory, Pasadena, California, pp. 1–11, November 15, 1989.

[3] K. Lambeck, *The Earth's Variable Rotation: Geophysical Causes and Consequences*, Cambridge, England: Cambridge University Press, 1980.

[4] C. F. Yoder, J. G. Williams, and M. E. Parke, "Tidal Variations of Earth Rotation," *J. Geophys. Res.*, vol. 86, no. B2, pp. 881–891, February 10, 1981.

[5] J. O. Dickey, S. L. Marcus, and J. A. Steppe, "An Investigation of the Earth's Angular Momentum Budget at High Frequencies" (abstract), *EOS Trans. Am. Geophys. Un.*, vol. 70, no. 43, p. 1055, October 24, 1989.

[6] J. A. Steppe, S. H. Oliveau, and O. J. Sovers, "Earth Rotation Parameters From DSN VLBI: 1990," *Earth Orientation and Reference Frame Determinations, Atmospheric Excitation Functions, Up to 1989 (Annex to the International Earth Rotation Service (IERS) Annual Report for 1989)*, Paris, France: Central Bureau of IERS–Observatoire de Paris, June 1990.

[7] Z. Altamimi, E. F. Arias, C. Boucher, and M. Feissel, "Earth Orientation Determinations: Some Tests of Consistency," in *Earth Rotation and Coordinate Reference Frames, IAG Symposium 105*, Edinburgh, Scotland, August 11, 1989.

[8] J. M. Davidson, C. L. Thornton, S. A. Stephens, G. Blewitt, S. M. Lichten, O. J. Sovers, P. M. Kroger, L. L. Skrumeda, J. S. Border, R. E. Neilan, C. J. Vegos, B. G. Williams, J. T. Freymueller, T. H. Dixon, and W. G. Melborne, *The Spring 1985 High Precision Baseline Test of the JPL GPS-Based Geodetic System: A Final Report*, JPL Publication 87-35, Jet Propulsion Laboratory, Pasadena, California, November 15, 1987.

[9] E. R. Swift, "Earth Orientation From GPS," presented at the Fifth Annual GPS Workshop, Jet Propulsion Laboratory, Pasadena, California, April 2, 1990.

[10] A. P. Freedman, J. O. Dickey, J. A. Steppe, L.-Y. Sung, and T. M. Eubanks, "The Short-Term Prediction of Universal Time and Length-of-Day Using Atmospheric Angular Momentum," in preparation for *J. Geophys. Res.*, 1991.

[11] S. M. Lichten and W. I. Bertiger, "Demonstration of Sub-Meter GPS Orbit Determination and 1.5 Parts in 10^8 Three-Dimensional Baseline Accuracy," *Bulletin Geodesique*, vol. 63, pp. 167–189, 1989.

[12] IERS, *International Earth Rotation Service (IERS) Annual Report for 1989*, Paris, France: Central Bureau of IERS–Observatoire de Paris, 1990.

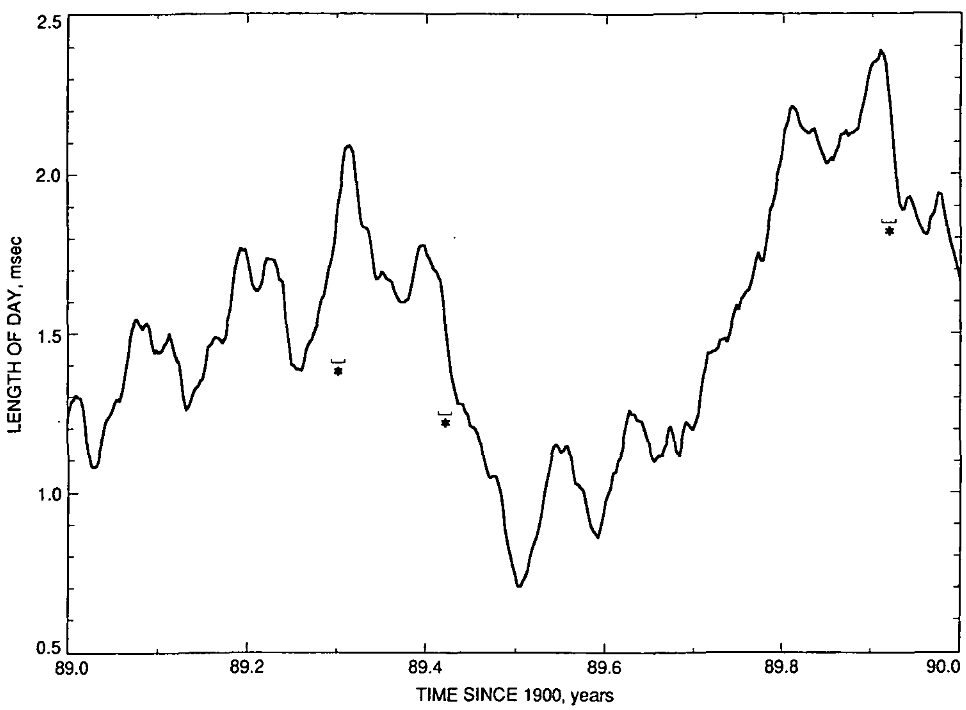

Fig. 1. Sample LOD time series. The asterisks mark selected time periods when LOD changed unusually rapidly.

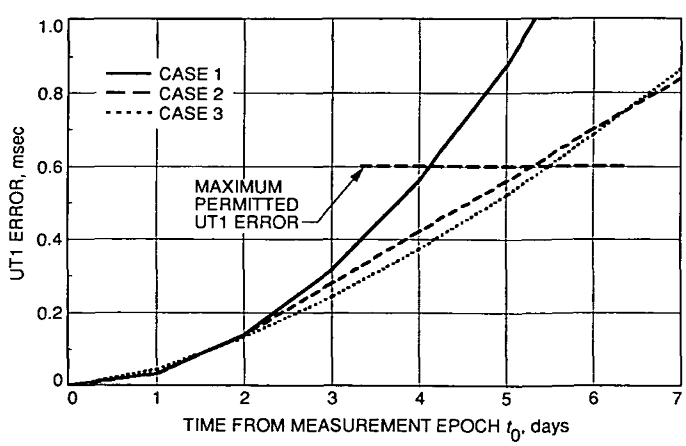

Fig. 2. Growth in UT1–UTC error as a function of time for three different LOD behaviors.

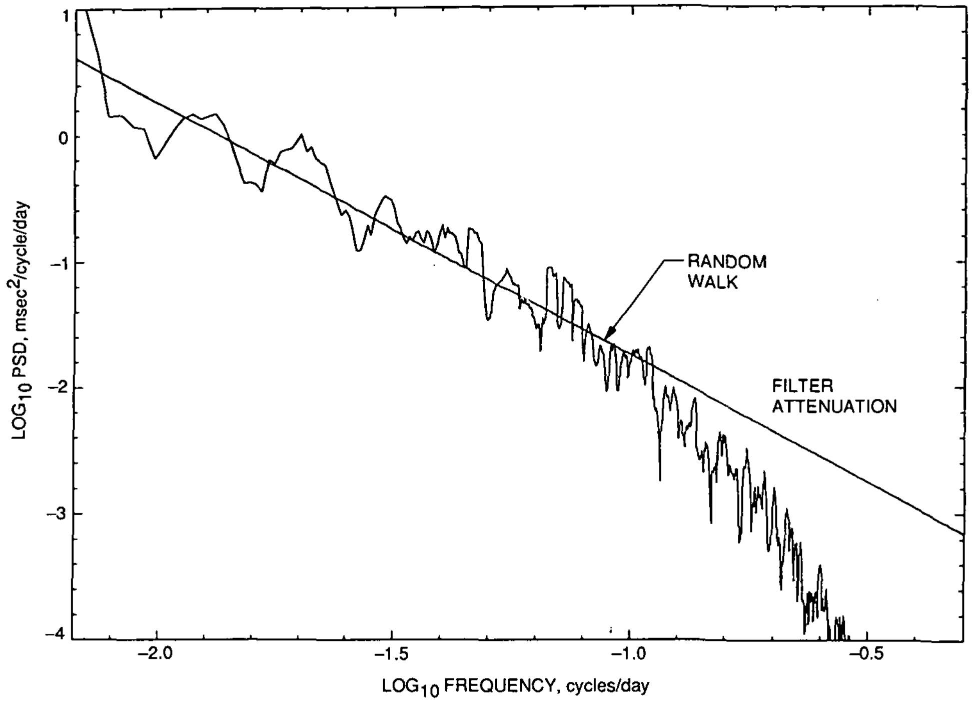

Fig. 3. PSD of smoothed multiyear LOD time series (after [5]).

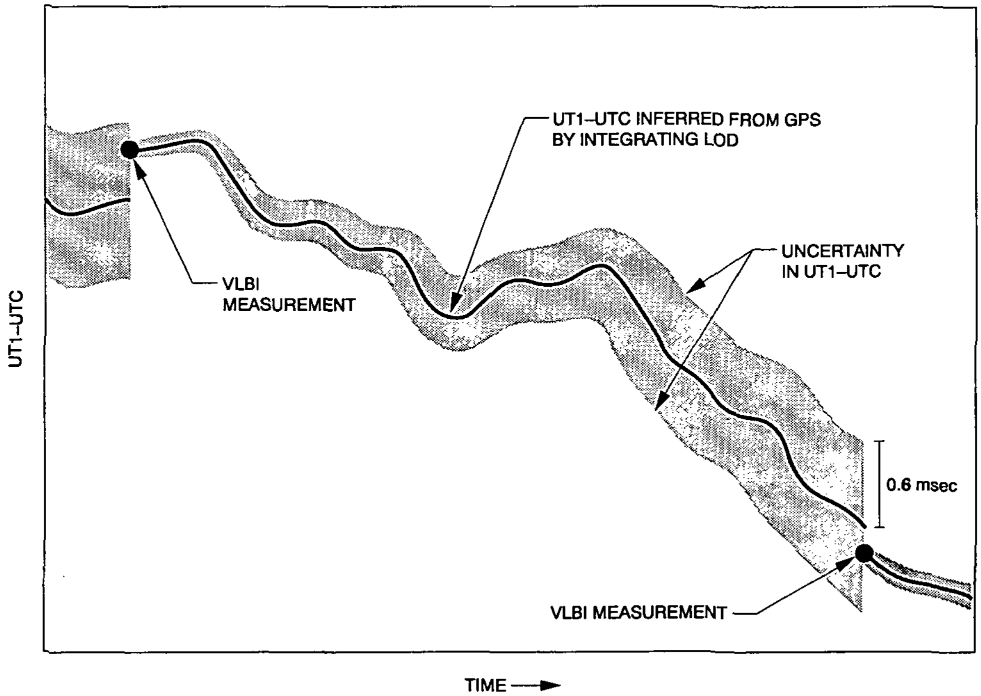

Fig. 4. The strategy for combining GPS and VLBI observations to achieve a high-precision, near-real-time UT1 time series (after [2]).

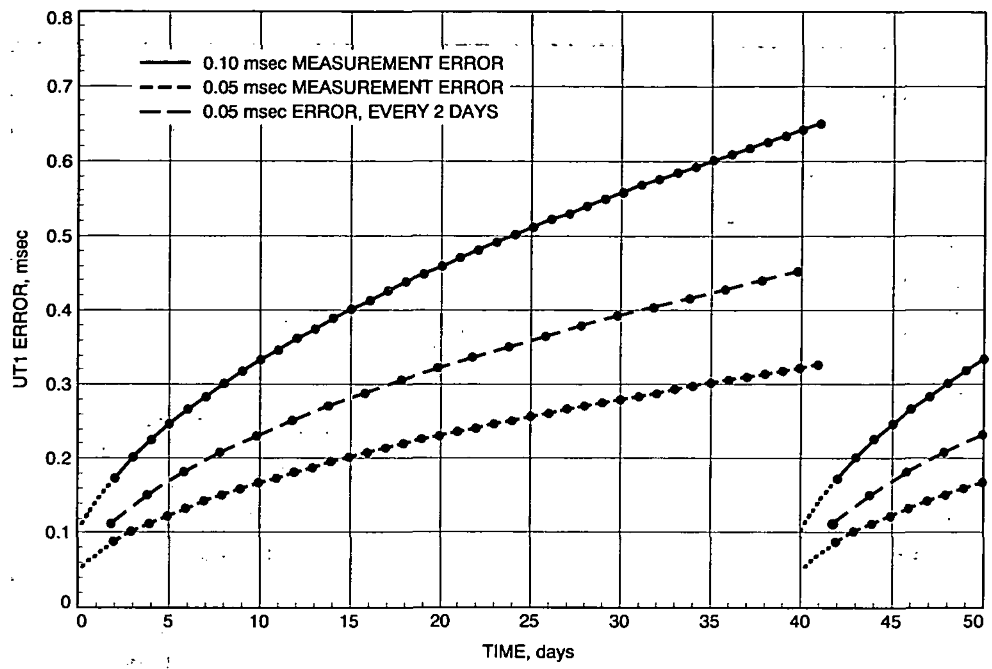

Fig. 5. Estimated one-sigma error in real-time UT1–UTC as a function of time for three different measurement scenarios. Lines are used to connect the points only; they do not represent the estimated errors at intermediate times. Dotted lines indicate retroactive uncertainty in UT1 following the VLBI measurement.

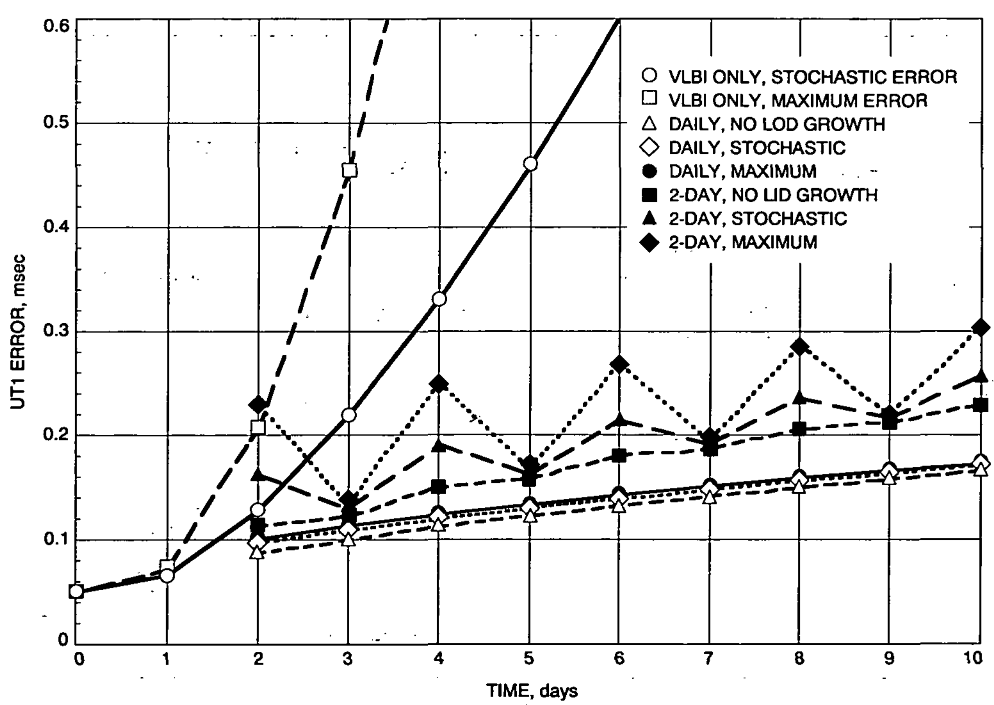

Fig. 6. Expected real-time one-sigma error in UT1–UTC when an LOD behavior model is included. Note that lines are used to connect the points only; they do not represent the estimated errors at intermediate times.

Appendix

Modeling the UT1 Error Structure

The mathematical model for the evolution of UT1 error estimates shown in Figs. 5 and 6 is presented below. Measured data are assumed to consist of the following: $U_{m|0}$ (UT1–UTC at time t_0) with formal error $\sigma_{U|0}$, and LOD estimates ($\bar{L}_{m|i}$) at times $t_i, i = 0, \ldots, l$, with formal errors $\sigma_L|_i$. The true measurement errors in UT1 and LOD are denoted by $\epsilon_{U|0}$ and $\epsilon_{L|i}$, respectively. All times are normalized, i.e., $t_i = time(i)/L_0$. Recall that LOD estimates are actually estimates of $-\Delta UT1/\Delta t$ over a time interval Δt.

The predicted value of UT1 at t_n, where $t_n > t_l$, is

$$U_p(t_n) = U_{m|0} - \sum_{i=1}^{l} \bar{L}_{m|i-1}\,(t_i - t_{i-1}) - \int_{t_l}^{t_n} L'(\tau)d\tau \tag{A-1}$$

where $L'(\tau)$ is the predicted behavior of LOD at normalized time τ. In the absence of data, LOD is usually assumed to remain constant, i.e., $L'(\tau) = \bar{L}_{m|l}$ for $\tau \geq t_l$. The true value of UT1 is represented by

$$U(t_n) = U_0 - \int_{t_0}^{t_n} L(\tau)d\tau = [U_{m|0} + \varepsilon_{U|0}] - \left[\int_{t_0}^{t_l} L(\tau)d\tau + \int_{t_l}^{t_n} L(\tau)d\tau\right] \tag{A-2}$$

where U_0 and $L(\tau)$ denote the true UT1 and LOD values, respectively.

The true value of LOD may be modeled within the above equation as

$$\int_{t_0}^{t_l} L(\tau)d\tau = \sum_{i=1}^{l} \int_{t_{i-1}}^{t_i} L(\tau)d\tau = \sum_{i=1}^{l} \bar{L}\,|_{i-1}(t_i - t_{i-1}) \tag{A-3}$$

where $\bar{L}\,|_{i-1}$ is the mean value of LOD over the interval t_{i-1} to t_i; it is assumed that $\bar{L}\,|_{i-1} = \bar{L}_{m|i-1} + \varepsilon_{L|i-1}$. The final term in Eq. (A-2), the effect of LOD variability following the last LOD measurement at time t_l, is modeled as

$$\int_{t_l}^{t_n} L(\tau)d\tau = \int_{t_l}^{t_n} [L'(\tau) + \eta(\tau)]\,dt = \int_{t_l}^{t_n} [\bar{L}_{m|l} + \eta(\tau)]\,d\tau \tag{A-4}$$

where the difference between the true and assumed LOD is

$$\eta(\tau) = \varepsilon_{L|l} + \int_{t_l}^{\tau} \omega(\mu)d\mu \tag{A-5}$$

In Eq. (A-5), the first term on the right is the LOD measurement error at t_l, while the final term in this expression emerges from the random-walk nature of LOD. The variable $\omega(\mu)$ describes a white-noise stochastic process whose integral over time generates the random-walk variations of LOD. Alternatively, the true LOD may be modeled as an analytic function of time, i.e., $\eta(\tau) = \varepsilon_{L|l} + f(\tau - t_l)$.

The error terms and white-noise behavior in the above equations may be described by the following set of expectation values:

$$\left.\begin{aligned}\langle \varepsilon_{U|0}\ \varepsilon_{U|0}\rangle &= (\sigma_{U|0})^2 \\ \langle \varepsilon_{L|i}\ \varepsilon_{L|j}\rangle &= (\sigma_L|_i)^2\delta_{ij} \\ \langle \omega(\mu)\omega(\rho)\rangle &= Q\ \delta\ (\mu-\rho)\end{aligned}\right\} \tag{A-6}$$

while all other expectation values of $\omega(\mu)$, $\varepsilon_{U|0}$, $\varepsilon_{L|i}$ and their products are assumed to be zero. In Eq. (A-6), δ_{ij} is the Kronecker delta, $\delta(\mu-\rho)$ is the Dirac delta function, and Q is the PSD of the white-noise process.

The error in UT1 as a function of time is given by

$$\varepsilon_U(t_n) \equiv U_p(t_n) - U(t_n) = -\varepsilon_{U|0} + \sum_{i=1}^{l}\left[\varepsilon_{L|i-1}(t_i - t_{i-1})\right] + \int_{t_l}^{t_n} \eta(\tau)d\tau \tag{A-7}$$

The variance of this error is

$$\begin{aligned}\mathrm{Var}(\varepsilon_U(t_n)) &= \langle \varepsilon_U(t_n)\varepsilon_U(t_n)\rangle \\ &= (\sigma_{U|0})^2 + \sum_{i=1}^{l}(\sigma_{L|i-1})^2(t_i - t_{i-1})^2 + \int_{\tau=t_l}^{t_n}\int_{\rho=t_l}^{t_n}\langle\eta(\tau)\ \eta(\rho)\rangle d\rho\ d\tau\end{aligned} \tag{A-8}$$

where most of the expectation values have been evaluated. The rightmost term in Eq. (A-8) consists of two parts, one corresponding to the error of the final LOD measurement, $\varepsilon_{L|l}$, and one corresponding to either the random-walk or analytic model for LOD. Thus

$$\int_{\tau=t_l}^{t_n}\int_{\rho=t_l}^{t_n}\langle\eta(\tau)\ \eta(\rho)\rangle\ d\rho\ d\tau = (\sigma_{L|l})^2(t_n - t_l)^2 + \left\{\begin{array}{c}\dfrac{Q(t_n - t_l)^3}{3} \\ \left[\int_{t_l}^{t_n} f(\tau - t_l)d\tau\right]^2\end{array}\right. \tag{A-9}$$

The upper quantity on the right corresponds to the random-walk generated uncertainty, and the lower to the uncertainty due to the modeled analytic behavior of LOD. Figure 5 is generated using the first two terms on the right-hand side of Eq. (A-8), while Fig. 6 utilizes all of Eqs. (A-8) and (A-9).