

NASA-CR-175799

(NASA-CR-195799) ANALYSIS OF CRUSTAL STRUCTURE OF VENUS UTILIZING RESIDUAL LINE-OF-SIGHT (LOS) GRAVITY ACCELERATION AND SURFACE TOPOGRAPHY DATA. A TRIAL OF GLOBAL MODELING OF VENUS GRAVITY FIELD USING HARMONIC SPLINE METHOD G3/91 0004062  
Final Technical Report (Woods Hole Oceanographic Inst.) 9 p

N94-29737  
Unclas

## FINAL TECHNICAL REPORT

Analysis of Crustal Structure of Venus Utilizing Residual Line-of-Sight (LOS)  
Gravity Acceleration and Surface Topography Data

NAG 2-768

A TRIAL OF GLOBAL MODELING OF VENUS GRAVITY  
FIELD USING HARMONIC SPLINE METHOD

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## 1. Introduction

To construct Venus gravity disturbance field ( also referred to as gravity anomaly in geological society ) with the spacecraft-observer line of sight (LOS) acceleration perturbation data, both global approach and local approach can be used. The global approach, e.g. spherical harmonic coefficients (Bills et. al.1987) and local approach, e.g. integral operator method (Barriot & Balmino, 1992), based on geodetic techniques are generally not the same, so that they must be used separately for mapping long wavelength features and short wavelength features. Harmonic spline, as an interpolation and extrapolation technique, is intrinsically flexible to both global and local mapping of a potential field. Theoretically, It preserves the information of the potential field up to the bound by sampling theorem regardless whether it is global or local mapping, and is never bothered with truncation errors. A patch by patch construction of Venus gravity field at a constant altitude using harmonic spline method has been proceeded by Bowin et. al.(1985). In the present investigation we try for the global mapping.

Since the oblateness of Pioneer Venus orbit is extra large sharply reducing the magnitude of signals along trajectories away from the periapses, it is unlikely that globe modeling with such unevenly magnitudinous data will produce very accurate short wavelength gravity features of the planet. Nevertheless, there are other reasons that warrant this trial. First, harmonic spline, the technique itself, needs modifications both theoretically and numerically in order to deal with a fairly large data set of single component measurements, and this has never been done before. Secondly, the only previous global modeling of a potential field with harmonic spline was the downward continuation of the earth surface magnetic field to the core mantle boundary (Shure et. al. 1982, Parker & Shure 1982), in which case the data was constantly distributed with respect to the radius of the earth. The present study will tell us how the uneven radius distribution of data of a harmonic field affects the modeling. This type of information will serve as a upper bound of the radial effect for the future processing of the data from Magellan mission where the orbit is supposed to be nearly round.

The emphasis of this report will be on the improvement of harmonic spline methodology. In the following sections we describe the new basis functions used in this study, then present a singlar value decomposition(SVD) based modification to Parker & Shure's(1982) (hereafter referred to PS) numerical procedure, and finally show some of the preliminary results.

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## 2. Basis functions for single component incomplete vector data

Harmonic spline was originally developed for a set of complete vector measurements of the gradient of a harmonic field which is regular at infinity (Shure et al. 1982 hereafter referred to as SPB). At each observation location there are three components of measurements, and the subset of basis indexed by the location contains three elements each of which corresponds to one component of measurement ( see SPB for detail). In space geodetic problems, more often we have only one component of measurement at each observation location like the LOS acceleration perturbation data. Under such circumstance the three element sub-basis still works though it may not be theoretically optimal. Bowin et. al. (1985) used such basis set to construct their Venus gravity map within each patch of area at several constant altitudes.

It is easy to prove that the optimal representation of the gradient of a harmonic field in the sense of minimizing certain norms can be achieved with a basis set that has a single element corresponding to each observation location if only a single component of measurement for the gradient is available at each observation location. In fact, the general theory by SPB does not require that there must be three components at each point. Therefore we can proceed strictly in parallel to SPB. Details referred to SPB here we simply brief the outcome. Notations except stated are also referred to SPB.

Let  $\hat{n}_j$  be a unit direction vector at  $j$  th observation location with its Cartesian components

$$(n_j^1, n_j^2, n_j^3)$$

By regarding each element  $\mathbf{g}_j$ , ( $j=1,2,\dots,N$ ) selected from the Hilbert space  $\bar{H}$  being the element of a functional for the gradient that gives value of  $\gamma_j$ , which is the  $\hat{n}_j$  direction component of the gradient of the harmonic field measured at the  $j$  th location, we have the norm minimizing element  $\mathbf{B}_0$  in  $\bar{H}$  such that

$$\mathbf{B}_0 = \sum_{j=1}^N \alpha_j \mathbf{g}_j \quad (1)$$

and the gradient of the harmonic field  $\mathbf{B}(\mathbf{r})$  is modeled as

$$\mathbf{B}(\mathbf{r}) = \sum_{j=1}^N \alpha_j \mathbf{G}_j(\mathbf{r}) \quad (2)$$

where

$$\mathbf{G}_j(\mathbf{r}) = \nabla \sum_{l,m} \frac{c}{l+1} \hat{n}_j \cdot \frac{\partial}{\partial \hat{n}_j} \left\{ \left( \frac{c}{r_j} \right)^{l+1} Y_l^m(\theta_j, \phi_j) \right\} \left( \frac{c}{r} \right)^{l+1} Y_l^m(\theta, \phi)$$

For comparison The  $\Gamma$  matrices in three different cases, SPB, Bowin et. al. (1985), and this report are listed here in the same order

$$\Gamma(I, i, J, j)$$

$$\Gamma(I, J, j) = \sum_{i=1}^3 \Gamma(I, i, J, j) n_j^i$$

$$\Gamma(I, J) = \sum_{i=1}^3 \sum_{j=1}^3 \Gamma(I, i, J, j) n_i n_j$$

where  $I, J = 1, 2, \dots, N$  represent observation locations, and  $i, j = 1, 2, 3$  the Cartesian components of vectors.

Equally important to the optimal nature of the new basis is the fact that the elimination of two basis elements for each observation location may save as much as two thirds in computer memory for equal number of basis locations without losing, at least in theory, resolution power, this can be seen from the expressions of the last two  $\Gamma$  matrices. This tremendous saving of memory makes it possible to use larger number of basis locations or even completely non-depleted basis for modeling (the concept of depleted basis is referred to PS). Roughly speaking, the minimum resolvable wavelength by harmonic spline for evenly distributed basis points over a sphere is twice the length between each adjacent basis locations (sampling theorem). This means that we can increase the resolution power up to a factor of  $\sqrt{3}$  with the same computer capacity.

### 3. SVD based numerical algorithm

From numerical point of view, the new basis introduced here is more susceptible to rounding errors than the other two. Particularly when directions of the single component data are biased. This is because of that the element corresponding to each observation location in the new basis is direction oriented, while for SPB and Bowin et al. (1985) bases three elements corresponding to each observation location make the bases totally neutral in terms of orientation. Our numerical experiments with Pioneer Venus LOS acceleration data using PS's QR decomposition based algorithm has shown that for data distributions of one point at every  $6^\circ \times 6^\circ$  grid and denser the numerical process failed at Cholesky factorization with the non-depleted and very close to non-depleted new bases, clearly due to the rounding errors in QR decomposition. To solve this problem we modified PS's algorithm by means of singular value decomposition (SVD).

Without losing generality, let us consider the minimization problem posed by PS. ✓

$$\min_{\beta} \left[ \beta^T H \beta + \frac{1}{\lambda} (|K\beta - \gamma|^2 - S^2) \right] \quad (3)$$

$$|K\beta - \gamma|^2 = S^2$$

where  $\lambda$  is the Lagrange multiplier,  $S^2$  is the given squared misfit,

$$\beta = (\beta_1, \beta_2, \dots, \beta_L)^T$$

is the parameter column to be determined (equivalent to  $\alpha$  parameters in eqn. (1) and (2)),

$$\gamma = (\gamma_1, \gamma_2, \dots, \gamma_N)^T$$

is the observation data column,

$$H_{ij} \quad i=1, 2, \dots, L, \quad j=1, 2, \dots, L$$

are the elements of a positive definite basis matrix  $\mathbf{H}$  (depleted or non-depleted),

$$K_{ij} \quad i=1, 2, \dots, N, \quad j=1, 2, \dots, L$$

are the elements of observation matrix  $\mathbf{K}$  with the property

$$\mathbf{H} (\mathbf{K}^T \mathbf{K}) = (\mathbf{K}^T \mathbf{K}) \mathbf{H}$$

Introducing SVD decomposition of  $\mathbf{K}$  (e.g. Lawson & Hanson 1974)

$$\mathbf{K} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T \quad (4)$$

where  $\mathbf{U}$  is a  $N \times N$  orthogonal matrix,  $\mathbf{V}$  is a  $L \times L$  orthogonal matrix,  $\mathbf{\Sigma}$  is a  $N \times L$  generalized diagonal matrix with none negative diagonal elements  $\omega_1, \omega_2, \dots, \omega_L$ . Following the logic of PS we can complete from (3) and (4) an iteration process for solving  $\beta$  and  $\lambda$ .

$$\left( \mathbf{D} + \frac{1}{\lambda} \mathbf{\Sigma}^2 \right) \mathbf{X} = \frac{\mathbf{\Sigma}^T \mathbf{d}}{\lambda} \quad (5)$$

$$S^2 = S_{\min}^2 + \lambda^2 \mathbf{X}^T \mathbf{D}^2 \mathbf{X} \quad (6)$$

$$\lambda_{j+1} = \lambda_j - \frac{S^2(\lambda_j) - S_{\text{desired}}^2}{\frac{\partial S^2}{\partial \lambda_j}} \quad j=0,1,2, \dots \quad (7)$$

$$\frac{\partial S^2}{\partial \lambda} = 2 \mathbf{X}^T \mathbf{D} \left[ \mathbf{D} + \frac{1}{\lambda} \mathbf{\Sigma}^2 \right]^{-1} \mathbf{D} \mathbf{X} \quad (8)$$

$$\lambda_0 = \left[ \frac{S^2 - S_{\min}^2}{\mathbf{d}^T \mathbf{\Sigma}^{-1} \mathbf{D} \mathbf{\Sigma}^{-2} \mathbf{D} \mathbf{\Sigma}^{-T} \mathbf{d}} \right]^{\frac{1}{2}} \quad (9)$$

where subscript  $j$  indicates the iteration times, and

$$S_{\min}^2 = \gamma^T \gamma - \mathbf{d}^T \mathbf{d} \quad (10)$$

$$\mathbf{d} = \mathbf{U}^T \gamma \quad (11)$$

$$\mathbf{D} = \mathbf{V}^T \mathbf{H} \mathbf{V} \quad (12)$$

$$\mathbf{X} = \mathbf{V}^T \beta \quad (13)$$

The starting trial value of the Lagrange multiplier  $\lambda$  can be calculated from (9) after the initial SVD decomposition, then Cholesky factorization or some equivalent method should be used to solve for  $\mathbf{X}$  from eqn. (5) and to perform the iteration until the calculated misfit falls into the tolerance bound for the desired misfit  $S_{\text{desired}}$ . Finally, (13) is solved for  $\beta$ . As a test we repeated, using SVD, the calculations for several patch of areas over Venus, previously done with the QR algorithm. For well conditioned matrices  $\mathbf{K}$ , SVD's results are in good agreement with QR's as they should. For ill

conditioned matrices  $\mathbf{K}$ , SVD succeeded in one out of total five cases that failed with QR. It seems to be a little advantage of SVD over QR, but not much.

The real advantage appears when the basis is not depleted. In this situation,

$$L = N$$

$$\mathbf{K} = \mathbf{H}$$

$$\mathbf{U} = \mathbf{V}$$

$$\mathbf{D} = \Sigma$$

where  $\Sigma$  hence  $\mathbf{D}$  is strictly diagonal. Diagonalization of  $\mathbf{D}$  greatly simplifies the whole process and makes it unnecessary to use the Cholesky factorization or anything similar. The new algorithm for a non-depleted basis is as simple as follows.

$$x_i = \frac{d_i}{\lambda + \omega_i} \quad i=1, 2, \dots, N \quad (5a)$$

$$S^2 = \lambda^2 \mathbf{X}^T \mathbf{X} \quad (6a)$$

$$\lambda_{j+1} = \lambda_j - \frac{S^2(\lambda_j) - S_{desired}^2}{\frac{\partial S^2}{\partial \lambda_j}} \quad j=0,1,2, \dots \quad (7a)$$

$$\frac{\partial S^2}{\partial \lambda} = 2\lambda \mathbf{X}^T \begin{pmatrix} \frac{\omega_1}{\lambda + \omega_1} & & & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot \\ 0 & & & \frac{\omega_N}{\lambda + \omega_N} \end{pmatrix} \mathbf{X} \quad (8a)$$

$$\lambda_0 = \left[ \frac{S_{desired}^2}{\mathbf{d}^T \Sigma^{-1} \mathbf{d}} \right]^{\frac{1}{2}} \quad (9a)$$

where  $x_i$  and  $d_i$  denote the  $i$  th components of  $\mathbf{X}$  and  $\mathbf{d}$  respectively.

After initial singular value decomposition the rest is just a simple algebra. The algorithm of (5a) through (9a) can be analogous to the damping least square algorithm with  $\lambda$  as the damping factor. Since  $\lambda$  has always to be positive (SPB), it will keep rounding errors from being amplified by some very small  $\omega_i$  s and guarantee the convergence of the iteration process. In another counterpart test to the one mentioned above, four out of five QR failed calculations have been successfully through at very fast convergence rate. Another QR failed case is with a non-depleted basis not applicable to the new algorithm. In another numerical experiment we picked up the starting value of  $\lambda$ ,  $\lambda_0$ , randomly along the positive side of the number axis, and found that

iterations never failed to converge no matter how ill conditioned the matrix  $K$  would be.

One should be reminded, however, that a guaranteed convergence by no means leads to a guaranteed resolution in the solution. Ill conditioning, as it does to the least square solutions, reduces the resolution power by smoothing off short wavelength signals. This can be clearly explained by the expression of (5a) and (8a). With extremely small  $\omega_i$  the denominators are effectively a constant  $\lambda$ , and information contained in these small  $\omega_i$  is smeared off from the solution. Detailed discussions over ill conditioning is essentially the same as that for least square problems which can be found in from e.g. Lawson & Hanson (1974) and other monographs on least square adjustment.

Calculations in the rest of this paper are all performed on non-depleted basis with the greatly simplified SVD algorithm.

#### 4. Some preliminary result of global modeling of Venus gravity

Pioneer Venus orbiter covers Venus surface effectively between -45 and +75 latitude. Below -45 latitude also has some data coverage but not as regular as the major covered region between -45 and 75 latitude. Our so called global modeling is literally referred to this major covered region, though the calculation has extended to -80 to +80 latitude. Detailed description of the acquisition of the data set has been given in a number of previous studies ( Sjogren et.al, 1980, Williams et.al, 1983, Mottinger et.al, 1985, Bills et.al, 1987 and so on ). Reported here are Venus radial gravity anomaly and geoid anomaly at 300 km altitude (Fig. 5 and Fig. 6) modeled from LOS acceleration perturbation data at a reduced density of angular distribution of each grid point representing a  $4^\circ \times 4^\circ$  area.

Totally we presently are using 2923 points angularly regularly distributed over the major covered region. This is a great sacrifice of the data set containing as many as 145,000 data points, but is the limit that we can handle for the non-depleted basis with our present computer facility. The data points are selected from orbits ranging from number 111-117, and 418-602 previously used by Bowin et.al (1985) in their patch by patch modeling. The radial distribution of data is illustrated by the altitude versus latitude plot for the orbit number 565 (Fig. 1), and the radial variation from orbit to orbit is shown in Fig. 2 where data points of many orbits are plotted. These low altitude orbits (150-350 km at periapsis) suffered more from the atmospheric drags than the high altitude orbits (950-1350 km at periapsis) used by Mottinger et.al (1985) to form their tenth-degree and tenth-order model. On the other hand, low altitude data contain more detailed gravity features than the high altitude data. Bills et.al (1987) used low altitude data in addition to high altitude one to produce their eighteenth degree and eighteenth order model.

Under perfect condition the shortest wavelength resolvable by the data distribution would be near 1000 km equivalent to about 40 spherical harmonic degree. Practically this goal can not be achieved due to observation errors and other limitations. Aside from the errors in the data itself, there are two effects that will diminish the resolution and introduce spurious information. One is the orientation of the LOS data. Since the orbital radius of Pioneer Venus is extremely small compare to the distance

between Venus and the Earth, the LOS data are very oriented latitudinally. As discussed earlier, this worsens the condition of the observation matrix  $\mathbf{K}$  ( or  $\mathbf{H}$  identical to  $\mathbf{K}$  ). Another effect is the radial reduction of the gravitational signal at high altitude.

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