NASA Contractor Report 195299

IN-24
4774
80P

Multiple Concentric Cylinder Model (MCCM) User's Guide

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April 1994

Prepared for
Lewis Research Center
Under Contract NAS3-26571

National Aeronautics and
Space Administration

(NASA-CR-195299) MULTIPLE CONCENTRIC CYLINDER MODEL (MCCM) USER'S GUIDE Final Report (Virginia Univ.) 80 p

N94-30125

Unclas

G3/24 0004774

MULTIPLE CONCENTRIC CYLINDER MODEL (MCCM)

USER'S GUIDE

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PREFACE

A user's guide for the computer program **mccm.f** is presented in this report. The program is based on a recently developed solution methodology for the inelastic response of an arbitrarily layered, concentric cylinder assemblage under thermo-mechanical loading which is used to model the axisymmetric behavior of unidirectional metal-matrix composites in the presence of various microstructural details. These details include the layered morphology of certain types of ceramic fibers, as well as multiple, fiber/matrix interfacial layers recently proposed as a means of reducing fabrication-induced, and in-service, residual stresses. The computer code allows efficient characterization and evaluation of new fibers and/or new coating systems on existing fibers with a minimum of effort, taking into account inelastic and temperature-dependent properties and different morphologies of the fiber and the interfacial region. It also facilitates efficient design of engineered interfaces for unidirectional metal matrix composites.

Notice: The **mccm.f** code is being made available strictly as a research tool. Neither the authors of the code nor NASA-Lewis Research Center assume liability for application of the code beyond research needs. Any questions or related items concerning this computer code can be directed to Mr. Todd O. Williams or Professor Marek-Jerzy Pindera at the Civil Engineering & Applied Mechanics Department, University of Virginia, Charlottesville, VA 22903 (Tel: 804-924-1040, e-mail: tow2a@virginia.edu or marek@virginia.edu).

Acknowledgements: The support for this work was provided by the NASA-Lewis Research Center through the contract **NAS3-26571**. The authors thank Dr. Steven M. Arnold of the NASA-Lewis Research Center, the technical monitor of this contract, for his valuable suggestions and comments in the course of this investigation and the preparation of this user's guide.

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1.0 INTRODUCTION

The multiple concentric cylinder model, **MCCM**, is a micromechanical model developed to investigate the effect of microstructural details on the inelastic response of unidirectional metal matrix composites under axisymmetric thermo-mechanical loading. The microstructural details include the layered morphology of such ceramic fibers as the SiC SCS-6 fiber used in titanium matrix composites (DiCarlo, 1988; Lerch et al., 1988; Wawner, 1988; Ning and Pirouz, 1991), the presence of a carbon coating around certain types of fibers, as well as the presence of an interface or interphase layer between the fiber and the matrix. The interfacial layer(s) can arise naturally due to chemical reactions between the fiber and the matrix (Wawner and Gundel, 1992), or can be introduced deliberately in order to minimize residual fabrication stresses in advanced metal matrix composites such as SiC/Ti (Arnold et al., 1990,1992). Due to the lack of matrix ductility, the large mismatch in the thermal expansion coefficients of the fiber and matrix phases, and the high processing temperature exhibited by SiC/Ti composites, these residual stresses can be sufficiently large so as to produce different types of micro-cracks in the fiber or at the fiber/matrix interface (Brindley, et al., 1990,1992; Larsen et al., 1990; McKay et al., 1991). By introducing compliant or compensating layers (those with a higher thermal expansion coefficient than the matrix) between the fiber and matrix phases, Arnold and co-workers (1990,1992) have demonstrated that the circumferential stress at the fiber/matrix interface can be reduced, thereby decreasing or eliminating the possibility of radial microcracking.

This report describes the use of the computer code **mccm.f** based on the multiple concentric cylinder model. The computer code can be used to generate the effective behavior of a unidirectional composite under arbitrary axisymmetric thermo-mechanical loading in terms of macroscopic stresses and strains from which the appropriate instantaneous properties can be calculated. In addition, pointwise distributions of the stress, strain, and displacements fields within each layer of the concentric cylinder assemblage can be determined. The solution methodology employed to generate these quantities utilizes a novel analytical technique for axisymmetric elastoplastic boundary-value problems of arbitrarily layered composite cylinders recently developed by Pindera and co-workers (1992,1993a,b). This solution technique, briefly outlined in Sections 1.1 through 1.4, combines elements of the **local/global stiffness matrix formulation** originally developed for efficient analysis of elastic multi-layered media, and Mendelson's iterative method of **successive elastic solutions** for elastoplastic boundary-value problems (Bufler, 1991; Pindera, 1991; Mendelson, 1983). Since the solution methodology is analytical rather than numerical (e.g., finite-element or finite-difference schemes), there is no need to generate meshes or grids every time the geometrical details or morphology of the fiber or the interfacial region are changed. Different configurations are efficiently handled by changing a few lines in the input file of the **mccm.f** computer code that can be executed on a personal computer/work station. This feature should make the computer program attractive for use by the materials scientist, designer and analyst alike. The construction of the input data file is simple and will be further simplified in a forthcoming version of the code by a user-friendly interface which will allow the user to specify the input parameters in an interactive fashion through menu-driven data entry.

The versatility of the developed computer code described in this user's guide, based on the **MCCM** solution methodology, has been demonstrated in the following investigations (Pindera et al., 1992, 1993; Williams et al., 1993):

- Effect of fiber morphology on residual stresses.
- Effect of multiple compliant/compensating layers on residual stresses.
- Effectiveness of graded interfacial layers in reducing residual stresses.

In summary, the developed computer code allows characterization and evaluation of new fibers and/or new coating systems on existing fibers with a minimum of effort, taking into account temperature-dependent properties and different morphologies of the fiber and the interfacial region. It should also facilitate efficient design of engineered interfaces for unidirectional metal matrix composites.

1.1 Analytical Model

The analytical model is based on a long, cylindrical assemblage of an arbitrary number of concentric cylinders or shells perfectly bonded to each other, Figure 1. Hence the name multiple concentric cylinder model or **MCCM**. Each of the cylindrical shells is either elastic or inelastic. The elastic shells may be isotropic, transversely isotropic, or orthotropic (radially or circumferentially), while the inelastic shells are taken as initially isotropic and are modeled using either time-independent incremental plasticity with the Prandtl-Reuss flow rule and isotropic hardening or one of two unified viscoplasticity theories. These include the Bodner-Partom and the Robinson viscoplasticity models. In addition, a user-defined inelastic constitutive model is available.

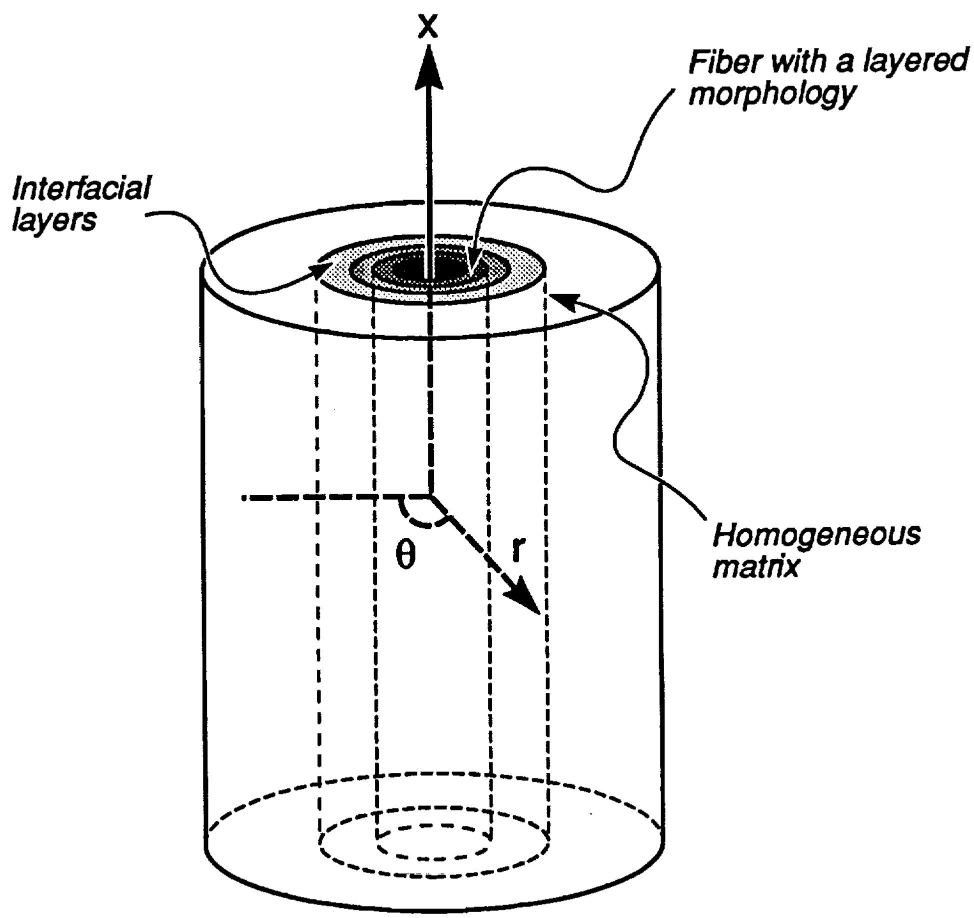


Figure 1. Multiple concentric cylinder assemblage.

For problems in cylindrical coordinates, the stress-strain equations for an orthotropic layer in the presence of axisymmetric thermo-mechanical loading and inelastic effects, and thus absence of shear strains, are given by,

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{rr} \end{Bmatrix} = \begin{bmatrix} C_{xx} & C_{x\theta} & C_{xr} \\ C_{x\theta} & C_{\theta\theta} & C_{\theta r} \\ C_{xr} & C_{\theta r} & C_{rr} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} - \varepsilon_{xx}^{in} - \alpha_{xx}(T - T_0) \\ \varepsilon_{\theta\theta} - \varepsilon_{\theta\theta}^{in} - \alpha_{\theta\theta}(T - T_0) \\ \varepsilon_{rr} - \varepsilon_{rr}^{in} - \alpha_{rr}(T - T_0) \end{Bmatrix} \tag{1}$$

In the above, the C_{ij}'s $(i, j = x, r, \theta)$ are the elastic stiffness matrix elements referred to the $x - r - \theta$ coordinate system, ε_{xx}, $\varepsilon_{\theta\theta}$, ε_{rr} are total strains, ε_{xx}^{in}, $\varepsilon_{\theta\theta}^{in}$, ε_{rr}^{in} are inelastic strains, and $\alpha_{xx}(T - T_0)$, $\alpha_{\theta\theta}(T - T_0)$, $\alpha_{rr}(T - T_0)$ are thermal strains, with T_0 and T denoting the reference and current temperature, respectively. For transversely isotropic materials, where the $r - \theta$ plane is the plane of isotropy, the following relations hold: $C_{x\theta} = C_{xr}$, $C_{\theta\theta} = C_{rr}$ and $\alpha_{\theta\theta} = \alpha_{rr}$, with similar additional relations for a fully isotropic material whose elastic behavior is defined by two independent material constants, say C_{xx} and $C_{x\theta}$.

The stiffness elements C_{ij}'s are related to the familiar engineering constants as follows:

$$C_{xx} = (1 - \nu_{r\theta}\nu_{\theta r})E_{xx}/\Delta\,,\;\; C_{rr} = (1 - \nu_{x\theta}\nu_{\theta x})E_{rr}/\Delta\,,\;\; C_{\theta\theta} = (1 - \nu_{xr}\nu_{rx})E_{\theta\theta}/\Delta$$

$$C_{xr} = (\nu_{xr} + \nu_{x\theta}\nu_{\theta r})E_{rr}/\Delta\,,\;\; C_{x\theta} = (\nu_{x\theta} + \nu_{xr}\nu_{r\theta})E_{\theta\theta}/\Delta\,,\;\; C_{\theta r} = (\nu_{r\theta} + \nu_{x\theta}\nu_{rx})E_{\theta\theta}/\Delta$$

where $\Delta = 1 - \nu_{xr}\nu_{rx} - \nu_{r\theta}\nu_{\theta r} - \nu_{x\theta}\nu_{\theta x} - 2\nu_{rx}\nu_{\theta r}\nu_{x\theta}$, $\nu_{\theta x} = E_{\theta\theta}/E_{xx}\nu_{x\theta}$, $\nu_{rx} = E_{rr}/E_{xx}\nu_{xr}$, and $\nu_{\theta r} = E_{\theta\theta}/E_{rr}\nu_{r\theta}$. These engineering constants are specified by the user for a given layer in the input data file **mccm.data** to the program **mccm.f** in a manner described in Section 3.0. When specifying engineering constants for orthotropic, elastic layers (such as those found in the complex microstructure of the SCS6 SiC fiber), care must be taken to ensure that the thermodynamic constraints on these constants are not violated (cf. Jones, 1975).

All the material parameters governing the response of the elastic and inelastic layers are functions of temperature. The geometrical model is thus general enough to allow one to model the actual morphologies of fiber and interfacial layers in sufficient detail.

1.2 Local/Global Stiffness Matrix Formulation of the MCC Model

The solution to the inelastic boundary-value problem for an arbitrarily layered concentric cylinder is formulated in terms of interfacial displacements as the basic unknown quantities using the concept of a local stiffness matrix (Bufler, 1971; Pindera, 1991). The local stiffness matrix relates the interfacial displacements of a given layer to the corresponding tractions, and is constructed from the solution for the radial displacement field within various types of layers based on the total stress-strain relations given by Equation (1) (Pindera et al., 1992,1993a). By assembling the individual stiffness matrices into a global stiffenss matrix in a manner that will be described subsequently, the interfacial continuity conditions and the external boundary conditions are identically satisfied, while the redundant equations that arise from the interfacial continuity conditions in the standard formulation are eliminated. This results in a nearly 50 percent reduction in the size of the system of simultaneous equations for an assemblage with a large number of layers. This reduction is important in elastoplastic problems for which the solution of the system of simultaneous equations that result from the application of interfacial continuity and boundary conditions must be obtained in an iterative fashion. Further, since the elements of the stiffness matrices for different types of elastic and inelastic layers have been provided in closed form, a given boundary-value problem does not have to be re-solved each time a particular concentric cylinder assemblage is considered. Different configurations are efficiently handled by assembling the global stiffness matrix in an appropriate fashion using the provided local stiffness matrices. The assembly of the global stiffness matrix is easily automated, facilitating addition of extra layers without any difficulty. These features make the method ideal for efficiently investigating the effects of various microstructural details on the axisymmetric response of metal matrix composites in the presence of inelastic effects.

The form of the local stiffness matrix equation for the kth layer in the state of generalized plane strain and in the presence of thermal and inelastic effects is given in Equation (2) below,

$$\begin{Bmatrix} -\sigma_{rr}^- \\ \sigma_{rr}^+ \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} w^- \\ w^+ \end{Bmatrix} + \begin{Bmatrix} k_{13} \\ k_{23} \end{Bmatrix} \varepsilon_{xx}^0 + \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} \Delta T + \begin{Bmatrix} g_1 \\ g_2 \end{Bmatrix} \tag{2}$$

where the superscripts "-" and "+" designate quantities at the inner and outer radii of the kth shell, and ε_{xx}^0 is the common axial strain for all layers. The elements $k_{11}^k, \ldots, k_{23}^k$ of the local stiffness matrix are functions of the geometry and elastic material properties of the kth layer. The thermal effects are represented by f_1^k and f_2^k, which are functions of the thermal expansion coefficients for the kth layer. The inelastic effects are represented by g_1^k and g_2^k, which are given in terms of the integrals of the inelastic strain distribution in the given layer that have the form

$$\int_{r_{k-1}}^{r_k} \sum_{i=x,\theta,r} \frac{(C_{ri} + C_{\theta i})}{C_{rr}} \varepsilon_{ii}^{in}(r') r' dr' \ , \qquad \int_{r_{k-1}}^{r_k} \sum_{i=x,\theta,r} \frac{(C_{ri} - C_{\theta i})}{C_{rr}} \varepsilon_{ii}^{in}(r') \frac{dr'}{r'} \tag{3}$$

Explicit expressions for the elements of the local stiffness matrix and the force vectors appearing in Equation (2) have been provided by Pindera et al. (1993a) for transversely isotropic and orthotropic, elastic layers, and isotropic, inelastic layers.

The interfacial displacements are determined by constructing a system of equations that satisfies the continuity of interfacial tractions and displacements at each interface within the concentric cylinder assemblage, the external boundary conditions specified in terms of the radial traction T_r at the outer radius, and the longitudinal force equilibrium specified in terms of the total axial load L_x. This system of equations is represented in terms of a global stiffness matrix which is assembled by superposing the local stiffness matrices of each layer given in Equation (2) along the main diagonal in the manner shown below in Equation (4) when the axial loading is given in terms of the total axial load L_x,

$$\begin{bmatrix} k_{22}^1 + k_{11}^2 & k_{12}^2 & 0 & \cdot & k_{23}^1 + k_{13}^2 \\ k_{21}^2 & k_{22}^2 + k_{11}^3 & \cdot & \cdot & \cdot \\ 0 & k_{21}^3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & k_{22}^n & k_{23}^n \\ \phi_{22}^1 + \phi_{11}^2 & \cdot & \cdot & \phi_{22}^n & \Sigma\psi_k \end{bmatrix} \begin{Bmatrix} w_1 \\ w_2 \\ \cdot \\ w_n \\ \varepsilon_{xx}^0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \cdot \\ \cdot \\ T_r \\ L_x \end{Bmatrix} - \begin{Bmatrix} f_2^1 + f_1^2 \\ \cdot \\ \cdot \\ f_2^n \\ \Sigma\Omega_k \end{Bmatrix} \Delta T - \begin{Bmatrix} g_2^1 + g_1^2 \\ \cdot \\ \cdot \\ g_2^n \\ \Sigma\Pi_k \end{Bmatrix} \tag{4}$$

and in Equation (5) when the axial loading is given in terms of the uniform axial strain ε_{xx}^0,

$$\begin{bmatrix} k_{22}^1 + k_{11}^2 & k_{12}^2 & 0 & \cdot & 0 \\ k_{21}^2 & k_{22}^2 + k_{11}^3 & \cdot & \cdot & \cdot \\ 0 & k_{21}^3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & k_{22}^n & 0 \\ \phi_{22}^1 + \phi_{11}^2 & \cdot & \cdot & \phi_{22}^n & -1 \end{bmatrix} \begin{Bmatrix} w_1 \\ w_2 \\ \cdot \\ w_n \\ L_x \end{Bmatrix} = \begin{Bmatrix} 0 \\ \cdot \\ \cdot \\ T_r \\ 0 \end{Bmatrix} - \begin{Bmatrix} k_{23}^1 + k_{13}^2 \\ \cdot \\ \cdot \\ k_{23}^n \\ \Sigma\psi_k \end{Bmatrix} \varepsilon_{xx}^0 - \begin{Bmatrix} f_2^1 + f_1^2 \\ \cdot \\ \cdot \\ f_2^n \\ \Sigma\Omega_k \end{Bmatrix} \Delta T - \begin{Bmatrix} g_2^1 + g_1^2 \\ \cdot \\ \cdot \\ g_2^n \\ \Sigma\Pi_k \end{Bmatrix} \tag{5}$$

The first n equations are obtained by enforcing the continuity of interfacial tractions and displacements (starting from the core identified by the superscript 1 and progressing outward), and the external boundary condition on the radial traction. The nth + 1 equation is obtained by imposing the longitudinal equilibrium condition. Explicit expressions for the elements of the last row, ϕ_{11}^k, ϕ_{22}^k, ψ_k, Ω_k, and Π_k have also been provided by Pindera et al. (1993a). As in the case of the elements g_1^k and g_2^k, Π_k is also given in terms of the integrals of the inelastic strain distribution in a given layer that have the form

$$\int_{r_{k-1}}^{r_k} \sum_{i=x,\theta,r} C_{xi}\varepsilon_{ii}^{in}(r')r'dr' \tag{6}$$

The construction of the global stiffness matrix is carried out automatically within the computer program when the given concentric cylinder configuration is specified by the user in a data file. The above system of equations is solved at each thermo-mechanical load increment in a manner outlined in the following section. The stress distributions in the individual regions of the concentric cylinder assemblage at the current state of loading are then obtained from the knowledge of the interfacial displacements.

1.3 Solution Procedure: Mendelson's Successive Approximations Technique

Since the elements of the inelastic force vector appearing on the right hand side of Equation (4) or (5) depend implicitly on the interfacial displacements, an incremental solution technique is employed within an iterative framework in solving for the interfacial displacements. The iteration is performed on the inelastic force vector consisting of the elements g_1^k, g_2^k and $\sum\Pi_k$. Since the elements of the global stiffness matrix and the thermal force vector are constant at a given temperature, only one inversion of the matrix for each sequence of iterations is required, and thus most of the computational effort lies in evaluating the integrals in Equations (3) and (6) at each iteration. This is carried out following the iterative scheme outlined by Mendelson (1983), and briefly outlined below.

For the given thermo-mechanical load increment, the inelastic strain at any point in each layer is expressed in terms of the strain from the preceding loading state plus an increment that results from the imposed load increment.

$$\varepsilon_{ij}^{in}(r)\,|_{current} = \varepsilon_{ij}^{in}(r)\,|_{previous} + d\varepsilon_{ij}^{in}(r) \tag{7}$$

The manner in which the inelastic strain increment in Equation (7) is calculated depends on the particular inelastic model chosen for the given layer as described in Appendix I (Section 8.1). The inelastic strain distribution in each layer is subsequently determined by calculating inelastic strains at twenty-one equally spaced radial locations after updating the plastic strains at these locations using Equation (7). The current values for the inelastic strains at these stations are then used in determining the integrals given in Equations (3) and (6), and thus the elements of the inelastic force vector in Equation (4) or (5). Updated values of the interfacial displacements are then obtained from these equations. With a knowledge of the current interfacial displacements and the axial strain ε_{xx}^{0}, solutions for the radial displacement $w_k(r)$ at any point within the given layer are obtained, from which radial and tangential total strains, and their corresponding stresses, are calculated. These are then used to obtain new approximations for the inelastic strain increments. The iterative process is terminated when the differences between two successive sets of inelastic strain increments are less than some prescribed value.

1.4 Convergence of the Iterative Solution

The convergence of the iterative solution technique employed within **mccm.f** depends on four factors, namely: 1) the constitutive model employed; 2) the size of the load increment for a given loading segment; 3) the number of iterations at a given load increment; and 4) the magnitude of error that can be tolerated between successive values of the inelastic strain increments during a sequence of iterations for a given load increment. The size of the load increment is governed by the initial and final values of the specified loading history parameters (temperature, external pressure and axial stress or strain) together with the number of increments into which a given loading segment is divided (see the manner of specifying the loading history in Section 3). The number of iterations and the error tolerance are specified by the user directly.

When the incremental plasticity theory is employed to calculate the inelastic strain increments in Equation (7) for a given applied load increment, the method of successive elastic solutions can be thought of as a purely spatial integration scheme for the rate-independent Prandtl-Reuss constitutive equations at every point throughout the cylinder assemblage. Williams and Pindera (1994) investigated the rates of convergence of this numerical technique for thermal loading situations and found the method to be robust even for large loading steps (50°F increments). In general, large loading increments require small error tolerances (on the order of 0.01 or 1%) and sufficiently large numbers of iterations at each load increment (on the order of 10). For smaller temperature increments, rapid convergence was achieved for as few as 4 iterations. The convergence rate also depends on the degree of plasticity exhibited by the given material. Thus, materials with small rates of hardening and low yield stresses generally require smaller load increments and larger numbers of iterations.

When a rate-dependent viscoplasticity theory is employed to calculate the inelastic strain increments, both point-wise time integration and spatial integration aspects have to be considered. In general, unified viscoplasticity theories are formulated in terms of first-order ordinary differential equations for the evolution of the inelastic strains that typically exhibit very stiff behavior. This stiff behavior requires that the time integration of these equations be carried out with great care, using a sufficiently small time increment that depends on the actual time integration scheme. A time-integration scheme coupled with the method of successive elastic solutions can be thought of as a predictor-corrector scheme, with each iteration for the given load increment providing a correction to the initial values of field variables at each point within the cylinder assemblage.

In the current version of the **mccm.f** computer code, an explicit forward Euler integration scheme, based on **a priori** known field quantities at the begining of a load increment, is employed to calculate the viscoplastic strain increments that result from the imposed thermomechanical load increment at every point within the assemblage. Therefore, the iterative scheme does not provide new information on the current field quantities with each successive iteration and only point-wise time integration aspects have to be considered. In order to ensure convergence at the local level, and thus the global level, with the presently available viscoplastic models and the integration scheme, very small time increments must be employed as illustrated in the provided examples. A forthcoming version of **mccm.f** will employ a Cauchy-Euler integration scheme for the viscoplastic models implemented within the code that will take full-advantage of Mendelson's iterative scheme. This coupled predictor-corrector integration scheme will allow the user to specify substantially larger time increments, resulting in faster execution times.

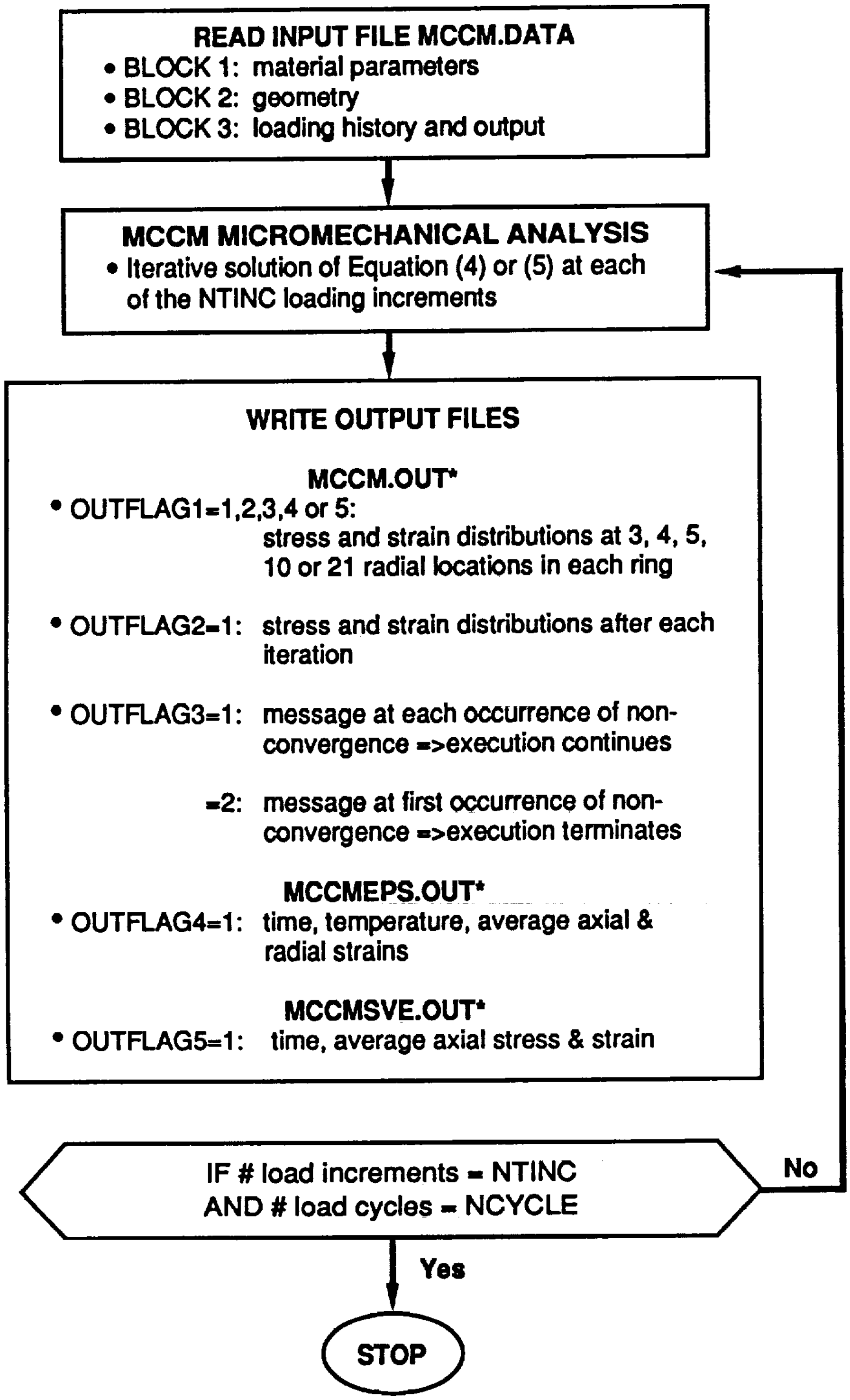
2.0 PROGRAM OVERVIEW

The current capabilities of the program **mccm.f** based on the outlined **MCCM** solution methodology are listed in Table I. The flow chart outlining the logical organization of the program is given in Figure 2. This section provides an overview of the program, including generic considerations required for successful implementation of the code.

Table I. Current available capabilities within **MCCM**.

Type	Description
Geometry	Arbitrarily layered, multiple concentric cylinder assemblage with an isotropic, or transversely isotropic, elastic core
Constitutive models[1]	**Elastic**: isotropic, transversely isotropic, orthotropic materials **Incremental Plasticity**: Prandtl-Reuss relations **Viscoplastic**: Bodner-Partom (with isotropic hardening) and Robinson (with kinematic hardening) unified theories for isotropic materials **Inelastic**: User defined, rate-dependent inelastic relations for isotropic materials
Integration schemes	Spatial: Successive elastic solutions (incremental plasticity) Time: Forward Euler integration technique (viscoplastic models)
Loading capabilities	Arbitrary axisymmetric thermo-mechanical loading, including monotonic and cyclic loads Load or strain controlled axial deformation
Modeling capabilities	Arbitrary number of layers and materials Modeling of layered fiber morphologies Modeling of interface regions around fibers
Predictive capabilities	Effective thermal expansion response Effective inelastic stress-strain response Internal stress and strain distributions within each layer

[1] Temperature-dependence of the material parameters is incorporated in all the constitutive models



*Quantities written to data files at every NTIC/NWRT load increment unless specified otherwise

Figure 2. Flow chart for the program **mccm.f.**

2.1 Memory Allocation

The program utilizes the INCLUDE statement located in the file **paraccm.h** to set three fundamental parameters which dimension the program's arrays. The program must be recompiled if these parameters are reset by the user. These parameters are:

NRING	= number of rings in the concentric cylinder assemblage (set to 10)
NMT	= number of materials for which properties are specified (set to 10)
NTEMP	= number of temperatures at which material properties are specified (set to 10)

There are several considerations with regard to these parameters that may make the use of the program more efficient. The number of rings specified by the parameter NRING must be set to the actual number of rings used in modeling the response of a given cylinder assemblage. NRING defines the size of the global stiffness matrix given in Equation (4) or (5) and thus an unnecessarily large number of rings will slow the program execution. In determining the number of rings that may be required, several considerations arise. The number of rings must, at a minimum, correspond to the number of different materials necessary to model the given system, as each layer is assigned only one material type. For instance, in the case of a fiber-interface-matrix cylinder assemblage with a distinct set of material properties in each of the three regions, the parameter NRING must be set to, at least, 3, Figure 3a. It may be necessary, however, to sub-divide a given cylindrical region, that may be modelled using one layer, into two or more layers, as shown in Figure 3b where the matrix region of the example given in Figure 3a now consists of three layers. In this case, NRING is set to 5. Sub-division of a region occupied by a given material into multiple layers is necessary when the region is relatively thick and undergoes plastic deformation. The plastic deformations typically occur initially in localized areas where large stresses and/or stress gradients are present, and increase with continued loading, accompanied by the expansion of the plastic zone. Typically, large stresses or stress gradients occur at the fiber/matrix interface, especially at very low volume concentrations, in the interfacial layers, or at the matrix/interfacial layer boundary. In such circumstances, a sufficient number of layers is required in those locations in order to accurately calculate the integrals of the plastic strain distribution appearing in Equations (3) and (6) that are used in solving for the interfacial radial displacements (see Equations (4) and (5)), and subsequently the stresses and strains in each layer. The number of layers employed to sub-divide a given region typically depends on the thickness of the region and the non-uniformity of the plastic strain distributions. Typically, thin interfacial layers can usually be modeled adequately by one or two layers since the plastic distributions in these layers are generally uniform and thus the integrals appearing in Equations (3)

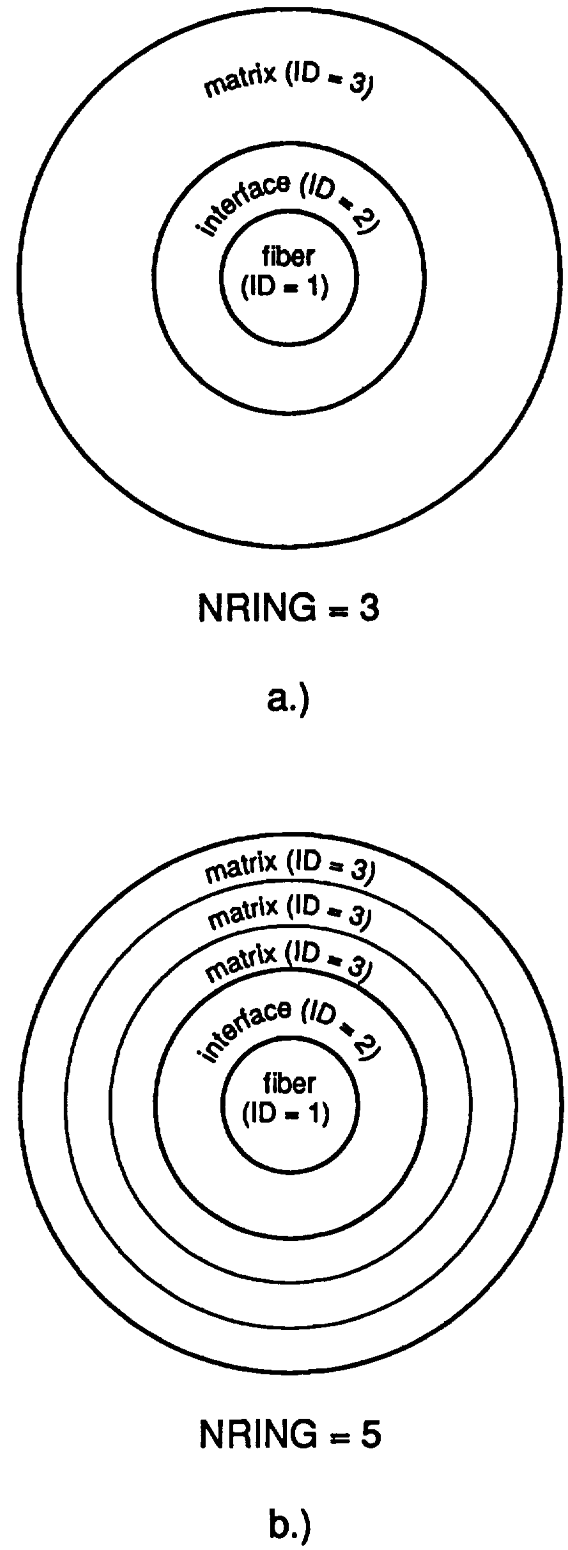


Figure 3. A fiber-interface-matrix cylinder assemblage with distinct material properties in each region: a.) three-region configuration (NRING = 3); b.) five-region configuration (NRING = 5) showing the outer matrix region further sub-divided into three layers.

and (6) can be calculated with sufficient accuracy using the pre-set twenty-one collocation points assigned to each layer. Thick matrix layers that undergo plastic deformation during imposed loading require three or more layers which should be appropriately dimensioned to capture the details of the plastic strain distribution with sufficient accuracy. In the case of layers that do not undergo plastic deformations or whose properties are elastic, only one layer per region with a distinct set of mechanical properties is necessary.

In specifying the parameter NMT, the maximum number of materials which may be required in a given application should be used. Not all of these materials need be used for a given case. A relatively large number of materials specified will affect the memory requirements of the program but will not significantly affect the execution time of the program.

The parameter NTEMP must be set to the actual number of temperatures at which the material properties are specified in the input file. The parameter NTEMP affects the memory and the accuracy with which temperature-dependent properties of the materials within the concentric cylinder assemblage are approximated. The properties are input at NTEMP temperatures and the program linearly interpolates between these temperatures to determine the current values used during the program's execution. This is done for all properties, that is, elastic, thermal, and inelastic. The minimum value for this parameter must be two and the corresponding input temperatures must differ. This minimum is usually only used for predicting the isothermal mechanical response of a system. An **extremely important point** is that the properties for each material must be specified at the same number of identical (NTEMP) input temperatures. Erroneous results will be generated if different temperatures are used to describe different materials in a given input data file. A forthcoming version of **mccm.f** will include the capability to specify material properties for different materials used in the same concentric cylinder assemblage at different temperatures.

2.2 Input/Output

The input data is read in from a file called **mccm.data** and the output is written to any or all of the three files **mccm.out, mccmsve.out** and **mccmeps.out**, see Figure 2. The input data is logically organized into three blocks, namely: the properties of different materials at different temperatures that may be assigned to various rings or cylinders within the multiple concentric cylinder assemblage (**Block 1**); the internal microstructure of the multiple concentric cylinder assemblage specified in terms of the material type and outer radius of the given ring (**Block 2**); and the loading history together with output control flags (**Block 3**). The applied loading is given in terms of the number and duration of loading segments, initial and final values of the load variables associated with each segment, and the number of load increments for each segment. The output file **mccm.out** contains the echo of the input data, and if indicated through the appropriate flag, the stress, strain and displacement profiles throughout the assemblage. The remaining output files are generated at the user's discretion through the use of appropriate flags. The output file **mccmsve.out** contains time and the average axial stress and strain. This file is, typically, generated if the user is interested in the axial stress-strain response of the concentric cylinder assemblage. The output file **mccmeps.out** contains time, temperature, and the average axial and radial strains, and is generated if the user is interested in the thermal or creep response of the concentric cylinder assemblage. The use of the various flags used in controlling the output is described in Table III in Section 3.3.

2.3 Termination of Program Execution

The execution of the program is terminated if incorrect input data essential to a successful execution is specified, or if the iterative scheme for the plastic strain distributions or the integration of the chosen viscoplastic constitutive equations produces non-convergent results. In the first category, the program will terminate automatically if the variable VPFLAG that defines the constitutive model for the individual rings in the assemblage is specified incorrectly (i.e., if it is not 1, 2, 3 or 4, see **Block 1** and Table II in Section 3.1), or if the variable LDFLAG that defines the type of axial loading for a given loading segment is not specified correctly (i.e., it is neither 1 nor 2, see **Block 3** and Table III in Section 3.3). The following messages are written to the output file **mccm.out** in these two instances:

NO CONSTITUTIVE MODEL SPECIFIED !! STOPPED EXECUTING

LDFLAG NOT SPECIFIED !! STOPPED EXECUTING

In the second category, the execution of the program can be terminated in three ways. First, the program will automatically terminate if the global stiffness matrix in Equation (4) or (5) becomes singular. Second, the program will terminate automatically if the effective inelastic strain increment $d\varepsilon_{eff}^{in} = \sqrt{2/3 d\varepsilon_{ij}^{in} d\varepsilon_{ij}^{in}}$ at any point within the concentric cylinder assemblage during the solution procedure exceeds 20% or 0.2. Such large increments typically indicate impending loss of convergence of the integration of the viscoplastic constitutive equations. In this case, a smaller load increment must be chosen. Third, the program execution can be terminated by the user at the occurrence of the first non-convergence condition by setting one of the output control flags described in Section 3.3.1 and Table III to an appropriate value. In this case, non-convergence is associated with the iterative scheme used for the solution of Equation

(4) or (5) when the classical plasticity constitutive model is used to calculate the inelastic strain increments. To remedy this problem, a smaller load increment, a greater number of iterations at each load increment, or larger error tolerance can be specified, see **Block 3**. The following messages are written to the output file **mccm.out** in the above three instances:

SINGULAR GSM !!! STOPPED EXECUTING

EXECUTION STOPPED, depeff > 0.2 (20%)

USER-SPECIFIED STOP OCCURRING FOR NON-CONVERGENCE

2.4 Examples

Detailed descriptions of the input and output files, together with examples that illustrate the structure of these files and the capabilities of the program, are given in the following sections and the appendices. The structure of the input and output files is presented in Sections 3 and 4, respectively, followed by four examples in Section 5 that illustrate the use of the four different constitutive models within **mccm.f**. In the first example, the evolution of residual stresses during fabrication cool-down is considered for a SiC/Ti composite using the incremental plasticity theory for the titanium matrix. The second example examines the effect of the cooling rate on the axial response of a Gr/Al composite using the Bodner-Partom theory for the aluminum matrix. In the third example, strain rate effects on the axial stress-strain response of a homogeneous copper alloy cylinder are investigated using the Robinson viscoplasticity model. Finally, the creep response of a B/Al composite is considered using a power-law creep model for the aluminum matrix.

3.0 INPUT REQUIREMENTS

The input data file **mccm.data** is organized into three distinct blocks, see Figure 2. The organization of the input data file and the variable names read by the program are given below.

3.1 Block 1: Material Properties

This block of data defines constituent material parameters for NMT materials at the same NTEMP number of identical temperatures.

		VPFLAG	constitutive model flag (see Table II)
			next line read only if VPFLAG=4
		USERPRP	number of user-defined inelastic material parameters
		DIRFLAG	material property input sequence flag (see Table II)
Repeat NMT Times			*begin sequential specification of different materials*
		ID	material identification number (1, 2, ..., NMT)
			begin material property input at a given temperature, starting with the highest temperature (DIRFLAG=1), or the lowest (DIRFLAG=2)
	Repeat NTEMP Times	TEMPP	temperature at which material properties are specified
		EXX, ERR, ETT	elastic Young's moduli (E_{xx}, E_{rr}, $E_{\theta\theta}$)
		VXR, VXT, VRT	Poisson's ratios (ν_{xr}, $\nu_{x\theta}$, $\nu_{r\theta}$)
		ALPXXP, ALPRRP, ALPTTP	instantaneous thermal expansion coefficients (α_{xx}, α_{rr}, $\alpha_{\theta\theta}$)
			IF VPFLAG=1 (Incremental Plasticity model) THEN
		YP, HSP	yield stress $\bar{\sigma}_y$; hardening slope for a bilinear elastoplastic material
			ELSE IF VPFLAG=2 (Bodner-Partom model) THEN
		Z0P, Z1P, D0P, BNP, BMP	Bodner-Partom viscoplastic model parameters (Z_0, Z_1, D_0, n, m)
			ELSE IF VPFLAG=3 (Robinson model) THEN
		RAP, RHP, RMP, RNP	Robinson viscoplastic model parameters (A, H, m, n)
		R0P, RQ0P, RT0P, RG0P	Robinson viscoplastic model parameters (R_0, Q_0, T_0, G_0)
		RK0P, RKP, RBETAP	Robinson viscoplastic model parameters (K_0, K, β)
			ELSE IF VPFLAG=4 (user-defined model) THEN
		USERPRPP(1)	user defined inelastic parameter
		USERPRPP(2)	user defined inelastic parameter
		.	.
		.	.
		USERPRPP(NUSERPRP)	user defined inelastic parameter
			END IF
			end of material property input at a given temperature
			end of material property input at NTEMP temperatures for a given material

3.1.1 Comments on variable specification in Block 1

The first block of data identifies the individual materials and their temperature-dependent elastic, thermal, and inelastic properties. The highest material identification number, ID, must coincide with the parameter NMT, and the number of temperatures at which material properties are specified must coincide with the parameter NTEMP and be the same for all materials in the input file as indicated previously in Section 2.1. As stated in this section, these variables are defined in the INCLUDE statement located in the file **paraccm.h**. The input of the elastic and thermal properties is the same for all constitutive relations. However, the input of the inelastic properties will change between the various constitutive relations depending on the choice of VPFLAG specified at the beginning of **Block 1**, see Table II. This variable controls the calculation of the inelastic strain increments appearing in Equation (7) needed in the solution of Equation (4) or (5). The inelastic strain increments may be calculated using one of the following models: incremental plasticity based on the Prandtl-Reuss relations; the Bodner-Partom unified viscoplasticity theory; the Robinson unified viscoplasticity theory; and a user defined, rate-dependent inelastic model. The appropriate equations and references are given in Appendix I (Section 8.1). If a user-defined inelastic theory is used, it must be remembered that the program currently handles the evolution of inelastic strains and back stresses only. Therefore, other types of evolutionary quantities must be included in the user-defined subroutine. An example of the construction of a subroutine for the user-defined inelastic model is given in Appendix II (Section 8.2).

Table II. Control flags used in **Block 1**.

Variable	Flag option	Description
VPFLAG	1	incremental plasticity theory with Prandtl-Reuss flow rule
	2	Bodner-Partom unified viscoplasticity theory
	3	Robinson's unified viscoplasticity theory
	4	user defined inelastic theory
DIRFLAG	1	material properties input from highest to lowest temperature
	2	material properties input from lowest to highest temperature

Note: In order to model an elastic layer, the inelastic parameters of any of the four available models (as specified by VPFLAG = 1 through 4) should be set to appropriate values. For instance, when VPFLAG = 1, a given material can be input as elastic by setting the yield stress YP to a very large value and the hardening slope HSP equal to the elastic Young's modulus EXX. Alternatively, inelastic effects in any one of the constitutive models can be suppressed by setting INDR to 1 (see Block 2 input data description).

It is important to note that only one type of an inelastic constitutive model is presently used for all the layers within a given concentric cylinder assemblage, since the variable VPFLAG is specified just once at the beginning of the block. Therefore, it is not possible to model one ring using the incremental plasticity theory (VPFLAG=1) for instance, while another ring is modeled using the Bodner-Partom theory (VPFLAG=2). However, different material parameters for the chosen inelastic constitutive model may be assigned to different rings. Thus an elastic material can be designated by VPFLAG=1 provided that the yield stress YP at the specified temperature is set to a large value and the hardening slope HSP is equal to the Young's modulus EXX. It is also possible to suppress the inelastic effects in a given ring through the variable INDR as will be explained in Section 3.2.

3.2 Block 2: Geometry

This block of data is used to define the microstructure by assigning various materials specified in **Block 1** to the associated rings in the concentric cylinder assemblage.

		begin specification of the concentric cylinder assemblage configuration, starting from the core and progressing outwards
Repeat NRING Times	PLYMT, INDR, R	PLYMT: material type for a given layer (chosen by specifying a material ID up to NMT) INDR: ring indicator which identifies a given ring as: INDR = 1: isotropic or transversely isotropic, elastic ring INDR = 2: isotropic, inelastic ring INDR = 3: orthotropic, elastic ring R: outer radius of the core or a layer
		end specification of the concentric cylinder assemblage configuration

3.2.1 Comments on variable specification in Block 2

The second block of data specifies the layering sequence of the concentric cylinder assemblage, whether the given ring is elastic or inelastic, and the dimension of each ring. The layering sequence is specified by assigning different material types entered in **Block 1** to the various rings through the variable PLYMT, starting with the core and progressing outwards. In assigning the material to a layer, PLYMT must correspond to one of the material identification numbers, ID, and must be less than or equal to NMT. **The program requires the core of the concentric cylinder assemblage to be an isotropic or transversely isotropic, elastic material.** Therefore, PLYMT in the first line must identify **either** an elastic, isotropic or transversely isotropic material, **or** an isotropic, elastoplastic material with the plastic effects suppressed through the variable INDR as will be explained subsequently. If the fiber of the investigated composite is inelastic, such as the tungsten fiber for instance, the above restriction can be circumvented by using a composite fiber consisting of a small elastic core surrounded by an inelastic sheath. The response of such a composite fiber will not differ significantly from that of the actual fiber if the radius of the core relative to the outer radius of the sheath is small and the elastic properties of the core the same as those of the inelastic sheath.

The variable INDR identifies the material type PLYMT assigned to the given ring as either isotropic or transversely isotropic, elastic (INDR=1), isotropic, inelastic (INDR=2), or orthotropic, elastic (INDR=3). Since the core must always be isotropic and elastic, INDR=1 in the first line of **Block 2**. The variable INDR must be always specified in a manner which is commensurate with the actual material properties of the given material type entered previously. In the case of isotropic, inelastic materials it is possible to suppress the inelastic effects in a given ring by assigning INDR=1 to that ring. Thus an isotropic inelastic material can be treated as isotropic elastic within a given ring or the core, providing the user with more flexibility.

Finally, the dimension of each ring is provided by specifying the outer radius, R, of the ring. For computational reasons, the outer radius of the concentric cylinder assemblage is normalized to 1.0, and the outer radii of each ring scaled accordingly.

3.3 Block 3: Loading History and Output

	NITER, ERROR	NITER: maximum number of iterations at a given loading increment; ERROR: convergence error tolerance
	NCYCLE	number of linear segments in the loading history (see Figure 4)
	LDFLAG	axial mechanical loading flag (see Table III)
	T0, TR0, AXX0	initial values of: temperature (T0); radial traction (TR0); and either average axial stress or strain (AXX0) depending on LDFLAG
		begin specification of loading history over a linear segment
Repeat	LDFLAG, DTIME, NTINC	axial mechanical loading flag (LDFLAG); the total time change for a linear loading segment (DTIME); and number of increments into which the segment is divided (NTINC)
NCYCLE	TEND, TREND, AXXEND	final values of: temperature (TEND); radial traction (TREND); and either average axial stress or strain (AXXEND) depending on LDFLAG
Times	NWRT	number of times specified output is written to a data file over a linear load segment (NTINC/NWRT must be an integer)
	OUTFLAG1, ... , OUTFLAG5	output control flags (see Table III)
		end specification of loading history over a linear segment

3.3.1 Comments on variable specification in Block 3

The final data block specifies the accuracy with which the solution of Equation (4) or (5) is generated, the loading history, and the desired output. The accuracy is determined by specifying the maximum number of iterations at each applied loading increment, and the error tolerance between successive values of the effective plastic strain increment at all points within the assemblage. ERROR is specified as a fractional number, therefore, if a convergence tolerance of 1% is required then ERROR=0.01. If the difference between successive values of the effective plastic strain increment is less then the value of ERROR at every point in the concentric cylinder assemblage then convergence is considered to have occurred. In this case, the program proceeds to the next load increment. If the maximum number of iterations assigned to NITER is reached and convergence does not occur, the program either proceeds to the next load increment, or is terminated, depending on the user-specified control flag described at the end of this section (see also Table III). It should be noted that when a viscoplastic constitutive model is employed to calculate the inelastic strain increments, NITER should be set to 1 since an explicit forward Euler integration scheme for the viscoplastic constitutive models is employed in the current version of the program, as mentioned in Section 1.4. If NITER is greater than 1, the iterative scheme will generate inelastic strain increments for the second iteration that are identical to those generated at the first iteration, automatically satisfying the convergence criterion.

The loading history is arbitrary and is specified in terms of NCYCLE segments, which may consist of any combination of axial, radial, and thermal loads varying linearly with time within the given segment, as illustrated in Figure 4. The axial deformation may be either strain or load controlled, the control mode being specified by LDFLAG. In the strain control, the strain quantities are given in terms of the actual strain values and not percentages. In the load control, the average axial stress, and not the axial force resultant, is specified. The axial deformation mode may be changed at the beginning of a new linear loading segment.

In specifying the loading history, initial or starting values of the applied temperature (T0), the radial pressure (TR0) and the average axial stress or strain (AXX0) are provided first, and just once. The actual loading history is given in terms of the axial deformation mode (LDFLAG), the total time for the given segment (DTIME), the desired number of load increments (NTINC) into which the given segment is divided, followed by the final values of the temperature (TEND), the radial traction (TREND) and the axial strain or stress (AXXEND) at the end of the segment. These variables are specified NCYCLE times. The initial or starting values of the load variables for the second, third, etc., cycles are the final load variables of the preceding cycle. In specifying the thermal aspects of the loading history, the starting and ending temperatures for the NCYCLE linear segments **must remain within the temperature range** for

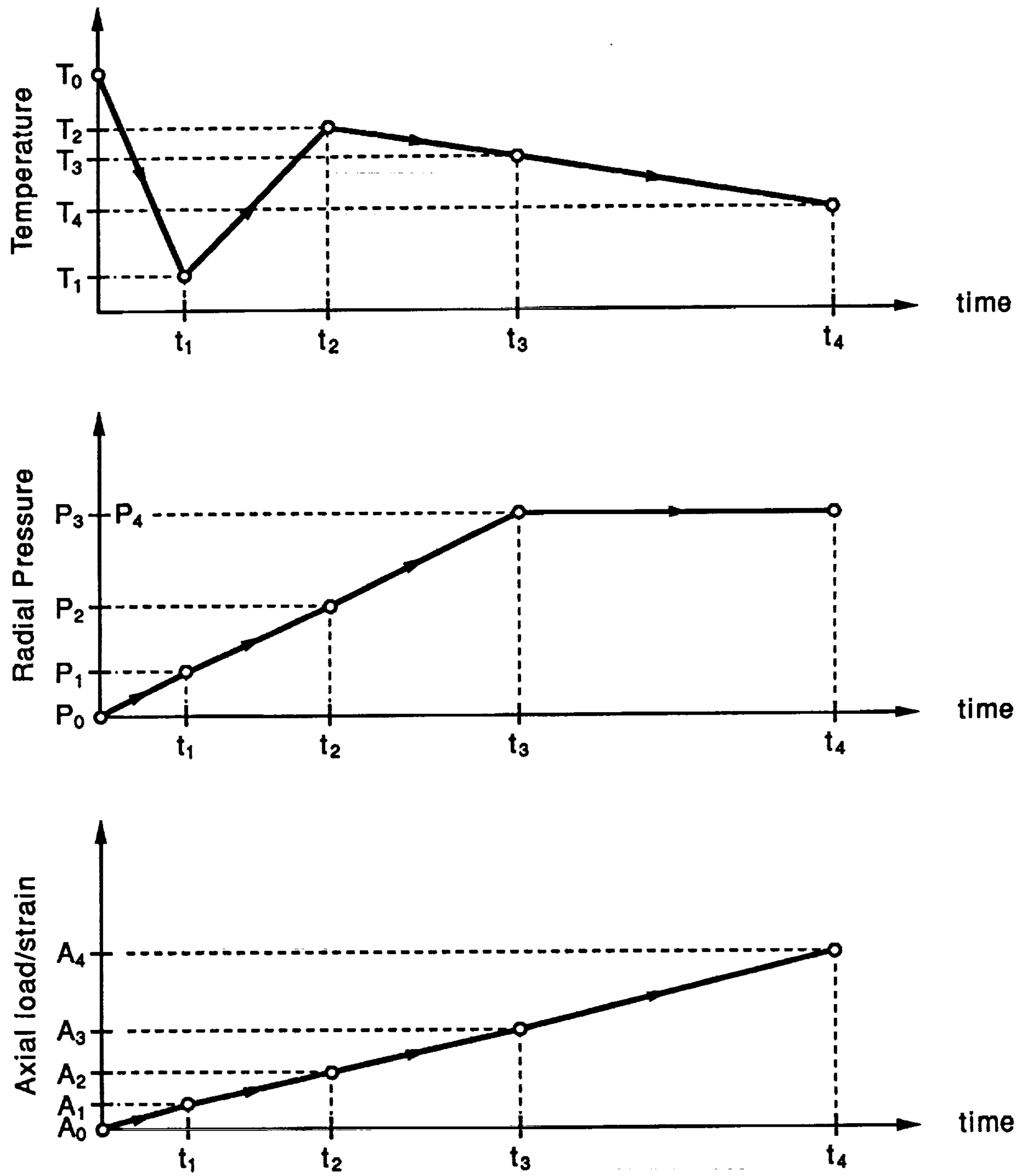


Figure 4. Loading history capability of **mccm.f**. NCYCLE = 4 in the above example.

which the material properties have been specified in **Block 1,** since the program will not extrapolate material properties beyond these limits. The program allows any temperature within the defined material parameter temperature range to be given as the starting, or initial stress-free, temperature for the loading history.

The generation of desired output data is controlled by NWRT and the flags OUTFLAG1 through OUTFLAG5. NWRT controls the number of times the specified output is written to any of the three output files during the execution of a given loading segment. Since the output can only be written to a data file at the end of a load increment, NWRT should always be chosen such that the ratio NTINC/NWRT, where NTINC is the number of load increments in a given cycle, produces an integer number. In this case, NWRT is the actual number of times that the specified data is written to an output file. The ratio NTINC/NWRT then represents the number of load increments between successive output data recordings.

Output control flags OUTFLAG1 through OUTFLAG5 provide the flexibility of generating specific output data in an economical manner, see Table III. OUTFLAG1 allows the user to generate information on the stress, and total and inelastic strain distributions throughout the composite cylinder assemblage at selected points along the loading history. In general, OUTFLAG2 and OUTFLAG3 are used for diagnostic purposes. OUTFLAG2 allows the user to monitor the values of the field variables after each iteration at the specified locations along the loading segment. A large amount of output data will, generally, be written to the **mccm.out** file if this flag is activated, and thus it should be set to 0 in most cases. OUTFLAG3 allows the user to closely monitor the convergence of the outlined solution technique after every load increment. This flag also controls the program execution if non-convergence is encountered at any point along the loading path. Finally, OUTFLAG4 and OUTFLAG5 are useful for investigating the response of a composite material under pure thermal and mechanical axial loading.

Table III. Control flags used in **Block 3**.

Variable	Flag option	Description
LDFLAG	1	load controlled axial deformation
	2	strain controlled axial deformation
OUTFLAG1	0	field variable profiles not written to **mccm.out**
	1	field variables written to **mccm.out** at 3 radial locations in each ring
	2	field variables written to **mccm.out** at 4 radial locations in each ring
	3	field variables written to **mccm.out** at 5 radial locations in each ring
	4	field variables written to **mccm.out** at 10 radial locations in each ring
	5	field variables written to **mccm.out** at 21 radial locations in each ring
OUTFLAG2	0	intermediate values of field variables not written to **mccm.out**
	1	field variables written to **mccm.out** after each iteration at the specified locations along the loading segment
OUTFLAG3	0	convergence conditions not written to **mccm.out**
	1	convergence conditions written to **mccm.out**, indicating at which steps convergence was not achieved; execution is not terminated
	2	convergence conditions written to **mccm.out**, indicating at which steps convergence was not achieved; execution terminates at first occurrance of nonconvergence
OUTFLAG4	0	output for **mccmeps.out** not generated
	1	time, temperature, axial strain, and average radial strain written to **mccmeps.out**
OUTFLAG5	0	output for **mccmsve.out** not generated
	1	time, average axial stress, and axial strain written to **mccmsve.out**

4.0 OUTPUT FILES

4.1 File mccm.out

The output file **mccm.out** is always generated, but may contain different information, depending on which of the flags OUTFLAG1 through OUTFLAG3 are activated. At a minimum, when the values of all these flags are set to zero, the file contains the echo of the input data. The echo consists of the properties of the NMT materials at the specified NTEMP temperatures, the concentric cylinder assemblage configuration, and the loading history. This information is written to the file **mccm.out** in the order, and according to the format, in which it is entered in **Blocks 1, 2** and **3**.

Additional information can be written to this file by setting the input values of the flags OUTFLAG1 through OUTFLAG3 to appropriate values (see Table III). Setting OUTFLAG1 to 1, 2, 3, 4 or 5 produces field variable (stress, radial displacement, plastic strain) distributions at either 3, 4, 5, 10 or 21 radial locations, respectively, within each ring, starting with the core and progressing outward. The program prints the following results every NTINC/NWRT increments according to the format (OUTFLAG1=1 in the example given below):

RING NO.	RADIUS	STRXX	STRRR	STRTT	W
1	...	...	...	...	...
1	...	...	...	...	...
1	...	...	...	...	...
.	...	...	...	...	...
.	...	...	...	...	...
.	...	...	...	...	...
NRING	...	...	...	...	...
NRING	...	...	...	...	...
NRING	...	...	...	...	...

RING NO.	EPXXP	EPRRP	EPTTP	STREFF	SIGEFF
1	...	...	...	...	...
1	...	...	...	...	...
1	...	...	...	...	...
.	...	...	...	...	...
.	...	...	...	...	...
.	...	...	...	...	...
NRING	...	...	...	...	...
NRING	...	...	...	...	...
NRING	...	...	...	...	...

where the variables appearing in the format headers have the following meaning:

STRXX	=	axial stress σ_{xx}
STRRR	=	radial stress σ_{rr}
STRTT	=	circumferential stress $\sigma_{\theta\theta}$
W	=	radial displacement w(r)
EPXXP	=	inelastic axial strain ε^p_{xx}
EPRRP	=	inelastic radial strain ε^p_{rr}
EPTTP	=	inelastic circumferential strain $\varepsilon^p_{\theta\theta}$
STREFF	=	effective stress calculated from the actual stress fields
SIGEFF	=	effective stress calculated from the effective stress-plastic strain curve

There are two ways of calculating the effective stress when the incremental plasticity model is chosen (VPFLAG = 1). STREFF is calculated according to the formula: $\bar{\sigma} = \sqrt{3/2 s_{ij} s_{ij}}$, normalized to the yield stress in uniaxial tension, where s_{ij} are the deviatoric stress components that are determined directly from the solution of Equation (4) or (5). SIGEFF, on the other hand, is calculated from the effective stress-plastic strain curve for an elastoplastic material with bilinear hardening that has the following representation: $\bar{\sigma} = \bar{\sigma}_y(T) + H^p(T)\bar{\varepsilon}^p$ (see Equation (A1.5), Appendix I). During plastic loading, the consistency condition requires that the stress vector remain on the yield surface. Therefore, by comparing STREFF and SIGEFF during elastoplastic deformation, the user can get an idea about the quality of the iterative solution technique employed in **mccm.f**. Ideally, these two quantities should be the same unless elastic unloading occurs at some point during the loading cycle. When one of the viscoplastic models is used (VPFLAG = 2 , 3 or 4), STREFF is still calculated as before, but SIGEFF has no meaning and zero is written in its place.

Setting OUTFLAG2 to 1 produces output identical to that when OUTFLAG1 is set to a non-zero value, except that the output is written after every iteration at the specified locations along the loading segment. Finally, setting OUTFLAG3 to 1 generates a message of the form:

```
NON-CONVERGENCE AT FOLLOWING LOADING STATE

        Time = *.********
        Temperature = *.********
        Radial traction = *.********
        Axial stress = *.********
        Axial strain = *.********
```

that informs the user that convergence has not been achieved at the indicated magnitudes of loading parameters, for the specified ERROR, load increment (defined by initial and final values of the applied load and NTINC), and the maximum number of iterations, NITER. This is written at every occurrance of non-convergence since setting OUTFLAG3 to 1 allows the program to continue executing. If, on the other hand, OUTFLAG3 is set to 3, the program will terminate at the first occurrence of non-convergence, and the following message, in addition to the message given above, will be written:

USER-SPECIFIED STOP OCCURRING DUE TO NONCONVERGENCE

As discussed previously, the problem of non-convergence can be corrected either by increasing the number of iterations, or by decreasing the load increment for the given error tolerance.

4.2 File mccmeps.out

The file **mccmeps.out** contains time, temperature, and average axial and radial strains. This file is generated by setting the output control flag OUTFLAG4 to 1. The information written to this file is typically generated when either pure thermal expansion response of a composite in the absence of mechanical loading, or creep response, is of interest.

TIME	TEMP	EPSXXAV	EPSRRAV
...	...	...	...
...	...	...	...
...	...	...	...

4.3 File mccmsve.out

The file **mccmsve.out** contains time, axial stress and strain. This file is generated by setting the output control flag OUTFLAG5 to 1. The information written to this file is typically generated when axial stress-strain response of a composite is of interest.

TIME	STRXXAV	EPSXXAV
...	...	...
...	...	...
...	...	...

5.0 ILLUSTRATIONS

Four examples are presented in this section that illustrate the construction of input files for each of the four inelastic constitutive options presently available within **mccm.f**. Also included are the resulting output files and pertinent graphical results. The material property data employed in these examples were obtained from the literature without changing the reported units. Thus the units in the first example are English units, whereas in the remaining three examples SI units were employed.

5.1 Example 1: Fiber/Elastoplastic Interface/Elastoplastic Matrix

The following example illustrates the construction of an input data file, the generated output file, and graphical results of the circumferential stress distributions for a SiC/Ti-24Al-11Nb unidirectional composite with one interfacial layer subjected to a cool-down from 1500°F to 75°F. The fiber is taken to be homogeneous, isotropic and elastic, and thus is modeled using a single core. The interfacial layer is also modeled using a single layer, whereas the surrounding matrix region is sub-divided into eight layers. Both the interfacial region and the matrix phase are taken to be isotropic and are modeled using the classical incremental plasticity (Prandtl-Reuss) equations. The properties of the silicon carbide fiber and the titanium matrix at six different temperatures are given in Table IV (Arnold et al., 1990), where the values of the thermal expansion coefficients are instantaneous and **not** secant quantities. With the exception of the Poisson's ratio, the mechanical properties of the interfacial layer are taken to be half of the matrix properties at the given temperatures, while the thermal expansion coefficient is twice that of the matrix. The fiber volume fraction is taken to be 40 percent. Based on the normalized radius for the entire concentric cylinder assemblage (set to 1.0), the radius of the fiber is 0.632, while the outer radius of the interfacial layer is 0.6952, or one tenth of the fiber radius (resulting in an interfacial volume fraction of 8 percent).

The total temperature drop of 1425°F is applied in 570 increments, producing a temperature change of 2.5°F per increment. The maximum number of iterations is set at 15, and the error tolerance is 0.01, or 1%. Since a time-independent incremental plasticity model is used, the total time specified for the applied temperature drop does not affect the response of the concentric cylinder assemblage and is arbitrarily set to 1.0. The output is written to the data file **mccm.out** three times during the thermal loading segment, or every 475 °F. The output written to the file **mccm.out**, in addition to the echo of the input data, consists of the stress and strain distributions. A convergence check is also included. No output is written to the files **mccmeps.out** and **mccmsve.out**.

Table IV. Material properties of homogeneous SiC fiber and titanium matrix.

Material properties	75 °F	392 °F	797 °F	1112 °F	1202 °F	1500 °F
Homogeneous SiC fiber						
Young's modulus (Msi)	58.00	58.00	58.00	58.00	58.00	58.00
Poisson's ratio	0.25	0.25	0.25	0.25	0.25	0.25
α (x 10^{-6}/°F)	1.96	2.01	2.15	2.33	2.38	2.50
Ti-24Al-11Nb matrix						
Young's modulus (Msi)	16.00	14.50	11.00	12.50	9.89	6.20
Poisson's ratio	0.26	0.26	0.26	0.26	0.26	0.26
α (x 10^{-6}/°F)	5.00	5.20	5.70	5.85	5.90	6.15
Yield stress (ksi)	53.89	59.00	53.70	42.20	39.10	24.00
Hardening slope (Msi)	3.33	0.44	0.32	0.19	0.097	0.00

The actual input data file **mccm.data** for this example is provided in Appendix III. Although each block of data, that is **Block 1**, **Block 2** and **Block 3**, is explicitly highlighted for clarity, these identifiers are not to be included in the actual input deck. The actual output data written to the file **mccm.out** is given in Appendix IV. The circumferential stress profiles generated at the three temperatures specified are illustrated in Figure 5. The numerical results given in Appendix IV indicate that at the temperature of 1025°F only the interfacial layer has deformed plastically. Since the thermal expansion coefficient of this layer is twice as high as that of the surrounding matrix, and the extent of plastic deformation relatively small, the circumferential stress in the interfacial region is initially higher than in the matrix phase. As the temperature is further decreased to 550°F, the interfacial layer continues to deform plastically while the surrounding matrix remains elastic. At this temperature therefore, the circumferential stress in the matrix phase is now higher than in the interfacial layer. Decreasing the temperature further to 75°F increases the circumferential stress in the surrounding matrix to the point that plasticity is allowed to initiate, which in turn slows the rate of the circumferential stress growth. As the matrix deforms plastically, the rate of plastic strain accummulation in the interfacial layer decreases, thereby increasing the rate of the circumferential stress growth in this region. Thus the final circumferential stress profile in the interfacial layer is now higher than that in the surrounding matrix.

As a final comment, it should be noted that no convergence warnings were generated for the specified maximum number of iterations, thermal increment size and error tolerance. As can easily be verified, decreasing the maximum number of iterations to 10 will generate a non-convergence warning at 1215°F. However, this will have negligible effect on the resulting stress profiles since very stringent convergence conditions have been implemented in the solution procedure of Equations (4) and (5), requiring convergence to occur simultaneously at every integration point in all the layers where plastic deformation occurs. The employed iterative scheme used in the solution of Equations (4) and (5) is sufficiently rugged to produce convergence at the lower temperatures despite occurrence of non-convergence at a few integration points at the higher temperature.

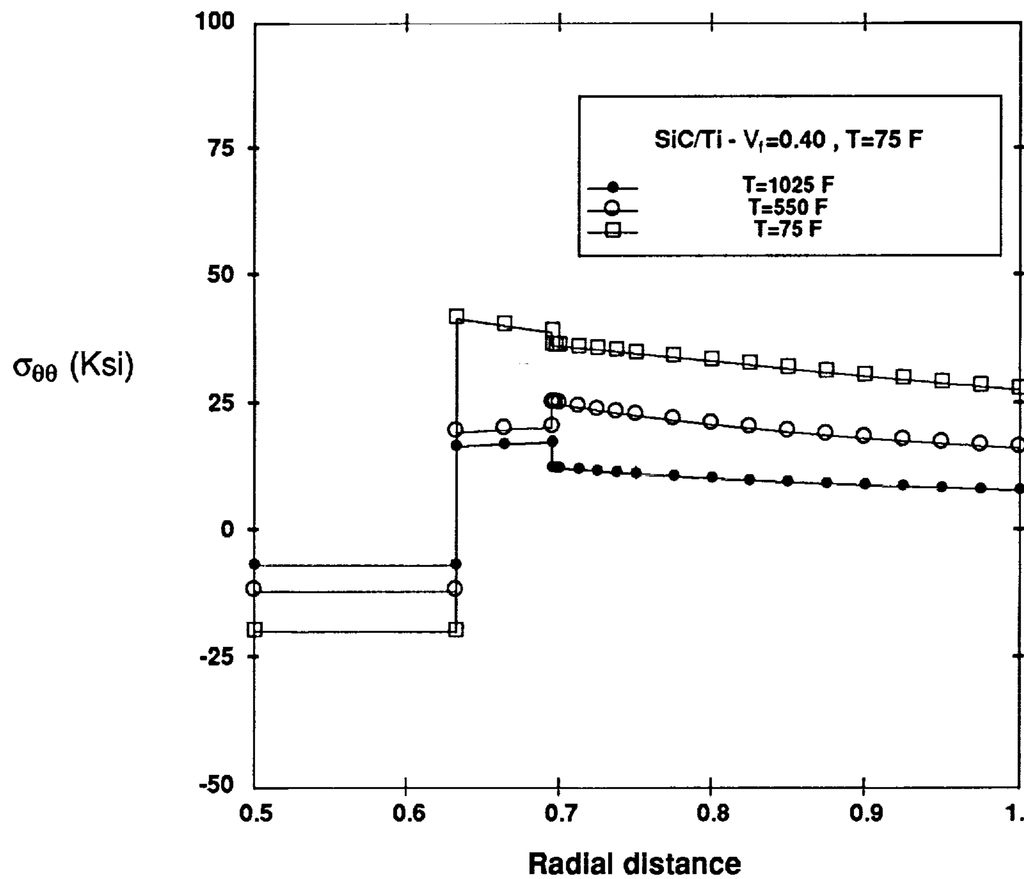


Figure 5. Circumferential stress distributions in a unidirectional SiC/Ti with an interfacial layer.

5.2 Example 2: Fiber/Viscoplastic (Bodner-Partom) Matrix

The following example illustrates the construction of an input data file, the generated output file, and graphical results of the axial strain as a function of temperature for a Gr/Al unidirectional composite without an interfacial layer subjected to a cool-down from 371°C to 21°C. The fiber is taken to be homogeneous, transversely isotropic and elastic, and thus is modeled using a single core. The surrounding aluminum matrix phase is taken to be isotropic and is modeled using the Bodner-Partom unified viscoplasticity theory. It is sub-divided into nine layers. The properties of the graphite fiber and the aluminum matrix at five different temperatures are given in Table V (Aboudi, 1991), where the values of the thermal expansion coefficients are instantaneous and not secant quantities. The fiber volume fraction is taken to be 30 percent. Based on the normalized radius for the entire concentric cylinder assemblage (set to 1.0), the radius of the fiber is 0.5477.

Table V. Material properties of homogeneous graphite fiber and aluminum matrix.

Homogeneous T-50 fiber						
E_A (GPa)	E_T (GPa)	G_A (GPa)	ν_A	ν_T	α_A ($\times 10^{-6}$/°C)	α_T ($\times 10^{-6}$/°C)
388.2	7.6	14.9	0.41	0.45	-0.68	9.74

Al 2024-T4 matrix					
Material properties	21 °C	149 °C	204 °C	260 °C	371 °C
Young's modulus (GPa)	72.40	69.30	65.70	58.40	41.50
Poisson's ratio	0.33	0.33	0.33	0.33	0.33
α ($\times 10^{-6}$/°C)	22.50	22.50	22.50	22.50	22.50
D_0^{-1} ($\times 10^{-4}$ sec)	1.00	1.00	1.00	1.00	1.00
Z_0 (MPa)	340.00	340.00	340.00	340.00	340.00
Z_1 (MPa)	435.00	435.00	435.00	435.00	435.00
m	300.00	300.00	300.00	300.00	300.00
n	10.00	7.00	4.00	1.60	0.55

The total temperature drop of 350°C is applied in 350 increments, producing a temperature change of 1.0°C per increment. The maximum number of iterations is set at 10, and the error tolerance is 0.01 or 1%. Since a time-dependent viscoplasticity model for the matrix phase is used, the total time specified for the applied temperature drop, which governs the local strain rates within the composite cylinder assemblage, does affect the response of the concentric cylinder assemblage. Thus three different cooling rates are employed to investigate the effect of this time dependence on the axial strain response, namely 0.5°C per second, 50°C per second, and 5000°C per second. The output is written to the data file **mccmeps.out** 35 times during the cool-down segment, or every 10°C. The output written to the file **mccm.out** consists only of the echo of the input data. A convergence check is also included.

The actual input data file **mccm.data** for this example is provided in Appendix V for the case with a cooling rate of 0.5°C per second. The remaining two data files differed from the file presented in Appendix V only by the different cooling rates applied. The actual output data written to the files **mccm.out** and **mccmeps.out** is given in Appendix VI. The axial strain as a function of temperature for the three cooling rates is illustrated in Figure 6. The graphical results indicate that no rate effects are observed in the axial strain response of the considered composite system until a temperature of 140°C is reached. Above this temperature, the residual stresses in the matrix phase are not sufficiently large to initiate substantial inelastic deformation. Below 140°C some rate effects are observed, with the stiffest response being observed for the highest cooling rate as expected. However, these rate effects are quite modest since in this temperature range the parameter that controls the rate sensitivity of the material, n, is quite large (Aboudi, 1991).

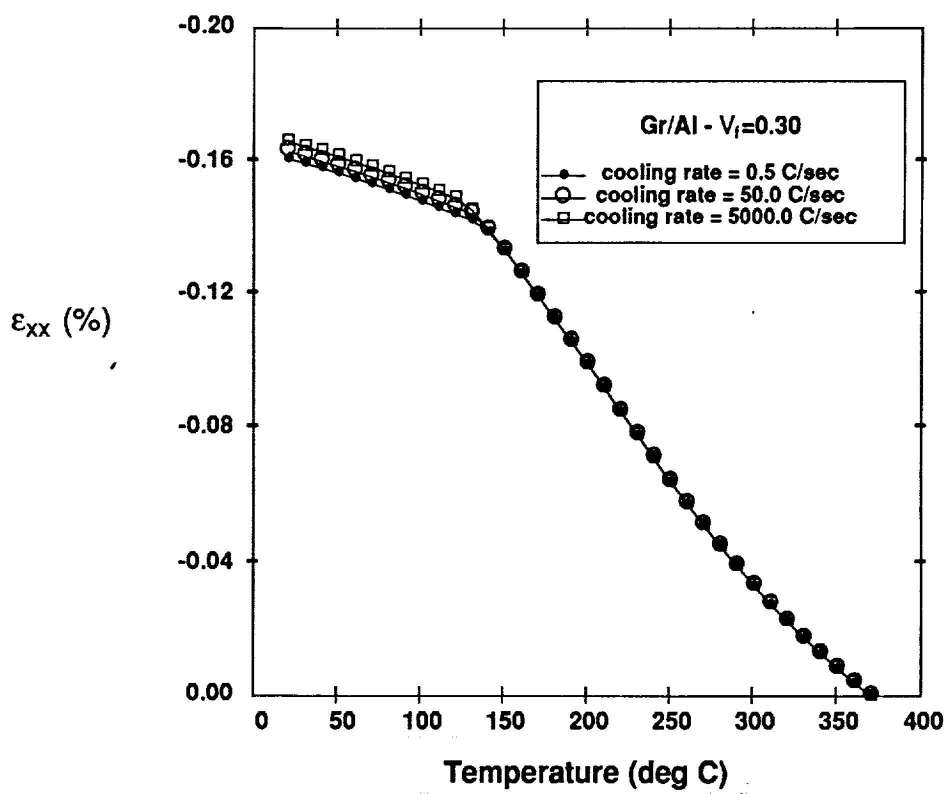


Figure 6. Axial strain as a function of temperature in a unidirectional Gr/Al.

5.3 Example 3: Homogeneous Cylinder (Monolithic Material)

The following example illustrates the construction of an input data file, the generated output file, and graphical results of the axial stress-strain response at room temperature (294°K) for a homogeneous copper alloy Narloy-Z cylinder subjected to an axial deformation. Robinson's unified viscoplasticity theory with kinematic hardening is used to model the response of the Narloy-Z matrix. The properties of the Narloy-Z matrix are given in Table VI at this temperature (Arnold, 1987). However, material properties at two different temperatures need to be specified in the input data file **mccm.data**, even though pure mechanical loading at a fixed temperature is applied. This is due to the manner in which temperature-dependent properties are interpolated within the program. Thus another set of properties at a different temperature than the one at which mechanical loading is applied must be specified to prevent division by zero when the properties at the current temperature are calculated within the program. For the sake of convenience, a slightly lower temperature (290°K) will be employed as the second temperature, with the same properties as those at 294°K.

Table VI. Material properties of copper alloy Narloy-Z matrix at 294°K.

Elastic parameters		
E (GPa)	ν	α ($\times 10^{-6}$/°C)
126.27	0.33	1.0

Inelastic parameters						
A (/hr)	H(GPa)	R_0 (MPa/hr)	Q_0 (°K)	T_0 (°K)	K_0 (MPa)	K(MPa)
138.5×10^{-6}	115.15	1.076×10^{-7}	40,000	811	15.543	50.182
$m = 3.86$ $n = 4.00$ $G_0 = 0.04$ $\beta = 0.846$						

As indicated previously, it is not possible for the core of the cylinder assemblage to exhibit inelastic effects. Since only one material is specified for the cylinder, a very small copper core is embedded in the copper matrix in which the inelastic effects are suppressed by assigning INDR=1 to the core. The surrounding viscoplastic matrix is sub-divided into nine layers.

The total axial deformation of 0.01 or 1% is applied in 2000 increments. The maximum number of iterations is set at 10, and the error tolerance chosen is 0.001 or 0.1%. Since a time-dependent viscoplasticity model for the matrix phase is used, the total time specified for the applied axial deformation does affect the response of the concentric cylinder assemblage. Thus three different axial deformation rates are employed to investigate the effect of this time dependence on the axial stress-strain response, namely 0.008 cm/cm per hour, 0.8 cm/cm per hour, and 80 cm/cm per hour. The output is written to the data file **mccmsve.out** 20 times during the loading segment, or every 0.0005 cm/cm. The output written to the file **mccm.out** consists of only the echo of the input data. A convergence check is also included.

The actual input data file **mccm.data** for this example is provided in Appendix VII for the case with the axial strain rate of 80 cm/cm per hour. The remaining two data files differed from the file presented in Appendix VI by the different strain rates. The actual output data written to the files **mccm.out** and **mccmeps.out** is given in Appendix VIII. The axial stress-strain response for the three cooling rates is illustrated in Figure 7, showing prounounced strain rate sensitivity. These results follow the trends reported by Arnold (1987). It should be noted that the presence of the elastic core has virtually no effect on the response of the cylinder.

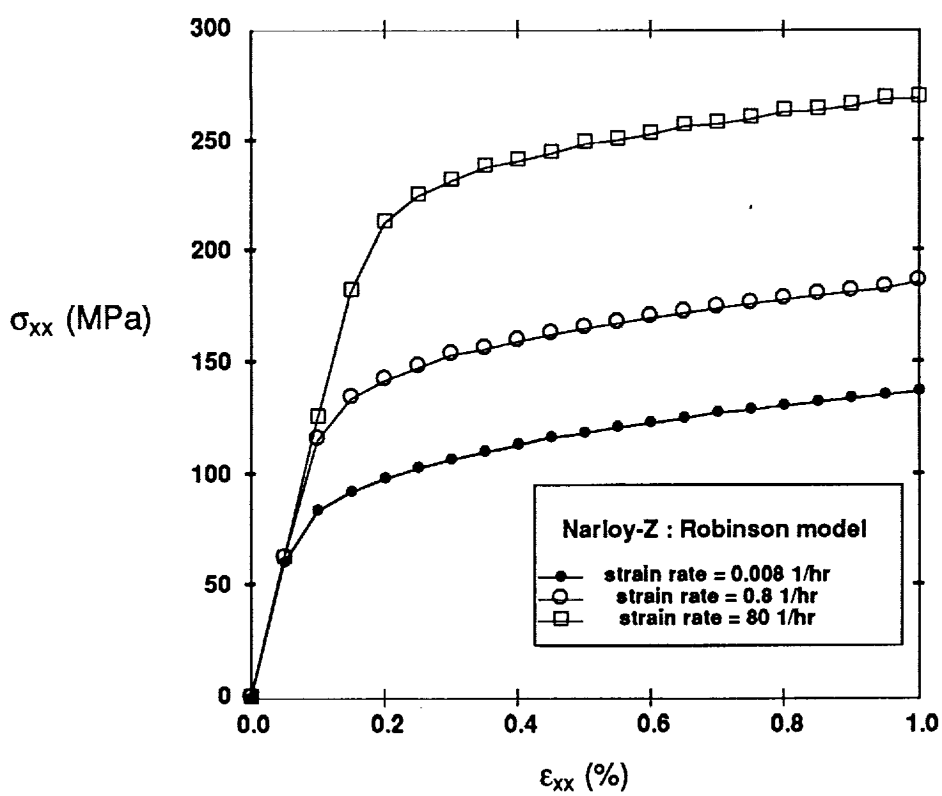


Figure 7. Axial stress-strain response of Narloy-Z as a function of strain rate.

5.4 Example 4: Fiber/Viscoplastic (Power-Law Creep) Matrix

The following example illustrates the construction of an input data file, the generated output file, and graphical results of the axial creep strain response for a B/Al unidirectional composite with different fiber volume concentrations subjected to a constant axial load at 121°C. The boron fiber is taken to be homogeneous, isotropic and elastic, and thus is modeled using a single core. The surrounding aluminum matrix is isotropic and exhibits elastic and a user-defined power-law creep response (Williams and Pindera, 1991) described in Appendix I. The properties of the boron fiber and the aluminum matrix are given in Table VII at this temperature, where the values of the thermal expansion coefficients are instantaneous and not secant quantities. The same properties will be employed at the second temperature (120°C) necessary for the execution of the program. The fiber volume fractions employed in the calculations are 0.25 and 0.50, and the results generated with these fiber concentrations are compared with the creep response of the pure aluminum matrix.

Table VII. Material properties of boron fiber and aluminum matrix.

Elastic parameters			
	E (GPa)	ν	α ($\times 10^{-6}$/°C)
Boron fiber	400.0	0.20	1.0
Al 2024-T4 matrix	70.3	0.34	1.0

Inelastic parameters for Al 2024-T4 aluminum matrix					
n_0	n_1 (1/GPa)	g_0 (1/GPa-hr)	g_1 (1/GPa^2-hr)	g_0' (1/GPa-hr)	g_1' (1/GPa^2-hr)
-0.0625	1.414	-0.1232×10^{-2}	0.799×10^{-2}	-1.298×10^{-2}	4.58×10^{-2}

In order to model the applied creep loading, the total axial stress is applied nearly instantaneously, increasing from zero to a final value in 0.00001 hours, and held at this level for 8 hours. 500 increments were employed during the first loading segment, and 8000 increments during the second. In order to compare the creep response of the three systems using a common basis, the magnitude of the creep load was adjusted for each system such that the initial strain response, immediately after the creep load was applied, was identical. Thus the creep stress for pure aluminum was 345 MPa, whereas for the B/Al composites with 0.25 and 0.50 fiber concentrations the creep stress levels were 750 and 1150 MPa, respectively. The maximum number of iterations is set at 10, and the error tolerance is 0.01. The output is written to the data file **mccmsve.out** once during the first loading segment, and 80 times during the second loading segment, or every 0.1 hour. The output written to the file **mccm.out** consists only of the echo of the input data. A convergence check is also included.

The actual input data file **mccm.data** for this example is provided in Appendix IX for the B/Al composite with 0.25 fiber volume fraction. The actual output data written to the files **mccm.out** and **mccmeps.out** is given in Appendix X. The axial creep strain response is illustrated in Figure 8. As expected, the presence of the stiff, elastic boron fiber relative to the aluminum matrix dramatically reduces the creep effects even for the system with a 25% fiber content. Increasing the fiber content to 50% virtually eliminates the pronounced creep observed in the bulk aluminum matrix.

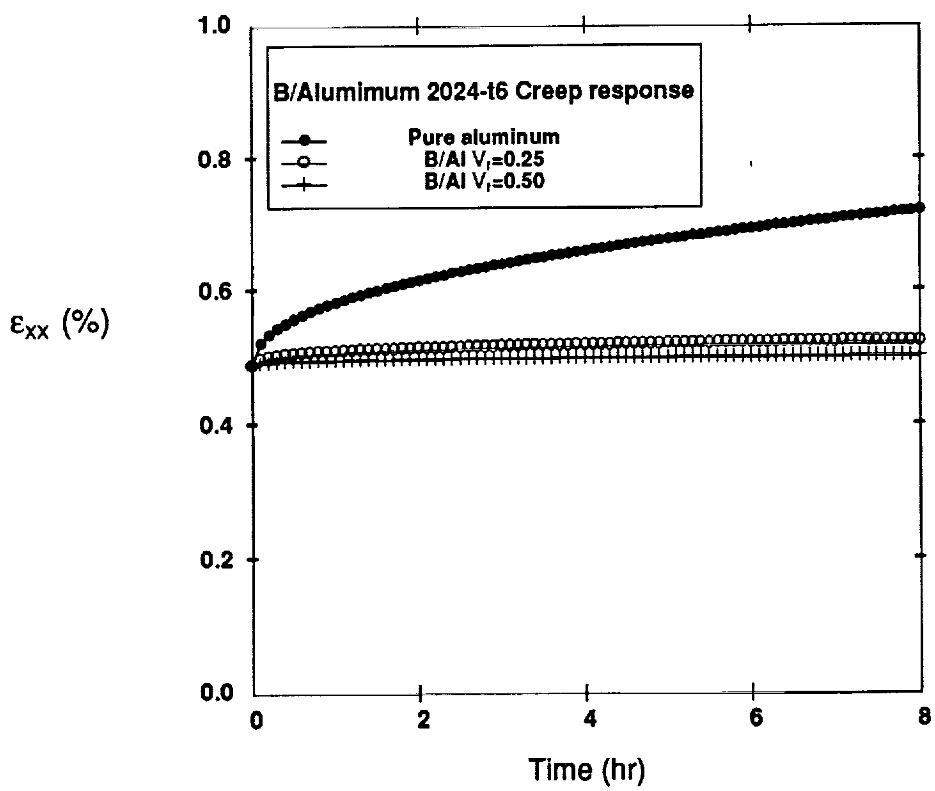


Figure 8. Total axial strain versus time for a unidirectional B/Al composite.

6.0 PLANS FOR FUTURE MODIFICATIONS OF MCCM.F

The following enhancements will be incorporated into the **mccm.f** computer code in order to make it a more flexible, efficient, and user-friendly design/analysis package:

- User-friendly interface for the construction of the input data files and execution of the program.
- Dynamic memory allocation for the parameters NRING, NMT and NTEMP in order to incorporate these variables directly into the input data file.
- Cauchy-Euler integration scheme for the implemented viscoplastic constitutive models.
- User-specified number of integration points for the evaluation of the integrals in Equations (3) and (6) for each ring.
- Capability to specify material properties for different materials used in the same concentric cylinder assemblage at different temperatures.

These enhancements will be implemented during the 1994 funding period (Phase II) of the contract.

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8.0 APPENDICES

8.1 Appendix I: Calculation of the Plastic Strain Increments

The plastic strain increments needed in updating the integrals of the plastic strain distributions that appear on the right hand side of Equation (4) or (5) can be calculated using one of the four different constitutive models presently available within **mccm.f**. These are the rate-independent incremental plasticity theory, rate-dependent Bodner-Partom and Robinson visco-plasticity theories, and one user-defined theory formulated in terms of inelastic strain rates. In the case of the three rate-dependent theories that are formulated in terms of inelastic strain rates, the inelastic strain increments are determined using a simple forward Euler integration formula once the local rates have been determined from the solution of the boundary-value problem.

8.1.1 VPFLAG = 1: Incremental plasticity

In the classical incremental plasticity theory, the plastic strain increment is derived from a von Mises yield condition of the form,

$$F = \frac{1}{2}\sigma'_{ij}\sigma'_{ij} - \frac{1}{3}\bar{\sigma}^2(\bar{\varepsilon}^p, T) = 0 \tag{A1.1}$$

where the effective yield stress $\bar{\sigma}$ is a function of both the effective plastic strain $\bar{\varepsilon}^p$ and temperature T. Using the associated flow rule, the plastic strain increment at any point in the matrix phase is thus,

$$d\varepsilon^p_{ij} = \frac{\partial F}{\partial \sigma'_{ij}} d\lambda = \sigma'_{ij} d\lambda \tag{A1.2}$$

where $d\lambda > 0$ for plastic loading, and $d\lambda \leq 0$ for neutral loading or unloading. The proportionality constant $d\lambda$ is obtained from a consistency condition that ensures that the stress vector remains on the yield surface during plastic loading, and is given in terms of the elastic stiffness elements, stresses, elastic strains and the strain-hardening characteristics (Pindera et al., 1993a,b). This form of the incremental plasticity equations was employed in previous investigations and found to yield generally good convergence. For materials with very low rates of strain-hardening however, difficulties can be encountered using this form of the incremental plasticity. To ensure convergence of the iterative scheme for a wide class of materials in a wide temperature range, so-called plastic strain-total strain plasticity relations were employed in the present investigation by rewriting Equation (A1.2) in terms of total strains without recourse to the stresses (Mendelson, 1983). In this formulation of the incremental plasticity equations, the plastic strain increments are now given in terms of so-called modified total strain deviators e_{ij},

$$d\varepsilon_{ij}^{p} = \frac{e_{ij}}{\bar{e}_{eff}}\, d\bar{\varepsilon}^{p} \tag{A1.3}$$

where $e_{ij} = \varepsilon_{ij} - 1/3\varepsilon_{kk}\delta_{ij} - \varepsilon_{ij}^{p}|_{previous}$, $\bar{e}_{eff} = \sqrt{2/3e_{ij}e_{ij}}$, and the effective plastic strain increment $d\bar{\varepsilon}^{p}$ is given by

$$d\bar{\varepsilon}^{p} = \bar{e}_{eff} - \frac{\bar{\sigma}}{3G} \tag{A1.4}$$

Herein, the elastoplastic stress-strain response of the matrix is taken to be **bilinear**, with the effective stress $\bar{\sigma}(\bar{\varepsilon}^{p}, T)$ given by,

$$\bar{\sigma}(\bar{\varepsilon}^{p}, T) = \bar{\sigma}_{y}(T) + H_{p}(T)\bar{\varepsilon}^{p} \tag{A1.5}$$

The implementation of these plastic strain-total strain plasticity relations is carried out in the same manner as the classical form. That is, the yield condition is first checked at each point within the elastoplastic material to determine whether the material continues to load elastically or whether it has yielded. If the material has yielded, then continued loading is ensured by $d\bar{\varepsilon}^{p} > 0$ and unloading by $d\bar{\varepsilon}^{p} \le 0$.

8.1.2 VPFLAG = 2: Bodner-Partom unified viscoplasticity theory

The Bodner-Partom theory currently available within **mccm.f** is limited to viscoplastic materials that exhibit isotropic hardening. While the theory, in general, models rate-dependent behavior of metals at elevated temperatures, it is particularly suitable for modeling rate-dependent plastic deformation at different loading rates. The model will predict creep effects at fixed applied loads, but is not as suitable for modeling creep-dominated behavior as the Robinson model described in the following section.

According to the Bodner-Parton theory, the viscoplastic strain rate is expressed as

$$\dot{\varepsilon}_{ij}^{p} = \Lambda\, s_{ij} \tag{A2.1}$$

where Λ is the flow rule function of the inelastic layer and s_{ij} are the deviatoric stress components, that is $s_{ij} = \sigma_{ij} - \sigma_{kk}\delta_{ij}/3$. The explicit form of the flow rule function is given by

$$\Lambda = D_0 \exp\{-\hat{n}\,[Z^2/(3J_2)]^n\} / \sqrt{J_2} \tag{A2.2}$$

where $\hat{n} = (n+1)/2n$, and $J_2 = s{\cdot}s/2$ is the second invariant of the deviatoric stresses. D_0 and n are inelastic parameters, and Z is a state variable given for an isotropic hardening material by

$$Z = Z_1 + (Z_0 - Z_1) \exp[-m\, W_p / Z_0] \tag{A2.3}$$

where W_p is the plastic work per unit volume.

The descriptions of the five parameters D_0, Z_0, Z_1, n and m in Equations (A2.2) and (A2.3) are given below.

D_0 = limiting strain rate in shear for large values of the second stress invariant J_2

Z_0 = initial value of the hardening variable Z which is related to the yield stress of the material in simple tension

Z_1 = saturation value of the hardening variable for large values of stresses

m = a parameter that controls the rate of work-hardening of the material

n = a parameter that controls the rate sensitivity of the material

More information regarding the meaning and physical interpretation of these parameters can be found in Bodner (1987) and Aboudi (1991).

8.1.3 VPFLAG = 3: Robinson's unified viscoplasticity theory

The Robinson unified viscoplastic theory is a potential-based theory. The full derivation of the normalized theory employed in **mccm.f** has been provided by Arnold (1987) and only the pertinent relations will be presented herein. Only kinematic hardening effects are included in the version of the theory incorporated into **mccm.f**.

According to the Robinson's theory, the flow law for the inelastic strain rates is given by

$$\dot{\varepsilon}_{ij}^{p} = \frac{AF^{n}\Sigma_{ij}}{\sqrt{J_2'}} \quad \text{when} \quad s_{ij}\Sigma_{ij} > 0 \quad \text{and} \quad F > 0 \tag{A3.1}$$

$$\dot{\varepsilon}_{ij}^{p} = 0 \qquad \text{when} \quad s_{ij}\Sigma_{ij} \leq 0 \quad \text{and} \quad F \leq 0 \ \text{or} \ F > 0$$

where $\Sigma_{ij} = s_{ij} - a_{ij}$, s_{ij} and a_{ij} are the deviatoric stress components and deviatoric back stress components, respectively, J_2' is the second invariant of Σ_{ij}, that is $J_2' = \Sigma \cdot \Sigma/2$, and the function $F = J_2'/K^2 - 1$ may be thought of as a Bingham-Prager threshhold function since the parameter K may be identified as a Bingham-Prager threshhold shear stress.

The evolution law for the back stress is given by

$$\dot{a}_{ij} = \frac{H}{G^{\beta}}\dot{\varepsilon}_{ij}^{p} - \frac{RG^{m-\beta}}{\sqrt{I_2}}a_{ij} \quad \text{when} \quad s_{ij}a_{ij} > 0 \ \text{and} \ G > G_o \tag{A3.2}$$

$$\dot{a}_{ij} = \frac{H}{G_o^{\beta}}\dot{\varepsilon}_{ij}^{p} - \frac{RG_o^{m-\beta}}{\sqrt{I_2}}a_{ij} \quad \text{when} \quad s_{ij}a_{ij} \leq 0 \ \text{and} \ G \leq G_o \ \text{or} \ G > G_o$$

where I_2 is the second invariant of a_{ij}, that is $I_2 = \mathbf{a} \cdot \mathbf{a}/2$, the function G is $G = I_2/K_o^2$, and the recovery term R is given by

$$R = R_0 \exp\left[Q_o\left(\frac{1}{T_o} - \frac{1}{T}\right)\right]$$

where the parameter Q_o is the activation energy for the material and T_o is a reference temperature.

In this model, there are eight material properties that characterize the response of a material; A, n, m, β, H, R, G_o, and K. The parameters A and n are constant and influence the hardening behavior of the inelastic strain rates. The parameters G_o, R, and m are related to the thermal recovery mechanism in the back stress while H and β influence the hardening behavior of the back stress. Of these terms, G_o, m, and H are constants. The value of K at the reference temperature T_o is denoted by K_o.

8.1.4 VPFLAG = 4: User defined, rate-dependent inelastic model

The user can provide his own rate-dependent, inelastic constitutive theory which allows calculation of the inelastic strain rate according to the following format:

$$\dot{\varepsilon}_{ij}^{p} = f_{ij}\,(\,t,\,T,\,\sigma_{kl}^{initial},\,\sigma_{kl}^{current},\,\alpha_{kl}^{initial},\,\alpha_{kl}^{current},\,\varepsilon_{kl}^{(initial)p},\,\varepsilon_{kl}^{(current)p}\,) \tag{A4.1}$$

As an illustration, consider the power-law creep model used to describe the creep response of an aluminum alloy (Al 2024-T4) at an elevated temperature of 121°C by Williams and Pindera (1991),

$$\begin{Bmatrix} \dot{\varepsilon}_{11}^{p} \\ \dot{\varepsilon}_{22}^{p} \\ \dot{\varepsilon}_{33}^{p} \end{Bmatrix} = \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{Bmatrix} g(\bar{\sigma})\, n(\bar{\sigma})\, t^{n-1} \tag{A4.2}$$

which is a modification of Norton's power law for steady-state creep, taking into account the nonlinear time dependence. In the above equation, $\bar{\sigma}$ is the effective stress defined as $\bar{\sigma} = \sqrt{3J_2}$, where J_2 is the second invariant of the stress deviators, and the functions $g(\bar{\sigma})$ and $n(\bar{\sigma})$ are given as follows:

$$n = n_0 + n_1\bar{\sigma} \tag{A4.3}$$

$$\begin{aligned} g &= 0, && \bar{\sigma} \le 207\text{MPa} \\ g &= g_0 + g_1\bar{\sigma}, && 207\text{ MPa} \le \bar{\sigma} \le 310\text{MPa} \\ g &= g_0' + g_1'\bar{\sigma}, && 310\text{ MPa} \le \bar{\sigma} \le 345\text{MPa} \end{aligned} \tag{A4.4}$$

An example of a user-constructed subroutine to calculate the inelastic strain increments based on the above model is provided in Appendix II.

8.2 Appendix II: Construction of a User-Defined Inelastic Constitutive Model Subroutine

The subroutine USERVP that calculates the strain rates based on the user-defined flow rule of the form given in Equations (A4.1) - (A4.4) is provided below.

```
C     *    *    *    *    *    *    *    *    *    *    *    *    *    *
C     *    *    *    *    *    *    *    *    *    *    *    *    *    *
C     *    *    *    *    *    *    *    *    *    *    *    *    *    *
      SUBROUTINE USERVP(INDR,ITER,PLYMT,NUSERPRP,STRXX,STRRR,STRTT,
     & STRXXI,STRRRI,STRTTI,AXX,ARR,ATT,AXXI,ARRI,ATTI,
     & EPSXX,EPSRR,EPSTT,EPIXXP,EPIRRP,EPITTP,USERPRP,TIME,TIMEINC,
     & TEMPC,TEMP,EPSXXR,EPSRRR,EPSTTR,AXXR,ARRR,ATTR)

C
C     This is the shell for the user defined inelastic subroutine. The
C     properties for the model are stored in the array USERPRP. These
C     properties are calculated using linear interpolation in the
C     subroutine USEPRP. The call statement passes the initial stress
C     and back stress states as well as the current time, the time
C     increment, the temperature increment, and the temperature change
C     from the starting temperature, the iteration, the material type
C     and the material indicator. No provision is made for passing the
C     drag or yield stress. If these quantities are required then they
C     must be updated within the user defined subroutine. The inelastic
C     strain rates and back stress rates calculated in this subroutine
C     are returned to the main program. The increments are subsequently
C     determined using a simple forward Euler formula.
C

      INCLUDE 'paraccm.h'

      INTEGER INDR(NRING),ITER
      INTEGER PLYMT(NRING),NUSERPRP
      INTEGER I,J,MT

      REAL*8 STRXX(NRING,21),STRRR(NRING,21),STRTT(NRING,21)
      REAL*8 STRXXI(NRING,21),STRRRI(NRING,21),STRTTI(NRING,21)
      REAL*8 AXX(NRING,21),ARR(NRING,21),ATT(NRING,21)
      REAL*8 AXXI(NRING,21),ARRI(NRING,21),ATTI(NRING,21)
      REAL*8 EPSXX,EPSRR(NRING,21),EPSTT(NRING,21)
      REAL*8 EPIXXP(NRING,21),EPIRRP(NRING,21),EPITTP(NRING,21)
      REAL*8 USERPRP(NMT,50)
      REAL*8 TIME,TIMEINC
      REAL*8 TEMPC,TEMP
      REAL*8 EPSXXR(NRING,21),EPSRRR(NRING,21),EPSTTR(NRING,21)
      REAL*8 AXXR(NRING,21),ARRR(NRING,21),ATTR(NRING,21)
C
C     START OF USER DEFINED CONSTITUTIVE RELATIONS
C     (e.g. Power-law creep model of Example 4)
C
      REAL*8 N0(NMT),N1(NMT)
      REAL*8 G0(NMT),G1(NMT),G0PRM(NMT),G1PRM(NMT)
      REAL*8 NU(NMT),YLDSIG(NMT),THRSIG(NMT)
      REAL*8 PMEAN,SXX,SRR,STT,STREFF
      REAL*8 NPWR,G,FT

      DO 25 I=1,NMT

       N0(I)=USERPRP(I,1)
```

```
      N1(I)=USERPRP(I,2)
      G0(I)=USERPRP(I,3)
      G1(I)=USERPRP(I,4)
      G0PRM(I)=USERPRP(I,5)
      G1PRM(I)=USERPRP(I,6)
      NU(I)=USERPRP(I,7)
      YLDSIG(I)=USERPRP(I,8)
      THRSIG(I)=USERPRP(I,9)

 25  CONTINUE

     DO 200 I=1,NRING

      IF(INDR(I).NE.2) GOTO 150
      MT=PLYMT(I)

      DO 100 J=1,21

       PMEAN=(STRXXI(I,J)+STRRRI(I,J)+STRTTI(I,J))/3.
       SXX=STRXXI(I,J)-PMEAN
       SRR=STRRRI(I,J)-PMEAN
       STT=STRTTI(I,J)-PMEAN
       STREFF=DSQRT((3./2.)*(SXX*SXX+SRR*SRR+STT*STT))

       NPWR=N0(MT)+N1(MT)*STREFF
       IF(STREFF.LT.YLDSIG(MT)) THEN
        G=0.0
       ELSE IF(STREFF.LT.THRSIG(MT)) THEN
        G=G0(MT)+G1(MT)*STREFF
       ELSE
        G=G0PRM(MT)+G1PRM(MT)*STREFF
       END IF
       FT=G*NPWR*TIME**(NPWR-1)

       EPSXXR(I,J)=FT*(STRXXI(I,J)-NU(MT)*(STRRRI(I,J)+STRTTI(I,J)))
       EPSRRR(I,J)=FT*(STRRRI(I,J)-NU(MT)*(STRXXI(I,J)+STRTTI(I,J)))
       EPSTTR(I,J)=FT*(STRTTI(I,J)-NU(MT)*(STRXXI(I,J)+STRRRI(I,J)))

 100  CONTINUE
 150 CONTINUE
 200 CONTINUE

     RETURN
     END
C     *    *    *    *    *    *    *    *    *    *    *    *    *    *
C     *    *    *    *    *    *    *    *    *    *    *    *    *    *
C     *    *    *    *    *    *    *    *    *    *    *    *    *    *

C*************************************************************************
C
C     DEFINITIONS OF VARIABLES IN USER DEFINED INELASTIC SUBROUTINE
C
C     INDR     - Indicates the material anisotropy type for a ring
C     ITER     - The current iteration number
C     PLYMT    - Indicates the material which composes a ring
C     NUSERPRP- The number of actual user defined material parameters
C                in the user's inelastic model
C     I,J      - Loop counters
C     MT       - Corresponds to PLYMT for the current ring
C
C     STRXX    - The axial component of the current stress vector
C     STRRR    - The radial component of the current stress vector
C     STRTT    - The tangential component of the current stress vector
C     STRXXI   - The axial component of the stress vector from the
```

```
C                     previously converged state
C       STRRRI  - The radial component of the stress vector from the
C                     previously converged state
C       STRTTI  - The tangential component of the stress vector from the
C                     previously converged state
C       AXX     - The axial component of the current back stress vector
C       ARR     - The radial component of the current back stress vector
C       ATT     - The tangential component of the current back stress vector
C       AXXI    - The axial component of the back stress vector in the
C                     previously converged state
C       ARRI    - The radial component of the back stress vector in the
C                     previously converged state
C       ATTI    - The tangential component of the back stress vector in the
C                     previously converged state
C       EPSXX   - The current total axial strain
C       EPSRR   - The current total radial strain
C       EPSTT   - The current total tangential strain
C       EPIXXP  - The total axial plastic strain from the previous state
C       EPIRRP  - The total radial plastic strain from the previous state
C       EPITTP  - The total tangential plastic strain from the previous state
C       USERPRP - The current values of the user defined inelastic properties
C       TIME    - The total time change
C       TIMEINC - The time increment
C       TEMPC   - The current temperature
C       TEMP    - The total temperature change from starting state
C       EPSXXRI - The axial inelastic strain rate from the previously
C                     converged state
C       EPSRRRI - The radial inelastic strain rate from the previously
C                     converged state
C       EPSTTRI - The tangential inelastic strain rate from the previously
C                     converged state
C       EPSXXR  - The current axial inelastic strain rate
C       EPSRRR  - The current radial inelastic strain rate
C       EPSTTR  - The current tangential inelastic strain rate
C       AXXR    - The current axial back stress rate
C       ARRR    - The currnet radial back stress rate
C       ATTR    - The current tangential back stress rate
C
C       DEFINITIONS FOR THE TERMS IN THE CREEP MODEL
C
C       N0      - The constant term in the power
C       N1      - The coefficient for the effective stress in the power
C       G0      - The constant term in the function G for effective
C                     stresses less than THRSIG
C       G1      - the coefficient for the effective stress in G for
C                     effective stresses less than THRSIG
C       G0PRM   - The constant term in the function G for effective
C                     stresses greater than THRSIG
C       G1PRM   - the coefficient for the effective stress in G for
C                     effective stresses greater than THRSIG
C       NU      - The Poisson's ratio of the material
C       YLDSIG  - The yield stress below which no inelastic deformation occurs
C       THRSIG  - The threshold or transition effective stress where
C                     the coefficients in G and NPWR change value
C       PMEAN   - The mean pressure
C       SXX     - The axial deviatoric stress
C       SRR     - The radial deviatoric stress
C       STT     - The tangential deviatoric stress
C       STREFF  - The effective stress
C       NPWR    - The power term
C       G       - The function G in the model
C       FT      - The function FT in the model
C
C**********************************************************************
```

8.3 Appendix III: Input File for Example 1

The input file **mccm.data** for the case described in Example 1 is given below. In this example, the parameters NRING, NMT and NTEMP in the INCLUDE statement were set to 10, 3 and 6, respectively. The highlighted text, not to be included in the input deck, identifies the three blocks of the input data, and the fiber, interface and matrix materials used in this example.

Block 1

```
1
1
1 -> Material # 1
1500.0
58.000D+06 58.000D+06 58.000D+06
 0.250D+00  0.250D+00  0.250D+00
 2.500D-06  2.500D-06  2.500D-06
10.000D+06 58.000D+06
1202.0
58.000D+06 58.000D+06 58.000D+06
 0.250D+00  0.250D+00  0.250D+00
 2.380D-06  2.380D-06  2.380D-06
10.000D+06 58.000D+06
1112.0
58.000D+06 58.000D+06 58.000D+06
 0.250D+00  0.250D+00  0.250D+00
 2.330D-06  2.330D-06  2.330D-06
10.000D+06 58.000D+06
 797.0
58.000D+06 58.000D+06 58.000D+06
 0.250D+00  0.250D+00  0.250D+00
 2.150D-06  2.150D-06  2.150D-06
10.000D+06 58.000D+06
 392.0
58.000D+06 58.000D+06 58.000D+06
 0.250D+00  0.250D+00  0.250D+00
 2.010D-06  2.010D-06  2.010D-06
10.000D+06 58.000D+06
  75.0
58.000D+06 58.000D+06 58.000D+06
 0.250D+00  0.250D+00  0.250D+00
 1.960D-06  1.960D-06  1.960D-06
10.000D+06 58.000D+06
2 -> Material # 2
1500.0
 3.100D+06  3.100D+06  3.100D+06
 0.260D+00  0.260D+00  0.260D+00
12.300D-06 12.300D-06 12.300D-06
12.000D+03  0.000D+06
1202.0
 4.945D+06  4.945D+06  4.945D+06
 0.260D+00  0.260D+00  0.260D+00
11.800D-06 11.800D-06 11.800D-06
19.550D+03  0.0485D+06
1112.0
 6.250D+06  6.250D+06  6.250D+06
 0.260D+00  0.260D+00  0.260D+00
11.700D-06 11.700D-06 11.700D-06
21.100D+03  0.095D+06
 797.0
 5.500D+06  5.500D+06  5.500D+06
```

```
 0.260D+00   0.260D+00   0.260D+00
11.400D-06 11.400D-06 11.400D-06
26.850D+03  0.161D+06
 392.0
 7.250D+06   7.250D+06   7.250D+06
 0.260D+00   0.260D+00   0.260D+00
10.400D-06 10.400D-06 10.400D-06
29.500D+03  0.220D+06
  75.0
 8.000D+06   8.000D+06   8.000D+06
 0.260D+00   0.260D+00   0.260D+00
10.000D-06 10.000D-06 10.000D-06
26.945D+03  1.665D+06
3 -> Material # 3
1500.0
 6.200D+06   6.200D+06   6.200D+06
 0.260D+00   0.260D+00   0.260D+00
 6.150D-06   6.150D-06   6.150D-06
24.000D+03  0.000D+06
1202.0
 9.890D+06   9.890D+06   9.890D+06
 0.260D+00   0.260D+00   0.260D+00
 5.900D-06   5.900D-06   5.900D-06
39.100D+03  0.097D+06
1112.0
12.500D+06 12.500D+06 12.500D+06
 0.260D+00   0.260D+00   0.260D+00
 5.850D-06   5.850D-06   5.850D-06
42.200D+03  0.187D+06
 797.0
11.000D+06 11.000D+06 11.000D+06
 0.260D+00   0.260D+00   0.260D+00
 5.700D-06   5.700D-06   5.700D-06
53.700D+03  0.322D+06
 392.0
14.500D+06 14.500D+06 14.500D+06
 0.260D+00   0.260D+00   0.260D+00
 5.200D-06   5.200D-06   5.200D-06
59.000D+03  0.441D+06
  75.0
16.000D+06 16.000D+06 16.000D+06
 0.260D+00   0.260D+00   0.260D+00
 5.000D-06   5.000D-06   5.000D-06
53.890D+03  3.333D+06
```

Block 2

```
  1  1  0.6320
  2  2  0.6952
  3  2  0.7000
  3  2  0.7250
  3  2  0.7500
  3  2  0.8000
  3  2  0.8500
  3  2  0.9000
  3  2  0.9500
  3  2  1.0000
```

Block 3

```
15  0.01
1
1
    1500.0    0.0    0.0
1   1.0  570
      75.0    0.0    0.0
3
1  0  1  0  0
```

8.4 Appendix IV: Output File for Example 1

The output file **mccm.out** generated by the input file **mccm.data** of Example 1 is given below (see Appendix III for the input file).

```
*************************************************
**                                             **
**     MULTIPLE CONCENCTRIC CYLINDERS MODEL    **
**                                             **
**                    MCCM                     **
**                                             **
**   DETERMINATION OF THE INELASTIC RESPONSE OF  **
** A MULTI-LAYERED COMPOSITE CYLINDER SUBJECTED **
**   TO AXISYMMETRIC THERMO-MECHANICAL LOADING  **
**                                             **
**                     BY                      **
**                                             **
**              TODD O. WILLIAMS               **
**             MAREK-JERZY PINDERA             **
**                                             **
**            UNIVERSITY OF VIRGINIA           **
**                                             **
**                                             **
** DEVELOPED FOR THE FATIQUE AND FRACTURE BRANCH**
**          NASA-LEWIS RESEARCH CENTER         **
**           UNDER CONTRACT NAS3-26571         **
**      DR. S. M. ARNOLD (CONTRACT MONITOR)    **
**                                             **
*************************************************

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--------------------------------------------------------


           ****** INPUT DATA ECHO ******


              MATERIAL SPECIFICATION


  Number of rings (NRING) = 10
  Number of materials (NMT) =  3
  Number of temperatures at which properties are specified (NTEMP) = 6
```

```
MATERIAL # 1

TEMPERATURE =   0.1500E+04

0.5800E+08  0.5800E+08  0.5800E+08
0.2500E+00  0.2500E+00  0.2500E+00
0.2500E-05  0.2500E-05  0.2500E-05
0.1000E+08  0.5800E+08

TEMPERATURE =   0.1202E+04

0.5800E+08  0.5800E+08  0.5800E+08
0.2500E+00  0.2500E+00  0.2500E+00
0.2380E-05  0.2380E-05  0.2380E-05
0.1000E+08  0.5800E+08

TEMPERATURE =   0.1112E+04

0.5800E+08  0.5800E+08  0.5800E+08
0.2500E+00  0.2500E+00  0.2500E+00
0.2330E-05  0.2330E-05  0.2330E-05
0.1000E+08  0.5800E+08

TEMPERATURE =   0.7970E+03

0.5800E+08  0.5800E+08  0.5800E+08
0.2500E+00  0.2500E+00  0.2500E+00
0.2150E-05  0.2150E-05  0.2150E-05
0.1000E+08  0.5800E+08

TEMPERATURE =   0.3920E+03

0.5800E+08  0.5800E+08  0.5800E+08
0.2500E+00  0.2500E+00  0.2500E+00
0.2010E-05  0.2010E-05  0.2010E-05
0.1000E+08  0.5800E+08

TEMPERATURE =   0.7500E+02

0.5800E+08  0.5800E+08  0.5800E+08
0.2500E+00  0.2500E+00  0.2500E+00
0.1960E-05  0.1960E-05  0.1960E-05
0.1000E+08  0.5800E+08


MATERIAL # 2

TEMPERATURE =   0.1500E+04

0.3100E+07  0.3100E+07  0.3100E+07
0.2600E+00  0.2600E+00  0.2600E+00
0.1230E-04  0.1230E-04  0.1230E-04
0.1200E+05  0.0000E+00

TEMPERATURE =   0.1202E+04

0.4945E+07  0.4945E+07  0.4945E+07
0.2600E+00  0.2600E+00  0.2600E+00
0.1180E-04  0.1180E-04  0.1180E-04
0.1955E+05  0.4850E+05

TEMPERATURE =   0.1112E+04
```

```
0.6250E+07  0.6250E+07  0.6250E+07
0.2600E+00  0.2600E+00  0.2600E+00
0.1170E-04  0.1170E-04  0.1170E-04
0.2110E+05  0.9500E+05

TEMPERATURE =  0.7970E+03

0.5500E+07  0.5500E+07  0.5500E+07
0.2600E+00  0.2600E+00  0.2600E+00
0.1140E-04  0.1140E-04  0.1140E-04
0.2685E+05  0.1610E+06

TEMPERATURE =  0.3920E+03

0.7250E+07  0.7250E+07  0.7250E+07
0.2600E+00  0.2600E+00  0.2600E+00
0.1040E-04  0.1040E-04  0.1040E-04
0.2950E+05  0.2200E+06

TEMPERATURE =  0.7500E+02

0.8000E+07  0.8000E+07  0.8000E+07
0.2600E+00  0.2600E+00  0.2600E+00
0.1000E-04  0.1000E-04  0.1000E-04
0.2694E+05  0.1665E+07


MATERIAL # 3


TEMPERATURE =  0.1500E+04

0.6200E+07  0.6200E+07  0.6200E+07
0.2600E+00  0.2600E+00  0.2600E+00
0.6150E-05  0.6150E-05  0.6150E-05
0.2400E+05  0.0000E+00

TEMPERATURE =  0.1202E+04

0.9890E+07  0.9890E+07  0.9890E+07
0.2600E+00  0.2600E+00  0.2600E+00
0.5900E-05  0.5900E-05  0.5900E-05
0.3910E+05  0.9700E+05

TEMPERATURE =  0.1112E+04

0.1250E+08  0.1250E+08  0.1250E+08
0.2600E+00  0.2600E+00  0.2600E+00
0.5850E-05  0.5850E-05  0.5850E-05
0.4220E+05  0.1870E+06

TEMPERATURE =  0.7970E+03

0.1100E+08  0.1100E+08  0.1100E+08
0.2600E+00  0.2600E+00  0.2600E+00
0.5700E-05  0.5700E-05  0.5700E-05
0.5370E+05  0.3220E+06

TEMPERATURE =  0.3920E+03

0.1450E+08  0.1450E+08  0.1450E+08
0.2600E+00  0.2600E+00  0.2600E+00
0.5200E-05  0.5200E-05  0.5200E-05
0.5900E+05  0.4410E+06
```

TEMPERATURE = 0.7500E+02

0.1600E+08	0.1600E+08	0.1600E+08
0.2600E+00	0.2600E+00	0.2600E+00
0.5000E-05	0.5000E-05	0.5000E-05
0.5389E+05	0.3333E+07	

CONCENTRIC CYLINDER CONFIGURATION

RING	MATERIAL	CONSTITUTIVE MODEL	OUTER RADIUS
1	1	Elastic	0.632000
2	2	Inelastic	0.695200
3	3	Inelastic	0.700000
4	3	Inelastic	0.725000
5	3	Inelastic	0.750000
6	3	Inelastic	0.800000
7	3	Inelastic	0.850000
8	3	Inelastic	0.900000
9	3	Inelastic	0.950000
10	3	Inelastic	1.000000

Inelastic model (VPFLAG = 1) : Classical Plasticity

LOADING PARAMETERS

Maximum number of iterations (NITER) = 15
Error tolerance (ERROR) = 0.1000E-01
Total number of loading segments (NCYCLE) = 1

****** OUTPUT RESULTS ******

THE LOADING INFORMATION FOR SEGMENT 1

Total time change (DTIME)= 0.1000E+01
Number of loading increments (NTINC) = 570

	TEMPERATURE	RAD. TRACT.	AXIAL STRESS
START	0.1500E+04	0.0000E+00	0.0000E+00
END	0.7500E+02	0.0000E+00	0.0000E+00
RATE	-0.1425E+04	0.0000E+00	0.0000E+00
INCREMENT	-0.2500E+01	0.0000E+00	0.0000E+00

```
Time =  0.3333E+00
Temperature =  0.1025E+04
Radial traction =  0.0000E+00
Axial strain = -0.1542E-02
Axial stress =  0.1312E-03
```

RING NO.	RADIUS	STRXX	STRRR	STRTT	W
1	0.0000E+00	-0.2668E+05	-0.6663E+04	-0.6663E+04	0.0000E+00
1	0.3160E+00	-0.2668E+05	-0.6663E+04	-0.6663E+04	-0.3510E-03
1	0.6320E+00	-0.2668E+05	-0.6663E+04	-0.6663E+04	-0.7020E-03
2	0.6320E+00	0.1605E+05	-0.6663E+04	0.1692E+05	-0.7020E-03
2	0.6636E+00	0.1767E+05	-0.5527E+04	0.1744E+05	-0.1065E-02
2	0.6952E+00	0.1918E+05	-0.4474E+04	0.1784E+05	-0.1428E-02
3	0.6952E+00	0.1776E+05	-0.4474E+04	0.1284E+05	-0.1428E-02
3	0.6976E+00	0.1776E+05	-0.4414E+04	0.1278E+05	-0.1438E-02
3	0.7000E+00	0.1776E+05	-0.4355E+04	0.1272E+05	-0.1447E-02
4	0.7000E+00	0.1776E+05	-0.4355E+04	0.1272E+05	-0.1447E-02
4	0.7125E+00	0.1776E+05	-0.4058E+04	0.1243E+05	-0.1494E-02
4	0.7250E+00	0.1776E+05	-0.3776E+04	0.1215E+05	-0.1542E-02
5	0.7250E+00	0.1776E+05	-0.3776E+04	0.1215E+05	-0.1542E-02
5	0.7375E+00	0.1776E+05	-0.3509E+04	0.1188E+05	-0.1589E-02
5	0.7500E+00	0.1776E+05	-0.3255E+04	0.1162E+05	-0.1636E-02
6	0.7500E+00	0.1776E+05	-0.3255E+04	0.1162E+05	-0.1636E-02
6	0.7750E+00	0.1776E+05	-0.2782E+04	0.1115E+05	-0.1728E-02
6	0.8000E+00	0.1776E+05	-0.2354E+04	0.1072E+05	-0.1820E-02
7	0.8000E+00	0.1776E+05	-0.2354E+04	0.1072E+05	-0.1820E-02
7	0.8250E+00	0.1776E+05	-0.1964E+04	0.1033E+05	-0.1910E-02
7	0.8500E+00	0.1776E+05	-0.1607E+04	0.9976E+04	-0.2000E-02
8	0.8500E+00	0.1776E+05	-0.1607E+04	0.9976E+04	-0.2000E-02
8	0.8750E+00	0.1776E+05	-0.1281E+04	0.9650E+04	-0.2089E-02
8	0.9000E+00	0.1776E+05	-0.9815E+03	0.9350E+04	-0.2177E-02
9	0.9000E+00	0.1776E+05	-0.9815E+03	0.9350E+04	-0.2177E-02
9	0.9250E+00	0.1776E+05	-0.7061E+03	0.9075E+04	-0.2263E-02
9	0.9500E+00	0.1776E+05	-0.4521E+03	0.8821E+04	-0.2350E-02
10	0.9500E+00	0.1776E+05	-0.4521E+03	0.8821E+04	-0.2350E-02
10	0.9750E+00	0.1776E+05	-0.2173E+03	0.8586E+04	-0.2435E-02
10	0.1000E+01	0.1776E+05	0.1091E-09	0.8369E+04	-0.2521E-02

RING NO.	EPXXP	EPRRP	EPTTP	STREFF	SIGEFF
1	0.0000E+00	0.0000E+00	0.0000E+00	0.2002E+05	0.1000E+08
1	0.0000E+00	0.0000E+00	0.0000E+00	0.2002E+05	0.1000E+08
1	0.0000E+00	0.0000E+00	0.0000E+00	0.2002E+05	0.1000E+08
2	0.1906E-02	-0.4062E-02	0.2156E-02	0.2316E+05	0.2316E+05
2	0.1710E-02	-0.3387E-02	0.1677E-02	0.2308E+05	0.2308E+05
2	0.1522E-02	-0.2810E-02	0.1289E-02	0.2301E+05	0.2301E+05
3	0.0000E+00	0.0000E+00	0.0000E+00	0.2023E+05	0.4538E+05
3	0.0000E+00	0.0000E+00	0.0000E+00	0.2015E+05	0.4538E+05
3	0.0000E+00	0.0000E+00	0.0000E+00	0.2007E+05	0.4538E+05
4	0.0000E+00	0.0000E+00	0.0000E+00	0.2007E+05	0.4538E+05
4	0.0000E+00	0.0000E+00	0.0000E+00	0.1970E+05	0.4538E+05
4	0.0000E+00	0.0000E+00	0.0000E+00	0.1935E+05	0.4538E+05
5	0.0000E+00	0.0000E+00	0.0000E+00	0.1935E+05	0.4538E+05
5	0.0000E+00	0.0000E+00	0.0000E+00	0.1902E+05	0.4538E+05
5	0.0000E+00	0.0000E+00	0.0000E+00	0.1871E+05	0.4538E+05
6	0.0000E+00	0.0000E+00	0.0000E+00	0.1871E+05	0.4538E+05
6	0.0000E+00	0.0000E+00	0.0000E+00	0.1816E+05	0.4538E+05
6	0.0000E+00	0.0000E+00	0.0000E+00	0.1768E+05	0.4538E+05
7	0.0000E+00	0.0000E+00	0.0000E+00	0.1768E+05	0.4538E+05
7	0.0000E+00	0.0000E+00	0.0000E+00	0.1725E+05	0.4538E+05
7	0.0000E+00	0.0000E+00	0.0000E+00	0.1688E+05	0.4538E+05
8	0.0000E+00	0.0000E+00	0.0000E+00	0.1688E+05	0.4538E+05

8	0.0000E+00	0.0000E+00	0.0000E+00	0.1655E+05	0.4538E+05
8	0.0000E+00	0.0000E+00	0.0000E+00	0.1626E+05	0.4538E+05
9	0.0000E+00	0.0000E+00	0.0000E+00	0.1626E+05	0.4538E+05
9	0.0000E+00	0.0000E+00	0.0000E+00	0.1600E+05	0.4538E+05
9	0.0000E+00	0.0000E+00	0.0000E+00	0.1577E+05	0.4538E+05
10	0.0000E+00	0.0000E+00	0.0000E+00	0.1577E+05	0.4538E+05
10	0.0000E+00	0.0000E+00	0.0000E+00	0.1557E+05	0.4538E+05
10	0.0000E+00	0.0000E+00	0.0000E+00	0.1539E+05	0.4538E+05

Time = 0.6667E+00
Temperature = 0.5500E+03
Radial traction = 0.0000E+00
Axial strain = -0.2977E-02
Axial stress = 0.3461E-03

RING NO.	RADIUS	STRXX	STRRR	STRTT	W
1	0.0000E+00	-0.5297E+05	-0.1177E+05	-0.1177E+05	0.0000E+00
1	0.3160E+00	-0.5297E+05	-0.1177E+05	-0.1177E+05	-0.6601E-03
1	0.6320E+00	-0.5297E+05	-0.1177E+05	-0.1177E+05	-0.1320E-02
2	0.6320E+00	0.1846E+05	-0.1177E+05	0.1968E+05	-0.1320E-02
2	0.6636E+00	0.2047E+05	-0.1026E+05	0.2021E+05	-0.2101E-02
2	0.6952E+00	0.2234E+05	-0.8863E+04	0.2061E+05	-0.2882E-02
3	0.6952E+00	0.3763E+05	-0.8863E+04	0.2544E+05	-0.2882E-02
3	0.6976E+00	0.3763E+05	-0.8745E+04	0.2532E+05	-0.2900E-02
3	0.7000E+00	0.3763E+05	-0.8628E+04	0.2521E+05	-0.2918E-02
4	0.7000E+00	0.3763E+05	-0.8628E+04	0.2521E+05	-0.2918E-02
4	0.7125E+00	0.3763E+05	-0.8040E+04	0.2462E+05	-0.3010E-02
4	0.7250E+00	0.3763E+05	-0.7482E+04	0.2406E+05	-0.3102E-02
5	0.7250E+00	0.3763E+05	-0.7482E+04	0.2406E+05	-0.3102E-02
5	0.7375E+00	0.3763E+05	-0.6952E+04	0.2353E+05	-0.3192E-02
5	0.7500E+00	0.3763E+05	-0.6448E+04	0.2303E+05	-0.3283E-02
6	0.7500E+00	0.3763E+05	-0.6448E+04	0.2303E+05	-0.3283E-02
6	0.7750E+00	0.3763E+05	-0.5512E+04	0.2209E+05	-0.3461E-02
6	0.8000E+00	0.3763E+05	-0.4663E+04	0.2124E+05	-0.3639E-02
7	0.8000E+00	0.3763E+05	-0.4663E+04	0.2124E+05	-0.3639E-02
7	0.8250E+00	0.3763E+05	-0.3890E+04	0.2047E+05	-0.3813E-02
7	0.8500E+00	0.3763E+05	-0.3184E+04	0.1976E+05	-0.3987E-02
8	0.8500E+00	0.3763E+05	-0.3184E+04	0.1976E+05	-0.3987E-02
8	0.8750E+00	0.3763E+05	-0.2538E+04	0.1912E+05	-0.4158E-02
8	0.9000E+00	0.3763E+05	-0.1945E+04	0.1852E+05	-0.4329E-02
9	0.9000E+00	0.3763E+05	-0.1945E+04	0.1852E+05	-0.4329E-02
9	0.9250E+00	0.3763E+05	-0.1399E+04	0.1798E+05	-0.4497E-02
9	0.9500E+00	0.3763E+05	-0.8956E+03	0.1748E+05	-0.4665E-02
10	0.9500E+00	0.3763E+05	-0.8956E+03	0.1748E+05	-0.4665E-02
10	0.9750E+00	0.3763E+05	-0.4306E+03	0.1701E+05	-0.4830E-02
10	0.1000E+01	0.3763E+05	0.2728E-09	0.1658E+05	-0.4996E-02

RING NO.	EPXXP	EPRRP	EPTTP	STREFF	SIGEFF
1	0.0000E+00	0.0000E+00	0.0000E+00	0.4121E+05	0.1000E+08
1	0.0000E+00	0.0000E+00	0.0000E+00	0.4121E+05	0.1000E+08
1	0.0000E+00	0.0000E+00	0.0000E+00	0.4121E+05	0.1000E+08
2	0.5552E-02	-0.1176E-01	0.6206E-02	0.3086E+05	0.3086E+05
2	0.5327E-02	-0.1048E-01	0.5157E-02	0.3060E+05	0.3060E+05
2	0.5114E-02	-0.9390E-02	0.4276E-02	0.3038E+05	0.3038E+05
3	0.0000E+00	0.0000E+00	0.0000E+00	0.4175E+05	0.5693E+05
3	0.0000E+00	0.0000E+00	0.0000E+00	0.4161E+05	0.5693E+05
3	0.0000E+00	0.0000E+00	0.0000E+00	0.4146E+05	0.5693E+05
4	0.0000E+00	0.0000E+00	0.0000E+00	0.4146E+05	0.5693E+05
4	0.0000E+00	0.0000E+00	0.0000E+00	0.4075E+05	0.5693E+05
4	0.0000E+00	0.0000E+00	0.0000E+00	0.4009E+05	0.5693E+05

5	0.0000E+00	0.0000E+00	0.0000E+00	0.4009E+05	0.5693E+05
5	0.0000E+00	0.0000E+00	0.0000E+00	0.3947E+05	0.5693E+05
5	0.0000E+00	0.0000E+00	0.0000E+00	0.3889E+05	0.5693E+05
6	0.0000E+00	0.0000E+00	0.0000E+00	0.3889E+05	0.5693E+05
6	0.0000E+00	0.0000E+00	0.0000E+00	0.3784E+05	0.5693E+05
6	0.0000E+00	0.0000E+00	0.0000E+00	0.3693E+05	0.5693E+05
7	0.0000E+00	0.0000E+00	0.0000E+00	0.3693E+05	0.5693E+05
7	0.0000E+00	0.0000E+00	0.0000E+00	0.3613E+05	0.5693E+05
7	0.0000E+00	0.0000E+00	0.0000E+00	0.3543E+05	0.5693E+05
8	0.0000E+00	0.0000E+00	0.0000E+00	0.3543E+05	0.5693E+05
8	0.0000E+00	0.0000E+00	0.0000E+00	0.3482E+05	0.5693E+05
8	0.0000E+00	0.0000E+00	0.0000E+00	0.3428E+05	0.5693E+05
9	0.0000E+00	0.0000E+00	0.0000E+00	0.3428E+05	0.5693E+05
9	0.0000E+00	0.0000E+00	0.0000E+00	0.3380E+05	0.5693E+05
9	0.0000E+00	0.0000E+00	0.0000E+00	0.3337E+05	0.5693E+05
10	0.0000E+00	0.0000E+00	0.0000E+00	0.3337E+05	0.5693E+05
10	0.0000E+00	0.0000E+00	0.0000E+00	0.3300E+05	0.5693E+05
10	0.0000E+00	0.0000E+00	0.0000E+00	0.3266E+05	0.5693E+05

Time = 0.1000E+01
Temperature = 0.7500E+02
Radial traction = 0.0000E+00
Axial strain = -0.4393E-02
Axial stress = -0.6867E-02

RING NO.	RADIUS	STRXX	STRRR	STRTT	W
1	0.0000E+00	-0.8384E+05	-0.1959E+05	-0.1959E+05	0.0000E+00
1	0.3160E+00	-0.8384E+05	-0.1959E+05	-0.1959E+05	-0.9506E-03
1	0.6320E+00	-0.8384E+05	-0.1959E+05	-0.1959E+05	-0.1901E-02
2	0.6320E+00	0.3889E+05	-0.1959E+05	0.4187E+05	-0.1901E-02
2	0.6636E+00	0.4036E+05	-0.1670E+05	0.4045E+05	-0.3008E-02
2	0.6952E+00	0.4172E+05	-0.1413E+05	0.3920E+05	-0.4115E-02
3	0.6952E+00	0.5032E+05	-0.1413E+05	0.3664E+05	-0.4115E-02
3	0.6976E+00	0.5050E+05	-0.1396E+05	0.3657E+05	-0.4142E-02
3	0.7000E+00	0.5067E+05	-0.1378E+05	0.3651E+05	-0.4170E-02
4	0.7000E+00	0.5067E+05	-0.1378E+05	0.3651E+05	-0.4170E-02
4	0.7125E+00	0.5155E+05	-0.1290E+05	0.3617E+05	-0.4310E-02
4	0.7250E+00	0.5238E+05	-0.1206E+05	0.3582E+05	-0.4450E-02
5	0.7250E+00	0.5238E+05	-0.1206E+05	0.3582E+05	-0.4450E-02
5	0.7375E+00	0.5317E+05	-0.1125E+05	0.3546E+05	-0.4587E-02
5	0.7500E+00	0.5392E+05	-0.1048E+05	0.3510E+05	-0.4724E-02
6	0.7500E+00	0.5392E+05	-0.1048E+05	0.3510E+05	-0.4724E-02
6	0.7750E+00	0.5531E+05	-0.9017E+04	0.3436E+05	-0.4991E-02
6	0.8000E+00	0.5656E+05	-0.7674E+04	0.3361E+05	-0.5257E-02
7	0.8000E+00	0.5656E+05	-0.7674E+04	0.3361E+05	-0.5257E-02
7	0.8250E+00	0.5768E+05	-0.6434E+04	0.3285E+05	-0.5514E-02
7	0.8500E+00	0.5867E+05	-0.5290E+04	0.3210E+05	-0.5772E-02
8	0.8500E+00	0.5867E+05	-0.5290E+04	0.3210E+05	-0.5772E-02
8	0.8750E+00	0.5956E+05	-0.4232E+04	0.3136E+05	-0.6023E-02
8	0.9000E+00	0.6035E+05	-0.3253E+04	0.3064E+05	-0.6273E-02
9	0.9000E+00	0.6035E+05	-0.3253E+04	0.3064E+05	-0.6273E-02
9	0.9250E+00	0.6105E+05	-0.2347E+04	0.2994E+05	-0.6518E-02
9	0.9500E+00	0.6167E+05	-0.1506E+04	0.2926E+05	-0.6763E-02
10	0.9500E+00	0.6167E+05	-0.1506E+04	0.2926E+05	-0.6763E-02
10	0.9750E+00	0.6222E+05	-0.7256E+03	0.2861E+05	-0.7004E-02
10	0.1000E+01	0.6270E+05	0.2692E-09	0.2799E+05	-0.7245E-02

RING NO.	EPXXP	EPRRP	EPTTP	STREFF	SIGEFF
1	0.0000E+00	0.0000E+00	0.0000E+00	0.6425E+05	0.1000E+08
1	0.0000E+00	0.0000E+00	0.0000E+00	0.6425E+05	0.1000E+08

1	0.0000E+00	0.0000E+00	0.0000E+00	0.6425E+05	0.1000E+08
2	0.7404E-02	-0.1572E-01	0.8320E-02	0.6003E+05	0.6003E+05
2	0.7268E-02	-0.1434E-01	0.7076E-02	0.5711E+05	0.5711E+05
2	0.7141E-02	-0.1315E-01	0.6012E-02	0.5463E+05	0.5463E+05
3	0.7948E-03	-0.1141E-02	0.3464E-03	0.5882E+05	0.5882E+05
3	0.7857E-03	-0.1123E-02	0.3372E-03	0.5874E+05	0.5874E+05
3	0.7767E-03	-0.1105E-02	0.3282E-03	0.5867E+05	0.5867E+05
4	0.7767E-03	-0.1105E-02	0.3282E-03	0.5867E+05	0.5867E+05
4	0.7309E-03	-0.1015E-02	0.2845E-03	0.5830E+05	0.5830E+05
4	0.6869E-03	-0.9325E-03	0.2456E-03	0.5796E+05	0.5796E+05
5	0.6869E-03	-0.9325E-03	0.2456E-03	0.5796E+05	0.5796E+05
5	0.6448E-03	-0.8558E-03	0.2110E-03	0.5764E+05	0.5764E+05
5	0.6044E-03	-0.7848E-03	0.1804E-03	0.5735E+05	0.5735E+05
6	0.6044E-03	-0.7848E-03	0.1804E-03	0.5735E+05	0.5735E+05
6	0.5292E-03	-0.6586E-03	0.1294E-03	0.5683E+05	0.5683E+05
6	0.4608E-03	-0.5509E-03	0.9014E-04	0.5638E+05	0.5638E+05
7	0.4608E-03	-0.5509E-03	0.9014E-04	0.5638E+05	0.5638E+05
7	0.3988E-03	-0.4591E-03	0.6023E-04	0.5599E+05	0.5599E+05
7	0.3430E-03	-0.3808E-03	0.3784E-04	0.5566E+05	0.5566E+05
8	0.3430E-03	-0.3808E-03	0.3784E-04	0.5566E+05	0.5566E+05
8	0.2928E-03	-0.3142E-03	0.2142E-04	0.5537E+05	0.5537E+05
8	0.2478E-03	-0.2575E-03	0.9696E-05	0.5512E+05	0.5512E+05
9	0.2478E-03	-0.2575E-03	0.9696E-05	0.5512E+05	0.5512E+05
9	0.2074E-03	-0.2091E-03	0.1656E-05	0.5490E+05	0.5490E+05
9	0.1713E-03	-0.1678E-03	-0.3529E-05	0.5471E+05	0.5471E+05
10	0.1713E-03	-0.1678E-03	-0.3529E-05	0.5471E+05	0.5471E+05
10	0.1391E-03	-0.1325E-03	-0.6516E-05	0.5455E+05	0.5455E+05
10	0.1102E-03	-0.1024E-03	-0.7827E-05	0.5441E+05	0.5441E+05

8.5 Appendix V: Input File for Example 2

The input file **mccm.data** for the case described in Example 2 is given below. In this example, the parameters NRING, NMT and NTEMP in the INCLUDE statement were set to 10, 2 and 5, respectively. The highlighted text, not to be included in the input deck, identifies the three blocks of the input data, and the fiber and matrix materials used in this example.

Block 1

```
2
1
1 -> Material # 1
371.0
388.200D+09    7.600D+09    7.600D+09
  0.410D+00    0.410D+00    0.450D+00
 -0.680D-06    9.740D-06    9.740D-06
  1.000D+00    1.000D+00    1.000D+00
  1.000D+00    1.000D+00
260.0
388.200D+09    7.600D+09    7.600D+09
  0.410D+00    0.410D+00    0.450D+00
 -0.680D-06    9.740D-06    9.740D-06
  1.000D+00    1.000D+00    1.000D+00
  1.000D+00    1.000D+00
204.4
388.200D+09    7.600D+09    7.600D+09
  0.410D+00    0.410D+00    0.450D+00
 -0.680D-06    9.740D-06    9.740D-06
  1.000D+00    1.000D+00    1.000D+00
  1.000D+00    1.000D+00
148.9
388.200D+09    7.600D+09    7.600D+09
  0.410D+00    0.410D+00    0.450D+00
 -0.680D-06    9.740D-06    9.740D-06
  1.000D+00    1.000D+00    1.000D+00
  1.000D+00    1.000D+00
 21.0
388.200D+09    7.600D+09    7.600D+09
  0.410D+00    0.410D+00    0.450D+00
 -0.680D-06    9.740D-06    9.740D-06
  1.000D+00    1.000D+00    1.000D+00
  1.000D+00    1.000D+00
2 -> Material # 2
371.0
 41.500D+09  41.500D+09  41.500D+09
  0.330D+00   0.330D+00   0.330D+00
 22.500D-06  22.500D-06  22.500D-06
340.000D+06 435.000D+06   1.000D+04
  0.550D+00 300.000D+00
260.0
 58.400D+09  58.400D+09  58.400D+09
  0.330D+00   0.330D+00   0.330D+00
 22.500D-06  22.500D-06  22.500D-06
340.000D+06 435.000D+06   1.000D+04
  1.600D+00 300.000D+00
204.4
 65.700D+09  65.700D+09  65.700D+09
  0.330D+00   0.330D+00   0.330D+00
 22.500D-06  22.500D-06  22.500D-06
340.000D+06 435.000D+06   1.000D+04
```

```
  4.000D+00 300.000D+00
148.9
 69.300D+09   69.300D+09   69.300D+09
  0.330D+00    0.330D+00    0.330D+00
 22.500D-06   22.500D-06   22.500D-06
340.000D+06 435.000D+06    1.000D+04
  7.000D+00 300.000D+00
 21.0
 72.400D+09   72.400D+09   72.400D+09
  0.330D+00    0.330D+00    0.330D+00
 22.500D-06   22.500D-06   22.500D-06
340.000D+06 435.000D+06    1.000D+04
 10.000D+00 300.000D+00
```

Block 2

```
  1   1   0.5477
  2   2   0.6000
  2   2   0.6500
  2   2   0.7000
  2   2   0.7500
  2   2   0.8000
  2   2   0.8500
  2   2   0.9000
  2   2   0.9500
  2   2   1.0000
```

Block 3

```
 10   0.01
 1
 1
    371.0    0.0    0.0
 1  700.0    350
     21.0    0.0    0.0
 35
 0   0   0   1   0
```

8.6 Appendix VI: Output Files for Example 2

The output file **mccm.out** generated by the input file **mccm.data** of Example 2 is given below (see Appendix V for the input file).

```
**************************************************
**                                              **
**    MULTIPLE CONCENCTRIC CYLINDERS MODEL      **
**                                              **
**                   MCCM                       **
**                                              **
**  DETERMINATION OF THE INELASTIC RESPONSE OF  **
** A MULTI-LAYERED COMPOSITE CYLINDER SUBJECTED **
**  TO AXISYMMETRIC THERMO-MECHANICAL LOADING   **
**                                              **
**                    BY                        **
**                                              **
**             TODD O. WILLIAMS                 **
**            MAREK-JERZY PINDERA               **
**                                              **
**           UNIVERSITY OF VIRGINIA             **
**                                              **
**                                              **
** DEVELOPED FOR THE FATIQUE AND FRACTURE BRANCH**
**          NASA-LEWIS RESEARCH CENTER          **
**           UNDER CONTRACT NAS3-26571          **
**     DR. S. M. ARNOLD (CONTRACT MONITOR)      **
**                                              **
**************************************************

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     or for damages resulting from use of this software.
---------------------------------------------------------

            ****** INPUT DATA ECHO ******


               MATERIAL SPECIFICATION


  Number of rings (NRING) = 10
  Number of materials (NMT) =  2
  Number of temperatures at which properties are specified (NTEMP) = 5
```

MATERIAL # 1

TEMPERATURE = 0.3710E+03

0.3882E+12	0.7600E+10	0.7600E+10
0.4100E+00	0.4100E+00	0.4500E+00
-.6800E-06	0.9740E-05	0.9740E-05
0.1000E+01	0.1000E+01	0.1000E+01
0.1000E+01	0.1000E+01	

TEMPERATURE = 0.2600E+03

0.3882E+12	0.7600E+10	0.7600E+10
0.4100E+00	0.4100E+00	0.4500E+00
-.6800E-06	0.9740E-05	0.9740E-05
0.1000E+01	0.1000E+01	0.1000E+01
0.1000E+01	0.1000E+01	

TEMPERATURE = 0.2044E+03

0.3882E+12	0.7600E+10	0.7600E+10
0.4100E+00	0.4100E+00	0.4500E+00
-.6800E-06	0.9740E-05	0.9740E-05
0.1000E+01	0.1000E+01	0.1000E+01
0.1000E+01	0.1000E+01	

TEMPERATURE = 0.1489E+03

0.3882E+12	0.7600E+10	0.7600E+10
0.4100E+00	0.4100E+00	0.4500E+00
-.6800E-06	0.9740E-05	0.9740E-05
0.1000E+01	0.1000E+01	0.1000E+01
0.1000E+01	0.1000E+01	

TEMPERATURE = 0.2100E+02

0.3882E+12	0.7600E+10	0.7600E+10
0.4100E+00	0.4100E+00	0.4500E+00
-.6800E-06	0.9740E-05	0.9740E-05
0.1000E+01	0.1000E+01	0.1000E+01
0.1000E+01	0.1000E+01	

MATERIAL # 2

TEMPERATURE = 0.3710E+03

0.4150E+11	0.4150E+11	0.4150E+11
0.3300E+00	0.3300E+00	0.3300E+00
0.2250E-04	0.2250E-04	0.2250E-04
0.3400E+09	0.4350E+09	0.1000E+05
0.5500E+00	0.3000E+03	

TEMPERATURE = 0.2600E+03

0.5840E+11	0.5840E+11	0.5840E+11
0.3300E+00	0.3300E+00	0.3300E+00
0.2250E-04	0.2250E-04	0.2250E-04
0.3400E+09	0.4350E+09	0.1000E+05
0.1600E+01	0.3000E+03	

TEMPERATURE = 0.2044E+03

0.6570E+11	0.6570E+11	0.6570E+11
0.3300E+00	0.3300E+00	0.3300E+00
0.2250E-04	0.2250E-04	0.2250E-04
0.3400E+09	0.4350E+09	0.1000E+05
0.4000E+01	0.3000E+03	

TEMPERATURE = 0.1489E+03

0.6930E+11	0.6930E+11	0.6930E+11
0.3300E+00	0.3300E+00	0.3300E+00
0.2250E-04	0.2250E-04	0.2250E-04
0.3400E+09	0.4350E+09	0.1000E+05
0.7000E+01	0.3000E+03	

TEMPERATURE = 0.2100E+02

0.7240E+11	0.7240E+11	0.7240E+11
0.3300E+00	0.3300E+00	0.3300E+00
0.2250E-04	0.2250E-04	0.2250E-04
0.3400E+09	0.4350E+09	0.1000E+05
0.1000E+02	0.3000E+03	

CONCENTRIC CYLINDER CONFIGURATION

RING	MATERIAL	CONSTITUTIVE MODEL	OUTER RADIUS
1	1	Elastic	0.547700
2	2	Inelastic	0.600000
3	2	Inelastic	0.650000
4	2	Inelastic	0.700000
5	2	Inelastic	0.750000
6	2	Inelastic	0.800000
7	2	Inelastic	0.850000
8	2	Inelastic	0.900000
9	2	Inelastic	0.950000
10	2	Inelastic	1.000000

Inelastic model (VPFLAG = 2) : Bodner-Partom viscoplasticity

LOADING PARAMETERS

Maximum number of iterations (NITER) = 10
Error tolerance (ERROR) = 0.1000E-01
Total number of loading segments (NCYCLE) = 1

****** OUTPUT RESULTS ******

THE LOADING INFORMATION FOR SEGMENT 1

```
Total time change (DTIME)=  0.7000E+01
Number of loading increments (NTINC) =      350
```

	TEMPERATURE	RAD. TRACT.	AXIAL STRESS
START	0.3710E+03	0.0000E+00	0.0000E+00
END	0.2100E+02	0.0000E+00	0.0000E+00
RATE	-0.5000E+02	0.0000E+00	0.0000E+00
INCREMENT	-0.1000E+01	0.0000E+00	0.0000E+00

The output file **mccmeps.out** generated by the input file **mccm.data** of Example 2 is given below (see Appendix V for the input file).

TIME	TEMP	EPSXXAV	EPSRRAV
0.2000E+00	0.3610E+03	-0.4026E-04	-0.2553E-03
0.4000E+00	0.3510E+03	-0.8318E-04	-0.5109E-03
0.6000E+00	0.3410E+03	-0.1287E-03	-0.7668E-03
0.8000E+00	0.3310E+03	-0.1767E-03	-0.1023E-02
0.1000E+01	0.3210E+03	-0.2273E-03	-0.1279E-02
0.1200E+01	0.3110E+03	-0.2803E-03	-0.1536E-02
0.1400E+01	0.3010E+03	-0.3356E-03	-0.1793E-02
0.1600E+01	0.2910E+03	-0.3933E-03	-0.2049E-02
0.1800E+01	0.2810E+03	-0.4533E-03	-0.2306E-02
0.2000E+01	0.2710E+03	-0.5155E-03	-0.2563E-02
0.2200E+01	0.2610E+03	-0.5799E-03	-0.2820E-02
0.2400E+01	0.2510E+03	-0.6448E-03	-0.3077E-02
0.2600E+01	0.2410E+03	-0.7113E-03	-0.3334E-02
0.2800E+01	0.2310E+03	-0.7797E-03	-0.3590E-02
0.3000E+01	0.2210E+03	-0.8498E-03	-0.3847E-02
0.3200E+01	0.2110E+03	-0.9217E-03	-0.4104E-02
0.3400E+01	0.2010E+03	-0.9926E-03	-0.4360E-02
0.3600E+01	0.1910E+03	-0.1059E-02	-0.4617E-02
0.3800E+01	0.1810E+03	-0.1127E-02	-0.4874E-02
0.4000E+01	0.1710E+03	-0.1195E-02	-0.5130E-02
0.4200E+01	0.1610E+03	-0.1265E-02	-0.5387E-02
0.4400E+01	0.1510E+03	-0.1335E-02	-0.5643E-02
0.4600E+01	0.1410E+03	-0.1398E-02	-0.5900E-02
0.4800E+01	0.1310E+03	-0.1447E-02	-0.6163E-02
0.5000E+01	0.1210E+03	-0.1467E-02	-0.6446E-02
0.5200E+01	0.1110E+03	-0.1487E-02	-0.6729E-02
0.5400E+01	0.1010E+03	-0.1505E-02	-0.7012E-02
0.5600E+01	0.9100E+02	-0.1523E-02	-0.7296E-02
0.5800E+01	0.8100E+02	-0.1540E-02	-0.7580E-02
0.6000E+01	0.7100E+02	-0.1557E-02	-0.7864E-02
0.6200E+01	0.6100E+02	-0.1574E-02	-0.8148E-02
0.6400E+01	0.5100E+02	-0.1589E-02	-0.8432E-02
0.6600E+01	0.4100E+02	-0.1604E-02	-0.8717E-02
0.6800E+01	0.3100E+02	-0.1619E-02	-0.9001E-02
0.7000E+01	0.2100E+02	-0.1633E-02	-0.9286E-02

8.7 Appendix VII: Input File for Example 3

The input file **mccm.data** for the case described in Example 3 is given below. In this example, the parameters NRING, NMT and NTEMP in the INCLUDE statement were set to 10, 1 and 2, respectively. The highlighted text, not to be included in the input deck, identifies the three blocks of the input data, and the matrix material used in this example.

Block 1

```
  3
  1
  1 -> Material # 1
 294.0
 126.273000D+00 126.273000D+00 126.273000D+00
   0.330000D+00   0.330000D+00   0.330000D+00
   1.000000D-06   1.000000D-06   1.000000D-06
   1.385000D-08   1.151465D+05   3.860000D+00  4.000
   1.075620D-07   4.000000D+04   8.110000D+02  0.040
   3.942461D+00   7.083925D+00   0.846070D+00
 290.0
 126.273000D+00 126.273000D+00 126.273000D+00
   0.330000D+00   0.330000D+00   0.330000D+00
   1.000000D-06   1.000000D-06   1.000000D-06
   1.385000D-08   1.151465D+05   3.860000D+00  4.000
   1.075620D-07   4.000000D+04   8.110000D+02  0.040
   3.942461D+00   7.083925D+00   0.846070D+00
```

Block 2

```
    1  1  0.000001
    1  2  0.000100
    1  2  0.100000
    1  2  0.200000
    1  2  0.300000
    1  2  0.400000
    1  2  0.500000
    1  2  0.700000
    1  2  0.850000
    1  2  1.000000
```

Block 3

```
   10     0.001
   1
   2
      294.0     0.0      0.0
   2    0.000125    2000
      294.0     0.0      0.01
   20
   1  0  0  0  1
```

8.8 Appendix VIII: Output Files for Example 3

The output file **mccm.out** generated by the input file **mccm.data** of Example 3 is given below (see Appendix VII for the input file).

```
**************************************************
**                                              **
**     MULTIPLE CONCENCTRIC CYLINDERS MODEL     **
**                                              **
**                    MCCM                      **
**                                              **
**   DETERMINATION OF THE INELASTIC RESPONSE OF **
** A MULTI-LAYERED COMPOSITE CYLINDER SUBJECTED **
**   TO AXISYMMETRIC THERMO-MECHANICAL LOADING  **
**                                              **
**                     BY                       **
**                                              **
**              TODD O. WILLIAMS                **
**             MAREK-JERZY PINDERA              **
**                                              **
**            UNIVERSITY OF VIRGINIA            **
**                                              **
**                                              **
** DEVELOPED FOR THE FATIQUE AND FRACTURE BRANCH**
**          NASA-LEWIS RESEARCH CENTER          **
**           UNDER CONTRACT NAS3-26571          **
**      DR. S. M. ARNOLD (CONTRACT MONITOR)     **
**                                              **
**************************************************

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     or for damages resulting from use of this software.
-------------------------------------------------------


            ****** INPUT DATA ECHO ******


               MATERIAL SPECIFICATION


  Number of rings (NRING) = 10
  Number of materials (NMT) =  1
  Number of temperatures at which properties are specified (NTEMP) = 2
```

MATERIAL # 1

TEMPERATURE = 0.2940E+03

0.1263E+06	0.1263E+06	0.1263E+06	
0.3300E+00	0.3300E+00	0.3300E+00	
0.1000E-05	0.1000E-05	0.1000E-05	
0.1385E-07	0.1151E+06	0.3860E+01	0.4000E+01
0.1076E-06	0.4000E+05	0.8110E+03	0.4000E-01
0.3942E+01	0.7084E+01	0.8461E+00	

TEMPERATURE = 0.2900E+03

0.1263E+06	0.1263E+06	0.1263E+06	
0.3300E+00	0.3300E+00	0.3300E+00	
0.1000E-05	0.1000E-05	0.1000E-05	
0.1385E-07	0.1151E+06	0.3860E+01	0.4000E+01
0.1076E-06	0.4000E+05	0.8110E+03	0.4000E-01
0.3942E+01	0.7084E+01	0.8461E+00	

CONCENTRIC CYLINDER CONFIGURATION

RING	MATERIAL	CONSTITUTIVE MODEL	OUTER RADIUS
1	1	Elastic	0.000001
2	1	Inelastic	0.000100
3	1	Inelastic	0.100000
4	1	Inelastic	0.200000
5	1	Inelastic	0.300000
6	1	Inelastic	0.400000
7	1	Inelastic	0.500000
8	1	Inelastic	0.700000
9	1	Inelastic	0.850000
10	1	Inelastic	1.000000

Inelastic model (VPFLAG = 3) : Robinson viscoplasticity

LOADING PARAMETERS

Maximum number of iterations (NITER) = 10
Error tolerance (ERROR) = 0.1000E-02
Total number of loading segments (NCYCLE) = 1

****** OUTPUT RESULTS ******

THE LOADING INFORMATION FOR SEGMENT 1

```
Total time change (DTIME)=  0.1250E+01
Number of loading increments (NTINC) =     2000
```

	TEMPERATURE	RAD. TRACT.	AXIAL STRAIN
START	0.2940E+03	0.0000E+00	0.0000E+00
END	0.2940E+03	0.0000E+00	0.1000E-01
RATE	0.0000E+00	0.0000E+00	0.8000E-02
INCREMENT	0.0000E+00	0.0000E+00	0.5000E-05

```
Time =  0.1250E+01
Temperature =  0.2940E+03
Radial traction =  0.0000E+00
Axial strain =  0.1000E-01
Axial stress =  0.1380E+03
```

RING NO.	RADIUS	STRXX	STRRR	STRTT	W
1	0.0000E+00	0.1434E+04	0.2599E+03	0.2599E+03	0.0000E+00
1	0.5000E-06	0.1434E+04	0.2599E+03	0.2599E+03	-0.1185E-08
1	0.1000E-05	0.1434E+04	0.2599E+03	0.2599E+03	-0.2369E-08
2	0.1000E-05	0.4057E+03	0.2599E+03	0.2838E+03	-0.2369E-08
2	0.5050E-04	0.1385E+03	0.4938E+00	0.5068E+00	-0.2418E-06
2	0.1000E-03	0.1385E+03	0.4986E+00	0.5019E+00	-0.4812E-06
3	0.1000E-03	0.1385E+03	0.4986E+00	0.5019E+00	-0.4812E-06
3	0.5005E-01	0.1380E+03	-0.3234E-06	0.3290E-06	-0.2410E-03
3	0.1000E+00	0.1380E+03	-0.8083E-07	0.8260E-07	-0.4814E-03
4	0.1000E+00	0.1380E+03	-0.8083E-07	0.8260E-07	-0.4814E-03
4	0.1500E+00	0.1380E+03	-0.3550E-07	0.3714E-07	-0.7221E-03
4	0.2000E+00	0.1380E+03	-0.1961E-07	0.2124E-07	-0.9629E-03
5	0.2000E+00	0.1380E+03	-0.1961E-07	0.2124E-07	-0.9629E-03
5	0.2500E+00	0.1380E+03	-0.1226E-07	0.1389E-07	-0.1204E-02
5	0.3000E+00	0.1380E+03	-0.8266E-08	0.9894E-08	-0.1444E-02
6	0.3000E+00	0.1380E+03	-0.8264E-08	0.9896E-08	-0.1444E-02
6	0.3500E+00	0.1380E+03	-0.5856E-08	0.7486E-08	-0.1685E-02
6	0.4000E+00	0.1380E+03	-0.4292E-08	0.5923E-08	-0.1926E-02
7	0.4000E+00	0.1380E+03	-0.4291E-08	0.5923E-08	-0.1926E-02
7	0.4500E+00	0.1380E+03	-0.3220E-08	0.4851E-08	-0.2166E-02
7	0.5000E+00	0.1380E+03	-0.2453E-08	0.4085E-08	-0.2407E-02
8	0.5000E+00	0.1380E+03	-0.2452E-08	0.4085E-08	-0.2407E-02
8	0.6000E+00	0.1380E+03	-0.1453E-08	0.3087E-08	-0.2889E-02
8	0.7000E+00	0.1380E+03	-0.8510E-09	0.2484E-08	-0.3370E-02
9	0.7000E+00	0.1380E+03	-0.8526E-09	0.2483E-08	-0.3370E-02
9	0.7750E+00	0.1380E+03	-0.5456E-09	0.2176E-08	-0.3731E-02
9	0.8500E+00	0.1380E+03	-0.3161E-09	0.1946E-08	-0.4092E-02
10	0.8500E+00	0.1380E+03	-0.3156E-09	0.1947E-08	-0.4092E-02
10	0.9250E+00	0.1380E+03	-0.1396E-09	0.1771E-08	-0.4453E-02
10	0.1000E+01	0.1380E+03	-0.1648E-11	0.1633E-08	-0.4814E-02

RING NO.	EPXXP	EPRRP	EPTTP	STREFF	SIGEFF
1	0.0000E+00	0.0000E+00	0.0000E+00	0.1174E+04	0.0000E+00
1	0.0000E+00	0.0000E+00	0.0000E+00	0.1174E+04	0.0000E+00
1	0.0000E+00	0.0000E+00	0.0000E+00	0.1174E+04	0.0000E+00
2	0.8208E-02	-0.5331E-02	-0.2877E-02	0.1354E+03	0.0000E+00
2	0.8906E-02	-0.4454E-02	-0.4452E-02	0.1380E+03	0.0000E+00
2	0.8906E-02	-0.4453E-02	-0.4453E-02	0.1380E+03	0.0000E+00
3	0.8906E-02	-0.4453E-02	-0.4453E-02	0.1380E+03	0.0000E+00

3	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
3	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
4	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
4	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
4	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
5	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
5	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
5	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
6	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
6	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
6	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
7	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
7	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
7	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
8	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
8	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
8	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
9	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
9	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
9	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
10	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
10	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00
10	0.8907E-02	-0.4454E-02	-0.4454E-02	0.1380E+03	0.0000E+00

The output file **mccmeps.out** generated by the input file **mccm.data** of Example 3 is given below (see Appendix VII for the input file).

TIME	SIGXXAV	EPSXXAV
0.6250E-01	0.6110E+02	0.5000E-03
0.1250E+00	0.8469E+02	0.1000E-02
0.1875E+00	0.9318E+02	0.1500E-02
0.2500E+00	0.9899E+02	0.2000E-02
0.3125E+00	0.1036E+03	0.2500E-02
0.3750E+00	0.1075E+03	0.3000E-02
0.4375E+00	0.1108E+03	0.3500E-02
0.5000E+00	0.1139E+03	0.4000E-02
0.5625E+00	0.1173E+03	0.4500E-02
0.6250E+00	0.1192E+03	0.5000E-02
0.6875E+00	0.1216E+03	0.5500E-02
0.7500E+00	0.1237E+03	0.6000E-02
0.8125E+00	0.1258E+03	0.6500E-02
0.8750E+00	0.1284E+03	0.7000E-02
0.9375E+00	0.1297E+03	0.7500E-02
0.1000E+01	0.1315E+03	0.8000E-02
0.1062E+01	0.1332E+03	0.8500E-02
0.1125E+01	0.1349E+03	0.9000E-02
0.1188E+01	0.1365E+03	0.9500E-02
0.1250E+01	0.1380E+03	0.1000E-01

8.9 Appendix IX: Input File for Example 4

The input file **mccm.data** for the case described in Example 4 is given below. In this example, the parameters NRING, NMT and NTEMP in the INCLUDE statement were set to 10, 2 and 2, respectively. The highlighted text, not to be included in the input deck, identifies the three blocks of the input data, and the fiber and matrix materials used in this example.

Block 1

```
4
9
1
1 -> Material # 1
121.0
 400.000D+00        400.000D+00        400.000D+00
     0.200D+00          0.200D+00          0.200D+00
     1.000D-06          1.000D-06          1.000D-06
     1.000D+00
     1.000D+00
     1.000D+00
     1.000D+00
     1.000D+00
     1.000D+00
     1.000D+00
     1.000D+00
     1.000D+00
120.0
 400.000D+00        400.000D+00        400.000D+00
     0.200D+00          0.200D+00          0.200D+00
     1.000D-06          1.000D-06          1.000D-06
     1.000D+00
     1.000D+00
     1.000D+00
     1.000D+00
     1.000D+00
     1.000D+00
     1.000D+00
     1.000D+00
     1.000D+00
2 -> Material # 2
121.0
  70.300D+00         70.300D+00         70.300D+00
     0.340D+00          0.340D+00          0.340D+00
     1.000D-06          1.000D-06          1.000D-06
    -0.6250D-01
     1.4140D+00
    -0.1232D-02
     0.7990D-02
    -1.2980D-02
     4.5800E-02
     0.3400D+00
     0.2070D+00
     0.3100D+00
120.0
  70.300D+00         70.300D+00         70.300D+00
     0.340D+00          0.340D+00          0.340D+00
     1.000D-06          1.000D-06          1.000D-06
    -0.6250D-01
     1.4140D+00
    -0.1232D-02
```

```
   0.7990D-02
  -1.2980D-02
   4.5800E-02
   0.3400D+00
   0.2070D+00
   0.3100D+00
```

Block 2

```
  1  1  0.5000
  2  2  0.5500
  2  2  0.6000
  2  2  0.6500
  2  2  0.7000
  2  2  0.7500
  2  2  0.8000
  2  2  0.8500
  2  2  0.9000
  2  2  1.0000
```

Block 2

```
10  0.01
2
1
   121.0    0.0    0.000
1  0.00001    500
   121.0    0.0    0.74957
1
0  0  1  0  1
1  8.00000  8000
   121.0    0.0    0.74957
80
0  0  1  0  1
```

8.10 Appendix X: Output Files for Example 4

The output file **mccm.out** generated by the input file **mccm.data** of Example 4 is given below (see Appendix IX for the input file).

```
**************************************************
**                                              **
**    MULTIPLE CONCENCTRIC CYLINDERS MODEL      **
**                                              **
**                    MCCM                      **
**                                              **
**   DETERMINATION OF THE INELASTIC RESPONSE OF **
** A MULTI-LAYERED COMPOSITE CYLINDER SUBJECTED **
**   TO AXISYMMETRIC THERMO-MECHANICAL LOADING  **
**                                              **
**                     BY                       **
**                                              **
**              TODD O. WILLIAMS                **
**             MAREK-JERZY PINDERA              **
**                                              **
**            UNIVERSITY OF VIRGINIA            **
**                                              **
**                                              **
** DEVELOPED FOR THE FATIQUE AND FRACTURE BRANCH**
**          NASA-LEWIS RESEARCH CENTER          **
**           UNDER CONTRACT NAS3-26571          **
**      DR. S. M. ARNOLD (CONTRACT MONITOR)     **
**                                              **
**************************************************


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                             or

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-------------------------------------------------------


            ****** INPUT DATA ECHO ******


               MATERIAL SPECIFICATION


  Number of rings (NRING) = 10
  Number of materials (NMT) =  2
  Number of temperatures at which properties are specified (NTEMP) = 2
```

MATERIAL # 1

TEMPERATURE = 0.1210E+03

```
0.4000E+03  0.4000E+03  0.4000E+03
0.2000E+00  0.2000E+00  0.2000E+00
0.1000E-05  0.1000E-05  0.1000E-05
0.1000E+01
0.1000E+01
0.1000E+01
0.1000E+01
0.1000E+01
0.1000E+01
0.1000E+01
0.1000E+01
0.1000E+01
```

TEMPERATURE = 0.1200E+03

```
0.4000E+03  0.4000E+03  0.4000E+03
0.2000E+00  0.2000E+00  0.2000E+00
0.1000E-05  0.1000E-05  0.1000E-05
0.1000E+01
0.1000E+01
0.1000E+01
0.1000E+01
0.1000E+01
0.1000E+01
0.1000E+01
0.1000E+01
0.1000E+01
```

MATERIAL # 2

TEMPERATURE = 0.1210E+03

```
0.7030E+02  0.7030E+02  0.7030E+02
0.3400E+00  0.3400E+00  0.3400E+00
0.1000E-05  0.1000E-05  0.1000E-05
-.6250E-01
0.1414E+01
-.1232E-02
0.7990E-02
-.1298E-01
0.4580E-01
0.3400E+00
0.2070E+00
0.3100E+00
```

TEMPERATURE = 0.1200E+03

```
0.7030E+02  0.7030E+02  0.7030E+02
0.3400E+00  0.3400E+00  0.3400E+00
0.1000E-05  0.1000E-05  0.1000E-05
-.6250E-01
0.1414E+01
-.1232E-02
0.7990E-02
-.1298E-01
0.4580E-01
0.3400E+00
```

0.2070E+00
0.3100E+00

CONCENTRIC CYLINDER CONFIGURATION

RING	MATERIAL	CONSTITUTIVE MODEL	OUTER RADIUS
1	1	Elastic	0.500000
2	2	Inelastic	0.550000
3	2	Inelastic	0.600000
4	2	Inelastic	0.650000
5	2	Inelastic	0.700000
6	2	Inelastic	0.750000
7	2	Inelastic	0.800000
8	2	Inelastic	0.850000
9	2	Inelastic	0.900000
10	2	Inelastic	1.000000

Inelastic model (VPFLAG = 4) : User defined model

LOADING PARAMETERS

Maximum number of iterations (NITER) = 10
Error tolerance (ERROR) = 0.1000E-01
Total number of loading segments (NCYCLE) = 2

****** OUTPUT RESULTS ******

THE LOADING INFORMATION FOR SEGMENT 1

Total time change (DTIME)= 0.1000E-04
Number of loading increments (NTINC) = 500

	TEMPERATURE	RAD. TRACT.	AXIAL STRESS
START	0.1210E+03	0.0000E+00	0.0000E+00
END	0.1210E+03	0.0000E+00	0.7496E+00
RATE	0.0000E+00	0.0000E+00	0.2355E+06
INCREMENT	0.0000E+00	0.0000E+00	0.4710E-02

THE LOADING INFORMATION FOR SEGMENT 2

Total time change (DTIME)= 0.8000E+01
Number of loading increments (NTINC) = 8000

	TEMPERATURE	RAD. TRACT.	AXIAL STRESS
START	0.1210E+03	0.0000E+00	0.7496E+00
END	0.1210E+03	0.0000E+00	0.7496E+00
RATE	0.0000E+00	0.0000E+00	0.3331E-15
INCREMENT	0.0000E+00	0.0000E+00	0.6661E-18

The output file **mccmeps.out** generated by the input file **mccm.data** of Example 4 is given below (see Appendix IX for the input file).

TIME	SIGXXAV	EPSXXAV
0.1000E-04	0.7496E+00	0.4898E-02
0.1000E+00	0.7496E+00	0.4991E-02
0.2000E+00	0.7496E+00	0.5020E-02
0.3000E+00	0.7496E+00	0.5040E-02
0.4000E+00	0.7496E+00	0.5056E-02
0.5000E+00	0.7496E+00	0.5068E-02
0.6000E+00	0.7496E+00	0.5078E-02
0.7000E+00	0.7496E+00	0.5087E-02
0.8000E+00	0.7496E+00	0.5095E-02
0.9000E+00	0.7496E+00	0.5102E-02
0.1000E+01	0.7496E+00	0.5109E-02
0.1100E+01	0.7496E+00	0.5114E-02
0.1200E+01	0.7496E+00	0.5120E-02
0.1300E+01	0.7496E+00	0.5125E-02
0.1400E+01	0.7496E+00	0.5129E-02
0.1500E+01	0.7496E+00	0.5133E-02
0.1600E+01	0.7496E+00	0.5137E-02
0.1700E+01	0.7496E+00	0.5141E-02
0.1800E+01	0.7496E+00	0.5145E-02
0.1900E+01	0.7496E+00	0.5148E-02
0.2000E+01	0.7496E+00	0.5152E-02
0.2100E+01	0.7496E+00	0.5155E-02
0.2200E+01	0.7496E+00	0.5158E-02
0.2300E+01	0.7496E+00	0.5161E-02
0.2400E+01	0.7496E+00	0.5164E-02
0.2500E+01	0.7496E+00	0.5166E-02
0.2600E+01	0.7496E+00	0.5169E-02
0.2700E+01	0.7496E+00	0.5172E-02
0.2800E+01	0.7496E+00	0.5174E-02
0.2900E+01	0.7496E+00	0.5177E-02
0.3000E+01	0.7496E+00	0.5179E-02
0.3100E+01	0.7496E+00	0.5181E-02
0.3200E+01	0.7496E+00	0.5184E-02
0.3300E+01	0.7496E+00	0.5186E-02
0.3400E+01	0.7496E+00	0.5188E-02
0.3500E+01	0.7496E+00	0.5190E-02
0.3600E+01	0.7496E+00	0.5192E-02
0.3700E+01	0.7496E+00	0.5194E-02
0.3800E+01	0.7496E+00	0.5196E-02
0.3900E+01	0.7496E+00	0.5198E-02
0.4000E+01	0.7496E+00	0.5200E-02
0.4100E+01	0.7496E+00	0.5202E-02
0.4200E+01	0.7496E+00	0.5204E-02
0.4300E+01	0.7496E+00	0.5206E-02
0.4400E+01	0.7496E+00	0.5208E-02
0.4500E+01	0.7496E+00	0.5209E-02

0.4600E+01	0.7496E+00	0.5211E-02
0.4700E+01	0.7496E+00	0.5213E-02
0.4800E+01	0.7496E+00	0.5214E-02
0.4900E+01	0.7496E+00	0.5216E-02
0.5000E+01	0.7496E+00	0.5218E-02
0.5100E+01	0.7496E+00	0.5219E-02
0.5200E+01	0.7496E+00	0.5221E-02
0.5300E+01	0.7496E+00	0.5222E-02
0.5400E+01	0.7496E+00	0.5224E-02
0.5500E+01	0.7496E+00	0.5225E-02
0.5600E+01	0.7496E+00	0.5227E-02
0.5700E+01	0.7496E+00	0.5228E-02
0.5800E+01	0.7496E+00	0.5230E-02
0.5900E+01	0.7496E+00	0.5231E-02
0.6000E+01	0.7496E+00	0.5232E-02
0.6100E+01	0.7496E+00	0.5234E-02
0.6200E+01	0.7496E+00	0.5235E-02
0.6300E+01	0.7496E+00	0.5236E-02
0.6400E+01	0.7496E+00	0.5238E-02
0.6500E+01	0.7496E+00	0.5239E-02
0.6600E+01	0.7496E+00	0.5240E-02
0.6700E+01	0.7496E+00	0.5242E-02
0.6800E+01	0.7496E+00	0.5243E-02
0.6900E+01	0.7496E+00	0.5244E-02
0.7000E+01	0.7496E+00	0.5245E-02
0.7100E+01	0.7496E+00	0.5246E-02
0.7200E+01	0.7496E+00	0.5248E-02
0.7300E+01	0.7496E+00	0.5249E-02
0.7400E+01	0.7496E+00	0.5250E-02
0.7500E+01	0.7496E+00	0.5251E-02
0.7600E+01	0.7496E+00	0.5252E-02
0.7700E+01	0.7496E+00	0.5253E-02
0.7800E+01	0.7496E+00	0.5254E-02
0.7900E+01	0.7496E+00	0.5256E-02
0.8000E+01	0.7496E+00	0.5257E-02

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
April 1994

3. REPORT TYPE AND DATES COVERED
Final Contractor Report

4. TITLE AND SUBTITLE
Multiple Concentric Cylinder Model (MCCM) User's Guide

5. FUNDING NUMBERS
WU-505-63-12
C-NAS3-26571

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of Virginia
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8. PERFORMING ORGANIZATION REPORT NUMBER
E-8689

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135-3191

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA CR-195299

11. SUPPLEMENTARY NOTES
Project Manager, Steve M. Arnold, Structures Division, organization code 5220, (216) 433-3334.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified - Unlimited
Subject Categories 24 and 39

12b. DISTRIBUTION CODE

13. ABSTRACT ***(Maximum 200 words)***

A user's guide for the computer program mccm.f is presented in this report. The program is based on a recently developed solution methodology for the inelastic response of an arbitrarily layered, concentric cylinder assemblage under thermo-mechanical loading which is used to model the axisymmetric behavior of unidirectional metal-matrix composites in the presence of various microstructural details. These details include the layered morphology of certain types of ceramic fibers, as well as multiple, fiber/matrix interfacial layers recently proposed as a means of reducing fabrication-induced, and in-service, residual stresses. The computer code allows efficient characterization and evaluation of new fibers and/or new coating systems on existing fibers with a minimum of effort, taking into account inelastic and temperature-dependent properties and different morphologies of the fiber and the interfacial region. It also facilitates efficient design of engineered interfaces for unidirectional metal matrix composites.

14. SUBJECT TERMS
Metal-matrix composites; Thermoelastic; Inelastic; Concentric cylinder; Computer program; Residual stress

15. NUMBER OF PAGES
80

16. PRICE CODE
A05

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	

National Aeronautics and
Space Administration
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