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NASA
Technical
Memorandum

NASA TM-108453

ROOT-SUM-SQUARE STRUCTURAL STRENGTH
VERIFICATION APPROACH

By Henry M. Lee

Structures and Dynamics Laboratory
Science and Engineering Directorate

April 1994

NASA
National Aeronautics and
Space Administration

George C. Marshall Space Flight Center

REPORT DOCUMENTATION PAGE

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	April 1994	Technical Memorandum

4. TITLE AND SUBTITLE

Root-Sum-Square Structural Strength Verification Approach

5. FUNDING NUMBERS

6. AUTHOR(S)

H.M. Lee

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

National Aeronautics and Space Administration
Washington, DC 20546

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

NASA TM-108453

11. SUPPLEMENTARY NOTES

Prepared by Structures and Dynamics Laboratory, Science and Engineering Directorate

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified—Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

Utilizing a proposed fixture design or some variation thereof, this report presents a verification approach to strength test space flight payload components, electronics boxes, mechanisms, lines, fittings, etc., which traditionally do not lend themselves to classical static loading. The fixture, through use of ordered Euler rotation angles derived herein, can be mounted on existing vibration shakers and can provide an innovative method of applying single axis flight load vectors. The versatile fixture effectively loads protoflight or prototype components in all three axes simultaneously by use of a sinusoidal burst of desired magnitude at less than one-third the first resonant frequency. Cost savings along with improved hardware confidence are shown to be the potential, with the end product being an efficient way to verify experiment hardware for both random vibration and strength.

14. SUBJECT TERMS

root-sum-square (RSS), strength test verification, ordered Euler rotations

15. NUMBER OF PAGES

48

16. PRICE CODE

NTIS

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	Unlimited

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ROOT-SUM-SQUARE STRUCTURAL STRENGTH VERIFICATION APPROACH

INTRODUCTION

At present the vast majority of payload experiments, electronic boxes, control packages, mechanisms, etc., designed for space flight are random vibration tested one axis at a time (x,y,z). In the majority of these cases, no strength verification testing is done because these items generally do not easily lend themselves to classical static testing. Such testing, when feasible, involves applying static loadings to the proposed flight hardware in all three axes at the same time. Unless a centrifuge facility is available, there is not another clear engineering solution. For this reason, hardware in the categories outlined above are generally strength verified by analysis only. This fact has somewhat forced many of the NASA field Centers to utilize what is commonly referred to as the controversial "untested factors of safety." At MSFC these factors are 1.25 and 2.00 for yield and ultimate, respectively. They are even larger at some other Centers. Strength testing along with analysis has always been the preferred method of verification for space flight hardware, not only from a requirements standpoint but also because it is a more secure engineering approach. If tested factors could be used at MSFC, then yield and ultimate would be 1.10 and 1.4, respectively. Several other NASA Centers have partially circumvented the problem by attempting to strength verify all such flight hardware on the vibration shake table, but again they are unable to place loads on their components in all three axes simultaneously. They must settle for a form of verification involving either greater than limit load random vibration testing or low frequency sinusoid testing one axis at a time (x,y,z). A more exact, yet practical solution is certainly a desirable goal.

The approach proposed in this report involves manufacturing what will be called a "root-sum-square (RSS) loads fixture" (fig. 1). The versatile fixture would be built and mounted on existing vertically and horizontally oriented vibration shake tables. It would provide the capability of accomplishing ordered Euler angle rotations such that a single expected flight load vector can be applied to protoflight or prototype hardware in all three axes with one test. Thus, the term RSS, which refers to root-sum-square as defined by the equation

$$\text{RSS acceleration} = \sqrt{A_X^2 + A_Y^2 + A_Z^2} \; ,$$

is used to describe the proposed fixture. At MSFC the RSS acceleration could be the summation of single axis random and the low frequency transient loads as described in NASA technical memorandum TM-86538,[1] or as dictated by program requirements. This fixture could be used prior to the normal random vibration verification testing of the National Space Transportation System (NSTS) or the expendable launch vehicle (ELV) payloads, electronic boxes, experiments, control packages, mechanisms, lines, and valves. It will be shown that by calculating a typical single RSS vector flight load set, applying a test factor of ≥ 1.20 (per NSTS 14046 requirement) for flight hardware and ≥ 1.40 for protoflight, and subjecting the component to a sinusoidal burst of that magnitude at $\leq 1/3$ the first resonant frequency, the component could be considered strength verified by both analysis and test. If desired, strain gauges could be secured to key areas on the structure so that correlation with analytical models is accomplished. In addition, historically MSFC has found that it always learns more about the hardware when it is subjected to testing rather than analysis only.
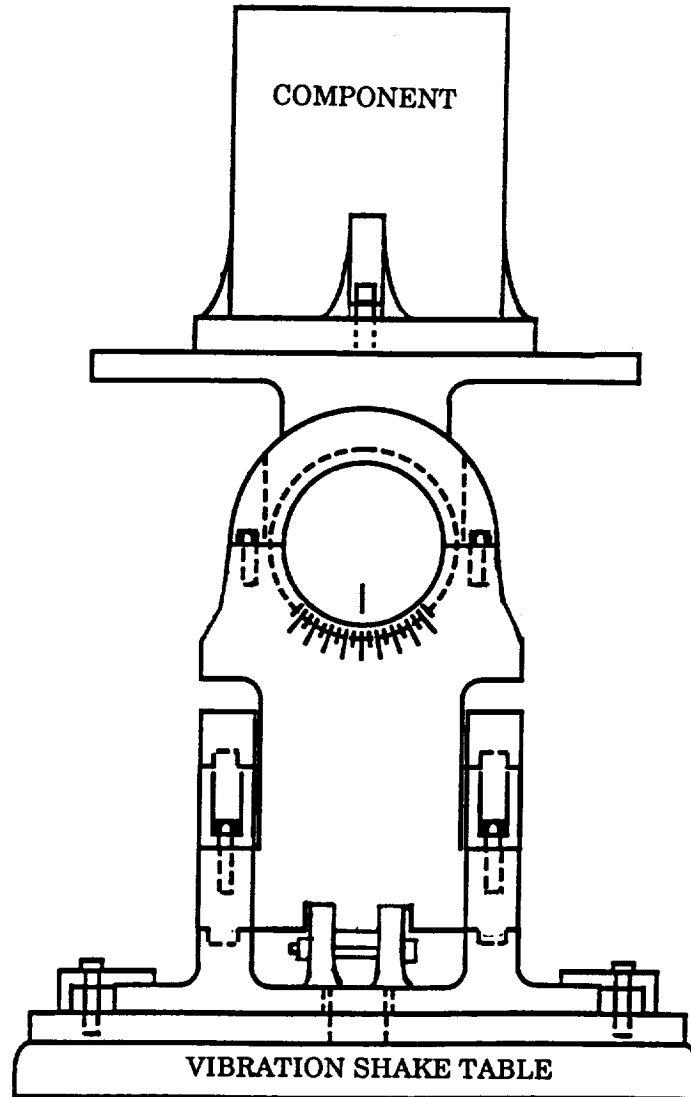
COMPONENT

VIBRATION SHAKE TABLE

Figure 1. RSS loads fixture concept.

POTENTIAL FIXTURE DESIGN

Figures 2 through 11 depict engineering sketches of how such a device might look. Additional views of the fixture are shown in appendix A. The primary capability feature is that a component mounted to this fixture could be rotated about all three axes until the desired single acceleration vector is established. This fact alone makes it possible to statically load the component in all three axes simultaneously with one test.
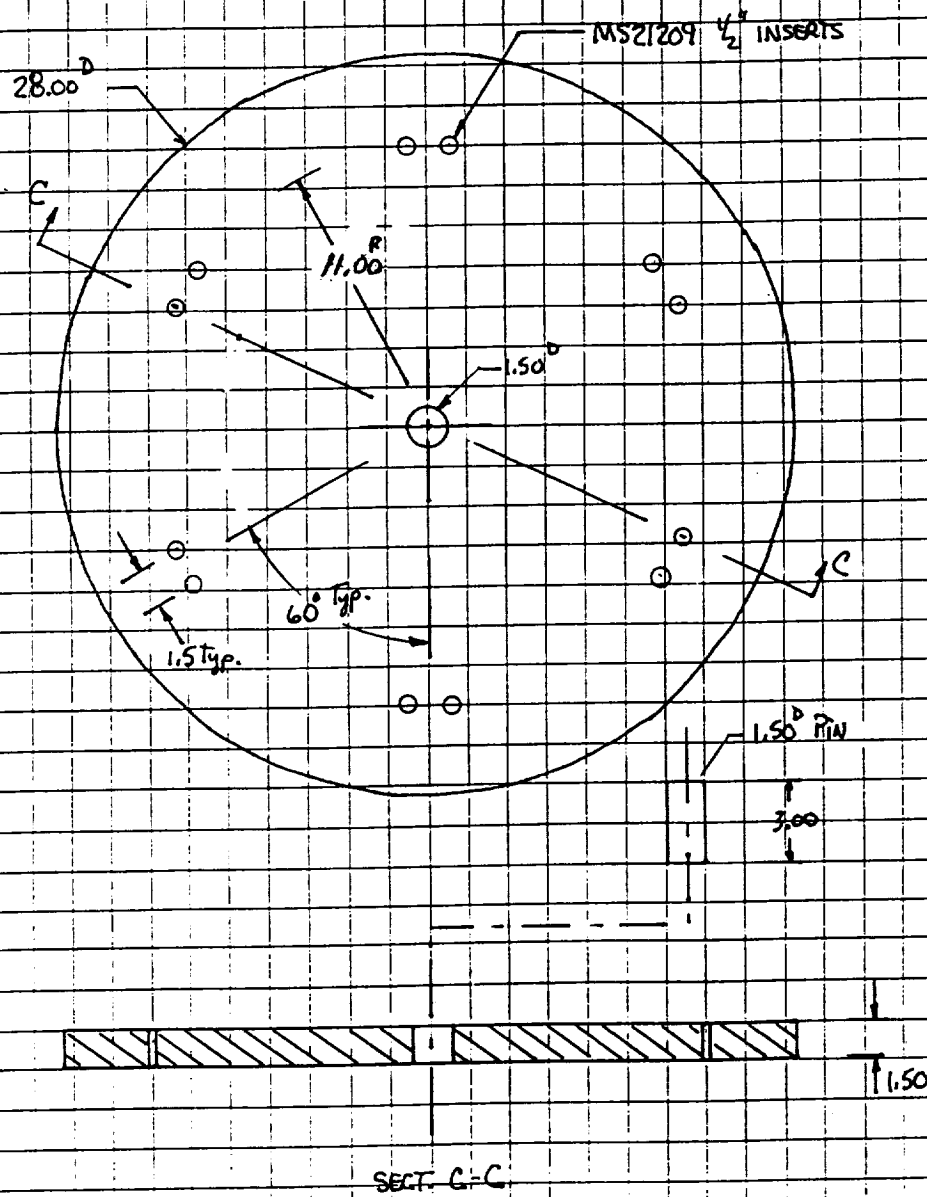
2

FRONT VIEW　　　　　　　**SIDE VIEW**

Component Mount Fixture

Bushing Block

Shear Bars

θxz Position Fixture

Bushing Block

Clamps

Shear Bars

θyz Position Fixture

θxy Position Base

Z

X　　　　　　　　　　Y

Figure 2. Exploded view.

3

Θ_{xy} POSITION BASE

28.00 D

MS21209 ½" INSERTS

C

11.00 R

1.50 D

60° Typ.

1.5 Typ.

C

1.50 D PIN

3.00

1.50

SECT. C-C

Figure 3. Θ_{xy} position base.

4

TITLE__RSS__FIXTURE__CONCEPT_____

From Page No.__

Θ_{xy} POSITION CLAMP

.5625 D

.75

1.50

3.00

2.00

2.00

NAS145B ½" bolts

1.00

2.50

(Θ_{yz} Position Fixture)

(Θ_{xy} Position Base)

1.50

To Page No._____

| Witnessed & Understood by me, | Date | Invented by HENRY M. LEE | Date 8/93 | Tol. ±.010" |

Figure 4. Θ_{xy} position clamp.

TITLE _RSS FIXTURE CONCEPT_____

Θ_{YZ} _POSITION FIXTURE_

MS21209 "$\frac{1}{2}$" INSERTS

1.25

8.00

2.50

2.00

1.50

1.75

1.50

.875

1.75

3.25^R

3.00^R

1.25

11.00

1.00

1.75^D
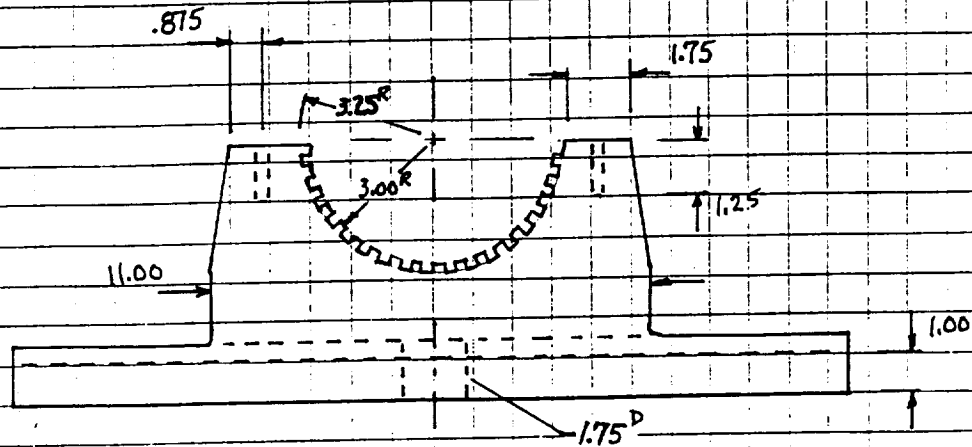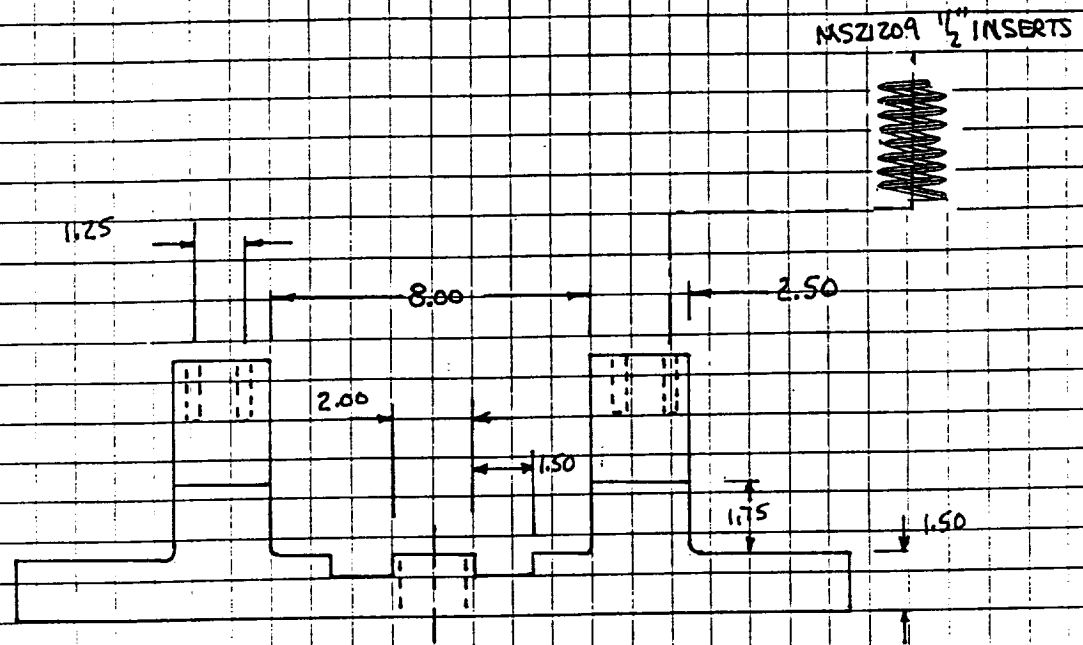
| Witnessed & Understood by me. | Date | Invented by HENRY M. LEE | Date 8/93 | ToL. $\pm$.010" |

Figure 5. Θ_{yz} position fixture.

6

TITLE RSS FIXTURE CONCEPT

Θ_{YZ} POSITION FIXTURE

21.00 D

1.50 w x .5 d slots

Witnessed & Understood by me.	Date	Invented by HENRY M. LEE	Date 8/93	ToL ±.0/0"

Figure 6. Θ_{yz} position fixture—plan view.

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BUSHING BLOCK
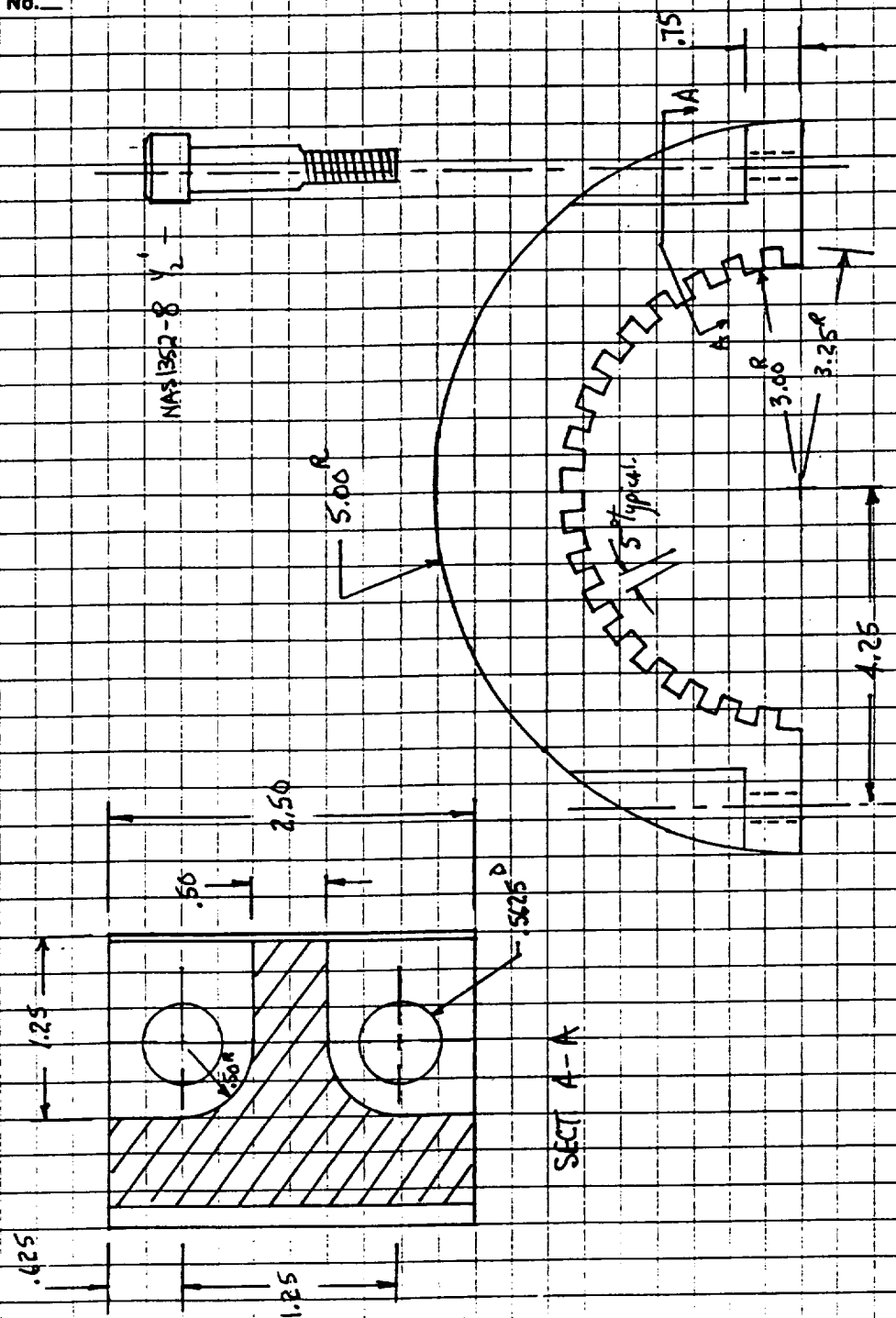
NAS 1352-8 Y₂

5.00 R

.75

A

R.3

3.00 R

3.25 R

5° typical

4.25

2.50

.50

1.25

.5625 D

.50 R

SECT A-A

.425

1.31

To Page No._____

Figure 7. Bushing block.

8

From Page No.___

Θ_{yz} & Θ_{xz} POSITION SHEAR BAR

NAS1958 ½" bolt

3.10

1.48

5.00

2.50

1.50

3.00

1.50

.5625

1.50

11.00

To Page No.____

| Witnessed & Understood by me. | Date | Invented by HENRY M. LEE | Date 8/93 | ToL. ±.010" |

Figure 8. Θ_{yz} and Θ_{xz} position shear bar.
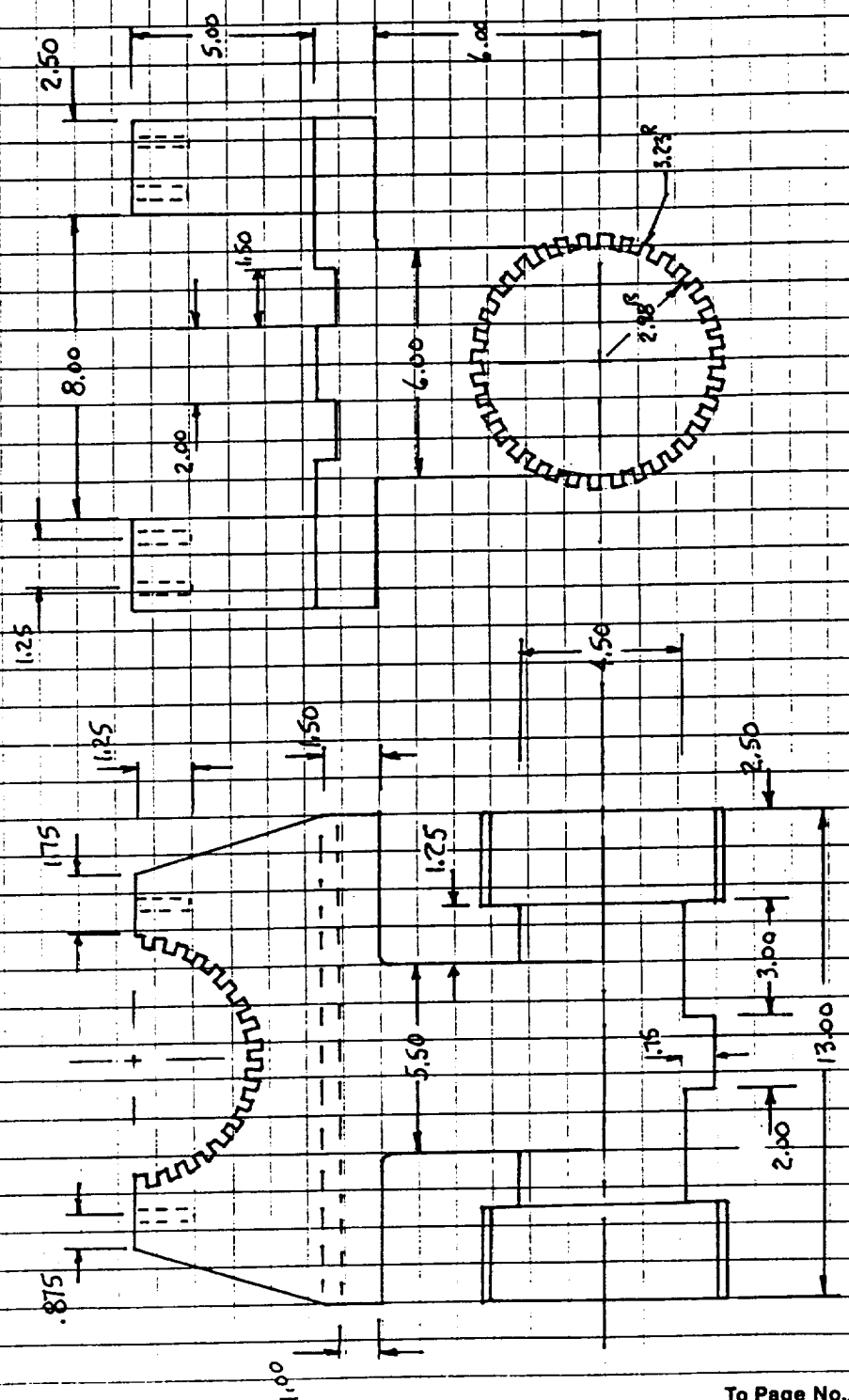
9

Figure 9. Θ_{xz} position fixture.

TITLE RSS FIXTURE CONCEPT

Project No._____
Book No._____

From Page No.___

Θ_{xz} POSITION FIXTURE

13.00

13.00

1.50w x .5d Slots.

To Page No.____

| Witnessed & Understood by me. | Date | Invented by HENRY M. LEE | Date 8/93 | ToL. ±.010" |

Figure 10. Θ_{xz} position fixture—plan view.

Figure 11. Component mount fixture.

In order to effectively design any test fixture, requirements must first be instituted. The following design requirements were placed on the potential test tool:

1. The experiment or component to be strength tested must weigh no more than 100 lb.

2. The maximum RSS acceleration load factor must be no more than 50 G's.

3. The experiment or component to be tested must have a center of gravity no more than 32 inches above the shaker attach plate.

4. The factor of safety against ultimate shall be 5.0.

5. The factor of safety against yield shall be 3.0.

In order to prove that the design is a structurally feasible one, a stress analysis is accomplished utilizing the design requirements as input. Such an analysis has been performed and can be reviewed in appendix B of this report. Once the design actually becomes hardware, it must also be subjected to some form of test verification. One effective way to do this is as follows:

1. Static proof test the fixture to two times the design requirements while it is in the fully extended position ($\Theta_{xy} = \Theta_{yz} = \Theta_{xz} = 0.0$).

2. Perform a sine sweep from 5 to 50 Hz at 1 octave per minute to 0.25 G's in each axis with a 100-lb rigid mass mounted to the fixture while in its fully extended position.

3. Sine burst test the fixture and rigid mass at $1/3$ the first resonant frequency. This would be done with the rigid mass and fully extended fixture mounted to a horizontal shake table.

4. Sine burst test as above but in several other rotated positions.

ORDERED EULER ROTATION FORMULAS

The calculation of an RSS single load vector is certainly an elementary accomplishment, however, just knowing the magnitude and direction of this vector is not enough. One must determine the three ordered angle rotations it will take to place the hardware in the right location such that the component to be strength tested is loaded as desired. The following pages effectively show how these rotations determine the position of any point on the component. The three transformations allow us to develop three equations. These equations solve for the ordered rotation angles ($\Theta_{xy}, \Theta_{yz}, \Theta_{xz}$) in terms of the desired final apparent angles ($\Psi_{xy}, \Psi_{yz}, \Psi_{xz}$) which were set when the RSS vector is calculated.

I. ROTATION TRANSFORMATION [X to Y about Z]:

Z, Z'

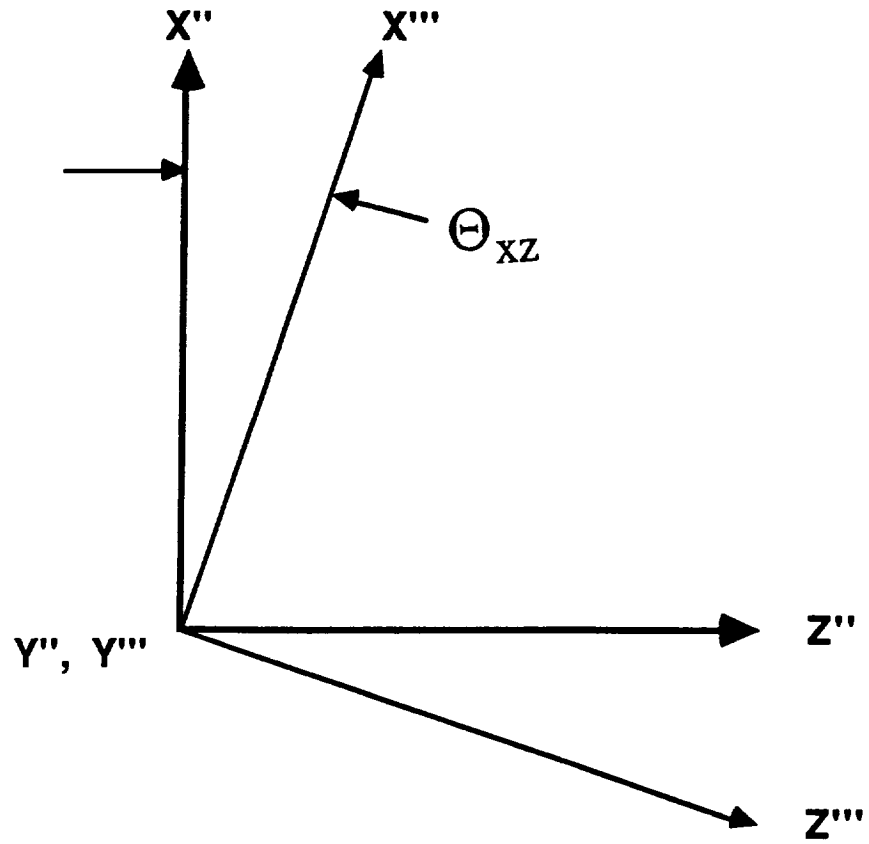**Y'**

Y

Θ_{xy}

X

X'
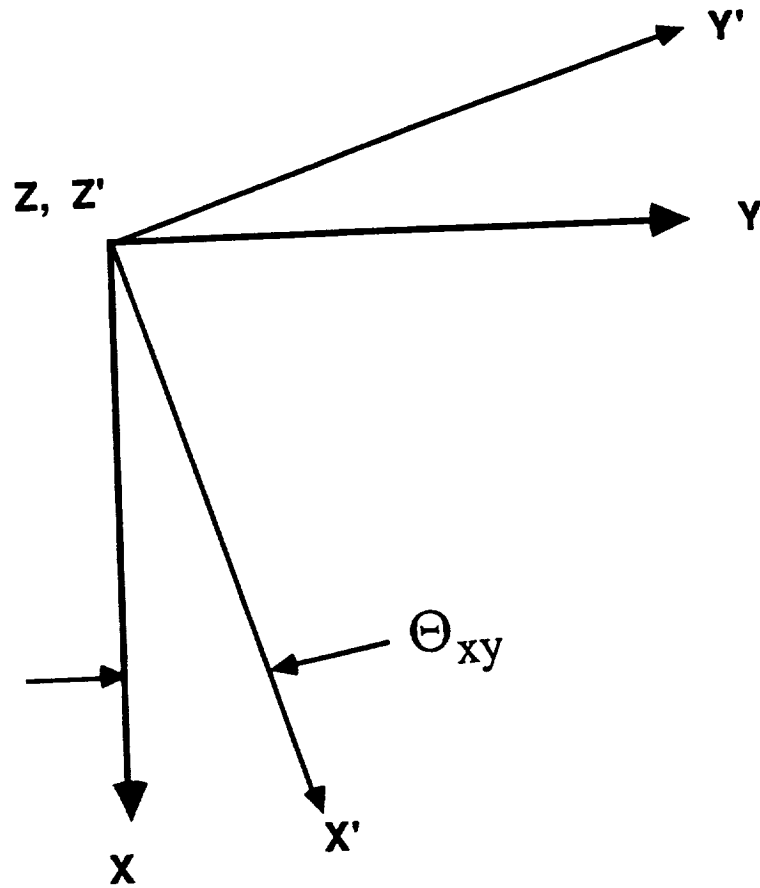
$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{bmatrix} \cos \Theta_{xy} & -\sin \Theta_{xy} & 0 \\ \sin \Theta_{xy} & \cos \Theta_{xy} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} X' \\ Y' \\ Z' \end{Bmatrix} .$$

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II. ROTATION TRANSFORMATION: [Y' to Z' about Y']
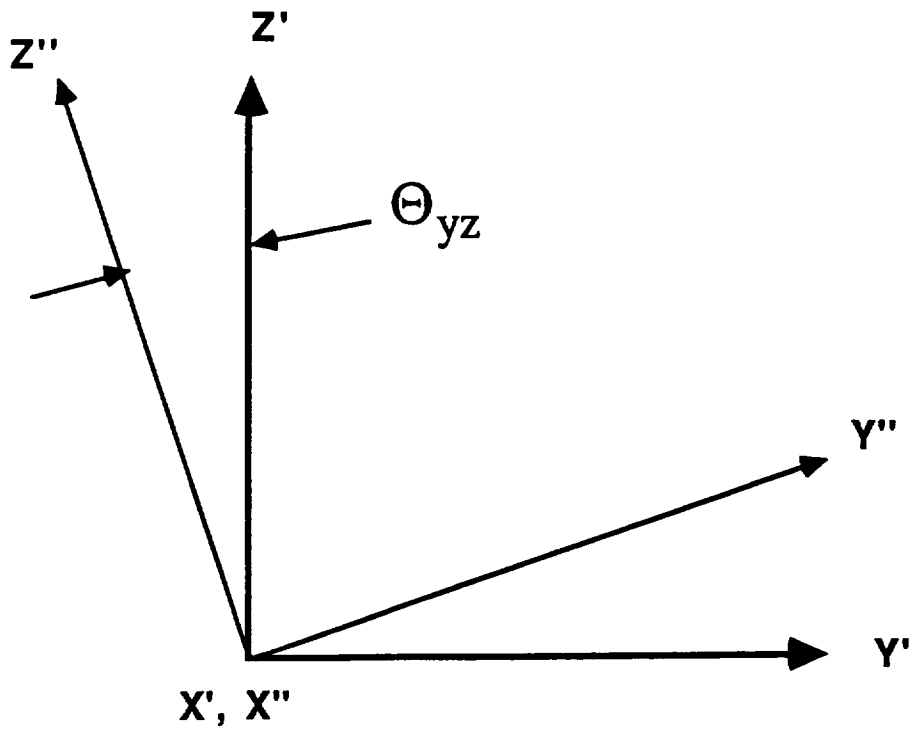
$$\begin{Bmatrix} X' \\ Y' \\ Z' \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Theta_{yz} & -\sin\Theta_{yz} \\ 0 & \sin\Theta_{yz} & \cos\Theta_{yz} \end{bmatrix} \begin{Bmatrix} X'' \\ Y'' \\ Z'' \end{Bmatrix} .$$

III. TRANSFORMATION ROTATION: [X'' to Z'' about Y'']

$$\begin{Bmatrix} X'' \\ Y'' \\ Z'' \end{Bmatrix} = \begin{bmatrix} \cos \Theta_{xz} & 0 & -\sin \Theta_{xz} \\ 0 & 0 & 0 \\ \sin \Theta_{xz} & 0 & \cos \Theta_{xz} \end{bmatrix} \begin{Bmatrix} X''' \\ Y''' \\ Z''' \end{Bmatrix} \cdot$$

IV. MATRIX MULTIPLICATION:

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{bmatrix} \text{trans.} \\ \text{matrix} \\ \text{I.} \end{bmatrix} \begin{bmatrix} \text{trans.} \\ \text{matrix} \\ \text{II.} \end{bmatrix} \begin{bmatrix} \text{trans.} \\ \text{matrix} \\ \text{III.} \end{bmatrix} \begin{Bmatrix} X''' \\ Y''' \\ Z''' \end{Bmatrix} \ ,$$

which becomes

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = [T] \begin{Bmatrix} X''' \\ Y''' \\ Z''' \end{Bmatrix} .$$

This transformation matrix $[T]$ relates the initial coordinates of a point (X, Y, Z) to the final coordinates (X''', Y''', Z''') after three consecutive ordered rotations $\Theta_{xy}, \Theta_{yz}, \Theta_{xz}$.

V. DETERMINATION OF ORDERED ROTATION ANGLES:

A. $\sin \Theta_{xy} \cos \Theta_{xz} - \cos \Theta_{xy} \sin \Theta_{yz} \sin \Theta_{xz} = (\tan \Psi_{xy})(\cos \Theta_{xy} \cos \Theta_{xz} + \sin \Theta_{xy} \sin \Theta_{yz} \sin \Theta_{xz})$,

B. $+\sin \Theta_{xy} \sin \Theta_{xz} + \cos \Theta_{xy} \sin \Theta_{yz} \cos \Theta_{xz} = (\tan \Psi_{yz})(\cos \Theta_{yz} \cos \Theta_{xz})$,

C. $+\cos \Theta_{xy} \sin \Theta_{xz} - \sin \Theta_{xy} \sin \Theta_{yz} \cos \Theta_{xz} = (\tan \Psi_{xz})(\cos \Theta_{yz} \cos \Theta_{xz})$.

Three equations and three unknowns (Θ_{xy}, Θ_{yz}, Θ_{xz}) which can be solved through trigonometric substitution:

$$\tan \Theta_{xy} = \frac{(\tan \Psi_{xz})(\tan \Psi_{yz}) + (\tan \Psi_{xy})(\tan \Psi_{yz})^2 + (\tan \Psi_{xy})}{(\tan \Psi_{xz})^2 + (\tan \Psi_{xy})(\tan \Psi_{yz})(\tan \Psi_{xz}) + 1} \ , \qquad \text{Equation 1}$$

$$\tan \Theta_{yz} = \frac{(\tan \Psi_{yz}) - (\tan \Psi_{xz})(\tan \Theta_{xy})}{(\tan \Theta_{xy})(\sin \Theta_{xy}) + (\cos \Theta_{xy})} \ , \qquad \text{Equation 2}$$

$$\tan \Theta_{xz} = \frac{(\tan \Psi_{xz})(\cos \Theta_{yz})}{(\cos \Theta_{xy})} + (\tan \Theta_{xy})(\sin \Theta_{yz}) \ , \qquad \text{Equation 3}$$

where:

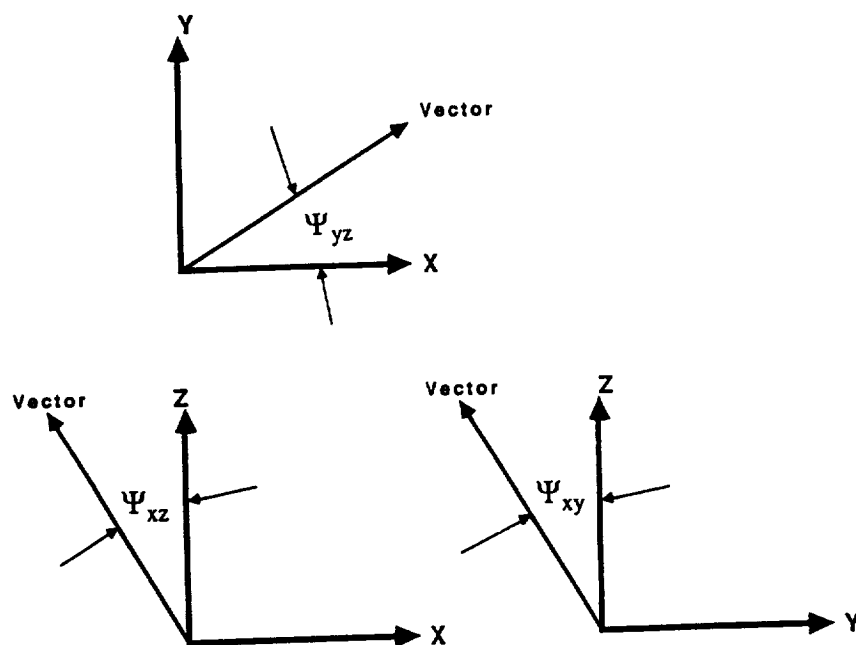Ψ_{xy} = apparent angle that acceleration vector makes in X–Y plane

Ψ_{yz} = apparent angle that acceleration vector makes in Y–Z plane

Ψ_{xz} = apparent angle that acceleration vector makes in X–Z plane

Θ_{xy} = unknown Euler angle – rotation about Z axis

Θ_{yz} = unknown Euler angle – rotation about X' axis

Θ_{xz} = unknown Euler angle – rotation about Y'' axis .

CALCULATION OF ACTUAL LOAD SET

The load set generated in this section assumes that the MSFC single-axis random philosophy is being used to establish strength loadings for flight hardware. This philosophy is less conservative than some other accepted approaches, but has been proven to be realistic for NSTS hardware in the launch phase. The drawback to the philosophy is that a large number of load cases will be generated for the strength analysis. Other more conservative ways that produce fewer load cases may become more attractive in the future when strength testing is feasible as with this fixture.

RSS SINGLE VECTOR FLIGHT LOAD SET

AXIS	QUASI-STATIC LOAD	RANDOM LOAD
X	$\pm S_1$	$\pm R_1$
Y	$\pm S_2$	$\pm R_2$
Z	$\pm S_3$	$\pm R_3$

There are 24 possible load cases using this standard set of loads as described in NASA TM-86538 "Design and Verification Guidelines for Vibroacoustic and Transient Environments." Utilizing a vibration shake table reduces this to 12 cases, since 2 cases are accomplished at the same time (i.e.; $[(S_1+R_1), S_2, S_3]$ and $[-(S_1+R_1), -S_2, -S_3]$).

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Another way of potentially reducing the load cases in the set would be to perform a pretest analysis and determine the most critical loads for the specific hardware. For typical experiments and electronic boxes, only four load cases would generally be needed. This is because the peak loading will occur on the four corners. The setup time in any case would be minimal and should take no more than 1 to 2 days to do 4 to 12 cases.

Assuming that a vertical shake table is utilized for these tests, the apparent angles (Ψ_{xy}, Ψ_{yz}, Ψ_{xz}) must be altered to align the acceleration vector with the shaker axis (Z). That means that the rotation vector, as shown in figure 12, is different from the acceleration vector. This is easily done by putting a minus ($-$) sign in front of the X and Y coordinates. The generic set of load cases for any hardware is shown below.

GENERIC LOAD SET

LOAD CASE	X	Y	Z
1	$+(S_1+R_1)$	$+S_2$	$+S_3$
2	$+(S_1+R_1)$	$+S_2$	$-S_3$
3	$+(S_1+R_1)$	$-S_2$	$+S_3$
4	$+(S_1+R_1)$	$-S_2$	$-S_3$
5	$+S_1$	$+(S_2+R_2)$	$+S_3$
6	$+S_1$	$+(S_2+R_2)$	$-S_3$
7	$+S_1$	$-(S_2+R_2)$	$+S_3$
8	$+S_1$	$-(S_2+R_2)$	$-S_3$
9	$+S_1$	$+S_2$	$+(S_3+R_3)$
10	$+S_1$	$+S_2$	$-(S_3+R_3)$
11	$+S_1$	$-S_2$	$+(S_3+R_3)$
12	$+S_1$	$-S_2$	$-(S_3+R_3)$

where:

	CASES 1–4	CASES 5–8	CASES 9–12
$\Psi_{xy} =$	$\tan^{-1}\left(\dfrac{S_2}{(S_1+R_1)}\right)$	$\tan^{-1}\left(\dfrac{(S_2+R_2)}{S_1}\right)$	$\tan^{-1}\left(\dfrac{(S_2)}{S_1}\right)$
$\Psi_{yz} =$	$\tan^{-1}\left(\dfrac{(S_2)}{S_3}\right)$	$\tan^{-1}\left(\dfrac{(S_2+R_2)}{S_3}\right)$	$\tan^{-1}\left(\dfrac{S_2}{(S_3+R_3)}\right)$
$\Psi_{xz} =$	$\tan^{-1}\left(\dfrac{(S_1+R_1)}{S_3}\right)$	$\tan^{-1}\left(\dfrac{(S_1)}{S_3}\right)$	$\tan^{-1}\left(\dfrac{S_1}{(S_3+R_3)}\right)$.
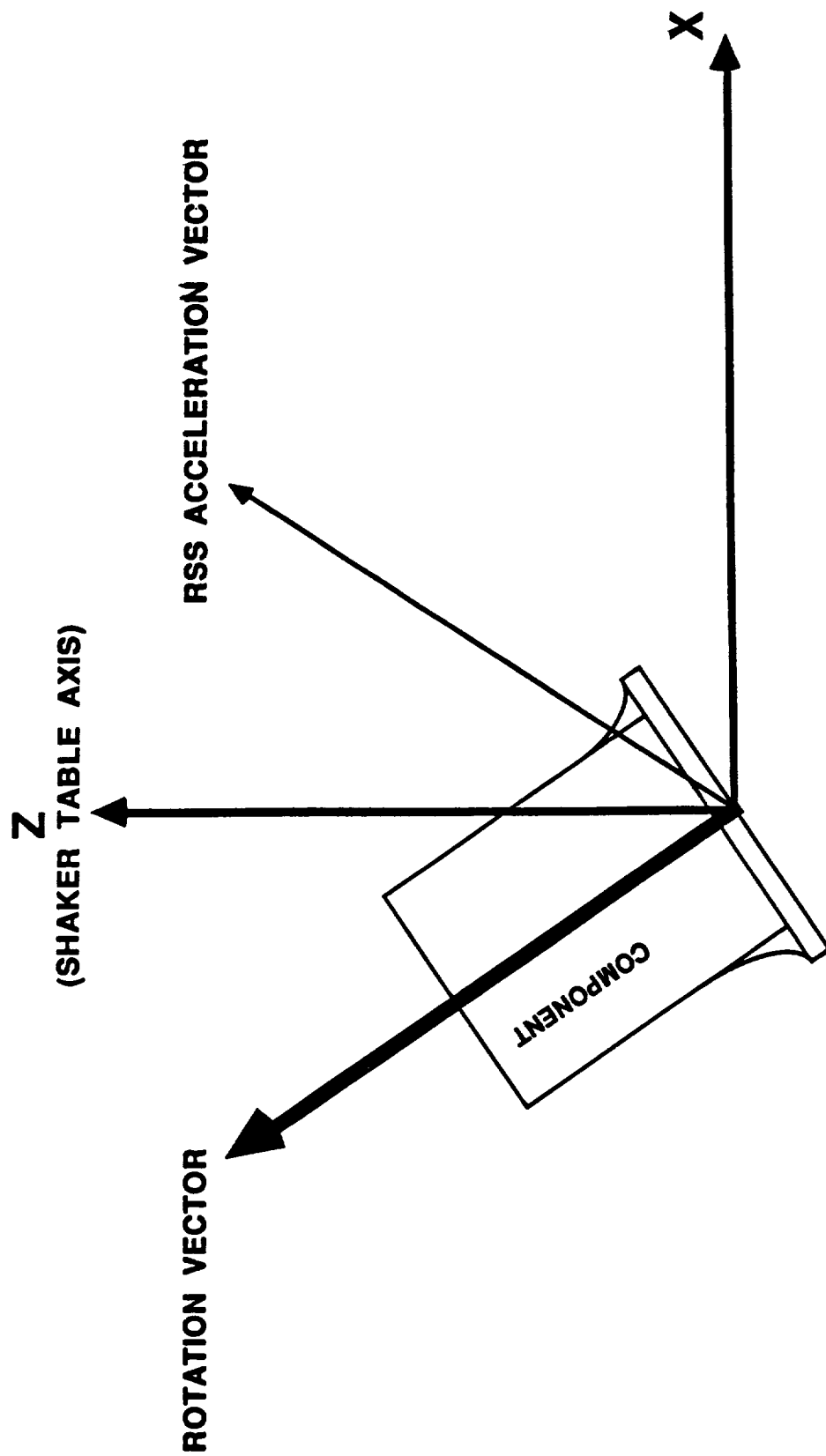
RSS Acceleration Vector is developed first, by knowing the accelerations in each axis $\left[\text{i.e.;} = \sqrt{(S_1+R_1)^2+S_2^2+S_3^2}\right]$. Since the shaker table axis is the Z axis, the component to be tested must be aligned with the Rotation Vector.

Figure 12. Hardware orientation.

CONCLUSIONS

This report has sought to present a possible scenario by which space flight hardware and associated components, which are not easily static strength tested, can be tested. The approach includes the design of a fixture which can be attached to vibration shake tables and can be rotated about all three axes such that any acceleration load vector can be achieved. Also outlined are the formulas necessary to calculate the magnitude of the three ordered Euler angles used to align the test component such that a single RSS load factor may be applied. In addition, a sample load set is generated for a typical component to be test verified per the MSFC philosophy. The fixture, too, is shown to be verified by analysis, and a suggested approach to test verification is also presented.

Though it would not be a major impact, this approach could reduce the weight required to develop experiments, since the ultimate factor of safety would be reduced from 2.0 to 1.4. For primary load carrying members, it could be a 30-percent weight savings which translates to material costs and overall payload weight reduction. Secondly, because of intercenter controversy over whether experiment hardware should require strength testing in order to be fully verified, and whether the so called "untested factors of safety" should be used, a significant amount of engineering time could be saved through a consistent utilization of tested factors of safety with this approach. The Rotator Chair developed by Johnson Space Center (JSC) for the IML-1 Spacelab mission absorbed some 300 engineering man-hours before resolution of a random vibration test verification approach. Other NASA Centers such as Goddard Space Flight Center (GSFC) and the Jet Propulsion Laboratory (JPL) are continually faced with the problem of strength test verification for such components. As stated previously, they have resorted to methods such as random vibration over-testing (one axis at a time), below resonance sine burst over-tests (one axis at a time), and, in a few cases, expensive centrifuge testing. The proposed methodology would bring a *unification* to the entire agency on how strength testing is accomplished for hardware that is difficult to apply static loadings to such as electronic boxes, experiments, control packages, mechanisms, lines, and valves, etc. The real beauty of the proposed RSS loads fixture testing philosophy is that random vibration testing is already a verification requirement for most of this hardware. The RSS loads fixture will attach to existing vibration shake tables, and testing would be accomplished during planned random testing with almost no schedule impact. The end product would be an efficient method for verifying experiment hardware for both random vibration and strength.

The specific cost savings from an implementation of this approach might include:

- **REDUCED MATERIAL REQUIREMENTS**
  Lower ultimate factor of safety can potentially reduce the quantity and cost of materials by 10 to 30 percent since reduced sizes for individual piece parts will be required to distribute generally lower loads (i.e., 2.0 × expected flight loads versus 1.4 × expected flight loads).

- **LOWER OVERALL COMPONENT WEIGHTS**
  Lower ultimate factor of safety also implies the possibility of lower overall component weights. This fact does not imply a decrease in safety, since the vast majority of components are designed with redundant load paths (i.e.; multiple fasteners, etc.). The proposed approach now makes the development of nonmetallic hardware more appealing since such composites require strength testing to additionally verify material processes and effects of possible damage. Lower weights always translate into savings because of the cost of placing them into orbit.

- **IMPROVED HARDWARE CONFIDENCE**
  Verifying hardware through testing has always provided the agency with a greater measure of confidence in performance and safety. Though it is somewhat intangible, the cost savings accomplished through testing has been evident in every program NASA has undertaken. With this in mind, the question should be, "what will be the cost if a component fails structurally in flight ?"

  Reliability will be increased through test verification. The cost savings would probably be intangible. Experience with more complex primary structure indicates that only 30 percent of such achieved test loads successfully (i.e.; failure before factor of safety × design limit load was reached).

- **UNIFICATION ACROSS THE AGENCY**
  Unification across the agency would be the end result of this approach. It would greatly reduce the engineering costs associated with the continuous debates over development of rationale for no test criteria and disagreement over the most correct method to strength test flight hardware for verification. An example of the magnitude of hardware involved is seen in the fact that there are some 40 to 50 experiments with associated electrical and fluid interfaces for a typical Spacelab module mission. A pallet mission can easily have 15 to 20 major experiments (telescopes, sensors, etc.) with numerous electronic boxes, recorders, power supplies, antennae, etc. in support of each. The Advanced X-Ray Astrophysics Facility-S (AXAF-S) payload had scheduled some 50 such components mounted on the dewar and avionics panels. Space station also promises to be configured such that great numbers of components will be involved in operations as well as experiments.

  As involvement of multiple NASA Centers on payloads has increased, interaction has surfaced many historical differences in philosophical approaches to structural verification.

- **APPLICABLE TO CONTRACTORS**
  This proposed, consistent engineering approach would be directly applicable to all contractors. Any program contractor currently accomplishing random vibration testing on its hardware could perform the strength testing. There would, in turn, be no disagreement on the strength verification process.

  As in the previous item, a unification throughout the contractor world would prevent costly disagreements over verification. Again disagreement costs are related directly to engineering labor costs.

22

REFERENCE

1. NASA MSFC Component Analysis Branch: "Design and Verification Guidelines for Vibroacoustic and Transient Environments." NASA TM-86538, March 1986.

APPENDIX A

ADDITIONAL FIXTURE VIEWS

θxy POSITION BASE
Side

θxy POSITION BASE

CLAMP

θ yz POSITION FIXTURE

SHAKER TABLE

26
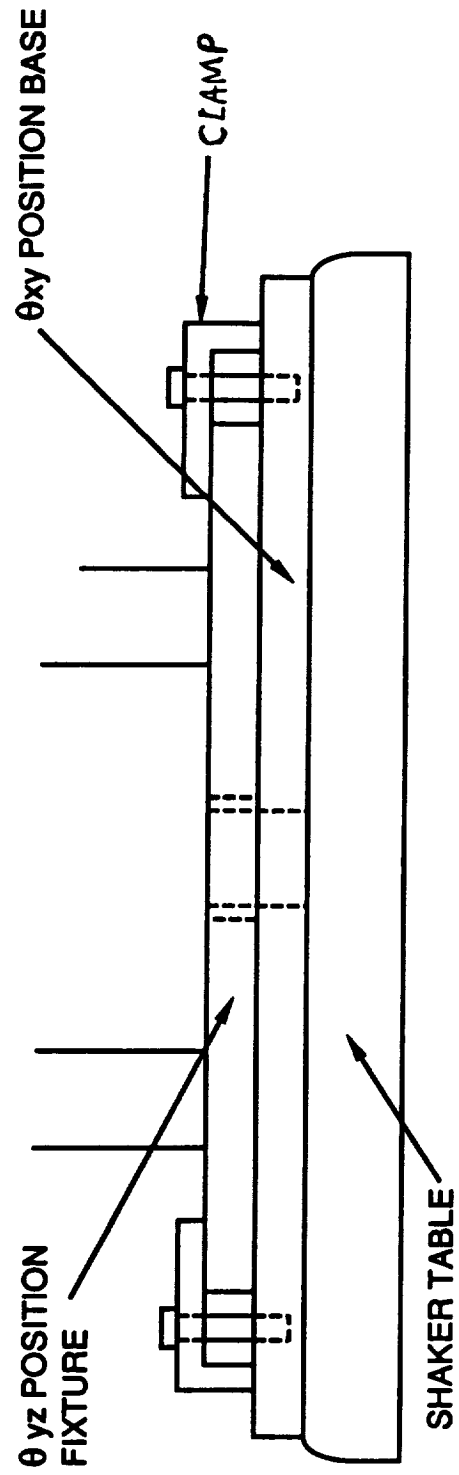
θxy Position Base
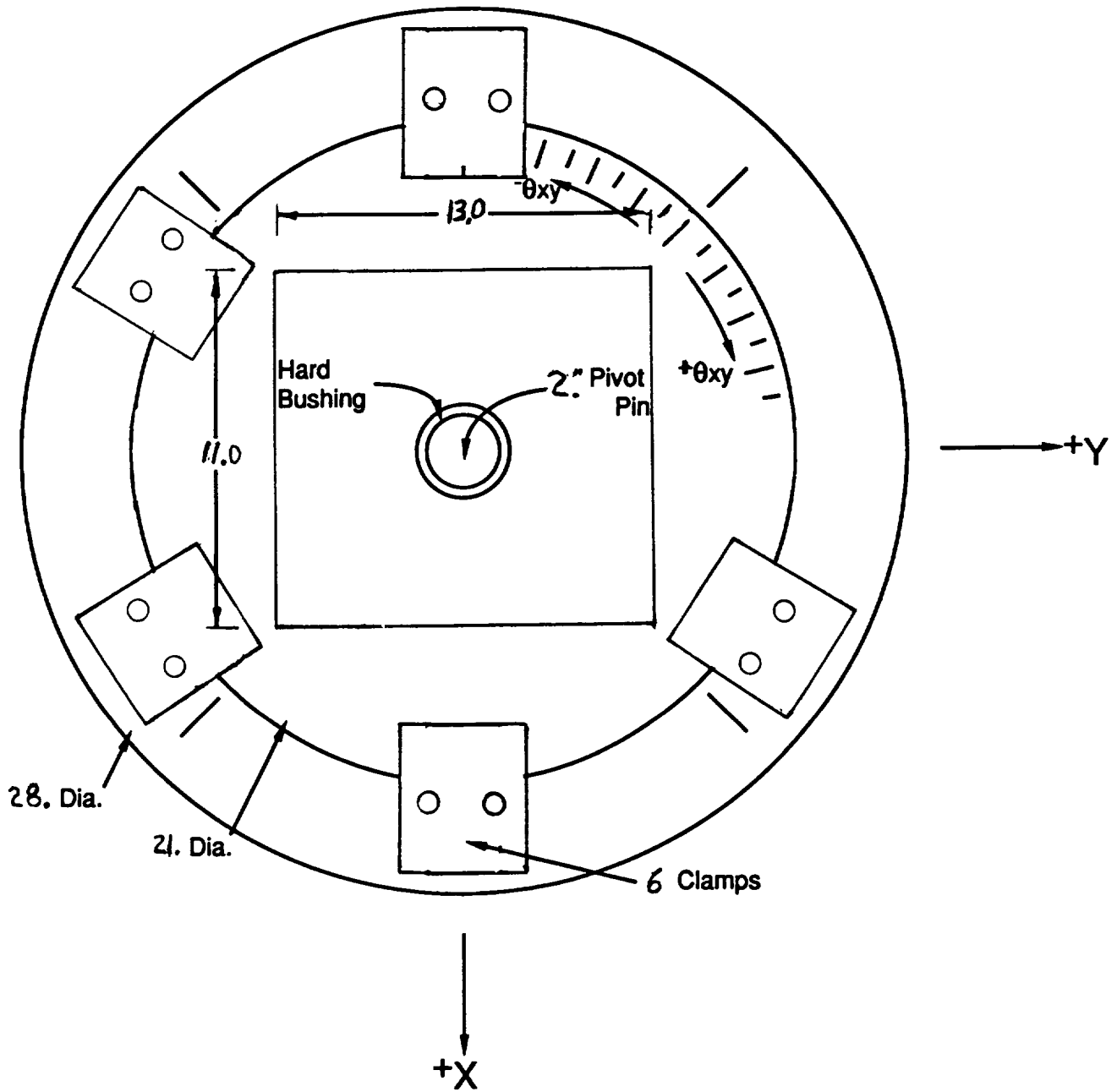
θyz Position Fixture
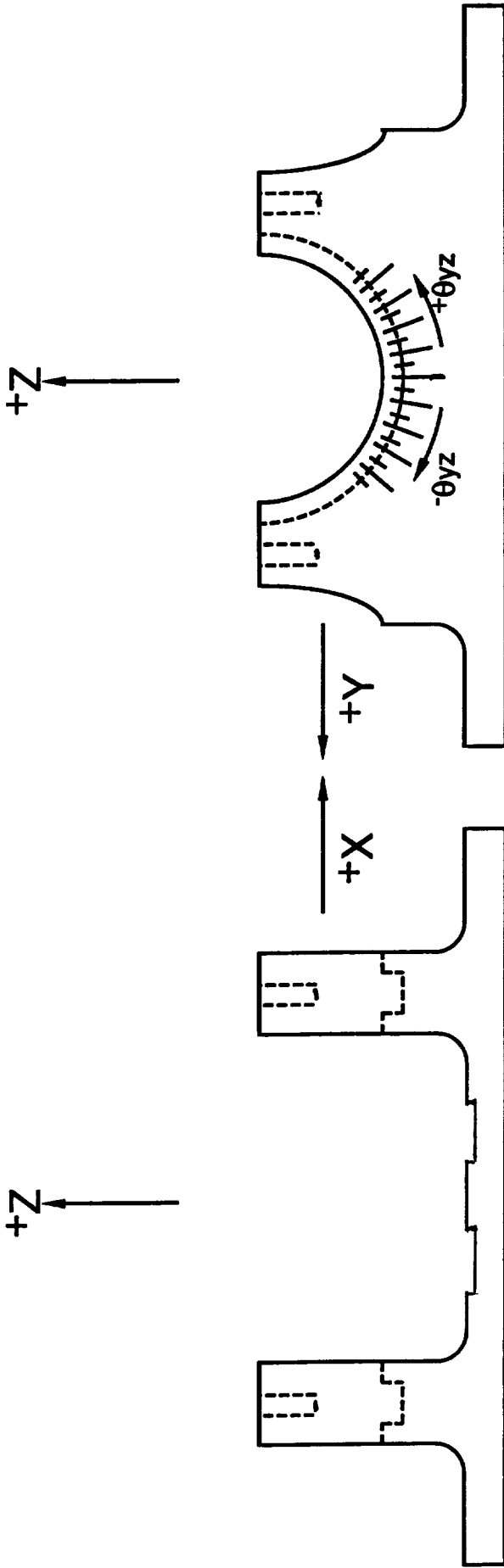
θxz Position Fixture

Component Mount Fixture

+Z

+Z

+Y

+X

30
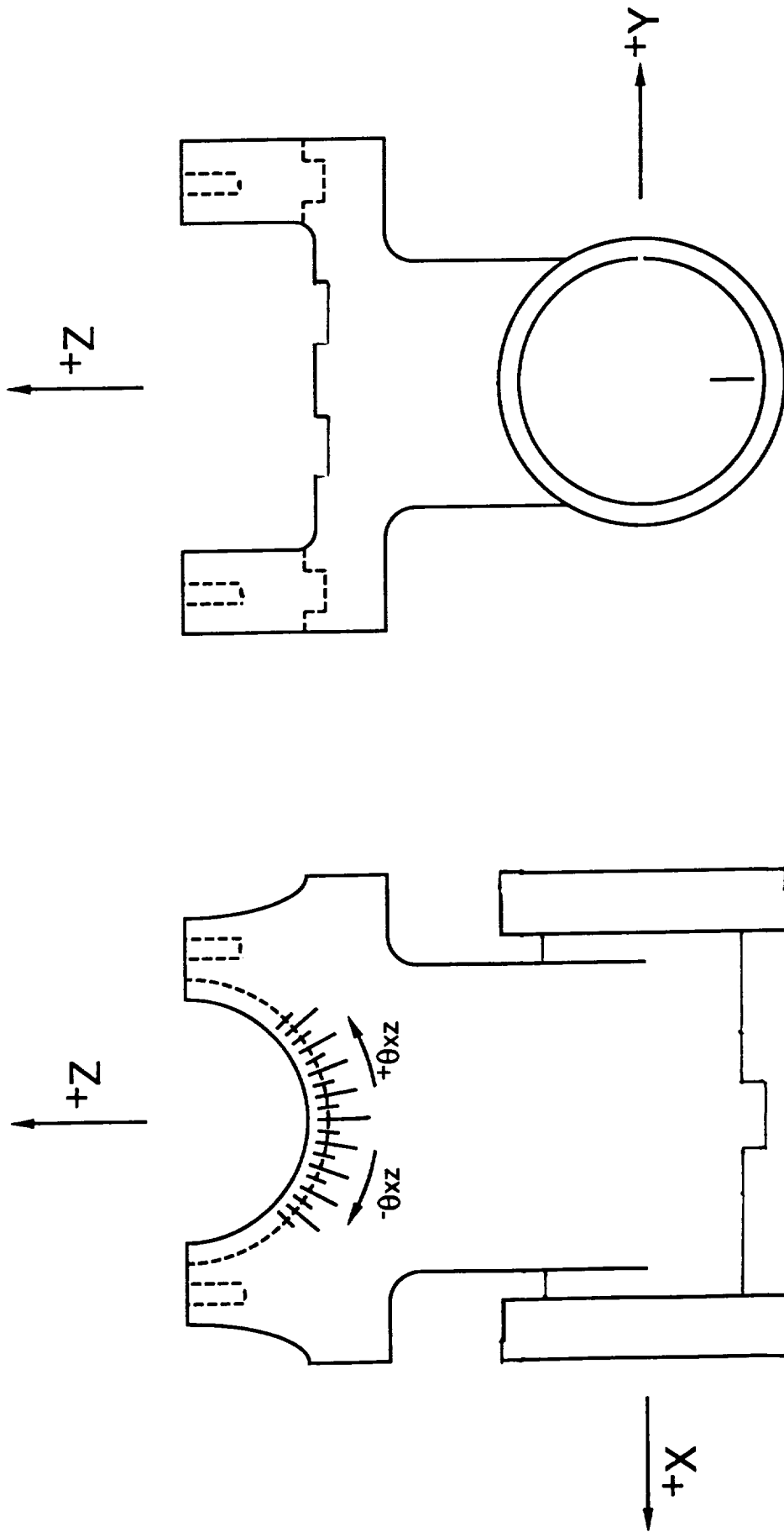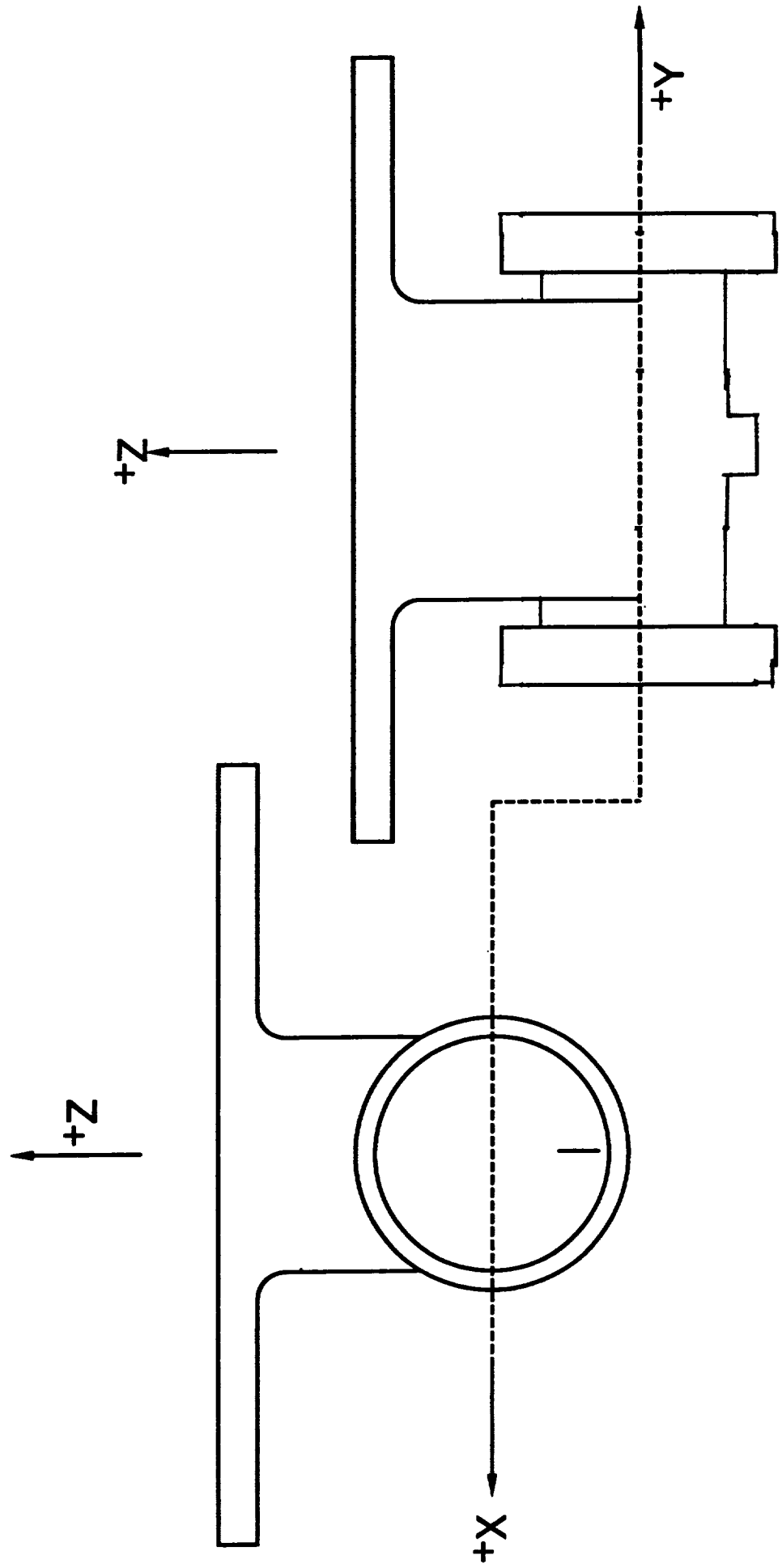
APPENDIX B

STRENGTH ANALYSIS

DESIGN LOAD CASES

The first design load assumption is that the fixture is fully extended; (i.e., $\Theta_{xy} = \Theta_{yz} = \Theta_{xz} = 0.0$) and is loaded laterally (x or y axis).

Utilizing the weight and center of gravity data on page 32, the shear and bending moment distributions were calculated along the length of the fixture. The figures on pages 33 and 34 show these distributions. These data assume a 100 lb component with a center of gravity at 32 inches above the shaker interface and a 50 G lateral loading factor.

The shaker to fixture interface loads are:

$$V = 50 \text{ g} \times [W_{fixture} + W_{component}]$$

$$= 50 \, [376{+}100] = 23{,}800 \text{ lb} \ .$$

$$M = 50 \text{ g} \times [W_{fix} \times CG_{fix} + W_{comp} \times CG_{comp}]$$

$$= 50 \, [376{\times}10.18{+}100{\times}32.0] = 351{,}385 \text{ in-lb} \ .$$

The structural factors of safety utilized in this analysis are:

$$FOS_{ult} = 5.0$$

$$FOS_{yld} = 3.0 \ .$$

MATERIALS

- 301 SS SERIES 1/2 HARD STEEL

$FTU = 141$ ksi	$FSU = 77$ ksi
$FTY = 92$ ksi	$FBRY = 167$ ksi .

- 6061-T6 AL

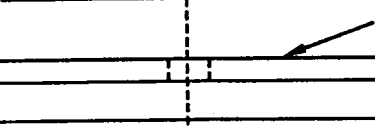|	
$FTU = 38$ ksi	$FSU = 25$ ksi
$FTY = 34$ ksi	$FBRY = 60$ ksi .
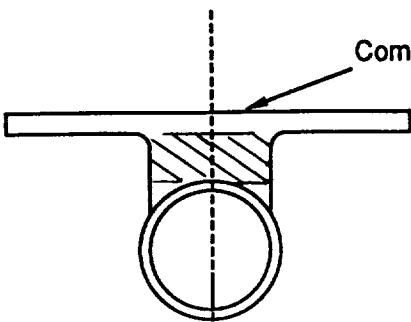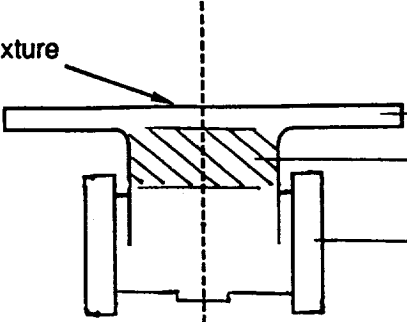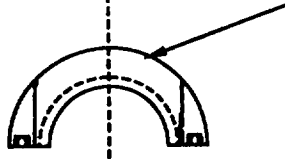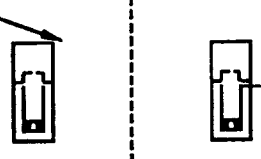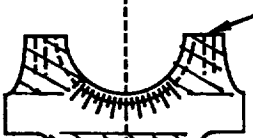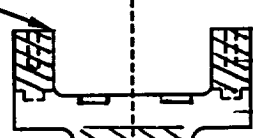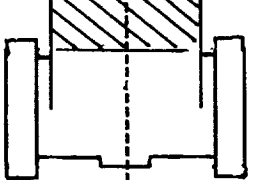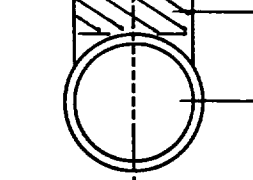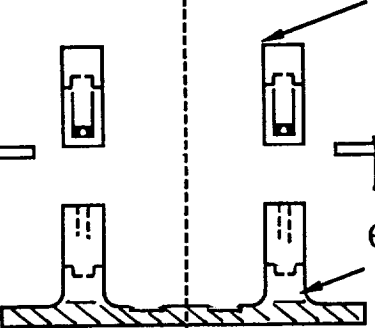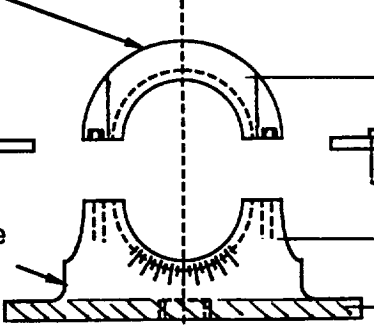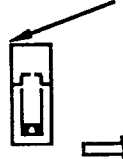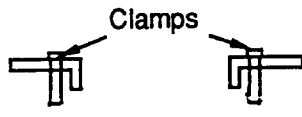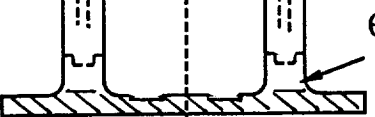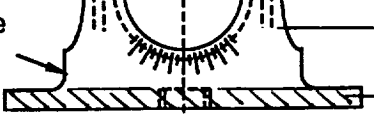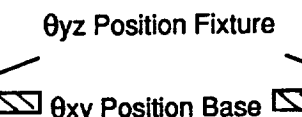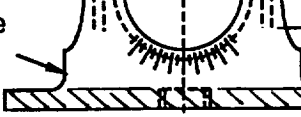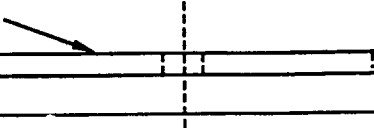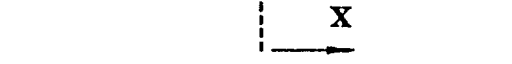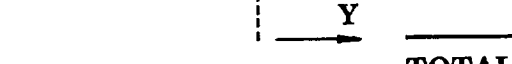
WEIGHT & CENTER OF GRAVITY

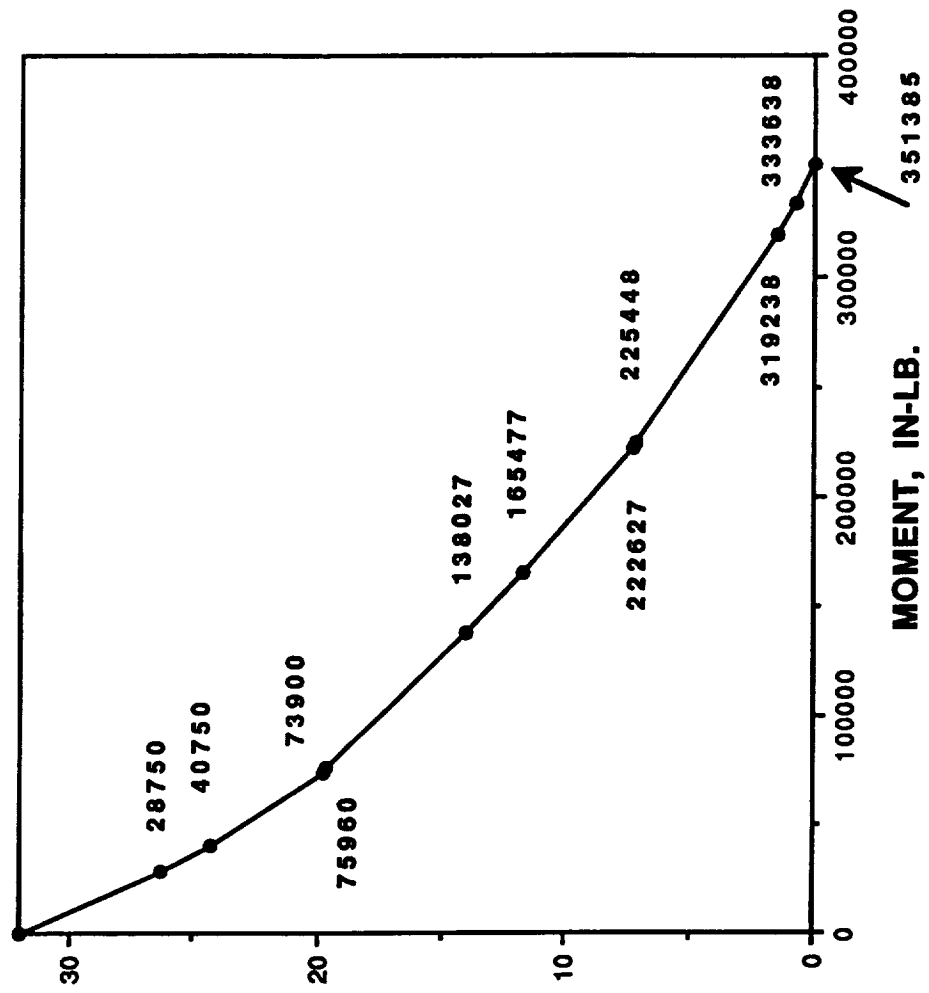| Component	WEIGHT, lb.	C.G., in.
Component Mount Fixture	32	26.25
	10	24.25
	40	19.75
Bushing Block	10	22.35
θxz Position Fixture	27	19.65
	25	14.00
	10	11.75
	40	7.25
Bushing Block	10	9.85
Clamps		
θyz Position Fixture	28	7.15
θxy Position Base	52	1.50
	92	.75
TOTAL	**376**	**10.18**

FRONT VIEW   SIDE VIEW

X   Y   Z

32
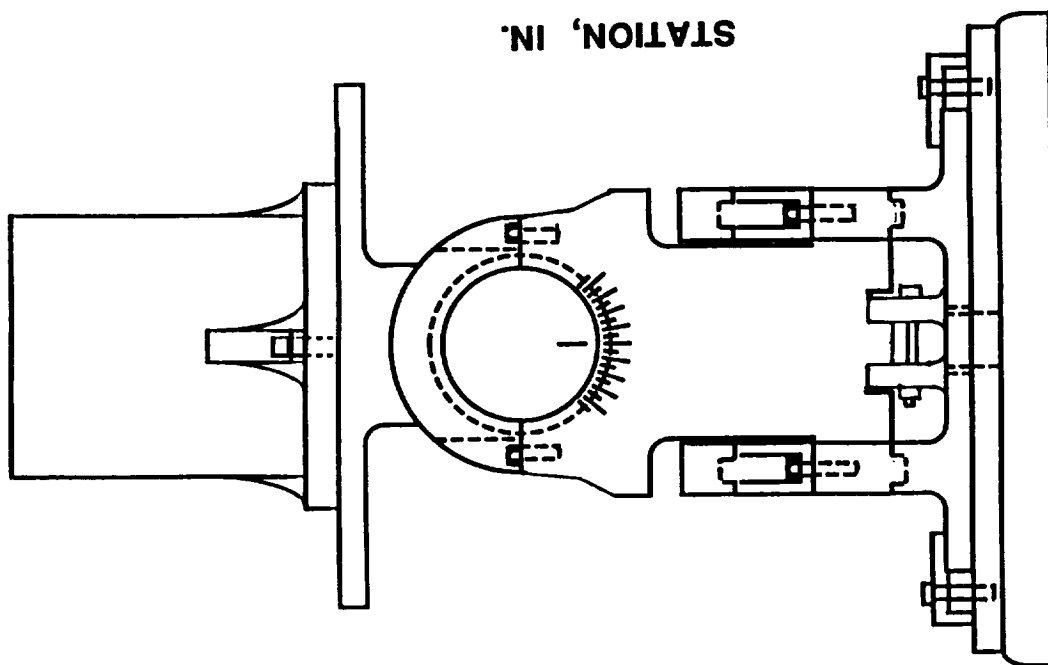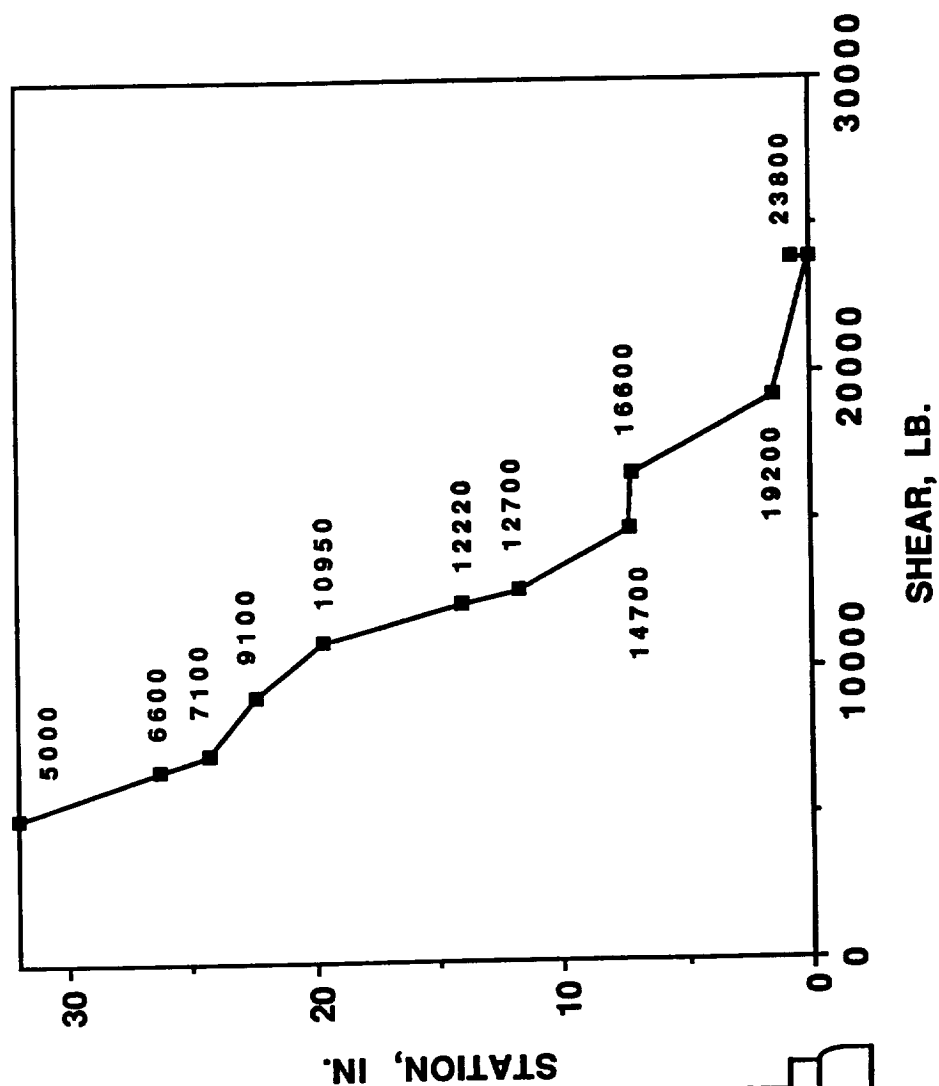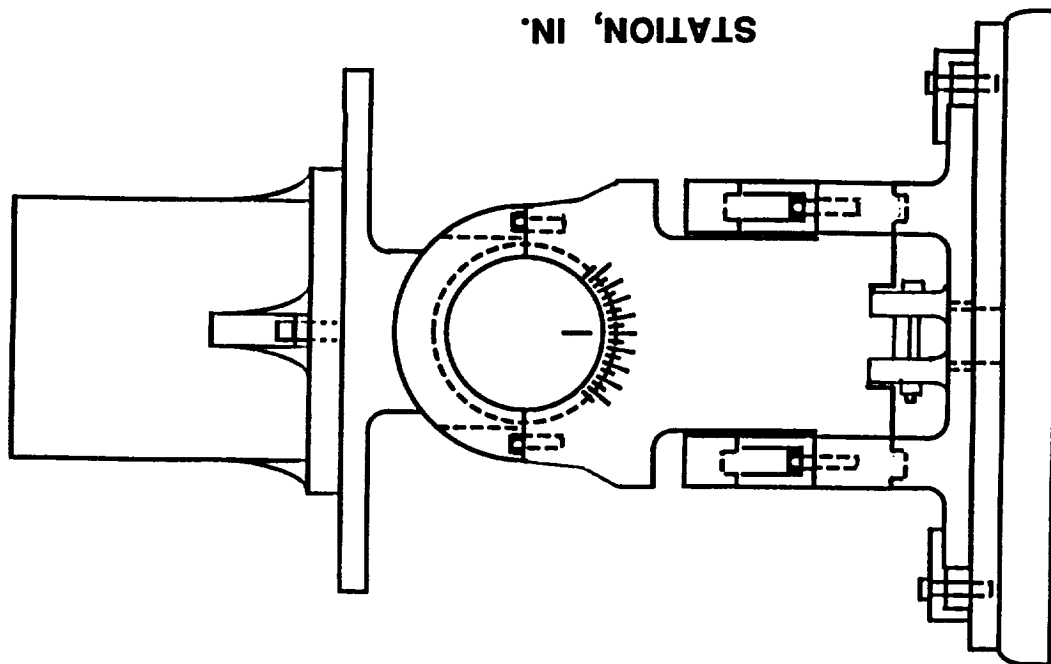
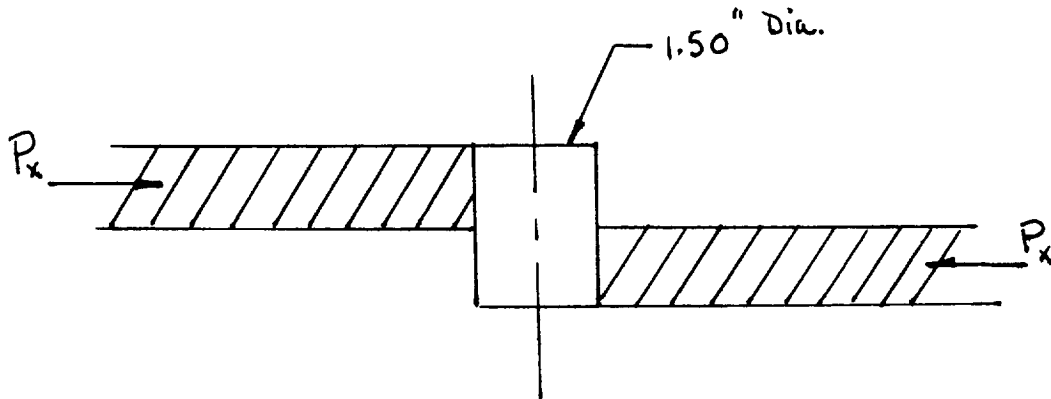SHEAR vs. STATION

MOMENT vs. STATION

MARGIN OF SAFETY

The margin of safety for a part is calculated as:

$$\text{Positive Margin of Safety (M.S.)} \equiv \frac{\text{Capability}}{(FOS)(\text{Design Limit Load})} - 1 \geq 0.0 \ .$$

(1) Θ_{xy} POSITION FIXTURE ROTATOR PIN

301 SS SERIES 1/2 HARD STEEL

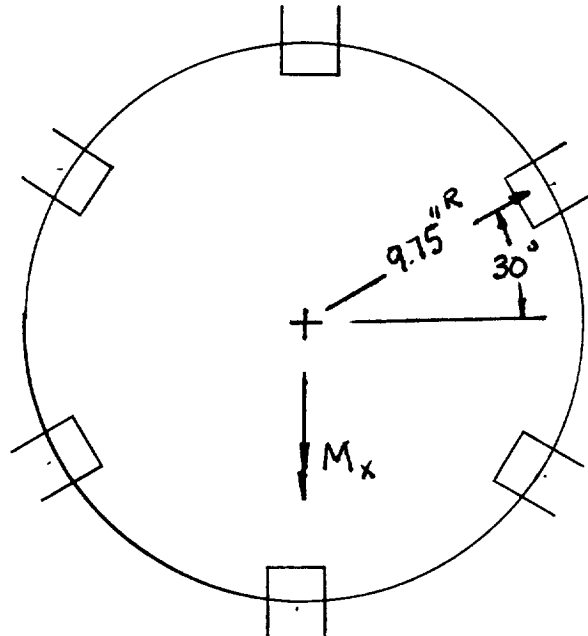$$\tau_{xy} = \frac{P_x}{A} = \frac{23,800}{\pi(0.75)^2} = 13,468 \text{ lb/in}^2$$

$$M.S._{sh} = \frac{77,000}{5(13,468)} - 1 = \underline{+0.14 \quad \text{ok}}$$

$$\sigma_{br} = \frac{23,800}{(1.5)(1.5)} = 10.578 \text{ lb/in}^2$$

$$M.S._{br} = \frac{60,000}{5(10,578)} - 1 = \underline{+0.13 \quad \text{ok}}$$

(2) Θ_{xy} CLAMPS (6)

301 SS SERIES 1/2 HARD STEEL

$$P_m = \frac{M_x}{4(9.75)(\cos 30°)}$$

$$= \frac{351,385}{4(9.75)(0.866)} = 10,404 \text{ lb} \ .$$

Fasteners NAS 1958 1/2-inch–20 A286

$V_{sh} = 21,200 \text{ lb}$	$P_{tu} = 30,900 \text{ lb}$
$A_t = 0.171 \text{ in}^2$	$A_{sh} = 0.196 \text{ in}^2$
$F_{ty} = 150 \text{ ksi}$	$F_{ty} = 180 \text{ ksi}$

$$P_{ext} = \frac{P_m}{2} = 5,202 \text{ lb}$$

$$P_{bolt} = PLD_{max} + (FOS)(n\phi)P_{ext}$$

$$\phi = \frac{k_b}{k_b+k_a} \quad ; \quad k_b = \frac{(3\times10^7)(0.196)}{1.75} = 3.36\times10^6 \text{ lb/in}$$

$$k_1 = \frac{(1\times10^7)(0.5^2-0.25^2)\pi}{1.0} = 17.671\times10^6 \text{ lb/in}$$

$$k_2 = \frac{(1\times10^7)(1)}{1.5} = 6.67\times10^6$$

$$k_3 = \frac{(3\times10^7)(1)}{1.5} = 2.0\times10^6 \text{ lb/in}$$

$$k = \frac{k_2+k_3}{2} = 4.33\times10^6 \text{ lb/in}$$

$$\frac{1}{k_{avg}} = \frac{1}{4.33\times10^6} + \frac{1}{17.67\times10^6} \quad ; \quad k_{avg} = 3.48\times10^6 \text{ lb/in}$$

$$\phi = \frac{3.36}{3.36+3.48} = 0.49 \quad ; \quad \eta = 1/2$$
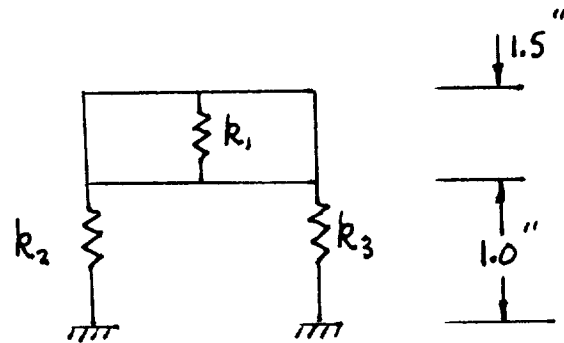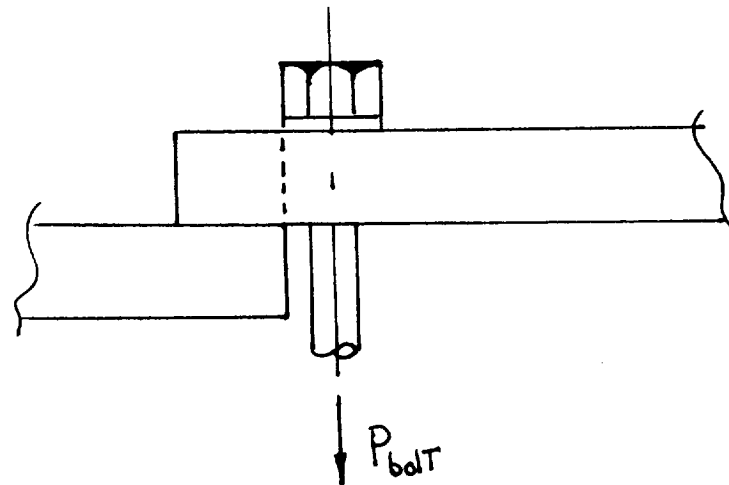
$$P_{bolt\,u} = 1.25(0.65)(150,000)(0.171)+5(0.49)(0.5)(5,202)$$
$$= 20,840+6,372 = 27,121 \text{ lb}$$

$$M.S._{ult} = \frac{30,900}{27,212} - 1 = \underline{+0.13 \quad ok}$$

$$P_{bolt\,y} = 20,840+3(0.49)(0.5)(5,202) = 24,664 \text{ lb}$$

$$M.S._{yld} = \frac{(0.171)(150,000)}{24,664} - 1 = \underline{+0.04 \quad ok}$$

Shear on clamp

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$$\tau = \frac{2(27,212)}{3(1)} = 18,141 \text{ lb/in}^2$$

$$M.S._{sh} = \frac{77,000}{18,141} - 1 = \underline{+3.24} \quad \text{ok}$$
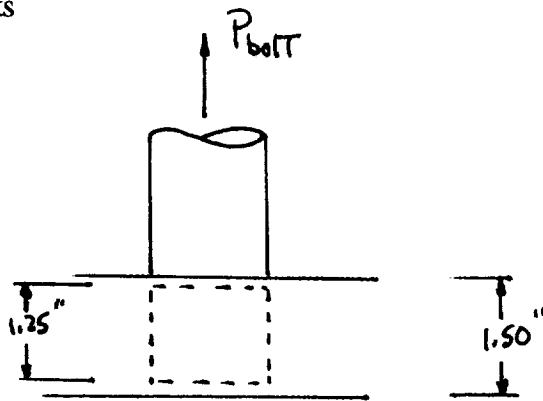
with P_{ext} only

$$\tau = \frac{2(5,202)}{3} = 3,468 \text{ lb/in}^2$$

$$M.S._{sh} = \frac{77,000}{5(3,468)} - 1 = \underline{+3.44} \quad \text{ok}$$

Shear failure of AL Θ_{xy} plate with clamp inserts
  MS 21209 1/2-inch into 6061-T6

$$A_{sh} \geq \pi(20)(1.25)(0.6)[1/40 + \ldots]$$
$$\sim 1.18 \text{ in}^2$$

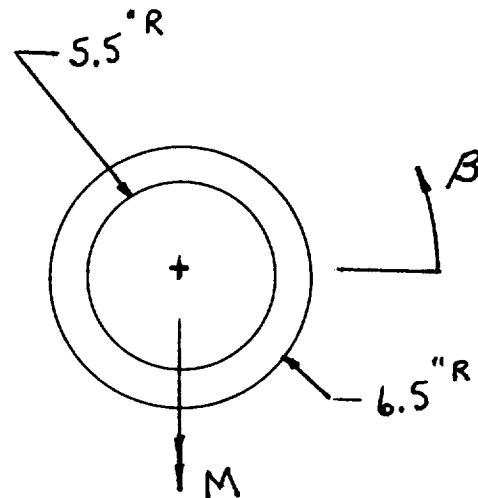$$\tau = \frac{27,212}{1.18} = 23,061 \text{ lb/in}^2$$

$$M.S._{sh} = \frac{25,000}{23,061} - 1 = \underline{+0.08} \quad \text{ok}$$

(3) Θ_{xy} Position Fixture
  –base plate 1.5 inch 6061-T6

$$P' = \frac{M \cos \beta}{\pi R^2}$$
$$= \frac{333,638}{\pi (5.5)^2}$$
$$= 3,511 \text{ lb/in}$$

$$\tau = \frac{P'}{t} = \frac{3,511}{1.5} = 2,340 \text{ lb/in}^2$$

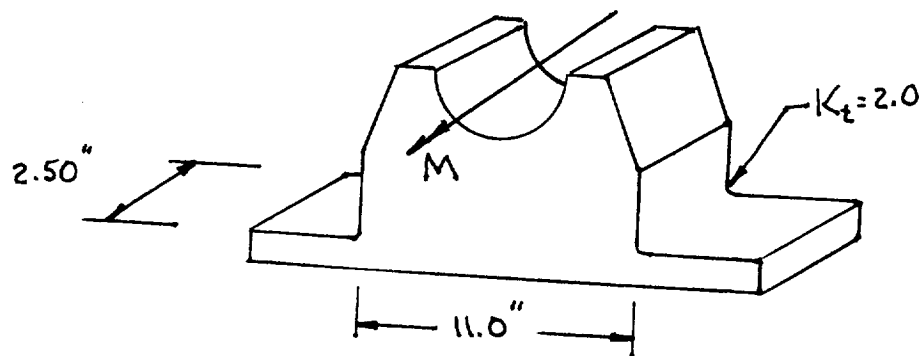$$M.S._{sh} = \frac{25,000}{5(2,340)} - 1 = \underline{+1.13} \quad \text{ok}$$

$$P'_{outer} = \frac{333,638}{\pi(6.5)^2} = 2,514 \text{ lb/in}$$

$$\sigma_b = \frac{6M_o}{t^2} = \frac{6(2,514)}{(1.5)^2} \text{ 6,704 lb/in}^2$$

$$M.S._u = \frac{38,000}{5(6,704)} - 1 = \underline{+0.13} \quad \text{ok}$$

–base plate to support intersection

$$M = \frac{333,638}{2} = 166,819 \text{ in–lb}$$
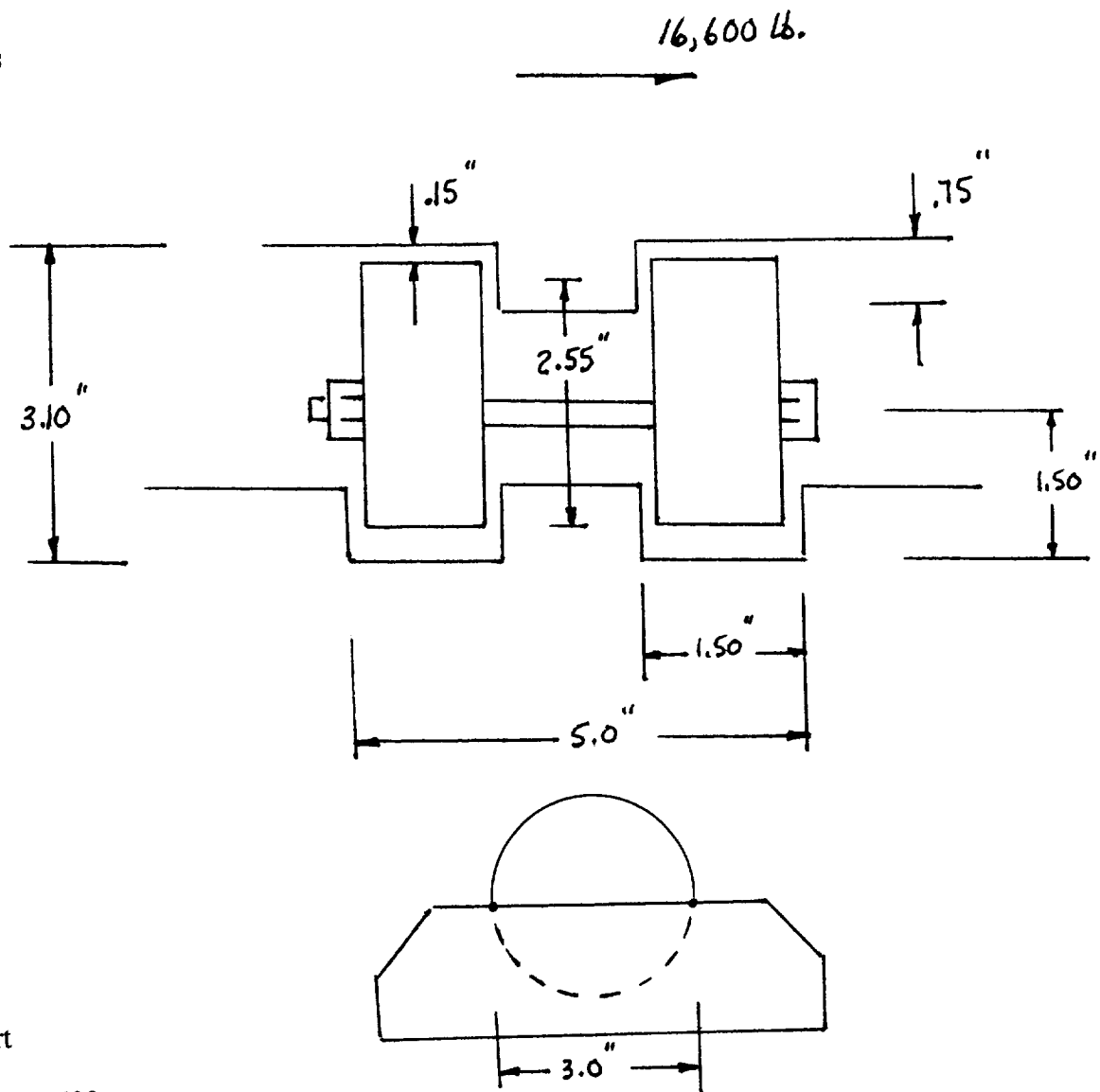
$$c = 5.5 \text{ in}$$

$$I = \frac{2.5(11)^3}{12} \text{ 277.3 in}^4$$

$$K_t = 2.0$$

$$\sigma_b = \frac{(166,819)(5.5)^2}{277.3} = 6,617 \text{ lb/in}^2$$

$$M.S._b = \frac{38,000}{5(6,617)} - 1 = \underline{+0.14} \quad \text{ok}$$

–Shear Bars

16,600 lb.

.15"   .75"

3.10"

2.55"

1.50"

1.50"

5.0"

3.0"

Θ_{yz} support

$$\tau = \frac{16,600}{2(3)} = 2,767 \ \text{lb/in}^2$$

$$M.S._{sh} = \frac{25,000}{5(2,767)} = \underline{+0.80} \quad \text{ok}$$

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–Fastener   **NAS 1958 $^1/_2$ in A286**

$$V_{ult} = 21,200 \text{ lb} \qquad FTU = 180 \text{ ksi}$$

$$P_{ult} = 30,900 \text{ lb} \qquad FTY = 150 \text{ ksi}$$

$$A_t = 0.171 \text{ in}^2 \qquad A_{sh} = 0.196 \text{ in}^2$$

$$P_b = PLD_{max} + (FOS)(n\phi)P_{ext}$$

$$PLD_{max} = 1.25(0.65)(150,000)(0.171) = 20,840 \text{ lb}$$

$$k_b = \frac{(3\times10^7)(0.196)}{5} = 1.176\times10^6 \text{ lb/in}$$

$$k_a = \frac{(3\times10^7)(0.43^2 - 0.281^2)\pi}{3} = 3.328\times10^6 \text{ lb/in}$$

$$\phi = \frac{1.176}{1.176+3.328} = 0.26 \quad ; \quad \eta = \frac{1}{2} \quad ; \quad P_{ext} = \frac{16,600}{4} = 4,150 \text{ lb}$$

$$P_{bu} = 20,240 + (5)(0.26)(0.5)(4,150) = 23,538 \text{ lb}$$

$$M.S._u = \underline{+0.31 \qquad ok}$$

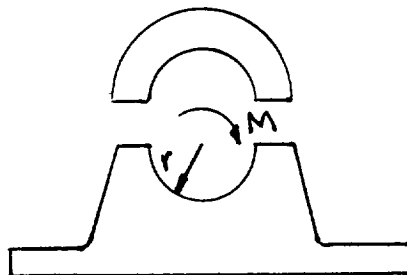$$P_{by} = 20,840 + (3)(0.26)(0.5)(4,150) = 22,458 \text{ lb}$$

$$M.S._y = \underline{+0.14 \qquad ok}$$

–Gear teeth

$$M = \frac{225,448}{2} = 112,724 \text{ in–lb}$$

$$N = 36 \text{ teeth}$$

$$r = 3.00 \text{ in (min)}$$

$$M_t = \frac{112,724}{36} = 3,131 \text{ in-lb/tooth}$$

$$F_t = \frac{3,131}{3} = 1,044 \text{ lb/tooth}$$

$$\tau_b = \frac{(1,044)}{2.5(0.261)} = 1,600 \text{ lb/in}^2$$

$$M.S._{sh} = \frac{25,000}{5(1,600)} - 1 = \underline{+2.12} \quad ok$$

$$\sigma_b = \frac{(1,044)(0.13)(0.1305)}{2.5(0.261)^3/12} = 4,782 \text{ lb/in}^2$$

$$M.S._b = \frac{38,000}{5(4,782)} - 1 = \underline{+0.58} \quad ok$$

(4) Bushing Block

$$M = 112,724 \text{ in-lb}$$

$$P_{ext} = \frac{M}{4(8.5)} = 3,315 \text{ lb/bolt}$$

–Fastener NAS 1352-8 $^1/_2$ in alloy steel

P_{tu} = 24,100 lb. P_{ty} = 16,870 lb.
A_t = 0.160 in^2 A_{sh} = 0.196 in^2
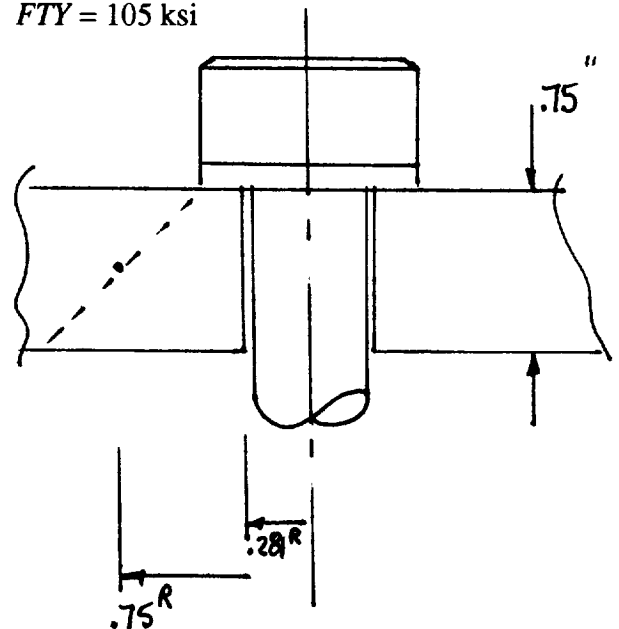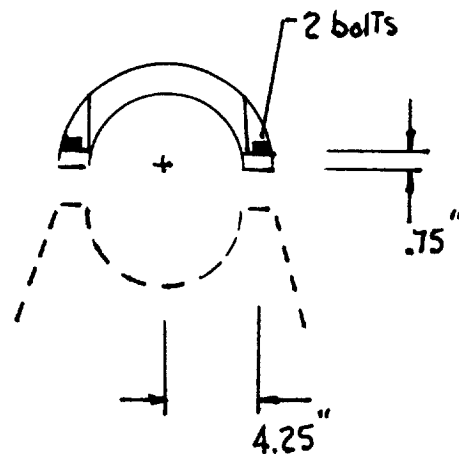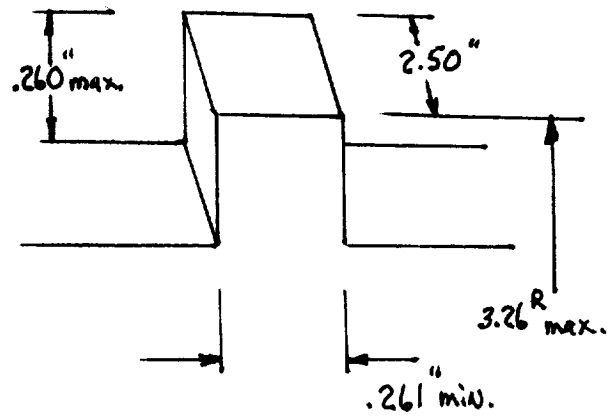FTU = 150 ksi FTY = 105 ksi

$$PLD_{max} = 1.25(0.65)(105,000)(0.16) = 13,650 \text{ lb}$$

$$k_b = \frac{3 \times 10^7 (0.196)}{0.75} = 7.84 \times 10^6 \text{ lb/in}$$

$$k_a = \frac{1 \times 10^7 (0.75^2 - 0.281^2)\pi}{0.75}$$
$$= 20.24 \times 10^6 \text{ lb/in}$$

$$\phi = \frac{7.84}{7.84 + 20.24} = 0.28$$

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$$P_{bu} = 13{,}650 + 5(0.5)(0.28)(3{,}315) = 15{,}970 \text{ lb}$$

$$M.S._u = \frac{24{,}100}{15{,}970} - 1 = \underline{+0.50 \quad ok}$$

$$P_{by} = 13{,}650 + 3(0.5)(0.28)(3{,}315) = 15{,}042 \text{ lb}$$

$$M.S. = \frac{16{,}870}{15{,}042} - 1 = \underline{+0.12 \quad ok}$$
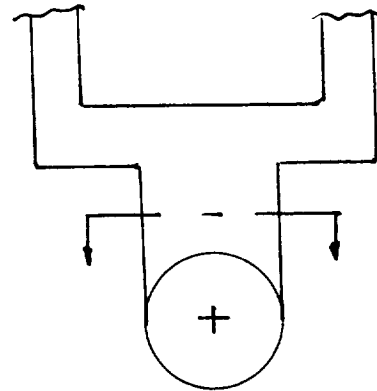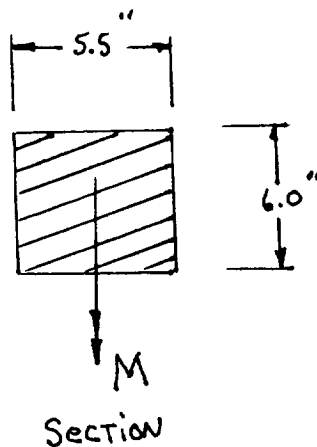
Θ_{yz} shear out

inserts MS21209 $^1/_2$ in D = 0.592 in

$$A_{sh} = \pi(13)(1.25)(0.592)\left[\frac{1}{2(13)} + ...\right] = 1.16 \text{ in}^2$$

$$\tau = \frac{15{,}970}{1.16} = 13{,}767 \text{ lb/in}^2$$

$$M.S._{sh} = \frac{25{,}000}{13{,}767} - 1 = \underline{+0.81 \quad ok}$$

Θ_{xz} body

Section

$M = 225{,}448$ in-lb

$$\sigma = \frac{(225{,}448)(2.75)}{6(5.5)^3/12} = 7{,}453 \text{ lb/in}^2$$

$$M.S. = \frac{38{,}000}{5(7{,}453)} - 1 = \underline{+0.02 \quad ok}$$

Θ_{xz} bushing block and component mount fixture have positive margins of safety by similarity and because of lower loads at these stations.

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APPROVAL

ROOT-SUM-SQUARE STRUCTURAL STRENGTH VERIFICATION APPROACH

By Henry M. Lee

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

J.C. BLAIR
Director, Structures and Dynamics Laboratory