Feed-Forward Control of Gear Mesh Vibration Using Piezoelectric Actuators

Gerald T. Montague
NYMA, Inc.
Engineering Services Division
Brook Park, Ohio

Albert F. Kascak
Vehicle Propulsion Directorate
U.S. Army Research Laboratory
Lewis Research Center
Cleveland, Ohio

and

Alan Palazzolo, Daniel Manchala, and Erwin Thomas
Texas A&M University
College Station, Texas

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FEED-FORWARD CONTROL OF GEAR MESH VIBRATION USING PIEZOELECTRIC ACTUATORS

Gerald T. Montague  
NYMA, Inc.  
Engineering Services Division  
Brook Park, Ohio 44142

Albert F. Kascak  
Vehicle Propulsion Directorate  
U.S. Army Research Laboratory  
Lewis Research Center  
Cleveland, Ohio 44135

and

Alan Palazzolo, Daniel Manchala, and Erwin Thomas  
Texas A&M University  
College Station, Texas 77843

SUMMARY

This paper presents a novel means for suppressing gear mesh-related vibrations. The key components in this approach are piezoelectric actuators and a high-frequency, analog feed-forward controller. Test results are presented and show up to a 70-percent reduction in gear mesh acceleration and vibration control up to 4500 Hz. The principle of the approach is explained by an analysis of a harmonically excited, general linear vibratory system.

INTRODUCTION

Transmissions can generate vibrations and noise by many mechanisms. Inherent in the transmission of motion with gears is the time-varying gear mesh stiffness as the number of teeth and position of load changes along the tooth profile. Many other factors can also contribute to the resultant level of vibration or noise found at the gear mesh frequency. Examples of these controllable factors include shaft misalignment or gear manufacturing errors, such as pitch runout or profile errors. Therefore, the gear meshing action can be minimized by proper profile manufacture, proper assembly, and tight tolerances on the support structure, but the gear mesh vibration or noise cannot be completely avoided.

Gear mesh vibrations are transmitted from the gear to the shaft and through the bearings to the case. A novel approach to reduce transmission vibrations is to apply a force with piezoelectric actuators at the bearing outer race. This method has evolved from the application of piezoelectric actuators for active vibration control of rotating machinery. Prior research has used piezoelectric actuators to control subsynchronous, synchronous, and transient rotor vibrations (Palazzolo et al., 1993).

Previous work has concentrated on developing an actuator with a small envelope exciter, high-frequency range, and high-force capability (Palazzolo et al., 1993). The configuration consisted of a piezoelectric actuator driving an input piston contained in a hydraulic line. The output piston applies forces to the shaft through a ball bearing.

This research applies piezoelectric actuators to control a high-frequency (4500 Hz) gear mesh vibration component. Active control of high-frequency vibration is in contrast to passive gear vibration control using...
viscoelastic damping materials (Ren et al., 1990). Gear vibration can be reduced by introducing coulomb damping to the radial motion of the rim (Botsford, 1980). A systematic approach to reduce gear noise by improving the gear train structure rather than by focusing on the excitation forces of individual gears has been presented by Ariga et al. (1992). Nagamatsu et al. (1979) performed a theoretical vibration analysis of a casing for a high-speed rotating machine using the reduced impedance method. Choy et al. (1993) developed a dynamic model to simulate gearbox vibration and used experimental results from a test rig to verify the analytical model. Umezawa et al. (1988) developed a method to estimate the vibration of encased gears by measuring exterior vibration. The work of Umezawa et al. (1988) demonstrated that the transfer function obtained from impact testing while the nonrotating gear was under a static torque can be used for the estimation of gear vibration. Gear vibration monitoring techniques were reviewed by Mathew et al., (1987) on a test rig. Mathew deliberately induced gear damage by eliminating lubrication, applying excessive torque, and introducing dust in the gearbox.

The objective of this research was to apply feed-forward control to a transmission test rig using piezoelectric actuators to reduce gear mesh vibration. The experimental research was conducted on a four-square spur gear transmission test rig at the NASA Lewis Research Center. Active control of gear mesh vibration was performed with various torque, speed, and actuator conditions. The details and results of these tests are presented herein.

**THEORY**

The operating principle for the feed-forward active vibration controller may be explained by considering the dynamic equilibrium equations for a general linear system

\[
[M] \ddot{X} + [C] \dot{X} + [K] X = F_D + F_C
\]  

(1)

where

- \([M]\) mass matrix
- \(\ddot{X}\) second derivative of displacement vector
- \([C]\) damping matrix
- \(\dot{X}\) first derivative of displacement vector
- \([K]\) stiffness matrix
- \(X\) displacement vector
- \(F_D\) external force vector (disturbance or gear mesh forces)
- \(F_C\) feed-forward control force vector

The gear mesh and control forces will be at the gear mesh frequencies; hence

\[
[M] \ddot{X} + [C] \dot{X} + [K] X = \hat{F}_D e^{i\omega t} + \hat{F}_C e^{i\omega t}
\]  

(2)

where

- \(\hat{F}_D\) complex vector of the disturbance force phasors
- \(i\) square root of \(-1\)
\( \omega \)  
gear mesh frequency

\( t \)  
time

\( \hat{F}_C \)  
complex vector of control force phasors

Setting

\[ X = \hat{X} e^{i\omega t} \]  (3)

where \( \hat{X} \) is a complex vector of the displacement phasors, yields

\[ (-\omega^2 [M] + i\omega [C] + [K]) \hat{X} = \hat{F}_D + \hat{F}_C \]  (4)

This may be solved for \( \hat{X} \) as

\[ \hat{X}_C = \hat{X}_o + \alpha \hat{F}_C \]  (5)

where

\( \hat{X}_C \)  
displacement vector phasor with control

\( \hat{X}_o = [\alpha] \hat{F}_D \)  
displacement vector phasor without control

\[ [\alpha] = (-\omega^2 [M] + i\omega [C] + [K])^{-1} \]  influence coefficient matrix

\( \alpha \)  
influence coefficient

Note the case where a single control force \( \hat{F}_{cj} \) is applied at degree of freedom \( j \) and the response is considered at a degree of freedom \( i \):

\[ \hat{X}_{ci} = \hat{X}_{oi} + \alpha_{ij} \hat{F}_{cj} \]  (8)

The control force that nulls the response at degree of freedom \( i \) is obtained by solving Eq. (8) with \( X_{ci} \) set to zero:

\[ F_{cj} = -X_{oi}/\alpha_{ij} \]  (9)

Alternatively, \( F_{cj} \) is the control force that makes

\[ \alpha_{ij} \hat{F}_{cj} = -\hat{X}_{oi} \]  (10)
The phase shifter-amplifier employed in the testing may vary $F_{cj}$ continuously in both the amplitude and the phase angle; hence, Eq. (10) may be approximately satisfied by tuning within the force limits of the actuator. In practice, $\hat{X}_{ci}$ in Eq. (8) is minimized by manually searching (tuning) through $F_{cj}$ values. Note that the influence coefficient $\alpha_{ij}$ does not need to be measured to minimize $\hat{X}_{ci}$. However, by applying a perturbation force, its measurement may expedite finding the optimum value for $F_{cj}$. In the literature, this latter approach is referred to as active or feed-forward balancing.

Additional displacement phasors may be nulled or minimized if multiple controllers are present. It is noteworthy that the displacements other than at degree of freedom $i$ in Eq. (8) may increase or decrease with the application of $F_{cj}$, as in influence coefficient flexible-rotor balancing. In practice, most displacements decrease, especially if a least-squares minimization procedure is employed.

**TEST FACILITY DESCRIPTION**

Figures 1 and 2 show a four-square spur gear facility at the NASA Lewis Research Center. The transmission is driven by a 7.45-kW (10-hp) dc motor rated at 3500 rpm. A ring feeder is used to secure the 96-tooth gear to the low-speed shaft, while a key-way is used to mount the 24-tooth gear to the high-speed shaft. There is a 0.875:1.0 gear tooth ratio from the motor to the input shaft and a 1:0.40 gear tooth ratio from the low-speed to the high-speed shaft. This gear configuration results in maximum shaft speeds of 4000 and 16 000 rpm.

The spur gears are American Gear Manufacturing Association (AGMA) class 9 with a 20° pressure angle, a diametral pitch of 16, and a face width of 22.4 mm (0.88 in.). The low-speed shaft is separated into two pieces bolted together at a slotted flange. Pretwisting the two-piece low-speed shaft allows different static preload torque to be locked into the shaft in or against the direction of rotation. A pretorque of 178 N·m (131 ft·lbs) circulates 74.6 kW (100 hp) through the system at a low shaft speed of 4000 rpm. A hydraulic system supplies lubrication to the bearings and gears through oil jets. An oil pressure regulator allows the gear lubrication to be adjusted or shut off. The rig is instrumented with 19 displacement probes, 8 accelerometers, 8 flowmeters, 8 pressure transducers, 6 thermocouples, 4 position encoders, and 1 sound pressure level meter.

Figure 2 shows the orientation of a piezoelectric actuator in the test rig. These piezoelectric actuators have a 3000-N (675-lbs) force capacity and 30-μm (0.00076-in.) stroke. The control force produced by the actuator is transmitted to the shaft via duplex precision angular contact ball bearings, which are fitted in adapters with a 0.102-mm (0.004-in.) radial clearance and a 19.1-mm- (0.75-in.-) thick steel rig casing.

**ACTUATOR RESPONSE AND SETUP**

Figure 3 shows the piezoelectric actuator that was used in the feed-forward control of the spur gear mesh vibration and the actuator’s specifications. Figure 4 shows a piezoelectric actuator free-tip frequency response and Fig. 5 shows the frequency response of the actuator mounted in the test rig. The actuators’ orientation relative to the gears and bearings is shown in Fig. 6. The actuators are mounted 20° clockwise from the vertical axis of each shaft and collinear with the spur gear tooth contact force. Cone washers were used to support the bearing and center the shaft in the housing. A small accelerometer is mounted at the actuator tip.

The block diagram representation of the control loop is shown in Fig. 7. A displacement probe gear tooth signal is sent to the analog phase shifter, which has a frequency and a phase adjustment dial. The phase shifter converts the gear tooth signal to the feed-forward signal, which is a sinusoidal signal at the same frequency. The phase is manually adjusted and a signal inverter is used to ensure a 360° adjustment range. The feed-forward
signal amplitude is manually adjusted through the isolation amplifier. An oscilloscope is used to monitor the gear tooth and feed-forward signals. Figure 8 shows the gear tooth, feed-forward, and one pulse per revolution signal on the oscilloscope. The feed-forward signal voltage and current are monitored by analog meters and then sent to the piezoelectric actuators.

A four-channel dynamic signal analyzer monitors the frequency response of four accelerometers mounted on the transmission casing (Fig. 2). Test conditions were adjusted to maximize the gear mesh vibration before applying the feed-forward control. The phase and amplitudes of the feed-forward signals are adjusted to minimize the gear mesh frequency component amplitudes.

**ROTATING RIG TEST RESULTS**

Feed-forward control using piezoelectric actuators was applied to the transmission rig under different torque, speed, and actuator conditions. Test speeds that produced high gear mesh components were chosen for the feed-forward tests (Tables I to III). Feed-forward control was tested using a single actuator on the low-speed shaft and using two in-phase actuators (one for each shaft). Figures 9 and 10 show the casing vibration spectra before and after the feed-forward loop was applied to the low-speed actuator. These figures show a 27-percent reduction in gear mesh vibration at 4450 Hz. The sound pressure spectra with and without low-speed shaft actuator feed-forward control shows a 22-percent reduction in the gear mesh vibration amplitude at 4650 Hz as seen in Figs. 11 and 12.

Figure 13 shows that gear mesh vibration and the actuator current increase as a function of the actuator voltage; Fig. 14 shows the effect of the feed-forward phase angle adjustment on the casing accelerometer A at a 4650-Hz gear mesh vibration and is compared with the no-control accelerometer A signal.

Table I presents the test conditions and the casing vibrations with and without control at four different speeds and two different torques using the low-speed shaft actuator only. As shown in the figure, gear mesh vibration was reduced as much as 74 percent. Casing accelerometer vibration spectra with and without feed-forward control for a one-actuator system with a +149-N·m (+110-ft·lb) preload torque are shown in Figs. 15 and 16 and in Table I. The results indicate very significant vibration reductions at A, B, and C. Table II presents the results for the condition in which a +194-N·m (143-ft·lb) preload torque was applied to a two-actuator system. Figures 17 and 18 and Table II show casing accelerometer vibration spectra with and without control for a two-actuator system with a +194-N·m (143-ft·lb) preload. These results also show very effective vibration control using the feed-forward approach.

Comparing a one-actuator system with a two-actuator system reveals a 74-percent vibration reduction with a +149-N·m (110-ft·lb) preload for the one-actuator system and a 76-percent reduction with a +194-N·m (143-ft·lb) preload for the two-actuator system. The vibration at some accelerometers actually increases with control (Table II). This result is similar to flexible rotor balancing and indicates that additional independently controlled actuators may be required to suppress vibrations at many locations. The vibration data of tests with different gear oil lubrication pressure is summarized in Table III. This test was performed to generate higher levels of gear mesh vibration; however, the results show little change in vibration amplitude; therefore feed-forward tests were not conducted.

**SUMMARY OF RESULTS**

Previous tests applied piezoelectric actuators to control rotor vibrations at low frequencies of 100 to 250 Hz. The results obtained in this study show that a piezoelectric actuator with a feed-forward control can
reduce the gear mesh vibration amplitude up to 75 percent for frequencies up to 4500 Hz. Experimental results showed a significant reduction in gear mesh vibration at different speeds and torques. Future work will incorporate independent control for more actuators, development of a digital feed-forward control, and the application of an actuator with an increased stroke.

REFERENCES


TABLE I—CASING ACCELEROMETER VIBRATION FOR
LOW-SPEED SHAFT ACTUATOR

[ Gear lubrication, 75 psig; torque, ±149 N·m (±110 ft·lb). ]

<table>
<thead>
<tr>
<th>Torque N·m</th>
<th>Shaft speed, rpm</th>
<th>Accelerometer</th>
<th>Gear mesh frequency, Hz</th>
<th>Gear mesh vibration, g</th>
</tr>
</thead>
<tbody>
<tr>
<td>-149</td>
<td>1600</td>
<td>A</td>
<td>2550</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
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<td>1943</td>
<td>A</td>
<td>3100</td>
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<td>B</td>
<td>3100</td>
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<tr>
<td></td>
<td>2267</td>
<td>A</td>
<td>3625</td>
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<td>2267</td>
<td>B</td>
<td>3625</td>
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<tr>
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<td>1829</td>
<td>B</td>
<td>2925</td>
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<tr>
<td></td>
<td>1942</td>
<td>B</td>
<td>3100</td>
<td>1.0</td>
</tr>
</tbody>
</table>

*Low-speed shaft.

TABLE II—GEAR MESH VARIATION
FOR TWO-ACTUATOR SYSTEM

[ Gear lubrication, 75 psig; preload torque, +194 N·m (143 ft·lb). ]

<table>
<thead>
<tr>
<th>Shaft speed, rpm</th>
<th>Accelerometer</th>
<th>Gear mesh frequency, Hz</th>
<th>Gear mesh vibration, g</th>
</tr>
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<tbody>
<tr>
<td>1942</td>
<td>A</td>
<td>3100</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>B</td>
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<td>B</td>
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<td>2344</td>
<td>A</td>
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<td>2.5</td>
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<tr>
<td>2628</td>
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<td>4200</td>
<td>4.6</td>
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<td>4200</td>
<td>4.8</td>
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<tr>
<td>2688</td>
<td>A</td>
<td>4300</td>
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<tr>
<td></td>
<td>B</td>
<td>4300</td>
<td>5.1</td>
</tr>
</tbody>
</table>

*Low-speed shaft.
TABLE III—GEAR MESH VIBRATION WITH FEED-FORWARD TO LOW-SPEED SHAFT ACTUATOR

<table>
<thead>
<tr>
<th>Shaft speed, rpm</th>
<th>Gear mesh frequency, Hz</th>
<th>Gear mesh vibration, g</th>
<th>Accelerometer</th>
<th>Gear lubrication, psig</th>
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<td></td>
<td></td>
<td></td>
<td>A</td>
<td>B</td>
</tr>
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<td>2550</td>
<td>0.8</td>
<td>1.6</td>
<td>0</td>
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<tr>
<td></td>
<td></td>
<td>1.0</td>
<td>1.1</td>
<td>175</td>
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<tr>
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<td>1.1</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.0</td>
<td>3.4</td>
<td>75</td>
</tr>
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<td>1.8</td>
<td>3.6</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>1.0</td>
<td>2.6</td>
<td>250</td>
</tr>
</tbody>
</table>

*Low-speed shaft.

Figure 1.—Open-casing test rig showing shafts.
Figure 2.—Piezoelectric actuator and casing accelerometers A to D.

Figure 3.—Piezoelectric actuator and specifications.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
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<tbody>
<tr>
<td>Operating voltage, V</td>
<td>-20 to +100 V</td>
</tr>
<tr>
<td>Admissible temperature range, °C</td>
<td>-20 to +80 °C</td>
</tr>
<tr>
<td>Mechanical preload, N</td>
<td>700 N</td>
</tr>
<tr>
<td>Nominal expansion at +100 V, μm</td>
<td>30 μm</td>
</tr>
<tr>
<td>Maximum pushing force, N</td>
<td>3000 N</td>
</tr>
<tr>
<td>Maximum pulling force, N</td>
<td>700 N</td>
</tr>
<tr>
<td>Electric capacitance, μF</td>
<td>14 μF</td>
</tr>
<tr>
<td>Stiffness, N/μm</td>
<td>100 N/μm</td>
</tr>
<tr>
<td>Resonant frequency, kHz</td>
<td>12 kHz</td>
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</table>
Figure 4.—Frequency response of piezoelectric actuator free-tip displacement to actuator input voltage.

Figure 5.—Frequency response of piezoelectric actuator mounted in test rig with a sine output of 0 to 6000 Hz.
Figure 6.—Application of force via piezoelectric actuators.

Figure 7.—Feed-forward control loop.
Figure 8.—Oscilloscope traces of gear tooth, feed-forward control, and one pulse per revolution signals.
Figure 9.—Casing vibration spectrum as function of frequency without feed-forward control for accelerometers A to D. Gear mesh frequency, 4450 Hz; torque, −149 N·m (−110 ft·lb); gear lubrication, 75 psig.

Figure 10.—Casing vibration spectrum as function of frequency with low-speed shaft actuator feed-forward control for accelerometers A to D. Gear mesh frequency, 4450 Hz; torque, −149 N·m (−110 ft·lb); gear lubrication, 75 psig.
Figure 11.—Sound pressure spectrum with low-speed shaft actuator feed-forward control. Gear mesh frequency, 4650 Hz; torque, –149 N·m (–110 ft·lb); gear lubrication, 75 psig.
Figure 12.—Sound pressure spectrum without low-speed shaft actuator feed-forward control. Gear mesh frequency, 4650 Hz; torque, -149 N-m (-110 ft-lb); gear lubrication, 75 psig.
Figure 13.—Gear mesh vibration and actuator current as function of actuator voltage for accelerometer A with low-speed shaft feed-forward control. Gear mesh frequency, 4650 Hz; phase angle, 240°; torque, −149 N·m (−110 ft·lb); gear lubrication, 75 psig.
Figure 14.—Gear mesh vibration as function of feed-forward control signal phase adjustment for the casing accelerometer A. Gear mesh vibration, 4650 Hz; torque, -149 N·m (-110 ft·lb); gear lubrication, 75 psig.

Figure 15.—Casing vibration as function of frequency without feed-forward control for accelerometers A to D. Gear mesh frequency, 3100 Hz; torque, +149 N·m (+110 ft·lb); gear lubrication, 75 psig.
Figure 16.—Casing vibration as function of frequency with low-speed shaft actuator feed-forward control for accelerometers A to D. Gear mesh frequency, 3100 Hz; torque, +149 N·m (-110 ft·lb); gear lubrication, 75 psig.

Figure 17.—Casing vibration as function of frequency without feed-forward control for accelerometers A to D. Gear mesh frequency, 3750 Hz; torque, +194 N·m (143 ft·lb); gear lubrication, 75 psig.
Figure 18.—Casing vibration as function of frequency with feed-forward control for a two-actuator system for accelerometers A to D. Gear mesh frequency, 3750 Hz; torque, +194 N·m (143 ft·lb); gear lubrication, 75 psig.
# Feed-Forward Control of Gear Mesh Vibration Using Piezoelectric Actuators

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**Authors**: Gerald T. Montague, Albert F. Kascak, Alan Palazzolo, Daniel Manchala, and Erwin Thomas

**Performing Organization**: NASA Lewis Research Center, Cleveland, Ohio 44135-3191 and Vehicle Propulsion Directorate, U.S. Army Research Laboratory, Cleveland, Ohio 44135-3191

**Sponsoring/Monitoring Agency**: National Aeronautics and Space Administration, Washington, D.C. 20546-0001 and U.S. Army Research Laboratory, Lewis Research Center, Cleveland, Ohio 44135-3191


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