Linear Stability Theory
and
Three-Dimensional
Boundary Layer Transition

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1977

The e^N method involves computation of the total amplification of the various instability modes and correlating the transition onset with the most amplified mode.

The general conclusion from various applications of the e^N method is that when fundamental physical effects are properly accounted for, then $N \approx O(9-11)$ is a good predictor of transition for low background disturbances.

The method can also be used to study the effect of various parameters (such as Mach number, pressure gradient, wall heat and mass transfer, etc.) have on transition. However, note the comments on the next page.

THE e^N METHOD FOR TRANSITION PREDICTION/LFC DESIGN

- IN LOW DISTURBANCE ENVIRONMENT, THE e^N METHOD CAN BE USED TO PARAMETERIZE THE EFFECT ON TRANSITION:

  - MACH NUMBER
  - PRESSURE GRADIENT
  - WALL TEMPERATURE
  - WALL MASS TRANSFER
  - SWEEP
  - FLOW HISTORY
  - BODY/STREAMLINE CURVATURE
  - BODY ROTATION/DYNAMICS
  - BLUNTNESS
  - FLOW CHEMISTRY
  - ANGLE OF ATTACK
  - REYNOLDS NUMBER(S)
  - SHOCK WAVES

1978

Linear Stability Theory

There are four different instability mechanisms which are important in the stability of boundary layers. These include TS/first mode, second mode, crossflow and Goertler. The second mode is relevant only at Mach numbers above about 4. The first mode further consists of two different mechanisms, namely viscous (such as TS waves) and inviscid instability due to the presence of generalized inflection points in compressible boundary layers or in flows with adverse pressure gradients.
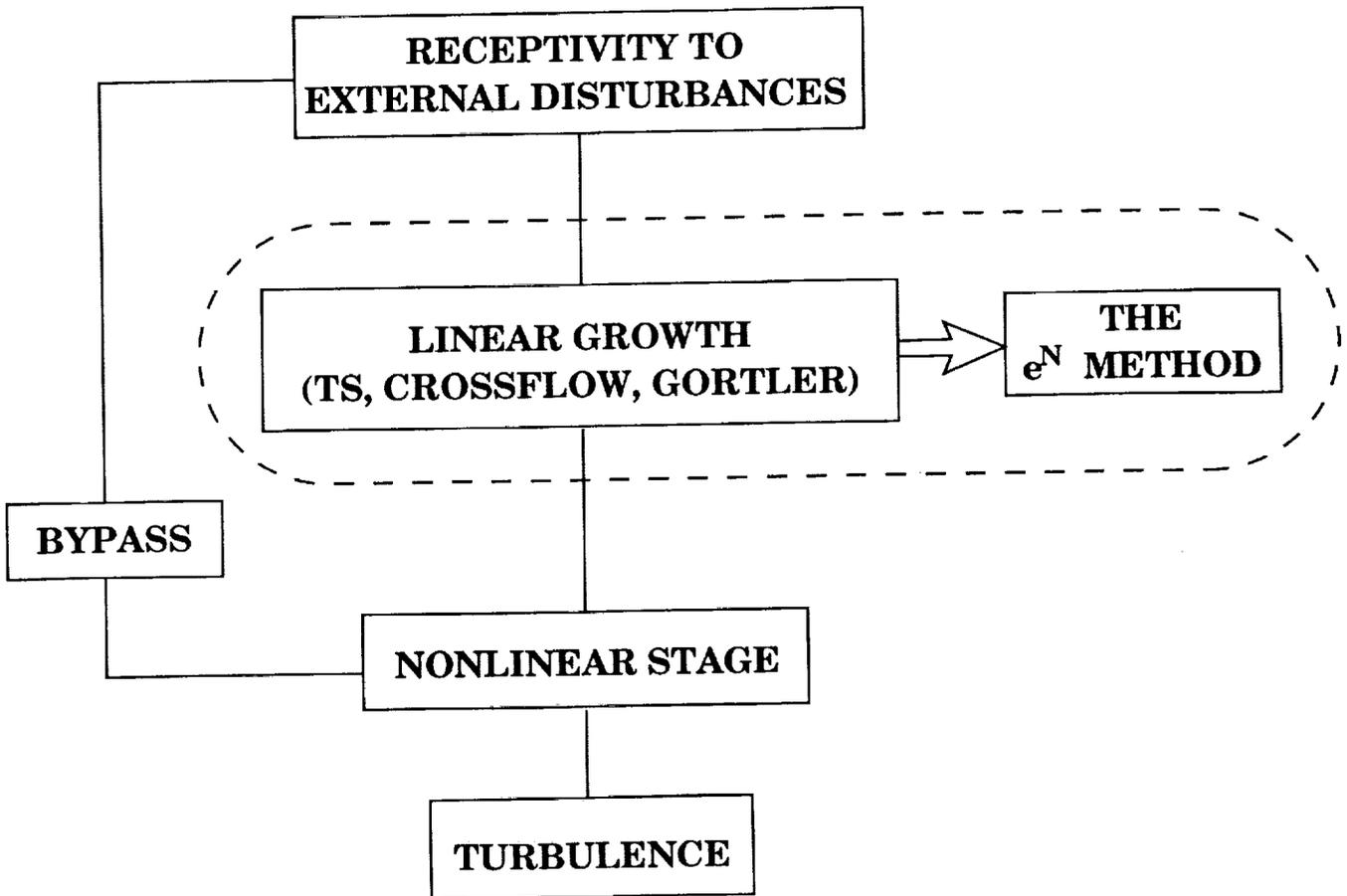
LINEAR STABILITY THEORY

FOUR DIFFERENT INSTABILITY MECHANISMS

- **FIRST MODE**

  - VISCOUS (TS TYPE)

  - INVISCID RAYLEIGH (DUE TO GENERALIZED INFLECTION POINT)

- **SECOND MODE**

  - INVISCID INSTABILITY DUE TO SUPERSONIC MEAN FLOW RELATIVE TO DISTURBANCE PHASE VELOCITY ($|U - C|/a > 1$)

- **CROSSFLOW**

  - INFLECTIONAL INSTABILITY OF THE CROSSFLOW VELOCITY PROFILE

  - PRESENT IN 3-D FLOWS (BODIES AT ANGLE OF ATTACK, ETC.)

- **GORTLER**

  - CENTRIFUGAL INSTABILITY DUE TO CONCAVE CURVATURE (BODY/STREAMLINE)

1979

Transition is a multi-step process involving receptivity (generation of instability waves), linear stability and non-linear breakdown to turbulence. Ideally, one needs to include all three stages in the transition prediction methodology. In this paper, however, we study some aspects of the linear growth of disturbances in both low and high speed boundary layers. In low disturbance environments, results of linear stability theory may be used to correlate the onset of transition with a wide range of parameters such as pressure gradient, Mach number, curvature, nose bluntness and wall temperature.

TRANSITION PROCESS

1980

One always has to keep in mind the limitations that the method is subject to. Since the method is based upon linear stability theory, it obviously cannot account for situations where transition is strongly influenced by factors such as elevated levels of external disturbances, distributed roughness and other non-linear interactions. Furthermore, the effects of parameters such as wall cooling on the secondary instability may be different than on the primary instability and, therefore, the effect on transition of a certain parameter may not be the same as on linear stability.

If good experimental data are available, then it is possible to parameterize these effects in the form of correlations. An example is the correlation developed by Mack [1] for low speed flows to account for the effect of turbulence level on the N-factor at transition.

e^N METHOD - CAUTION

- TRANSITION INFLUENCED BY

  - ELEVATED STREAM/WALL DISTURBANCE FIELDS (INCL. PARTICULATES)

  - DISTRIBUTED ROUGHNESS

  - COMBINATION OF NON-LINEAR DISTURBANCE MODES

  - ORGANIZED MEAN VORTICITY (VORTICES)

  - SHOCK WAVES (EMBEDDED/IMPINGING)

- e^N METHOD CANNOT ACCOUNT FOR THESE EFFECTS

  - EMPIRICAL CORRELATIONS POSSIBLE (E.G. $N = -8.43 - 2.4 \, \ell n \, Tu$, Tu IS TURBULENCE LEVEL, MACK (1977))

Crossflow Reynolds Number Criteria for High-Speed Flows

The value of the crossflow Reynolds number at transition for high-speed flows may be much higher than the upper limit of about 200 for incompressible flows. The value of 200 comes from the correlation of low-speed data, it is necessary to account for the compressibility effect in order to collapse the data from different Mach number flows. This may be achieved in various ways; for example, by defining an effective kinematic viscosity or by computing an effective length scale. Based upon some preliminary studies, we have found that an effective way to account for the compressibility effect is to rescale the characteristic length. Since the boundary-layer thickness δ varies (for adiabatic wall flows) with Mach number as:

$$\delta \propto 1 + \frac{\gamma-1}{2} M_e^2,$$

one way to scale out the Mach number effect is to reduce the crossflow characteristic length scale be a factor $1 + ((\gamma - 1)/2 \, M_e^2$. Thus the effective crossflow Reynolds number may be defined as:

$$\overline{Re}_{cf} = R_{cf}/(1 + \frac{\gamma - 1}{2} M_e^2) \tag{1}$$

The table below shows the values of $\overline{Re}_{cf}$ along the transition onset trajectory for the Mach 8 flow over a 7° half angle cone at 2° incidence. It can be seen that the maximum value of the scaled crossflow Reynolds number is of O(200), i.e., the same as for incompressible flows.

Experiments performed in the NASA Langley Mach 3.5 quiet tunnel show that the unscaled maximum crossflow Reynolds number at transition could be as high as 500-600. However, the scaled $\overline{Re}_{cf}$ from Eq. (1) would be of O(200). Similar results have been obtained for transition in supersonic flow past swept wings. Therefore, for compressible, adiabatic wall flows, it appears that Eq. (1) provides a reasonable *upper limit* for crossflow Reynolds number. Of course, transition may occur at lower $\overline{Re}_{cf}$ due to the influence of other instability mechanisms. The fact that Re_{cf} is much higher for supersonic flows also implies that compressibility has a stabilizing influence on crossflow instability.

Crossflow Reynolds number criteria
for high speed flows

¤ $Re_{cf} = \dfrac{U_n \, \delta_{0.1}}{\nu_e}$

¤ At low speeds correlations show that $Re_{cf} \approx 200$ represents an upper limit for laminar flow

¤ boundary layer thickness varies as:

$$\delta \propto 1 + \frac{\gamma - 1}{2} M_e^2$$

¤ scale out effect of Mach number by defining:

$$\overline{Re}_{cf} = \frac{Re_{cf}}{1 + \dfrac{\gamma - 1}{2} M_e^2}$$

¤ A range of data up to Mach 8 correlates with $\overline{Re}_{cf} \approx 200$

Mach 8 Flow Past a 7° Sharp Cone at 2° Incidence
Re/ft= 1 million

Values of Certain Parameters at the Estimated (N=10) Transition Location				
θ^o	x (ft)	Re_{cf}	$\overline{Re}_{cf}$	f(KHZ)
0	8	0	0	80
48	6	1382	144	40
68	4.7	1690	172	35
110	3.8	2220	213	30
132	3.8	2440	228	20

Linear Stability Calculations for 3-D Boundary Layers

The ability to predict, using analytical tools, the location of boundary-layer transition over aircraft-type configurations is of great importance to designers interested in laminar flow control (LFC). The e^N method has proven to be fairly effective in predicting, in a consistent manner, the location of the onset of transition for simple geometries in low disturbance environments. This method provides a correlation between the most amplified single normal mode and the experimental location of the onset of transition. Studies indicate that values of N between 8 and 10 correlate well with the onset of transition.

For most previous calculations, the mean flows have been restricted to two-dimensional or axisymmetric cases, or have employed simple three-dimensional mean flows (e.g., rotating disk, infinite swept wing, or tapered swept wing with straight isobars). Unfortunately, for flows over general wing configurations, and for nearly all flows over fuselage-type bodies at incidence, the analysis of fully three-dimensional flow fields is required.

In the remainder of this paper we discuss results obtained for the linear stability of fully three-dimensional boundary layers formed over both wing and fuselage-type geometries, and for both high and low speed flows. When possible, transition estimates from the e^N method are compared to experimentally determined locations.

The stability calculations are made using a modified version of the linear stability code COSAL. Mean flows have been computed using both Navier-Stokes and boundary-layer codes.

1984

Linear stability calculations

3D Boundary layers

Low speed flows

¤ Ellipsoid of revolution of fineness ratio 6:1

Mach number = 0.13

Reynolds number = 6.6×10^6

Angle of attack = 10 degrees

Boundary-layer was computed using analytic metric coefficients and edge velocity conditions

¤ Cessna Fuselage

Re/ft=1.3 million

Mach number = 0.27

Comparison with experimental of data of Vijgen.

¤ Flat plate/cylinder configuration

Re/ft = 800,000

$U_\infty = 125.4 ft/sec$

Effects of both adverse and favorable pressure gradients

TS and crossflow instability

1985

Linear stability calculations

3D boundary layers

High speed flows

¤ **Analytic Forebody**

Mach number = 2.0

Angle of attack = 2 degrees

Boundary layer edge conditions computed using space marching Euler option of CFD code GASP

¤ **F16XL Laminar Flow Control Glove**

Mach number = 1.6

Mean flow computed by V. Iyer using Navier-Stokes code CFL3D

¤ **Dagenhart model for NASA Langley "quiet tunnel"**

Mach number = 3.5

Mean flow computed by V. Iyer using Navier-Stokes code CFL3D.

1986

Geometry and Coordinate System for Prolate Spheroid

The linear stability of the fully three-dimensional boundary-layer formed over a 6:1 prolate spheroid at $10°$ is investigated using the linear stability code COSAL. For this case, both Tollmien-Schlichting (TS) and crossflow disturbances are relevant in the transition process. The predicted location of the onset of transition using the e^N method compares favorably with experimental results of Meier and Kreplin [2]. Using a value of N=10, the predicted transition location is approximately 10% upstream of the experimentally determined location. Results also indicate that the direction of disturbance propagation is dependent on the type of disturbance, and consequently, on dimensional frequency. Results also indicate that $Re_{cf} = 180$ represents the upper limit for laminar flow (based on N=10).

Geometry and coordinate system for prolate spheroid.
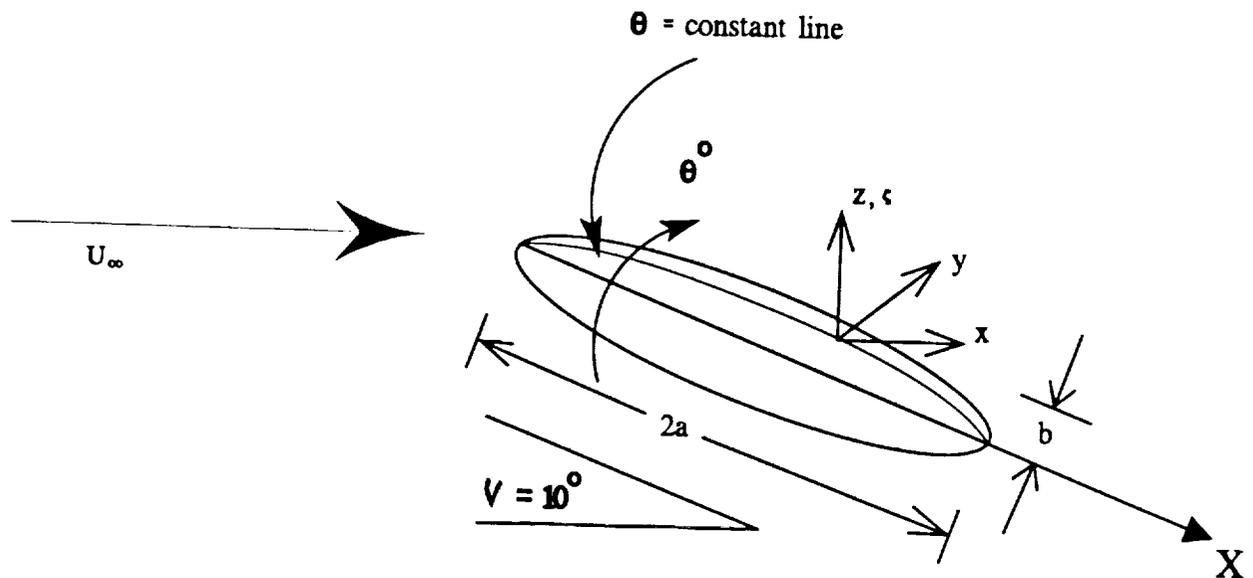
Contour Plot of Constant C_p on 6:1 Prolate Spheroid
M=0.132, angle of attack=10°, Re=6.6×10^6

The analytic inviscid velocity distribution and metric coefficients were used in the solution of the boundary-layer equations. Here we present a contour plot of the distribution of C_p over the ellipsoid. Note that an adverse pressure gradient is encountered at approximately $\xi = -0.9$ on the leeward symmetry line and $\xi = 0.9$ on the windward symmetry line (where $-1.0 \leq \xi \leq 1.0$). This suggests that transition on the leeward symmetry line may take place much sooner than transition on the windward symmetry line, since boundary layers usually become highly unstable in regions of adverse pressure gradient.

Contour plot of constant C_p on 6:1 prolate spheroid
M=0.132, $v = 10°$ and Re = 6.6 × 10^6

1988
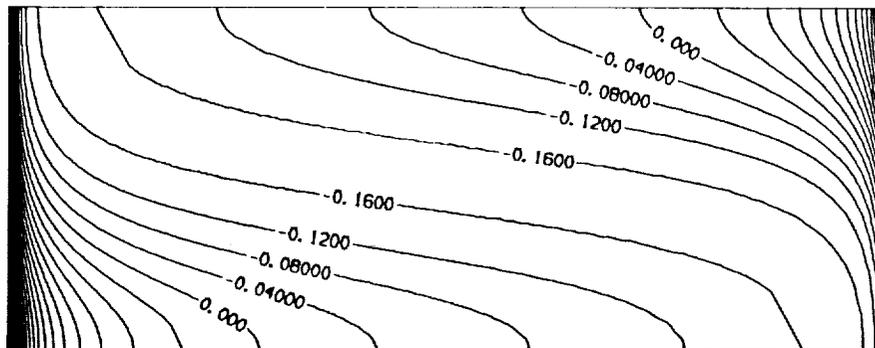
Contour Plot of Crossflow Reynolds Number

The above figure indicates the boundary-layer computational domain, and also shows contours of constant Crossflow Reynolds numbers. The cross-hatched area has been excluded from the domain of the boundary-layer calculation (due to separation). Also indicated is the location of the initial separation point. Since transition takes place upstream of this point, the exclusion of the region is of no consequence here. The figure indicates a rapid increase in crossflow Reynolds number as the separation point is approached. This results from an increase in the crossflow length scale as the region of adverse pressure gradient is encountered near the leeward symmetry line. Note the occurence of a local minimum in the crossflow Reynolds number just upstream of the initial separation point.

Contour plot of crossflow Reynolds number

Initial Separation Point   **Excluded Region**

1989
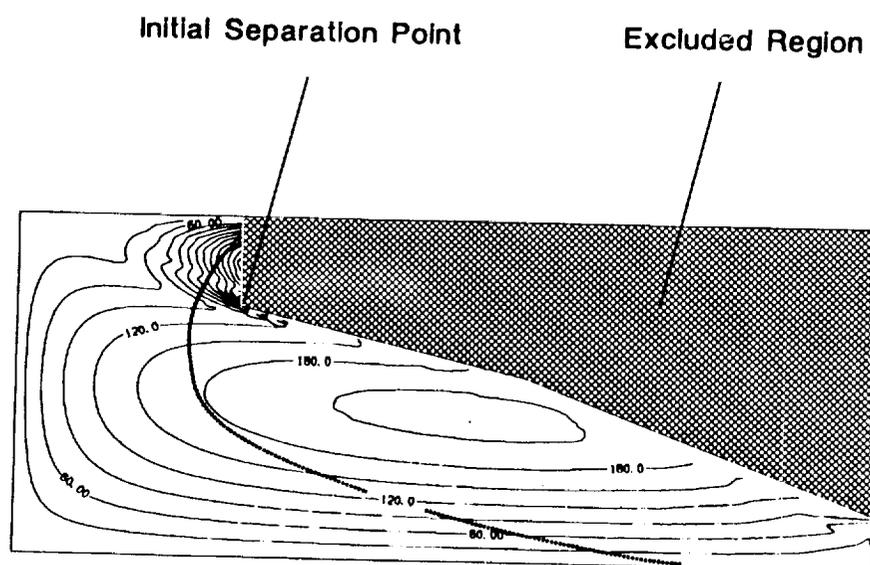
Comparison of Theoretical (based on N=10) and Experimentally Determined Locations for the Onset of Transition

The transition front obtained using COSAL is compared with the experimental results of Meier and Kreplin [2]. Transition was assumed to occur at N=10. The overall agreement between theory and experiment is good. Near the windward edge ($\theta = 0^{\circ}$), where two-dimensional TS-type disturbances are responsible for transition, the predicted location of transition is about 10% downtream of the experimental results. For the flowfield at $\theta > 20^{\circ}$, for which instabilities are predominately of the crossflow type, the predicted transition front occurs approximately 10% upstream of the experimental results. The present results might be improved if the displacement thickness were taken into account when calculating the inviscid solution. In addition, the disturbances originating at higher values of θ follow highly curved trajectories, so that wavefront curvature effects may be important. If these effects were included, they would act in a stabilizing manner, and thus tend to shift the computed transition front downstream.

Comparison of theoretical and experimentally determined locations (Meier and Kreplin) for the onset of transition. Theoretical calculations based on a value of n=10.

1990
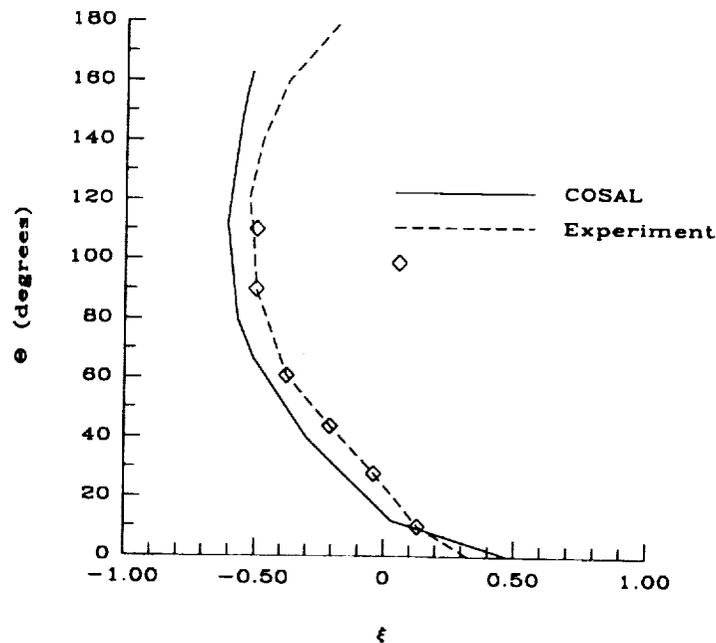
Cessna Forebody Configuration
Typical Inviscid Grid

The linear stability of the fully three-dimensional boundary-layer formed over a general aviation fuselage at 0° incidence is investigated. The free stream velocity was taken as $279 ft/sec$ and the free stream temperature as $T_\infty = 472^o$ R. The unit Reynolds number was 1.3 million. The location of the onset of transition was estimated using the N-factor method. The results are compared with existing experimental data [3] and indicate N-factors of 8.0 on the side of the fuselage and 3.0 near the top. Considerable crossflow exists along the side of the (asymmetric) fuselage, which significantly alters the unstable modes present in the boundary layer. The value of 3.0 along the top may be due to surface waviness, as suggested in Ref [3], where stability calculations using the axisymmetric analog method were performed.
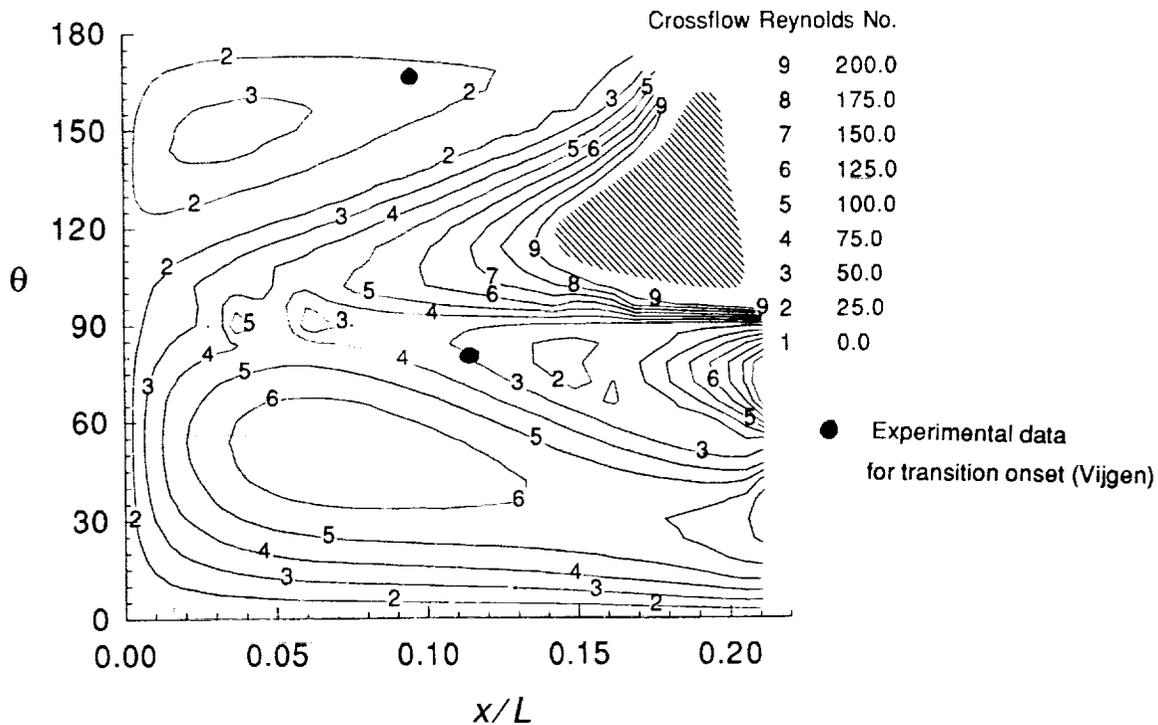
Cessna forebody configuration.
Typical inviscid grid.
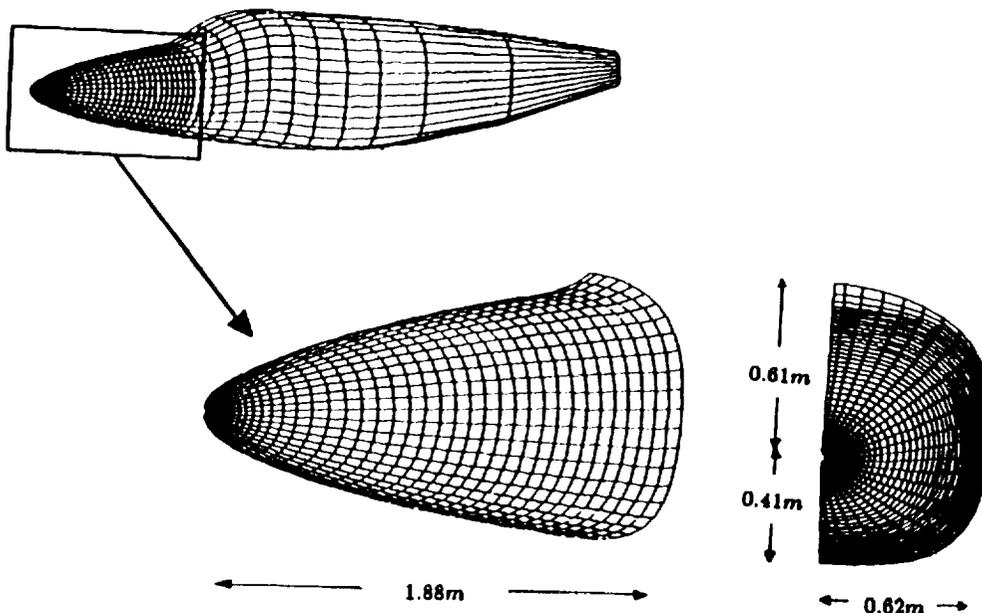
Crossflow Reynolds Number Distribution
for Cessna Fuselage

N-factors computed using linear stability theory are compared with experimentally determined transition location as given in Vijgen [3]. The contours were obtained from a series of calculations originating along neutral curves (for specific frequencies) at successive circumferential locations. Results for the frequencies which first reach N=9 are plotted. These frequencies varies from 1000 Hz in regions of relatively high crossflow, to 1800 Hz in regions of relatively low crossflow. In addition, since the "envelope method" is used, the disturbances which are evaluated at each successive streamwise location represent the most unstable mode. Whether of not this corresponds to the evolution of an actual disturbance within a boundary layer is unknow. The experimental data points, at streamwise points corresponding to transitional flow, are indicated on the figure. The detection of transition onset was determined through surface hot-film anemometry [3]. We also computed a maximum value of N=3.0 at the location of the upper experimental data point. This corresponded to a higher frequency than those which first resulted in N=9.
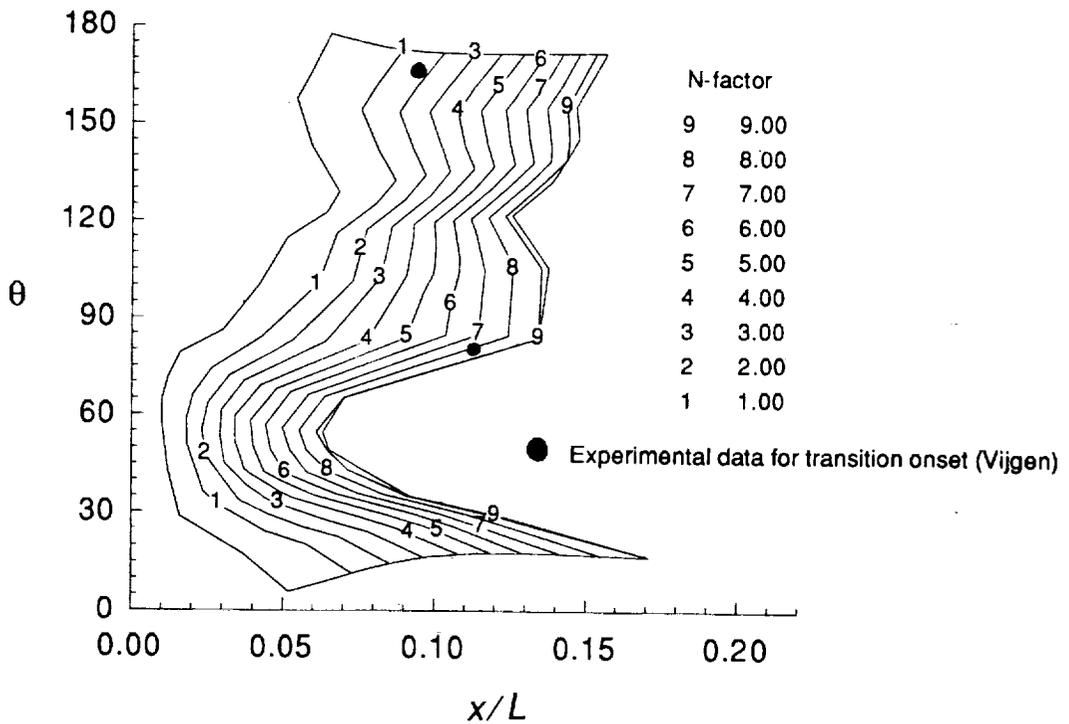
Crossflow Reynolds number distribution
for Cessna fuselage.
Contours levels over 200 omitted.

Crossflow Reynolds No.

9	200.0
8	175.0
7	150.0
6	125.0
5	100.0
4	75.0
3	50.0
2	25.0
1	0.0

● Experimental data
for transition onset (Vijgen)

Cessna Fuselage
Contours of Constant N-Factors
Re/ft=1.3 million, 0° Incidence

See discussion for previous slide.

## Cessna fuselage
Re/ft=1.3 million, angle of attack = 0°
Contours of constant N-factors.

N-factor	
9	9.00
8	8.00
7	7.00
6	6.00
5	5.00
4	4.00
3	3.00
2	2.00
1	1.00

● Experimental data for transition onset (Vijgen)

Crossflow Reynolds Number Distribution for Analytic Forebody

The linear stability of the Mach 2.0 flow over an analytic forebody configuration [4] is investigated. In this case, both first mode and crossflow instabilities are present in the boundary layer. Crossflow Reynolds numbers reach values of over 1000. From the correlation presented earlier, at Mach 2.0, one would expect $Re_{cf} \approx 360$ to represent an upper limit for possible laminar flow. N-factor calculations reveal that along the upper portion of the body, the transition process is likely to be crossflow dominated, since N reaches values of 10 when the crossflow Reynolds number reaches approximately 350. (Note that traces shown in any of the remaining figures represent disturbance trajectories which begin at N=1 and terminate at N=10.) Over the lower portion of the body, the value of the crossflow Reynolds number is in the range of 50-150 at the location where N=10. In this location we conclude that the most amplified disturbances reveal characteristics intermediate between crossflow and first mode instabilities.
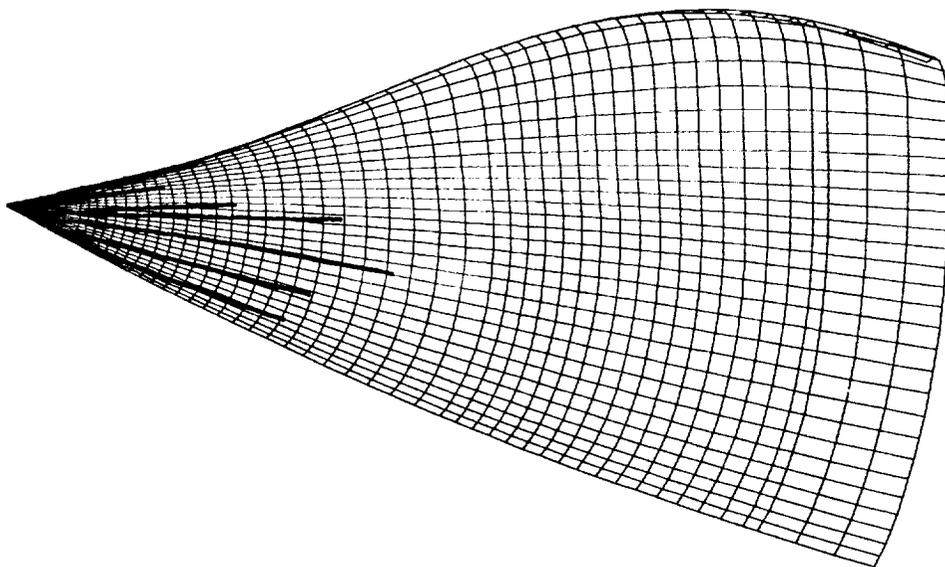
Crossflow Reynolds number distribution for Analytic Forebody. Contour levels above 1,000 omitted.

Crossflow Reynolds No.

1000
900
800
700
600
500
400
300
200
100
0

Analytic Forebody

Mach 2.0, 40,000 ft.

1994
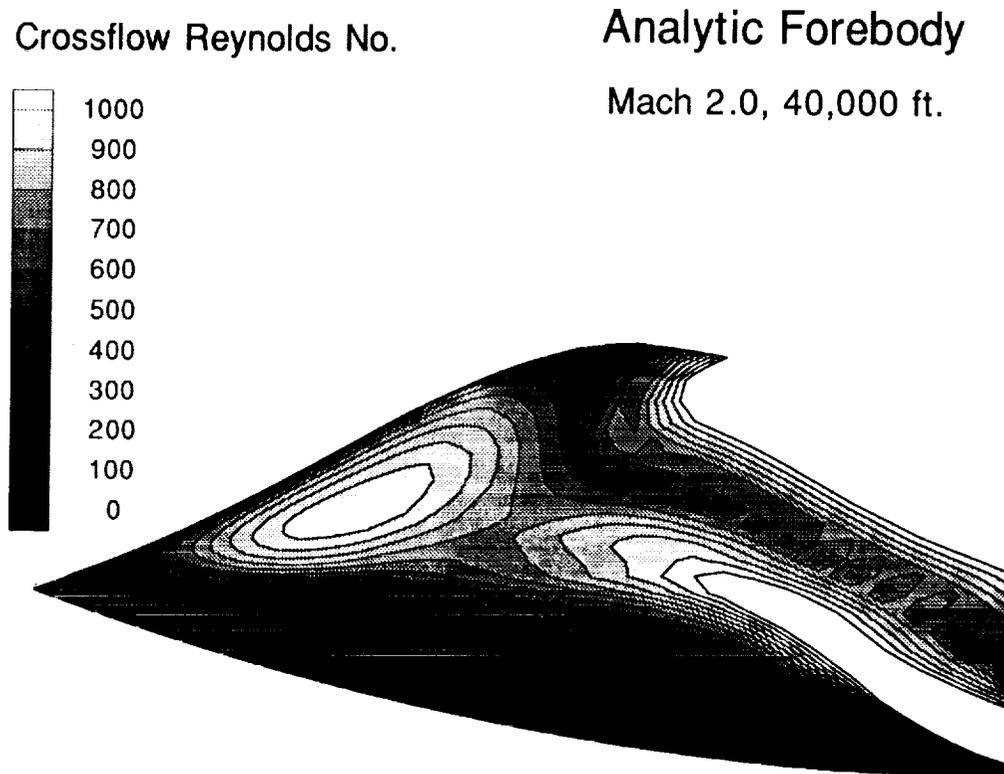
N-Factor Calculations for Analytic Forebody

See discussion for previous slide.

N-factor calculations for Analytic Forebody
Mach 2.0, Altitude=40,000 ft.

N-Factor Calculations for Swept Leading Edge Model for use in LARC Mach 3.5 Quiet Tunnel

Stability calculations for the flow over a highly swept leading edge model to be used for transition studies in the NASA Langley Mach 3.5 Low-disturbance Pilot Tunnel have been performed. The model is a representation of the leading edge of a laminar flow control wing for the F16-XL aircraft [5]. The traces shown in the figure represent disturbances of 40,000 Hz, and the wave angles and wavelengths (not shown) indicate the disturbances are primary of the crossflow type. Additional calculations perfomed for stationary disturbances resulted in maximum values of $N \approx 6$ at the end of the body.

N-factor calculations for swept leading edge model to be utilized for transition studies in LARC Mach 3.5 Quiet Tunnel.

U

1996
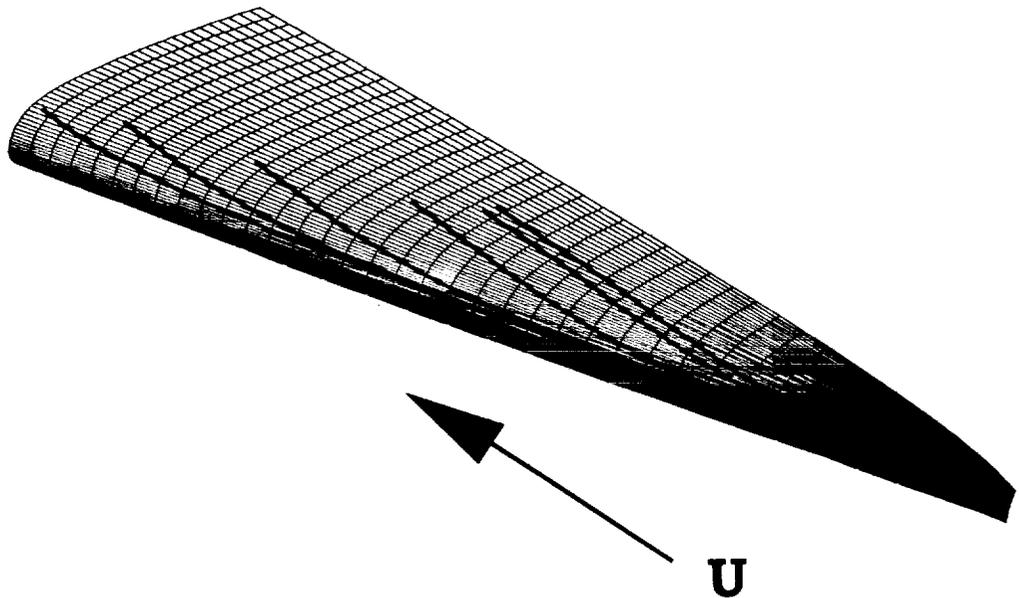
N-Factor Calculations for Laminar Flow Control Glove on F16XL Aircraft

Linear stability/N-factor calculations for the laminar flow glove region for the F16XL fighter aircraft, both with and without boundary-layer suction, have been performed. The results indicate that suction has a stabilizing influence on the boundary layer. The Mach number was 1.6, which indicates an upper limit on the crossflow Reynolds number of ≈ 300. Contours of constant $Re_{cf} \approx 300$ correlate very well with values of N=10 from linear stability theory. To completely laminarize the glove region, surface contouring and/or additional suction will be required.

N-factor calculations for laminar flow control glove on F16-XL aircraft.

Without suction

U

With suction
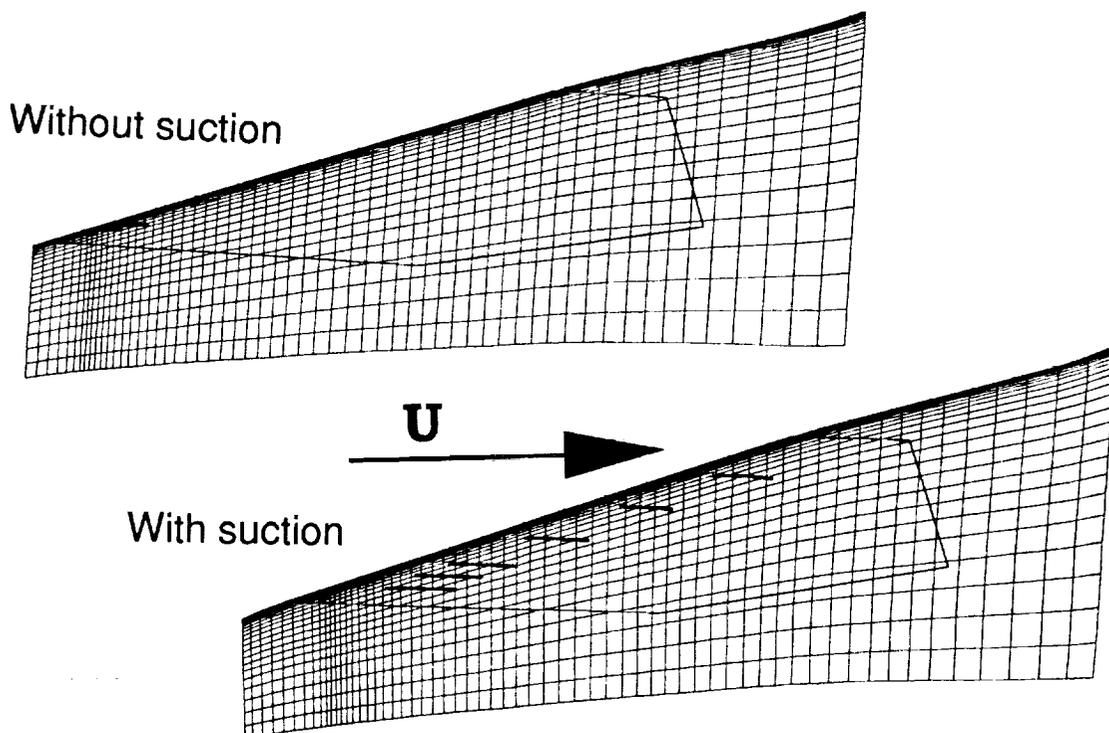
Summary/Conclusions

¤ Completed stability calculations for
Low Speed:
Ellipsoid at incidence
General aviation fuselage
High Speed:
Analytic forebody
Leading edge configuration
F16XL laminar flow control glove area

¤ Linear stability theory/e^N method offers a viable means towards estimating the location of the onset of transition over a wide speed range for both swept-wing and fuselage-type configurations.

¤ Effects of disturbance fields, surface roughness/waviness, etc. not accounted for but may be important (i.e. low value of N on top of Cessna fuselage).

¤ For high-speed flows, compressibility corrections allow for the use of a crossflow Reynolds number criterion in establishing an upper limit for laminar flow.

1998

References

1. Mack, L.M., "Transition Prediction and Linear Stability Theory," in AGARD Conference Proceedings No. 224, pp. 1-22, NATO, Paris.

2. Meier, H.U. and Kreplin, H., "Experimental Investigation of the Boundary Layer Transition and Separation on a Body of Revolution," Zeitscrhrift Fur Flugwissenschaften and Weltraumforschung, Vol. 4, March/April 1980, pp. 65-71.

3. Vijgen, P., Incompressible Boundary-Layer Transition Flight Experiments Over a Nonaxisymmetric Fuselage Forebody and Comparisons with Laminar Boundary-Layer Stability Theory," Doctor of Engineering Dissertation, University of Kansas, December, 1988.

4. Townsend, J.C., Howell, D.T., Collins, I.K. and Hayes, C., "Surface Pressure Data on a Series of Analytic Forebodies at Mach Numbers From 1.7 to 4.5 and Combined Angles of Attack and Slideslip," NASA TM 80062, June 1979.

5. Dagenhart, J.R., Private Communication, Experimental Flow Physics Branch, NASA Langley Research Center, Hampton, VA.

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