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Analysis of Wavelet Technology for NASA Applications

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RECENT ADVANCES IN WAVELET TECHNOLOGY

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1 INTRODUCTION AND OVERVIEW

In this paper I want to report on some recent developments in wavelet technology and, in particular, how it relates to some of the research activities at NASA. First, I want to indicate the nature of our research effort at Rice University in this direction. We have developed over the last four years a **Computational Mathematics Laboratory (CML)** housed in the **Computer and Information Technology Institute (CITI)** at Rice. This laboratory has as its primary focus research in the theory and applications of wavelets and more generally multiscale phenomena in mathematics, science and engineering. The researchers in the CML are:

- R. O. Wells, Jr., Professor of Mathematics (Rice), Director of CML
- C. S. Burrus, Professor of Electrical and Computer Engineering (Rice) and Director of CITI
- W. W. Symes, Professor and Chairman of Computational and Applied Mathematics (Rice)
- Roland Glowinski, Cullen Professor of Mathematics, University of Houston
- 4 Post Doctoral Fellows and 5 Graduate Students
- *Principal Support:* ARPA, NASA, Aware, Inc., Texas Instruments, Texas Higher Education Coordinating Board

Wavelet research has been developing rapidly over the past five years, and in particular in the academic world there has been significant activity at Rice, Yale, MIT, Delaware, Brown, S. Carolina, Washington Univ., Minnesota, Dartmouth, and numerous other universities. In the industrial world, there has been developments at Aware, Inc., Lockheed, Martin-Marietta, TRW, Kodak, Exxon, and many others. The government agencies supporting wavelet research and development include ARPA, ONR, AFOSR, NASA, and many other agencies. The recent literature in the past five years includes a recent book [6] which is an index of citations in the past decade on this subject, and it contains over 1,000 references and abstracts.

2 WAVELET MATHEMATICS

Fundamentally, wavelets are a new type of function which provide an excellent orthonormal basis for functions of one or more variables. They provide a localized basis, and can represent square-integrable functions, but also constant and, more generally, polynomial functions in a locally finite manner.

In 1988 Daubechies' fundamental paper on wavelets [1] appeared. In this paper we find for the first time a parametrized family of orthonormal systems of functions in $L^2(\mathbb{R})$ with certain important complementary properties. Each system of functions has the following properties:

- each system is generated from a scaling function $\varphi(x)$ and a wavelet function $\psi(x)$ by rescalings (by powers of an integer) and translations (e.g., $\varphi_{j,k}(x) := 2^{j/2}\varphi(2^j x - k)$ and $\psi_{j,k}(x) := 2^{j/2}\psi(2^j x - k)$). The wavelet system

$$\{\psi_{0,k}(x), \psi_{j,k}(x), \quad j, k \in \mathbb{Z}, \quad j \geq 0\} \quad (1)$$

is an orthonormal basis for $L^2(\mathbb{R})$ and more general functions as well (including constants and higher order polynomials, depending on the wavelet system chosen).

- each element in a given system has compact support and is continuous or can be chosen to be smooth up to a given finite order and by the rescaling above, the supports of the basis functions becomes very small for large scaling index j .
- There are fast algorithms for computing the coefficients of the expansion of a given digitized (sampled function). This is the *discrete wavelet transform* (from the sampled function to the wavelet expansion coefficients), and it is an $O(N)$ algorithm.
- The classical discrete Fourier and cosine transforms appear as a special case of the general discrete wavelet transform (DWT)

- The discrete wavelet transform is parallelizable and can be implemented on massively parallel machines as well as can be designed into specialized VLSI chips (e.g., for digital video editing).

In general a scaling function and corresponding wavelet function satisfy the *scaling equation*

$$\varphi(x) = \sum_{k=0}^{2g-1} a_k \varphi(2x - k) \quad (2)$$

and the corresponding *wavelet defining equation*

$$\psi(x) := \sum_{k=0}^{2g-1} b_k \varphi(2x - k), \quad (3)$$

where the coefficients of the scaling equation a_k must satisfy linear and quadratic constraints of the form:

$$\sum a_k = 2, \quad (4)$$

$$\sum a_k a_{k+2l} = 2\delta_{l,0}, \quad (5)$$

and where $b_k := (-1)^{k+1} a_{2g-1-k}$.

One of the powers of wavelet technology is the ability to choose the defining coefficients for a given wavelet system to be best adapted to the given problem. Daubechies developed in her original paper [1] specific families of wavelet systems which had maximal vanishing moments of the ψ function and which were very good for representing polynomial behavior. In Figure 1 we see the corresponding Daubechies scaling and wavelet function for the case of 4 coefficient ($g = 2$) where

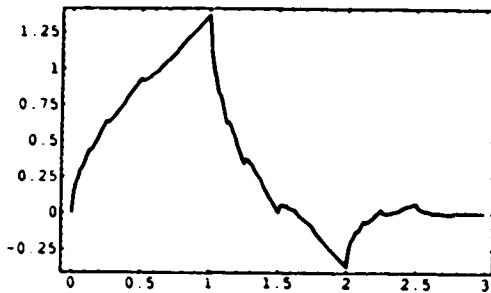
$$\{a_0, a_1, a_2, a_3\} = \left\{ \frac{1+\sqrt{3}}{4}, \frac{3+\sqrt{3}}{4}, \frac{3-\sqrt{3}}{4}, \frac{1-\sqrt{3}}{4} \right\} \quad (6)$$

and

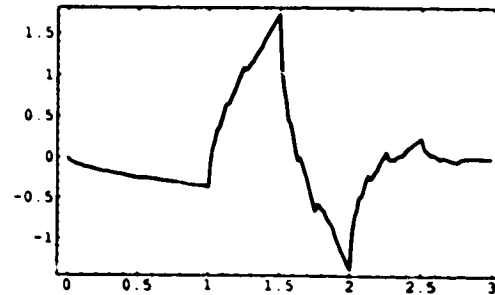
In Figure 2 we see the contrast between the Fourier representation and wavelet representation for a given example of a transitory signal, and that the wavelet representation does provide a superior representation for this particular example.

3 WAVELET MULTISCALE REPRESENTATION OF DATA

If we consider such a wavelet system, and assume that there is a certain amount of smoothness (C^2 , for instance), then we can try to use these functions as basis elements for representing discrete data at different scales.

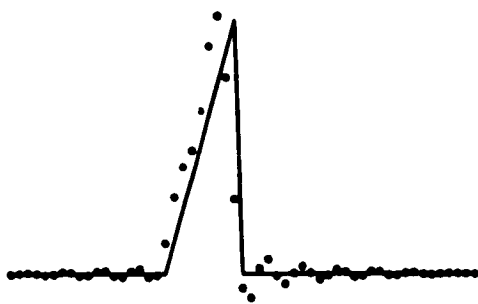


Scaling Function

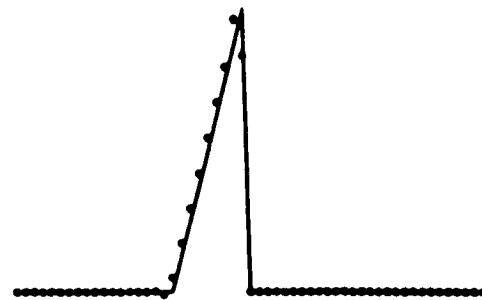


Wavelet Function

Figure 1: On the left is the 4-coefficient Daubechies scaling function and on the right is the corresponding wavelet function



27-term Fourier



27-term Wavelet

Figure 2: Comparison of a wavelet and Fourier representation of a transient signal

Namely, if we let for fixed $J \in \mathbb{N}$,

$$\tilde{f}(x) = \sum_k c_{Jk} \varphi_{Jk}(x), \quad (7)$$

where c_{Jk} represents a sampling of a given function $f(x)$ at the points $x = k/2^J$, then $\tilde{f}$ is a smooth wavelet interpolation of our original sampled $f(x)$ at the scale J (or, what is the same thing, on a mesh with mesh size $h = 1/2^J$). Mallat [5] showed that from the scaling equations defining φ and ψ one can reexpress $\tilde{f}$ in terms of scaling and wavelet functions at coarser scales, namely:

$$\tilde{f}(x) = \sum_k c_{Jk} \varphi_{Jk}(x) = \sum_k c_{0k} \varphi_{0k}(x) + \sum_k \sum_{j=0}^{J-1} d_{jk} \psi_{jk}(x). \quad (8)$$

In (8) we see that the left hand side (LHS) represents the data at a single "fine" scale J , while the right hand side (RHS) gives a multiscale representation of the data at the coarser scales $\{0, 1, \dots, J-1\}$. The Mallat transform consists of mapping the coefficients at the single scale on the LHS of (8) to the multiscale coefficients on the RHS of (8), and conversely (inverse Mallat transform). This transform consists of convolution with the filters which define the scaling and wavelet functions along with downsampling (and upsampling for the inverse transform).

4 IMAGE COMPRESSION AND TELECOMMUNICATIONS TECHNOLOGY

A major application of wavelets to technology has been in the area of data compression. The following list indicates the breadth of this application area. In each case the compression ratios indicated are what is roughly currently available, and are all products of Aware, Inc., of Cambridge, Mass., which is the leading commercial supplier of wavelet-based compression algorithms, in the form of software, chips, and plug-in boards for various application areas. Moreover, the compression ratio indicates compression to a version of the original signal which is indistinguishable from the original signal for the purposes at hand, and has been verified and tested by the industry experts in that given area. As one example, audio compression, listed at 8:1 compression ratio, has the property that the human ear cannot normally distinguish the compressed signal from the original, and the compression algorithm uses information about how the ear perceives sound and at what frequency scales.

- Audio compression - high fidelity at 8:1
- Still-image compression 20:1 (BW), 100:1 (Color)

- Seismic compression 20:1
- Radiology images 20:1
- Fingerprint images 25:1
- Video compression (color) 140:1

The basic idea in a compression algorithm in all of the above examples is to represent the digitized signal in terms of a wavelet expansion (the coefficients of this expansion will be the Discrete Wavelet Transform). Using a statistical analysis of the data type involved one carries out a systematic dropping of bits of these wavelet expansion coefficients at specific scales (this is the quantization process) to represent the same signal effectively with less bits, and an additional lossless compression is then applied to the result, which can then be either transmitted or archived. To recover the signal, one reverses the process with the exception of the quantization step, as those bits cannot be recovered. For further details about this compression process in the context of images, see, e.g., [12], and more information about specific technologies in all of the areas above is available, in particular, from Aware, Inc. in Cambridge, Mass.

One important feature of all of these algorithms is that one can download a compressed signal (or even an uncompressed signal represented in terms of its DWT), at any desired scale to obtain "snapshots" of the data, and download additional information later (or in the case of audio, to increase the fidelity at a later time). This technology is undergoing rapid development at the present time, and there is still much to be learned and understood in terms of modeling these compression ideas.

A second important area in which the DWT has played an important role is that of Asymmetric Data Subscriber List (ADSL) technology. This is the basic copper wire twisted-pair communications link between American homes and their telephone companies. The spectral bands of this communication link are divided into three regions, the lowest being POTS ("Plain Old Telephone Service"), the second being a band for sending conventional digital data (linking computers for instance), and the high end of the band is reserved for digital video communication. The problem was that this was such a noisy channel that it was difficult to send video signals over this band in a meaningful manner. Very recently, Aware, Inc. announced a partnership with Analog Devices (a second Boston area company) to build transceivers which will implement such video communication in an effective manner, and this will be marketed to the telephone industry by an Alliance involving this partnership plus Westel, Newbridge, and AT&T, all of whom are involved in various aspects of the telecommunication industry. The technical report [8] which will appear soon in the proceedings of the International Communications Conference to be held in New Orleans in 1994 gives further information about this new advance in wavelet communications technology.

5 WAVELET-BASED NUMERICAL SOLUTIONS OF DIFFERENTIAL EQUATIONS

The wavelet representation of a sampled function of the form (7) allows one to use the scaling functions at a given scale (in this case at the arbitrary scale J corresponding to a mesh size of $h = 1/2^J$) as finite-element or Galerkin-type basis elements in a discrete approximation to some continuous problem (e.g., solving a partial differential equation numerically). In a number of recent papers these ideas have been carried out for various types of elliptic boundary value problems [11,10,9,2,4]. In addition one can use the multiscale representation of data as given in (8) to implement multigrid iterative schemes for solving such elliptic boundary problems where the solution by direct methods or by iterative methods at a single scale is prohibitive. In particular, one obtains an efficient multiscale algorithm for solving the model problem involving Laplace's equation for a domain with a very general boundary [3]. Moreover, a second model problem involving anisotropic coefficients in two dimensions with periodic boundary values admits a robust multigrid algorithm whose condition number is independent of the mesh size and of the anisotropy parameter [7]. In these multigrid applications of wavelets to numerical analysis the linear Mallat algorithms (transform from single scale to multiscale and conversely) play a major role. They allow one to map simply from one adjacent scale to another in a very effective manner, and that, along with the implicit orthogonality (and hence lack of redundancy) is one of the keys to their success in this applications (which is also true in their application to digital signal processing).

References

- [1] I. Daubechies. Orthonormal bases of compactly supported wavelets. *Comm. Pure Appl. Math.*, 41:906-966, 1988.
- [2] R. Glowinski, J. Periaux, M. Ravachol, T. W. Pan, R. O. Wells, Jr, and X. Zhou. Wavelet methods in computational fluid dynamics. In et al. M. Y. Hussaini, editor, *Algorithmic Trends in Computational Fluid Dynamics*, New York, pp. 259-276, 1993. Springer-Verlag.
- [3] R. Glowinski, A. Rieder, R. O. Wells, and X. Zhou. A wavelet multigrid preconditioner for Dirichlet boundary value problems in general domains. Computational Mathematics Laboratory TR93-06, Rice University, 1993.
- [4] Roland Glowinski, T. W. Pan, R. O. Wells, and Xiaodong Zhou. Wavelet and finite element solutions for the Neumann problem using fictitious domains. Technical Report 92-01, Rice University, 1992. Computational Mathematics Laboratory.

- [5] S. Mallat. Multiresolution approximation and wavelet orthonormal bases of $L^2(\mathbb{R})$. *Trans. Amer. Math. Soc.*, 315:69-87, 1989.
- [6] S Pittner, J. Schneid, and C. W. Ueberhuber. *Wavelet Literature Survey*. Institute for Applied and Numerical Mathematics, Technical University Vienna, Wien, Austria, 1993.
- [7] A. Rieder, R. O. Wells Jr., and X. Zhou. A wavelet approach to robust multilevel solvers for anisotropic elliptic problems. *Applied and Computational Harmonic Analysis*, 1994. To appear.
- [8] Michael A. Tzannes, Marcos C. Tzannes, John Proakis, and Peter N. Heller. DMT systems, DWMT systems and digital filter banks. Conference Proceedings of ICC '94, New Orleans. IEEE, 1994. (Aware, Inc. Technical Report AD940308).
- [9] R. O. Wells and Xiaodong Zhou. Representing the geometry of domains by wavelets with applications to partial differential equations. In J. Warren, editor, *Curves and Surfaces in Computer Graphics III*, volume 1834, pages 23-33. SPIE, 1992.
- [10] R. O. Wells and Xiaodong Zhou. Wavelet solutions for the Dirichlet problem. Technical Report 92-02, Rice University, 1992. Computational Mathematics Laboratory.
- [11] R. O. Wells and Xiaodong Zhou. Wavelet interpolation and approximate solutions of elliptic partial differential equations. In R. Wilson and E. A. Tanner, editors, *Noncompact Lie Groups*. Kluwer, 1994. To appear. Proceedings of NATO Advanced Research Workshop.
- [12] W. Zettler, J. Huffman, and D. Linden. Application of compactly supported wavelets to image compression. In *Proc. SPIE: Image Processing Algorithms and Techniques*, volume 1244, pages 150-160. Intl. Soc. for Optical Engineering, 1990.

1. Introduction

The purpose of this grant was to introduce a broad group of researchers and administrators at NASA to the nature of the new, important, and fast evolving area of wavelet technology and at the same time to determine research and development areas at NASA JSC where wavelet technology might possibly play an important role in the future, and to determine possible future specific areas of collaboration between the Computational Mathematics Laboratory at Rice University and NASA JSC.

In this report we bring together several of the progress reports submitted periodically to the NASA liaison, Dr. Kumar Krishen, during the course of this grant, and give an overall summary of the progress towards the goals of the grant in the Conclusion of this report.

2. Briefings

Report of 11 August 1993

I am reporting to you on various briefings which have taken place over the past several months between NASA scientists at the Johnson Space Center and Members of the Computational Mathematics Laboratory (CML) at Rice University.

These briefings and the activities related to them are part of the planned activities by the Rice CML as outlined in the currently active NASA JSC grant to the Rice CML entitled: *Analysis of Wavelet Technology for NASA Applications*. This report is a progress report under the auspices of this grant.

Initially there were general briefings for a team of NASA scientists chosen by you. In a memo from you dated 15 April 1993 the Wavelet Technology Working Group (WTWG) was chosen:

Richard Juday/EE621

John Sunkel/EG241

Chien Li/EG3

Dave Homan/ER

Dean Glenn/IC

Tim Cleghorn/PT41

Dave Proctor/SE3

Kumar Krishen/IA4 Chairman

There were two general briefings given by myself for members of this team and other invited participants.

General Briefing 1: (4 May 93 at NASA JSC)

General Briefing 2: (7 May 93 at NASA JSC)

At these two briefings I outlined the general background of wavelet theory and its recent applications to and the major developments in the newly emerging wavelet technology. The major properties of the wavelet representation of functions included:

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Localization

Orthogonality

Multiscale representation

I showed in various examples how these three properties together allow a given signal (or function) in one or more dimensions to be represented in a wavelet series (similar to a Fourier series), and that one could manipulate the function in question by manipulating the coefficients in the expansion. In particular, for compression applications, one could delete some of the less relevant coefficients in a systematic manner so that the original image or function was still adequately represented by the remaining wavelet coefficients for the purpose at hand.

More generally, I indicated the research activities in digital signal processing and in the numerical solution of partial differential equations at the Rice CML, as well as wavelet technological developments on the commercial side at Aware, Inc., a leading commercial developer of wavelet technology. Copies of the transparencies used in the presentation were made available for the NASA staff. In addition, I showed samples of image compression of still images and of a video segment which had been developed by Aware, Inc., all of which illustrated the power of wavelet technology in the general area of data compression.

The next general briefing was a more specialized one concerned with numerical solutions of partial differential equations for members of the WTWG interested in this more specialized topic.

General Briefing 3: (15 June 93 NASA JSC)

This briefing was given by Professor Roland Glowinski of the University of Houston, who is a Member of the Rice CML and myself. We jointly presented an outline of recent work at the CML and at Aware, Inc. concerning the numerical solution of partial differential equations concerning: solutions of elliptic boundary value problems with very general boundaries, progress in the areas of solutions to Euler flow in the general study of turbulence (work of John Weiss at Aware, Inc.), developments in the implementation of multigrid (using the multiscale nature of the wavelet representation of functions) and of using the localization properties of wavelets to implement the numerical solution code on parallel computers. A film prepared by John Weiss at Aware, Inc. was shown which illustrated the dynamic Euler flow solution for various boundary condition.

The next four briefings were specific technical briefings with WTWG members and others on special topics. The purpose of these technical briefings was to allow the WTWG members to specify in some detail for the CML at Rice problem areas in which wavelet technology might be useful for them in their respective research and development activities at NASA JSC. This involved in each case a discussion with members of the CML and NASA personnel the nature of the current NASA activity, the techniques now being used in the given area, and the problem areas where the NASA scientists thought that the wavelet ideas, as exposed to them in the General Briefings earlier, might be helpful to them.

Technical Briefing 1: (23 July 1993 at Rice Univ.) *Computational Fluid Dynamic*

Participants: Chien Li (WTWG) NASA JSC, R. O. Wells, Jr. Roland Glowinski, Andreas Rieder, Xiao-dong Zhou, Jun Tian Rice CML and Rick Sanders of the Univ. of Houston

C. P. Li's presentation focused on problems in Computational Fluid Dynamics of interest to NASA JSC. From his presentation we can extract two classes of problems which deserve further investigation, namely:

Transonic flow of compressible inviscid and viscous fluids.

Such a flow is encountered during the taking off of the space shuttle. More specifically, the orbiter plus the shuttle rocket system is treated currently as two separate transonic problems with their own adapted mesh coordinate systems which are linked together in a specific ad hoc manner. The resulting flow problem is quite difficult due to the complicated geometry of the problem.

Hypersonic reentry of the orbiter and other space vehicles.

The equations modeling the reentry are either the compressible Euler or Navier-Stokes equations for compressible gases, coupled to chemical equations.

Both problems have been addressed by finite element, finite volume and finite difference methods. Concerning steady flow calculations, it is known that multigrid methods have been quite successful at computing compressible fluid flow via finite-difference, finite-element or finite-volume approximations. On the other hand wavelet approximations have built-in properties making them very suitable for multilevel solutions and adaptive refinement. In particular, it was noted that the wavelet methods being developed at the CML involve code which will work for very general boundary conditions, without having adapted elements near boundary curves, and that this might very well be highly suited for the complicated geometry of shuttle rocket system with its attached orbiter.

Dr. Richard Sanders of the University of Houston is one of the leading experts for the numerical simulation of highly compressible, reactive inviscid or viscous flow by finite-difference and finite-volume methods. Because of his expertise, he had been invited to participate in the briefing and was an active participant in the meeting. He is presently working on multigrid methods for the above-mentioned flow problems. As discussed at the meeting, a cooperative effort between the Rice CML and Sanders is being planned for future activities in this problem area. The extension to wavelet methods would be a natural extension of his previous work and his expertise would enhance the overall effort.

This was his first exposure to some of the details of the wavelet methods for multigrid and for general boundary value problems as was presented at the briefing by Rieder and Zhou, and he was very encouraging about the very genuine potential of this new approach to these problems with which he was quite familiar with from his own point of view.

Indeed, preliminary results by researchers at Aware, Inc. and the Rice CML show that wavelet approximations provide naturally (i.e., without upwinding) high quality solutions for flow with shock, shear layers, etc., suggesting that these methods have a high potential for the simulation of flow at high Reynolds number and to study, for example, the transition from the laminar to the turbulent regime.

In summary it was felt that it would be very worthwhile to develop a research effort in these two specific problem areas.

Technical Briefing 2: (23 July 1993) *Virtual Reality*

Participants: Tim Cleghorn (WTWG), Bowen Loftin and Gordon Johnson NASA JSC and R. O. Wells, Jr., Rice CML

The staff at NASA JSC concerned with NASA's virtual reality project demonstrated and explained their system. The overall system being developed involved sight, sound, temperature, and pressure sensations for the individual participant. The demo in operation concerned itself with fundamentally the visual aspect. Graphical images in binocular viewing elements were linked with computer graphics images created on Silicon Graphics workstations. Motions of the head and right hand would generate motion of the images due to feedback from sensors on the head and hand to the computer system and one could see the right hand as it performed actions in space (such as pushing on a button to open a panel). The action of the left hand simulated the motion of an astronaut in space due to the action of a rocket attachment to a space suit. The overall effect was very impressive, but there are still problems to be addressed.

In addition samples from Aware, Inc. of wavelet image compression for both still and video images was demonstrated for the NASA personnel who had not seen them before. There was a lengthy discussion of the possibilities for using wavelet representation for the images in the virtual reality pictures, to enhance the zooming capabilities, to allow for improved data management, and to perhaps significantly speed up the process to make it the virtual reality environment more realistic. At present there is a slight lag in the images due to the processing time. The possibility of using a wavelet transform hardware accelerator was also discussed in this context. The consensus was that there was a strong possibility that the new wavelet technology could help with the technical problems in this area. In addition, it was suggested that the Mission Operations Directorate (MOD) and the Mission Evaluation Room (MER) be contacted for possible use of wavelet technology in their ongoing operations due to the immense amount of incoming data they deal with on a regular basis.

Technical Briefing 3: (25 July 1993) *Digital Signal Processing*

Participants: Richard Juday (WTWG), Jim Dalles, Doug Holland, Speight Rimes, Denise Richlin, Terry Kelly, and Leonard Helley, Dave Pitts NASA JSC and R. O. Wells, Jr. Rice CML

Those present represented responsibilities for analyzing tests for communications systems and links, data compression, HDTV, algorithms, and printers and transmission of text and graphics. After an initial discussion of the major responsibilities of each of those present, a brief discussion of wavelet technology and the nature of wavelet data compression and the progress in this area was presented to all present. There was a general discussion of these topics with a number of technical questions. The comparisons with JPEG and other standards like MPEG II were discussed. A major focus was on data compression and on HDTV. After a general discussion there was an adjournment to another room to see the Aware, Inc. video compression demo. The consensus of the experts present was that they had not seen data compression at that level of compression (140:1) of such high quality. They were very interested in seeing NASA get more involved in understanding wavelet compression and seeing it possibly be used at NASA both for compression and scalable

control of various data types in the context of general data compression problems as well as specifically for HDTV.

Doug Holland suggested that it would be useful to talk to Dave Pitts and Chris Dailey who dealt with NASA images, especially photographic images and their digitization and archiving, as well as making some images available on-line. A meeting was immediately set up with Dave Pitts and a lengthy discussion of wavelet technology and images was held in his office. He has some 165,000 pictures to store, to digitize, and to make available on line as possible. Currently he has some 500 pictures digitized for the data base for which there are currently 3000 logins per month. He showed samples of CD-ROM discs with a number of NASA photos. The disks had been created by Kodak on a commercially available machine which automatically digitized the photos and created the CD-ROM disk from photographs provided by NASA. The disk included 100 images per CD-ROM disk, and included a compression ration of 4-1 with 5 different resolutions of each image possible. Currently he has 20 gigabytes of images on line. I showed him some image compression samples from Aware, Inc, and discussed the new Aware, Inc. CD-ROM Disk which compressed audio signals at a ratio of 8-1, and pointed out that image disks of a similar nature but with built in high-order compression, was feasible for the same purposes as the CD-ROM disk from Kodak. We had a long-ranging discussion of various ranges of problems and how wavelet technology might interact with it. He indicated he would be very interested in learning more about the possibilities of wavelet technology (which he had heard of previously) benefiting the problem areas in his specific area of responsibilities.

Technical Briefing 4: (29 July 1993) *Structural Engineering*

John Sunkel (WTWG), Tim Cao, James Smith, and Jim McMahon NASA JSC, R. and Glowinski and R. O. Wells, Jr. Rice CML

The three engineers gave an overview of their problem areas. Cao had the responsibility for the larger scale structural dynamics problems as a whole for the shuttle, and the other engineers had a family of more localized stress-strain problems which were the result of pointers which were a result of the global analysis. The general problems they considered involved 50,000 degrees of freedom, and required significant supercomputer time for their solution. During the discussion at the briefing, it appeared that structural engineers at NASA JSC largely rely on NASTRAN for their static and dynamic calculations. Problems of interest concern stress calculations in general, or at and near cracks, complex structures combining shells, plates, beams and rods. It is clear that wavelet methods have not reached the versatility and flexibility of finite-element methods that have been applied to structural calculations for about forty years, this being a major focus of aerospace engineering. In this field wavelet systems can offer their multiscale properties, their suitability to multilevel solution methods, and the ability to solve boundary value problems with very general and quite arbitrary shaped boundary regions in which the coding of the general solution is to a larger extent independent of the boundary shape. In particular it is felt that these multiscale and localization properties will make the wavelet methods well suited to crack and composite material investigations. On the other hand, due to the extreme variety of boundary conditions encountered in engineering applications, we think that the wavelet methodology can benefit from interacting with structural mechanics.

From the conversation with the NASA JSC specialists several model problems appeared worthy of further investigation. Among them the static and dynamics of flexible beams was mentioned in particular since these structures plays an important role in two-

dimensional statics (remote manipulator systems of the space station and of the space shuttle). In addition the problem of computing displacements and stresses for a stretched plate with a hole was considered as a good test case for feasibility studies and also to demonstrate the validity of the wavelet paradigm in view of structural mechanics applications.

Report of 4 November 1993

I am writing to summarize our three-way meeting on 12 October 1993 at NASA.

The purpose of the meeting was to discuss strategy for developing a joint research, development and implementation effort concerning wavelet technology involving Rice University's Computational Mathematics Laboratory (CML), Aware, Inc., and NASA. This meeting was a follow-up to the previous seven technical briefings carried out under the auspices of my current NASA grant and summarized in my memo of 11 August 1993 to Kumar Krishen, and a number of previous briefings at NASA JSC by Rice and Aware, Inc.

Huffstetler discussed various parts of NASA as a whole which might be interested in helping support wavelet technology research, development or implementation. This was in addition to the areas at Johnson Space Center which have already been identified and which were a part of the earlier briefings.

Langley, Ames and Lewis (Aeronautics): Ray Hook would be a principal contact, and this would be an area interested in many aspects of wavelet technology, including communications, compression and simulation (computational fluid dynamics, thermal profiles, and data extraction

Kennedy Space Center: There is a large amount of data describing various aspects of the launch process, and a serious effort at data compression for this effort could possibly have a major economic enhancement on the operation.

NASA Headquarters in Washington: John McCarthy, the Special Assistant to the Director of NASA (Goldin) for Technology was identified as the appropriate level to formulate a presentation and briefing in Washington with respect to wavelet technology for all of NASA. He currently has a budget of \$20M some of which has not been committed, and which could in principle be partially utilized for implementation of wavelet technology for NASA provided the justification for such a new venture could be appropriately made in the right forum.

Action Items:

1. Wells would initiate immediately two proposals from the CML at Rice on Wavelet-Based Digital Signal Processing and Wavelet-Based Numerical Solutions of Partial Differential Equations for the University Research Office at JSC in the order of magnitude of 60K per proposal which would be reviewed soon and with possible first of the year start dates.
2. There would be discussions with the different operations and research groups at NASA JSC for the joint development between CML and JSC for budgeting and formulating proposals for research and development proposals for specific targeted NASA problems in the areas of digital signal processing and partial differential equations. Richard Juday and Chien Li have agreed to be the NASA JSC representatives to spearhead this joint

venture in these two areas in conjunction with the existing NASA Wavelet Technology Working Group. The goal here would be to have two additional proposals in these two areas, also of about 60K per year per proposal.

3. Huffstetler would contact the various individuals at NASA Headquarters and the other space centers to set up the appropriate meetings to demonstrate the existing wavelet technology developed at Aware, Inc. for possible use at different parts of NASA. The plan would be to have such a briefing in Washington in late January or February. The purpose of the meeting would be to explore the possible implementation of existing technology developed by Aware, Inc. or possible adaptations of it for NASA environments. The goal would be to provide significant cost-savings and higher performance in bandwidth management activities in on-going NASA programs.

3. New Proposals

Digital Signal Processing

A proposal entitled "Wavelet Digital Signal Processing of Image Compression Algorithms" was submitted in November 1993 to NASA JSC with R. O. Wells, Jr. as Principal Investigator.

Partial Differential Equations

A proposal entitled "Wavelet-based Numerical Solutions of Partial Differential Equations" was submitted in November 1993 to NASA JSC with R. O. Wells, Jr. as Principal Investigator.

4. Workshops and Research

Report of 25 May 1994

As you requested I am sending you a progress report of the activities of the Computational Mathematics Laboratory since my last progress report to you on 4 November 1994.

There have been activities on two fronts:

1. Research Work
2. Briefings and Workshop Presentations at NASA.

Research Developments:

The work of our laboratory includes on using wavelets for solving problems of computational fluid dynamics, and the work of my graduate student concerning the computation of higher order wavelets of the coiflet type (with an equal number of vanishing moments (maximal) of both the scaling and wavelet functions), a generalization of the original wavelets of Daubechies.

The differential equations work is resulting in several technical reports which will be reported on in the Final Report in more detail. The basic progress consists of developing (both theoretically and with code) a new wavelet multigrid algorithm for elliptic boundary value problems with very general boundary and also for anisotropic elliptic problems with

periodic boundary conditions (this work was carried out by myself with Roland Glowinski, Xiao-dong Zhou, Andreas Rieder with partial support from the NASA grant).

The graduate student (Jun-Tian) has calculated the coiflets for several orders higher than currently is available, and is developing algorithms for calculating them to all orders.

Workshops and Briefings:

1. On 1 February 94 I presented a paper at the second annual NASA Dual-Use Workshop at JSC entitled "Recent Advances in Wavelet Technology" and a paper was submitted for the conference proceedings.
2. On 21 April 94 I presented a paper at the annual NASA Workshop on Automation and Robotics on "Wavelets and Technology".
3. On 21 April 94 I had a two-hour meeting with Paul Coan, Chief, Image Sciences Division, at JSC, about possible recent advances in wavelet compression and data management techniques. We discussed a number of areas of common interest concerning both still image and video compression and management problems. The end result of the meeting was an agreement that there were several areas in which we could work together, and he would attempt to outline some possible specific areas of future collaboration which would involve specific proposals to NASA.
4. On 9 June 94, I will be giving a lecture to the Galveston Bay Section of IEEE on wavelets and their applications to signal processing which will take place at NASA JSC.

6. Conclusion

The year-long interaction between Rice and University Researchers interested in wavelet research and the research and mission community at NASA JSC has proved to be very fruitful, and it is clear after numerous discussions that there are a number of areas in which there can and will be future collaborations between the Computational Mathematics Laboratory at Rice and NASA. The two proposals submitted in November are the first instances of such proposed collaboration, and after the discussion with Paul Coan and the concluding discussions with Kumar Krishen, it is evident that future proposals for joint collaboration are desirable and will have potential benefit for NASA and the broader scientific and technical community. The primary areas of common interest are reflected in the two specific proposals submitted to NASA last November in the areas of image analysis and numerical solutions of partial differential equations arising in computational fluid dynamics and structural mechanics. During the course of the interaction of the senior members of the CML (Wells and Glowinski) with the NASA staff there was additional interaction with the junior staff members of CML, and part of the NASA grant funding was used to help partially support post-doctoral and graduate student researchers working in the CML on the areas discussed above. The bibliography below contains references to several papers which were written during the course of this grant by CML researchers and which were partially supported by this grant.

7. Bibliography

The following list of papers appearing under the auspices of NASA-sponsored research is part of a longer series of Technical Reports issued by the Computational Mathematics

Laboratory. Further information on Technical Reports as well as selected Technical Reports themselves are available via anonymous ftp from cml.rice.edu in the directory /pub/reports.

R. Glowinski, A. Rieder, R. O. Wells, Jr., and X. Zhou, "A wavelet multilevel method for Dirichlet boundary value problems in general domains," submitted.

A. Rieder, R. O. Wells, Jr., and X. Zhou, "A wavelet approach to robust multilevel solvers for anisotropic elliptic problems," to appear in ACHA.

R. A. Gopinath, "Some Thoughts on Least-Squared Error Optimal Windows"

Peter Heller and R. O. Wells, Jr., "The spectral theory of multiresolution operators and applications," to appear in Proceedings of the Intl. Conference on Wavelets, Taormina, Italy, 1993. Academic Press, 1994.

Andreas Rieder and Xiaodong Zhou, "On the Robustness of the damped SV-cycle of the Wavelet Frequency Decomposition Multigrid Method"

J. E. Odegard and C. S. Burrus, "On the robustness of K-regular M-band wavelets"

R. A. Gopinath, "Discrete-time local trigonometric bases with applications"

R. A. Gopinath, "Factorization approach to time-varying filter banks and wavelets," in Proc. ICASSP, Adelaide, Australia, 1994. IEEE.

R. A. Gopinath, "Theory of time-varying filter bank trees and wavelets"

R. O. Wells, Jr., "Adaptive Wave Propagation Modeling," in Optical Engineering, 1994. To appear.

R. O. Wells, Jr., "Wavelets and Wave Propagation Modeling," in Proceedings of SPIE OE/Aerospace Sensing Conference, Orlando, 1994. To appear.

R. O. Wells, Jr., "Recent advances in wavelet technology," in proceedings of NASA Dual-Use Technology Conference, 1994. To appear.

8. Appendix : Paper for NASA Dual-Use Conference

The attached paper "Recent Advances in Wavelet Technology" was presented at the NASA JSC Dual-Use Technology Conference in February of 1994 and was published in its proceedings. It incorporates in summary form some of the aspects of the technical briefings described above.