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Non-Local Sub-Characteristic Zones of Influence in Unsteady Interactive Boundary-Layers

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Abstract

The properties of incompressible, unsteady, interactive, boundary layers are examined for the model hypersonic boundary layer of Brown *et al* (1974,1975) and internal flow past humps or, equivalently, external flow past short-scaled humps. Using a linear high frequency analysis, it is shown that the domains of dependence within the viscous sublayer may be a strong function of position within the sublayer and may be strongly influenced by the pressure displacement interaction, or the prescribed displacement condition. Detailed calculations are presented for the hypersonic boundary layer. This effect is found to carry over directly to the fully viscous problem as well as the nonlinear problem. In the fully viscous problem, the non-local character of the domains of dependence manifests itself in the sub-characteristics. Potential implications of the domain of dependence structure on finite difference computations of unsteady boundary layers are briefly discussed.

1. Introduction

The phenomena of unsteady separation and vortex eruption appear to involve a number of possible high Reynolds number structures, including: classical boundary layers and singularities (Van-Dommelen & Shen (1980), Smith & Burggraf (1985)), interactive boundary layers and interactive singularities (Brotherton-Ratcliffe & Smith (1987), Smith (1988)), and the introduction of inviscid effects and normal pressure gradients (Elliott *et al* (1983)). As noted above, at some point the vortex eruption may pass through a stage in which viscous-inviscid interaction becomes important, although it is now generally accepted that viscous-inviscid interaction does not suppress the ultimate boundary-layer singularity leading to vortex eruption. However, the interactive stage may certainly be one segment of the vortex eruption and could be used as a means of controlling the development of the singularity. This, of course, assumes that marginal states could be found and that some form of artificially induced interaction could be used to control those marginal states (a suggestion which seems reasonable given recent computations). It seems clear that computational methods may have to accommodate the viscous-inviscid interaction at some point in the vortex eruption and such interactions may include artificially induced interactions if an attempt is made to control the development of the unsteady flow. It is important, therefore, to understand any factors which may complicate the theory and computation of these interactive structures – which brings us to the crux of the present study.

In 1985, Smith & Burggraf showed that two-dimensional Tollmien-Schlichting waves can pass into a high frequency regime with elevated amplitudes in which the wave is predominantly inviscid. Viscous effects are confined to a sublayer which is governed by classical unsteady boundary layer equations and is driven by the slip velocity of the inviscid sublayer. As such, the viscous sublayer could admit the Van-Dommelen & Shen (1980) singularity and burst into the outer layers (see also Elliott *et al* (1983)). However, with this exception, the viscous sublayer is decoupled from the rest of the structure. The inviscid sublayer is found to be governed by nonlinear thin-layer Euler equations and it is this layer which feels the effect of the pressure-displacement interaction. For hypersonic flows, the predominantly inviscid flow is governed by a modified inviscid Burgers equation

$$A_T + (A-1)A_X = 0 \quad . \tag{1.1}$$

This equation also governs the unsteady initial value problem for high frequency, large amplitude, short-scaled waves introduced into the original viscous interactive boundary-layer. Additional implications of eqn. (1.1), especially as regards finite-time singularities, are addressed by Brotherton-Ratcliffe & Smith (1987) and Smith (1988).

From the Smith & Burggraf (1985) model, eqn. (1.1), several interesting observations can be made. Linear waves propagate upstream, finite-time shocks are inevitable at finite wave amplitudes, and at sufficiently high amplitudes the characteristics change direction. This suggests that the differencing of at least the streamwise convective term uu_x and pressure gradient, which produce the $(A-1)A_X$ term, is not a simple matter, but depends on an interplay between the deviation from a "mean flow profile" and the pressure displacement interaction.

Most finite difference methods for calculating high Reynolds number unsteady viscous flows tend to rely on upwinding schemes based on the sign of the streamwise velocity (see for example Keller (1978)). However, the above observations suggest that traditional differencing techniques may not be appropriate in some unsteady interactive boundary-layer computations. The present study indicates that in non-parallel, unsteady, hypersonic boundary layers – and in a wide variety of zero-displacement boundary layers – the zones of influence are determined by a subtle interplay between the convective effects, the pressure-displacement interaction and the nonparallel (possibly separated) flow – *throughout the entire viscous sublayer.*

2. Governing Equations

The principal problem to be addressed in this study is the linear unsteady flow superimposed upon an originally nonlinear steady hypersonic boundary layer. The equations governing a high Reynolds number unsteady interactive hypersonic boundary layer may be found in a number of studies, including: Brown *et al* (1974,1975) and Gajjar & Smith (1985). The governing equations, with the Prandtl transposition, are found to be: the conservation of mass equation,

$$U_X + V_Y = 0 \tag{2.1}$$

and the conservation of streamwise momentum equation,

$$U_T + UU_X + VU_Y = -P_X(X,T) + U_{YY} \quad , \tag{2.2}$$

with no-slip boundary conditions at the wall and

$$U(X,Y,T) \rightarrow Y + F(X) + A(X,T) \qquad \text{as } Y \rightarrow \infty . \tag{2.3}$$

The pressure displacement relation in the hypersonic flow of Brown *et al* (1974,1975) is simply P=–A. Here F(X) is a prescribed steady hump/indentation shape. For the unsteady internal flow of Smith (1976) and Duck (1979,1985), as well as the short-scale hump in a Blasius boundary layer (Smith *et al* (1981)), the P=–A law is replaced by A=0. The primary issue of interest here is the numerical integration of the above system of equations using a finite difference procedure. This issue, as will be shown in this study, may not be as simple as it might first appear.

3. High Frequency Limit

Suppose that a nonlinear steady state solution has been calculated for equations (2.1) through (2.3) – say a hump induced separation. This may be easily done using a variant of the Davis (1984) alternating direction explicit (ADE) method (see also Brotherton-Ratcliffe (1987)). A linear unsteady disturbance is superimposed upon this steady base solution:

$$[U,V] \sim [U_0,V_0](X,Y) + \bar{\epsilon}\,[\hat{u}_1,\hat{v}_1,](X,Y,T) + O(\bar{\epsilon}^2) \qquad (\bar{\epsilon} \ll 1) \ . \tag{3.1}$$

Equations (2.1) and (2.2) will be further simplified by letting the time scale become short, or

$$\frac{\partial}{\partial T} = \Omega \frac{\partial}{\partial t_0} \qquad (\Omega \gg 1) \ . \tag{3.2}$$

The reader will note that this approach follows very closely the work of Smith & Burggraf (1985), Tutty &

Cowley (1986), Brotherton–Ratcliffe & Smith (1987) and Smith (1988). The lower deck is found to break up into two distinct regions (I and III in Fig. 1). For purposes of discussion, and comparison with Smith & Burggraf (1985), an extra region, II, will be introduced. Region II can be derived as a limit of Region III. The entire structure is found to be confined to a neighborhood of a point X_0 within the lower deck, with

$$X = X_0 + \Omega^{-1} x_0 \quad , \tag{3.3}$$

and viscous effects are found to be confined to a thin classical Stokes layer, Region I, where $Y = \Omega^{-1/2}\overline{Y}$. The Stokes layer equations are solved subject to the no-slip boundary conditions at the wall and matching with Region III. It should be noted that Region I is decoupled from Region III. In Region III we take Y to be O(1) and the governing equations are found to be

$$\hat{u}_{1_{x_0}} + \hat{v}_{1_Y} = 0 \quad \text{and} \quad \hat{u}_{1_{t_0}} + U_0(Y)\hat{u}_{1_{x_0}} + U'_0(Y)\hat{v}_1 = -\hat{p}_{1_{x_0}} \quad , \tag{3.4a,b}$$

where $U_0(Y)$ is the local steady velocity profile. Upon integration of equation (3.4b), application of the tangency boundary condition, and the hypersonic pressure displacement relation, the entire problem is reduced to:

$$\hat{u}_{1_{t_0}} + U_0(Y)\hat{u}_{1_{x_0}} - U'_0(Y)\int_0^Y \hat{u}_{1_{x_0}}(x_0, \eta, t_0)d\eta = \hat{a}_{1_{x_0}} \quad , \tag{3.5}$$

subject to $\hat{u}_1 \to \hat{a}_1$ as $Y \to \infty$. Equation (3.5) will form the basis of the numerical calculations presented later in this study. It may be shown that in the limit as Y becomes large eqn. (3.5) reduces to the linear version of the Smith & Burggraf (1985) wave equation (1.1), but with a non-trivial displacement effect from region III:

$$\hat{a}_{1_{t_0}} + (F_0 + A_0 - 1)\hat{a}_{1_{x_0}} = -\delta \quad . \tag{3.6}$$

Further details of this structure may be found in Tutty & Cowley (1986) and Rothmayer (1990).

4. Zones of Influence for the Unsteady Linear and Nonlinear Viscous Sublayer

Although the numerical results of this study will be for the high frequency limit of section 3, this approach may be generalized to other cases. First consider an integrated form of equations (2.1) and (2.2) for the fully nonlinear problem:

$$U_T + UU_X - U_Y\int_0^Y U_X(X, \eta, T)d\eta = -P_X(X, T) + \underline{U_{YY}} \quad , \tag{4.1}$$

where P(X,T) is fixed from the appropriate pressure displacement law and the u-matching condition (2.3). Linearized problems may be considered, in which case, given an expansion of the form of eqn. (3.1), equation (4.1) becomes

$$u_T + U_0 u_X + \underline{uU_{0_X}} + \underline{V_0 u_Y} - U_{0_Y}\int_0^Y u_X(X, \eta, T)d\eta = -p_X(X, T) + \underline{u_{YY}} \quad , \tag{4.2}$$

subject to no-slip at the wall and $u(X, \infty, T) = a + f$ as $Y \to \infty$. The high frequency limit of Rothmayer (1990) results in a parallel flow approximation and neglecting viscous effects in the main portion of the boundary layer. Equivalently, the high frequency equation is simply equation (4.2) without the underlined terms, for linear flows, or equation (4.1) without the underlined terms, for nonlinear flows. Equation (4.2) may be evaluated on the Y grid $Y_j = (j-1)\Delta Y$ using, say, a central difference approximations for u_Y and u_{YY} and a trapezoidal rule quadrature for the integral in (4.2). As will be shown in section 5, equation (4.2) may be reduced to the form

$$\mathbf{u}_T + \mathbf{A}\mathbf{u}_X = \mathbf{B}\mathbf{u} + \mathbf{g} \quad \text{or} \quad [\hat{\mathrm{u}}_1]_{t_0} + \mathrm{A}[\hat{\mathrm{u}}_1]_{x_0} = 0 \tag{4.3a,b}$$

when a high frequency approximation is used (see eqn. (3.5), also Rothmayer (1990)) and the underlined terms in equation (4.2) are neglected. This equation assumes that a suitable pressure displacement relation has been used to relate the pressure to some combination of the velocities on the vertical grid (see Rothmayer (1990)). For example, the hypersonic interaction law gives $\hat{p}_1 = -\hat{u}_{1_{iN}}$ whereas the prescribed displacement law (A=0) may be used, in which case the high frequency approximation gives:

$$p_x = -g + \sum_{j=2}^{N-1} u_{j_x} \Delta Y \qquad \text{and} \qquad g(x,t) = f_t + (Y_N + A_0 + F_0 - 1/2) f_x \quad . \tag{4.4}$$

The vector $\mathbf{g}$ is given by $\mathbf{g} = [g \ldots g]^T$ while the matrix $\mathbf{A}$ is given in Rothmayer (1990) for the high frequency limit. The same analysis may be applied to equation (4.1), but now in terms of $\mathbf{U} = [U_{i1}, \ldots, U_{iN}]^T$ and the coefficient matrix $\mathbf{A}$ is a function of the solution. Equation (4.3a) is simply an N-dimensional wave equation which may be solved by standard means using the method of characteristics. Note that the viscous effect as well as the nonparallel effects (i.e. all underlined terms in (4.2)) only contribute to $\mathbf{B}$ and so do not affect the sub-characteristic analysis. The primary difference between the present study and the standard method of sub-characteristics is that the characteristics cannot be determined via a local analysis, due to the pressure displacement interaction P=–A (or the A=0 law). The eigenvalues of the coefficient matrix $\mathbf{A}$ can readily be calculated and satisfy $|\mathbf{A} - \lambda_i \mathbf{I}| = 0$, and the eigenvectors are found from $\mathbf{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i$. The actual method used in this study for finding the eigenvalues and eigenvectors is a nonlinear Newton-Raphson method (which is discussed in Rothmayer (1990)). It is not assumed that the eigenvectors have been normalized. Using classical methods (see John (1971)), a new solution vector $\mathbf{u} = \mathbf{V}\bar{\mathbf{u}}$ is defined, where $\mathbf{V}$ is an NxN matrix whose columns are the eigenvectors, i.e. $\mathbf{V} = [\mathbf{v}_1 \ldots \mathbf{v}_N]$. Substitution into equation (4.3a) and multiplication by the inverse of $\mathbf{V}$ diagonalizes eqn. (4.3a), and gives

$$\bar{\mathbf{u}}_T + \Lambda \bar{\mathbf{u}}_X = \bar{\mathbf{B}}\bar{\mathbf{u}} + \bar{\mathbf{g}} \quad , \tag{4.5}$$

where $\bar{\mathbf{B}} = \mathbf{V}^{-1}\mathbf{B}\mathbf{V}$, $\bar{\mathbf{g}} = \mathbf{V}^{-1}\mathbf{g}$ and Λ is the diagonal eigenvalue matrix $\Lambda = diag(\lambda_1, \ldots, \lambda_N)$. Equation (4.5) can be easily integrated along its characteristics for $\bar{\mathbf{u}}$, and then the $\bar{\mathbf{u}}$ can be converted back to $\mathbf{u}$. Direct integration of (4.3a) requires a differencing formulation which correctly incorporates the diagonalization transformation $\mathbf{u} = \mathbf{V}\bar{\mathbf{u}}$.

5. A Solution of the Non-Local High-Frequency Characteristic Problem

As in the preceding section, consider a quadrature of the integral in equation (3.5) on the vertical grid $Y_j = (j-1)\Delta Y$ with j=1,...,N. A trapezoidal rule will be employed here, although other quadratures may be used, in which case:

$$\hat{u}_{1_{j_{t_0}}} - \left(\frac{1}{2}\Delta Y U'_{0_j}\right)\hat{u}_{1_{1_{x_0}}} - \sum_{L=2}^{j-1} \Delta Y U'_{0_j} \hat{u}_{1_{L_{x_0}}} + \left(U_{0_j} - \frac{1}{2}\Delta Y U'_{0_j}\right)\hat{u}_{1_{j_{x_0}}} - \hat{u}_{1_{N_{x_0}}} = 0 \quad , \tag{5.1}$$

keeping in mind that $\hat{u}_{1_N} = \hat{a}_1$. Equation (5.1) is applied at the points j=3,...,N. At j=2 the summation from L=2,...,j–1 is removed from the equation, while at j=1 equation (5.1) is replaced by $\hat{u}_{1_{1_{t_0}}} - \epsilon\, \hat{u}_{1_{N_{x_0}}} = 0$. The above equations are just the single matrix wave-equation (4.3b) where $[\hat{\mathbf{u}}_1] = [\hat{u}_{1_1} \;\; \ldots \;\; \hat{u}_{1_N}]^T$ and the coefficient matrix A is given in Rothmayer (1990). The characteristic slopes, λ_j, are the eigenvalues of A, and satisfy $|\mathbf{A} - \lambda_i \mathbf{I}| = 0$, where $\lambda = \partial x_0 / \partial t_0$. The Riemann invariant Γ_j associated with the λ_j eigenvalue satisfies (see Chorin & Marsden (1979)):

$$A^T\left[\frac{\partial \Gamma_j}{\partial \hat{u}_{1_i}}\right] = \lambda_j\left[\frac{\partial \Gamma_j}{\partial \hat{u}_{1_i}}\right] \quad . \tag{5.2}$$

In general there are N eigenvalues, and it turns out that the first N-1 are roughly the slope of the U_0 velocity at a particular j gridpoint (for the particular solution being calculated in this study). The last eigenvalue, λ_N, does not appear to have any simply determined value and must be found via numerical computation. These eigenvalues have associated with them the Γ_j Riemann invariants, each of which is constant along the characteristic with slope λ_j. Therefore at each (x_0, t_0, Y) gridpoint a system of N equations in the N $\hat{u}_{1_j}$'s needs to be solved, namely

$$\sum_{i=1}^{N}\left(\frac{\partial \Gamma_j}{\partial \hat{u}_{1_i}}\right)\hat{u}_{1_i} = (\Gamma_j)_0 \quad , \tag{5.3}$$

where $(\Gamma_j)_0$'s are the Riemann invariants evaluated along the initial data plane. The equations (5.3) may be inverted to give:

$$[\hat{u}_1] = \left[\frac{\partial \Gamma_j}{\partial \hat{u}_{1_i}}\right]^{-1}\left[(\Gamma_j)_0\right] \quad . \tag{5.4}$$

This is effectively the diagonalization process of equation (4.5). Equation (5.4) gives the dependence of each $\hat{u}_{1_j}$ on the Riemann invariant Γ_j associated with the λ_j eigenvalue. Therefore the inverse of the Jacobian matrix of the Riemann invariants gives the domain of dependence of the streamwise velocity at each jth gridpoint in terms of the j characteristics with slope λ_j, providing that the Riemann invariant multiplying a given row element of the inverse is non-zero. In the general case, there is no reason to expect the domains of dependence to be simple.

For purposes of computation, an idealized flow will be considered. The velocity field is assumed to take the form:

$$U_0(Y) = Y + A\left(1 - e^{-Y}\right) \quad , \tag{5.5}$$

where A is taken as an independent parameter, in lieu of A_0+F. The wall shear stress is given by $\tau_w = U_0'(0) = 1 + A$, indicating that the flow is attached for A>-1 and separated for A<-1. The case A=0 gives an undisturbed sublayer. Velocity profiles for various values of A are shown in Figs. 2 through 4, along with the eigenvalue and eigenvector calculations. In addition, a new function, Θ_i, will be defined, which is the product of a row *element* in the inverse eigenvector matrix and the corresponding *element* of the Riemann invariant vector:

$$\Theta_i = \left[\frac{\partial \Gamma}{\partial \hat{u}_1}\right]^{-1}_{ij}\left[\Gamma_j\right]_0 \quad . \tag{5.6}$$

The results of these calculations are shown in Figs. 5 through 8, assuming constant perturbation velocities on the initial data plane. The results for A=0 are in agreement with the linear results of the Smith & Burggraf (1985) study. The solution at any vertical point in the grid depends only on the j=N eigenvalue which has slope -1. The results of the above calculations for an accelerated flow are shown in Figs. 6 and 7. Consider the schematic interpretation of Figs. 5 through 7, shown in Fig. 8. This figure is a qualitative interpretation only and is not meant to convey accurate quantitative data. At low amplitudes (i.e. A near 0) all of the characteristics point downstream with slope -1. As the flow is accelerated (i.e. A increasing) a region of dependence begins to emerge for small values of j, and the slope of the downstream-directed characteristic begins to decrease. A typical case, say A=5, now has the j=N eigenvalue competing with the eigenvalues which are approximately the velocities near the bottom of the boundary layer. Note that this only occurs in the outer portion of the boundary layer (i.e. for j large). The eigenvalues associated with small j's have characteristics which point upstream and so the tendency in the outer portion of the boundary layer is for the upstream directed characteristics to begin competing with, and eventually overtaking, the downstream directed characteristic. The overall picture is that the characteristics in the outer portion of the sublayer appear to change direction for increasing A, whereas the characteristics in the lower portion of the sublayer do not. This means that the characteristics are not pointing in the same direction throughout the sublayer when the change in the direction of the characteristics does occur, but vary with vertical position.

7. Conclusion and Implications for Finite Difference Computations

In this study it has been shown that the sub-characteristics in an unsteady interactive viscous flow are not simply determined by the convective velocity, but rather are fixed by an interplay between the convective terms, the pressure displacement interaction, and the nonparallel base flow. In addition, the sub-characteristics may vary throughout the entire viscous sublayer and may possess a complex structure. These results seem to be in accord with the work of Smith & Burggraf (1985) on nonlinear hypersonic waves, which suggests that the characteristics will change direction at sufficiently large disturbance amplitude. Is it worthwhile to perform the domain of dependence calculation before proceeding to the finite difference solution? This of course depends on the results for a particular case. A simple differencing scheme may correctly capture the physics of the problem in question. However, the present study indicates that it is possible for an unsteady flow to possess complicated domains of dependence, and hence to require complex differencing schemes. It is anticipated that problems related to incorrectly modeling the domains of dependence in a finite-difference method will manifest themselves either as a CFL (Courant-Friedrichs-Lewy) condition or as spurious oscillations in the solution.

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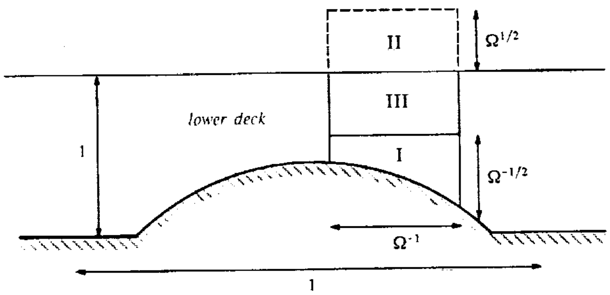

Figure 1. Asymptotic structure for a high frequency unsteady linear disturbance in a nonparallel viscous sublayer.

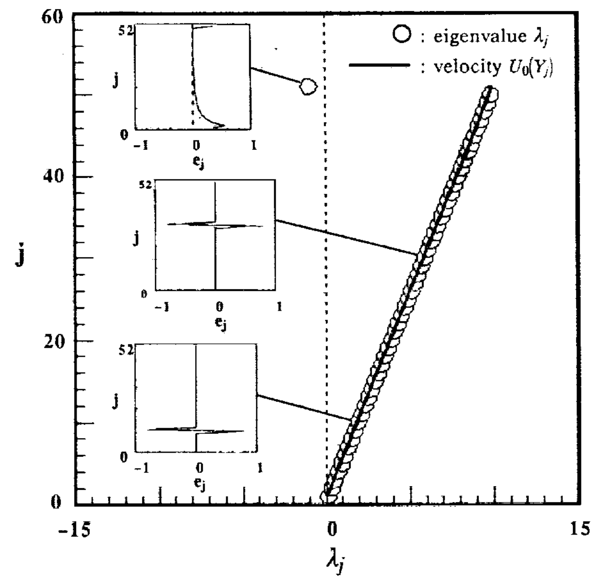

Figure 2. Eigenvalues, eigenvectors and the velocity profile for A=0 and N=51. e_j is the eigenvector associated with λ_j.

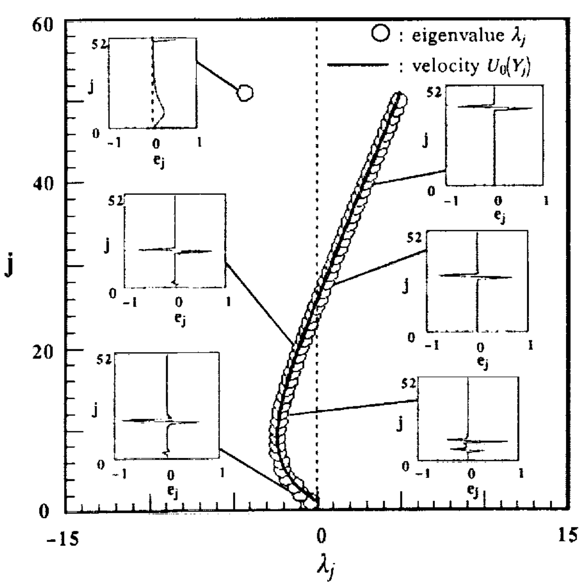

Figure 3. Eigenvalues, eigenvectors and the velocity profile for a model separated flow (A=-5, N=51).

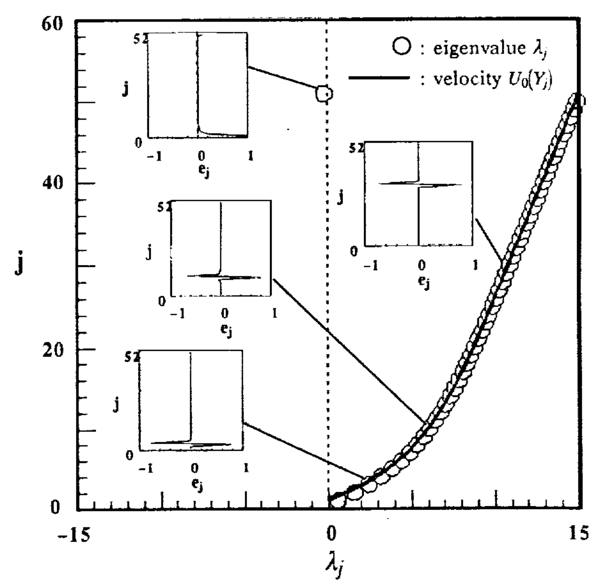

Figure 4. Eigenvalues, eigenvectors and the velocity profile for a model accelerated flow (A=5, N=51).

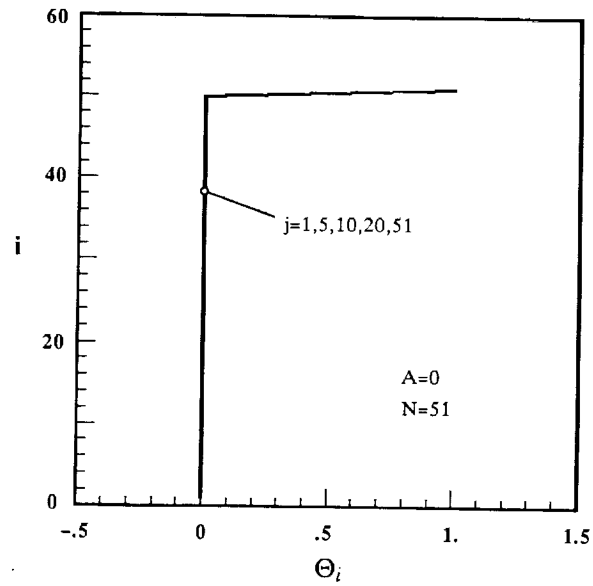

Figure 5. Domains of dependence for vertical positions in the grid and A=0.

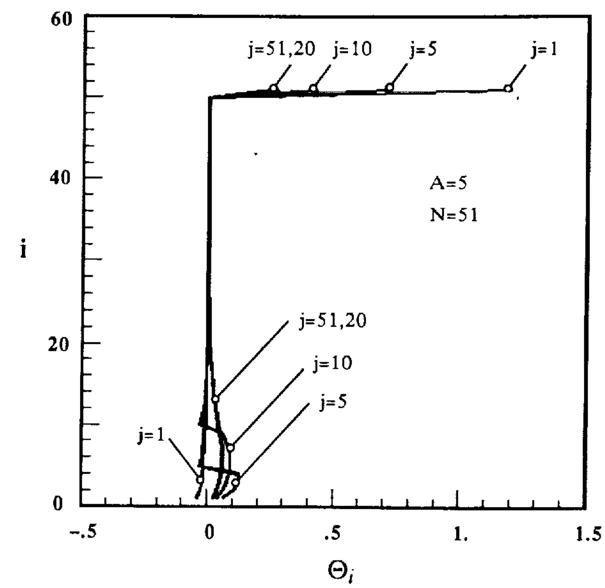

Figure 6. Domains of dependence for various vertical positions in the grid and A=5.

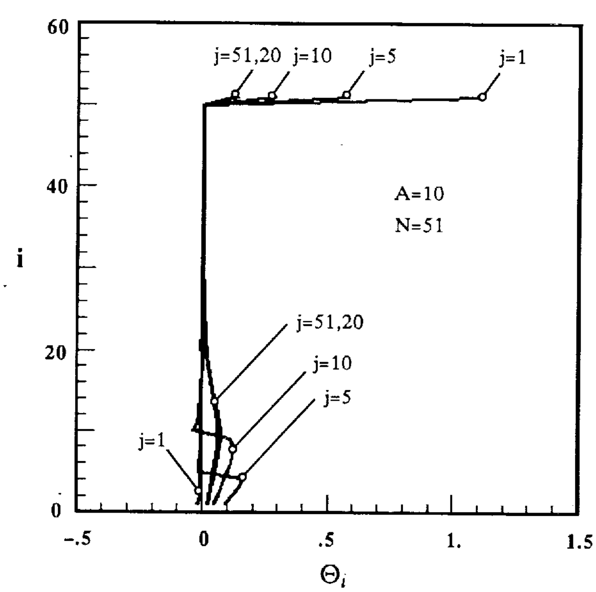

Figure 7. Domains of dependence for various vertical positions in the grid and A=10.

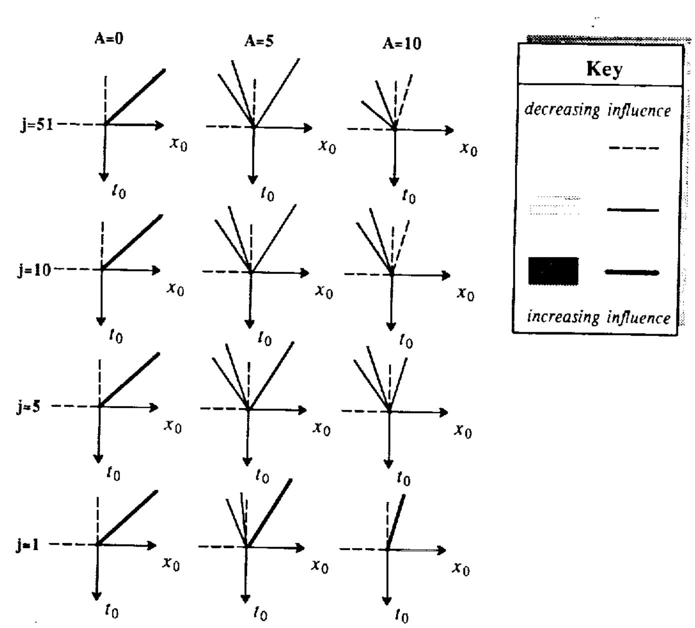

Figure 8. Schematic diagram of the domains of dependence given by Figs. 5 through 7. Results are qualitative only.