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ABSTRACT

Orbital sensitivity of the gravity field for high degree terms (> 30) is analyzed on satellites employed in the Goddard Mars Model GMM-1, complete in spherical harmonics through degree and order 50. The model is obtained from S-band Doppler data on Mariner 9 (M9), Viking Orbiter 1 (VO1), and Viking Orbiter 2 (VO2) spacecraft, which were tracked by the NASA Deep Space Network on 7 different orbits. Numerical integration theory was used to compute the orbits on approximately 270 orbital arcs with arc lengths varying up to 9 days. The orbits are highly eccentric with periods of one day for VO1 and VO2 and one-half day for M9 and with varying inclinations and periapsis altitudes. Because of these periods and the near daily rotation rate of Mars to Earth, the VO1 and VO2 orbits have resonant perturbations of all orders m and the M9 orbits have even order resonance with periods varying up to 50 days. Also V01 and V02 low orbits (300 km periapsis altitude) have deep resonance of high order with significant side-band perturbations. In our analysis orbital sensitivity is based upon the velocity perturbation of the gravity signal and is compared to the accuracy level (noise) of the Doppler tracking. The gravity signal spectrum used a Kaula power law of $13 \times 10^{-5}/l^2$ for coefficients of degree l, which was derived by Balmino et al.(1982).

Spectral perturbations of orbital velocity are obtained from numerical integration of the variational equations of the gravitational harmonics over 8-day arc lengths. These perturbations are also compared with those obtained from Kaula's analytical theory using a modified eccentricity function and a prorating technique for perturbations with periods greater than 8 days. Using numerical integration the velocity sensitivity for the spectrum by degree (rss for harmonics in a given degree) is obtained for the above orbits. The main sensitivity of the high degree terms is obtained from the VO1 and VO2 low orbits (300 km periapsis altitude), where significant spectral sensitivity is seen for all degrees out through degree 50. The velocity perturbations show a dominant effect at periapsis and significant effects out beyond the semi-latus rectum covering over 180° of the orbital groundtrack for the low altitude orbits. These low orbits also exhibit perturbations that have a spectral pattern of sensitivity for the high degree terms (>30) which vary by order as a function of the argument of periapsis (ω). Because of the wideband of periapsis motion covering nearly 180° in ω and $\pm 39°$ in latitude coverage, the VO1 300 km periapsis altitude orbit with inclination of 39° gave the dominant sensitivity in the GMM-1 solution for the high degree terms. Although the VO2 low periapsis orbit has a smaller band of periapsis mapping coverage, it strongly complements the VO1 orbit sensitivity for the GMM-1 solution with Doppler tracking coverage over a different inclination of 80°.

1. INTRODUCTION

This report analyzes the orbit sensitivity of Martian satellites with Doppler tracking for the high degree spectrum of spherical harmonics of the gravity field for GMM-1, Goddard Mars Model-1 (Smith et al., 1993), which is complete in harmonics through degree and order 50. It is shown that there exists strong orbital sensitivity for terms below degree 30 and hence attention here is focused on the high degree spectrum to help analyze the truncation level for which the gravity signal in the tracking data is exhausted for GMM-1.

The sensitivity analysis uses an a priori knowledge of the power spectrum of the Mars gravity field. Previous analyses of Mars gravity particularly the later spherical harmonic models (Christensen and Balmino, 1979; Christensen and Williams, 1979; and Balmino et al., 1982) and recent orbital sensitivity studies of Rosborough and Lemoine (1991) and Lemoine (1992) have facilitated the work of the present report.

The gravitational potential at spacecraft altitude, V_M, is represented in spherical harmonic form as

$$V_M(\bar{r}) = \frac{GM_M}{r} \sum_{l=0}^{N} \sum_{m=0}^{l} \left[\frac{r_M}{r}\right]^l P_{lm}(\sin\phi)\ [C_{lm} \cos m\lambda + S_{lm} \sin m\lambda] \tag{1}$$

where $\bar{\mathbf{r}}$ is the position vector of the spacecraft in areocentric coordinates, r is the radial distance from the center of mass of Mars to the spacecraft, ϕ and λ are the areocentric latitude and longitude of the spacecraft, r_M is the mean radius of the reference ellipsoid of Mars, GM_M is the gravitational constant for Mars, P_{lm} are the normalized associated Legendre functions of degree l and order m, C_{lm} and S_{lm} are the normalized spherical harmonic coefficients which were estimated from the tracking observations to determine the gravitational model, and N is the maximum degree representing the size (or resolution) of the field.

This report gives a more complete account of the sensitivity analysis than what was presented in the GMM-1 paper. It is organized in subsequent sections to describe the following material: (2) Satellite Orbital Characteristics and Doppler Tracking Data in GMM-1, (3) Spectral Sensitivity of the Gravity Signal, and (4) Summary.

2. SATELLITE ORBITAL CHARACTERISTICS AND DOPPLER TRACKING DATA IN GMM-1

2.1 Satellite orbit characteristics

The Mariner 9, Viking 1 (VO1) and Viking 2 (VO2) spacecraft were in highly eccentric orbits with periods of approximately 1 day for Viking 1 and 2 and 1/2 day for Mariner 9. The satellite orbit characteristics are summarized in Table 1. The orbital periods are nearly commensurate with the rotational period of Mars (24.623 hr) which produce dominant resonant perturbations (Kaula 1966) for all orders m of the Viking spacecraft (24 hour period) and for the even orders of Mariner 9 (12 hour period). The resonant periods range mostly from about 1 to 50 days for shallow resonant terms and also include deep (very long) resonant periods. The beat period, or fundamental resonant period, identifies the shift (or "walk") in successive ground tracks and is useful in mapping the orbital coverage over Mars (a plus sign represents an eastward "walk" and a negative sign for a westward "walk"). The beat period changes after each maneuver of the Viking 1 and 2 spacecraft (Snyder 1979). For example (as noted in Table 1) orbital trimming, producing significant orbital period changes, have occurred on Viking 1 for a close approach to Phobos on Jan. 22, 1977 and on Viking 2 for a close approach to Deimos on Sept. 26, 1977. Orbit maneuvers were made on VO2 on March 2, 1977 producing a very slow walk to synchronize with the Viking Lander (VL2). Strong resonant perturbations of long period were produced on VO1 commencing on Dec. 2, 1978 to provide a slow walk around the planet. Mission events such as leakages, attitude control jetting, and other phenomena that cause variations in the orbital periods are described by Snyder (1977; 1979).

The 300 km periapsis altitude orbits of VO1 and VO2 (hereafter simply referred to as VO1 and VO2 low orbits) provide the strongest contribution of data to the solution for the higher degree terms, particularly the VO1 low orbit with a range of about 180° for the argument of periapsis (ω) as compared to 28° for the VO2 low orbit. The observing period for the VO1 low orbit shown in Table 1 covers almost 2 years from 77-03-12 to 79-01-27, and the periapsis point varies in latitude from +39° to -39° during this period. The VO1 low orbit provides about a 9° ground track "walk" per revolution for the 1 1/2 year period from 77-07-01 to 78-12-02. This corresponds to a "near repeat" of the ground track for a 39 day period, except for the small motion of periapsis. After the 39 day near repeat period, the orbital ground track shifts by 1.8° from the previous repeat track which corresponds to a deep orbital resonance with a period of about 200 days. This produces, over a 200 day coverage, a global grid ($\pm$39° latitude) with approximately 1.8° ground track separations and provides for a high resolution recovery of the gravity field.

2.2. Data summary and characteristics

The data set consisted of 270 orbital arcs representing over 1100 days of S-band Doppler tracking data from the Mariner-9 and Viking-1 and -2 spacecraft, collected by the Deep Space Network between 1971-1978. These data, grouped by satellite periapsis altitude and inclination, are summarized in Table 2. In total over 230,000 observations were included in the GMM-1 solution.

The data consist of two-way S-band (2.2 GHz) Doppler measurements compressed to 60 seconds (1 minute data points). These 1 minute points have a precision of 1 mm/sec. Data far removed from periapsis, approximately greater than 12,000 km altitude, were compressed to 10 minute intervals and correspond to a precision of approximately 0.3 mm/sec.

All observations were collected by three DSN sites located at Goldstone (California), Madrid (Spain), and Canberra (Australia). They were processed in the differenced-range Doppler formulation taking into account relativistic bending due to the Sun (Moyer, 1971). Observations near satellite periapsis are most valuable for determining the gravity field and periapsis is generally observable by at least one of the DSN sites except when occulted by Mars.

Table 1 SATELLITE ORBIT CHARACTERISTICS
Including Beat Periods and Ground Track Walks

Satellite	Periapsis altitude (km)	Epoch * (Yr-Mo-Da)	Inclination (degrees), Eccentricity	Orbit period (hours)	Periapsis argmnt. (°), rate (°/day)	Nodal rate (°/day)	Beat period (days)	Walk per revolution (degrees)	Comments
Mariner 9	1500	71-11-16	64, 0.60	11.97	-0.02	-0.18	18	10	New orbit
		72-04-19							End of data
Viking 1	1500	76-06-21	38, 0.75	24.63	47, 0.17	-0.13	>1000	<1	Synchronous
		76-09-13		21.87	56		8	44	New orbit
		76-09-24		24.63	60		>1000	<1	Over lander
		77-01-22		22.99	79		14	25	Near Phobos
	300	77-03-12	39, 0.80	21.92	99 ,0.27	-0.21	8	43	New orbit
		77-03-24		23.50	102		21	17	New walk
		77-07-01		23.97	120		38	10 **	Dual station
		78-12-02		24.85	264		-129	-3	Near synch.
		79-01-27			270				End of data
Viking 2	1400	76-08-07	55, 0.76	27.31	72 ,0.05	-0.09	-10	-35	New orbit
		76-08-28		24.63	73		>1000	<1	Synchronous
	1500	76-10-02	75, 0.80	26.79	68, -0.05	-0.04	-13	-29	New inclin.
	800	76-12-21	80. 0.80	26.48	62, -0.08	-0.03	-15	-25	Low periaps.
	750	77-03-05		24.73	55		-215	-2	Over lander
	700	77-04-18		22.72	51		12	29	
		77-09-26		24.29	34		78	5	Near Deimos
	600	77-10-09		24.20	33		60	6	
	300	77-10-25		23.98	32, -0.10	-0.04	39	9	Low periaps.
		78-07-25			2				End of data

* Start epoch of the orbit parameters cited
** Secondary walk is about 2 degrees over approximately 200 day duration

Table 2
Summary of Data Used in Mars Gravity Model GMM-1
DSN Doppler Tracking Data (± 0.1 cm/sec)

Satellite	Altitude km.	Inclination degree	ARCS Total	ARCS Average Arc-Length (days)	ARCS Average Input RMS Residuals (cm/sec)	ARCS Average No. of Obs.	Total Number of Observations	Total no. of Days
Viking-1	1500	38.2	29	4.2	0.446	1082	31393	122
Viking-1	300	39.1	95	4.8	2.844	673	63977	425
Viking-2	1400	55.4	12	3.8	0.256	990	11878	46
Viking-2	1500	75.1	11	4.7	0.507	952	10467	52
Viking-2	778	80.1	54	3.8	0.210	655	35375	204
Viking-2	300	80.2	37	4.2	6.282	793	29355	155
Mariner-9	1500	64.4	32	4.3	0.212	1559	49878	138
Total			270				232323	1142

3. SPECTRAL SENSITIVITY OF GRAVITY SIGNAL

The spectral sensitivity of the gravity field is analyzed for the Mariner 9, VO1 and VO2 orbits. Velocity perturbation sensitivity for the high degree terms (>30) is the main area of interest and these perturbations are compared with a threshold level corresponding to the precision of the DSN Doppler signal. The DSN Doppler tracking when compressed to 1 minute data points has a precision of 1 mm/sec. The gravity signal for sensitivity analysis employed a form of Kaula's rule, namely $13 \times 10^{-5}/l^2$ for terms of degree l, which was obtained by Balmino et al. (1982) for the power spectrum of Mars. Sensitivity studies for the above Mars orbiters have been made by Rosborough and Lemoine (1991) and Lemoine (1992) for terms through degree 20. Analysis for the high degree terms is given here.

GMM-1 (Smith et al., 1993) employs numerical integration theory for the orbit computation over arc lengths varying up to 9 days. Arc lengths were restricted to 9 days because of errors in non-conservative force modeling and other possible systematic effects. Hence, our sensitivity analysis was also based upon gravitation perturbations obtained from numerical integration theory where we employed an 8 day orbital arc length as a basis for the analysis. However, some comparison of the numerical integrated perturbations will be made below with those obtained from the analytical perturbation theory.

3.1. Analytical Perturbation Theory

The theory is very useful to us for obtaining knowledge of the periods of the orbital perturbations. Kepler perturbations from Kaula's theory (1966) were computed by modifying the eccentricity function because of the highly eccentric orbits (Wagner,1990). The periods are derived for the Kepler spectral terms. A total velocity perturbation was obtained by a simplified projection of the rss of the amplitudes of the Kepler spectral terms for each harmonic of a given degree and order. This analysis represents perturbations over the lifetime repeat period of the orbit, thus long period spectral perturbations will require adjustment for short arcs as indicated below.

Since the orbital periods of Viking 1 and 2 are close to the near daily rotational period of Mars, the orbits have dominant resonant perturbations for all orders m with periods ranging from greater than 50 days for the low orders down to less than one day for the higher order terms. Similarly, with a one-half day period, M9 orbits have only even order resonant perturbations. The theory also shows on the VO1 and VO2 low orbits significant sidebands associated with long period (>200 days) deep resonant perturbations of order 38 with sideband periods varying up to 50 days. Generally perturbations with longer periods have larger amplitudes. Hence, in order to compare with numerical integrated perturbations over an 8-day arc length, analytical perturbations with periods greater than 8 days have been prorated to this length with a factor of 8/period. Also periods greater than 40 days have been excluded to avoid inflated perturbations for long period terms, particularly for deep resonance.

Some comparison of the analytical perturbations with those obtained from numerical integration will be presented in Section 3.3 showing the important effects of the periods of resonant perturbations on the arc lengths.

3.2. Numerically Integrated Perturbations

Numerically integrated perturbations have been obtained from the integration of the variational equations of the gravitational harmonics over a given length of orbital arc by use of our computer program Geodyn (Putney,1977). The integration yields the partial derivatives of the vector position and velocity components with respect to the harmonic coefficients and when scaled by the coefficient signal (Kaula's rule) they will give the linear perturbations. However these perturbations are referred to a reference orbit with osculating parameters at epoch (time at the start of the orbital arc). Hence the perturbations are adjusted to refer to a mean reference orbit over the arc by use of our computer program Erodyn (Englar et al.,1993). The mean reference orbit is obtained through a least squares orbit adjustment and this provides a reference orbit with a minimum rms velocity perturbation over the orbital arc length. Our velocity perturbation refers to the total vector value.

The spectrum by degree (rss of harmonics for a given degree) for the numerically integrated velocity perturbations is presented in Table 3 for orbits from Mariner 9, VO1 and VO2. Only the VO1 and VO2 low orbits at 300 km periapsis altitude in this table show significant sensitivities for terms above degree 30. The velocity spectrum by degree for each of these low orbits exceeds the DSN Doppler noise level of 1 mm/sec for all terms out through degree 50. This shows the signal is not exhausted at degree 50, and hence in order to account for the complete signal above the noise level additional harmonics are needed beyond degree 50 for the VO1 and VO2 low altitude orbits.

Sensitivity for the Mariner 9 and Viking 1 orbits both at 1500 km periapsis altitude shows that Mariner 9 has stronger perturbations than Viking 1 (at 1500 km altitude) because of its closer proximity on average to Mars with twice per day revolution and smaller eccentricity. Sensitivity for the Viking 2 orbits at 800 km periapsis altitude clearly shows that the lower altitude orbit of 800 km for VO2 has significantly larger perturbations for the higher degree spectrum and falls short of the DSN noise level of 1 mm/s at degree 26. Also the velocity perturbations for the low orbits of VO1 and VO2 in the 8-day arc length include only very small effects due to the deep resonant perturbations of order 38. However, the sidebands of the deep resonant order 38 also have appreciable perturbations with periods ranging from 5 to 40 days and this results in the upturn in the spectrum at degree 34 (as shown in Table 3) for the high degree terms of the VO2 low orbit.

Analytical perturbations give similar results for Table 3. For the VO2 low orbit the eccentricity function required expansion for a maximum q of 45 (Kaula, 1966). A result with significantly smaller perturbations was obtained using only a maximum q of 12 and was given previously in the GMM-1 gravity report. Analytical and numerical perturbations are compared in the next section.

Table 3 Spectral Sensitivity By Degree

USING A POWER RULE OF 13E-05/L**2
MARINER 9, VIKING 1 & 2 SAMPLED ORBITS *
VELOCITY PERTURBATIONS IN CM/SEC

PERIAPSIS ALTITUDE(KM):	VKG1 300	VKG2 300	VKG2 800	VKG1 1500	MRN9 1500
DEGREE	EPOCH 78-01-15	EPOCH 77-12-17	EPOCH 77-04-19	EPOCH 77-02-05	EPOCH 72-04-10
2	392.340	1384.119	463.300	182.252	870.212
3	513.386	646.823	121.944	75.628	148.293
4	291.038	352.520	51.552	31.407	121.843
5	188.431	201.811	24.616	13.240	46.511
6	124.383	119.637	13.308	5.654	22.566
7	84.726	72.912	7.888	2.722	10.294
8	58.478	45.987	5.084	1.549	4.611
9	40.855	30.242	3.657	0.842	2.202
10	28.804	20.753	2.984	0.410	1.083
11	20.512	14.752	3.128	0.242	0.548
12	14.778	10.764	3.585	0.161	0.286
13	10.801	8.023	3.595	0.087	0.151
14	8.027	6.112	3.156	0.045	0.081
18	2.910	2.560	1.112	0.007	0.008
22	1.297	1.404	0.315	0.001	0.001
26	0.676	0.989	0.092	0.000	0.000
30	0.401	0.945	0.041	0.000	0.000
34	0.268	1.359	0.019	0.000	0.000
36	0.230	1.527	0.012	0.000	0.000
38	0.204	1.510	0.007	0.000	0.000
40	0.187	1.324	0.005	0.000	0.000
42	0.180	1.047	0.003	0.000	0.000
46	0.187	0.518	0.001	0.000	0.000
50	0.199	0.213	0.000	0.000	0.000
ω (DEG):	175	29	51	80	332

* BASED ON 8 DAY ARC LENGTHS

3.3. Perturbations As a Function of Orbital Arc Length

The rms velocity perturbation for the harmonic spectrum of the VO1 low orbit is obtained for different arc lengths and the rss components by degree and order are plotted respectively in Figures 1a and 1b. The sensitivity for the one-day arc length is very small. These results are compared to the analytical perturbations (prorated to 8-days as indicated above). The analytical perturbation spectrum by degree and order is expected to follow the corresponding numerically integrated result for the 8-day arc length but tends to be conservatively smaller for degrees less than 30. The perturbation spectrum by order in Figure 1b shows that the numerical results tend to follow the analytical characteristic for the deep resonant sidebands (significant terms with periods of 5 to 50 days) of order 38 as the arc length increases to a 25-day span. The deep resonant perturbation with a period of 200 days would appear mostly as a secular rate over 25 days and hence would be absorbed in the mean motion during orbit convergence. These results show significant sensitivity for the high degree terms for arc lengths greater than 3 days when compared to the 1 mm/sec noise level of the Doppler signal.

Figure 1a. Spectral Sensitivity of Gravity Signal (by Degree).

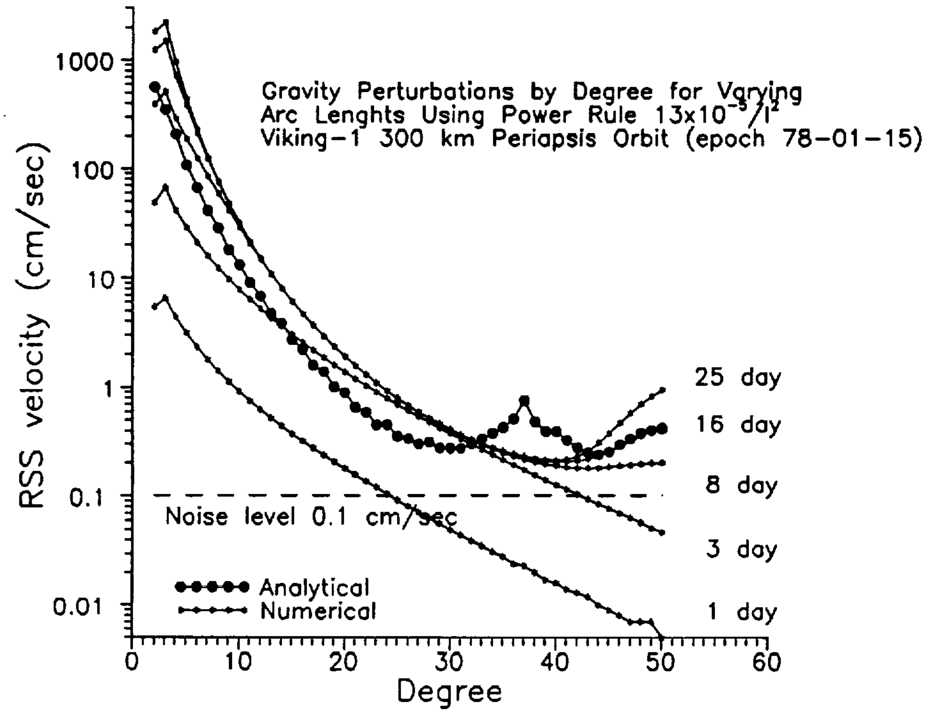
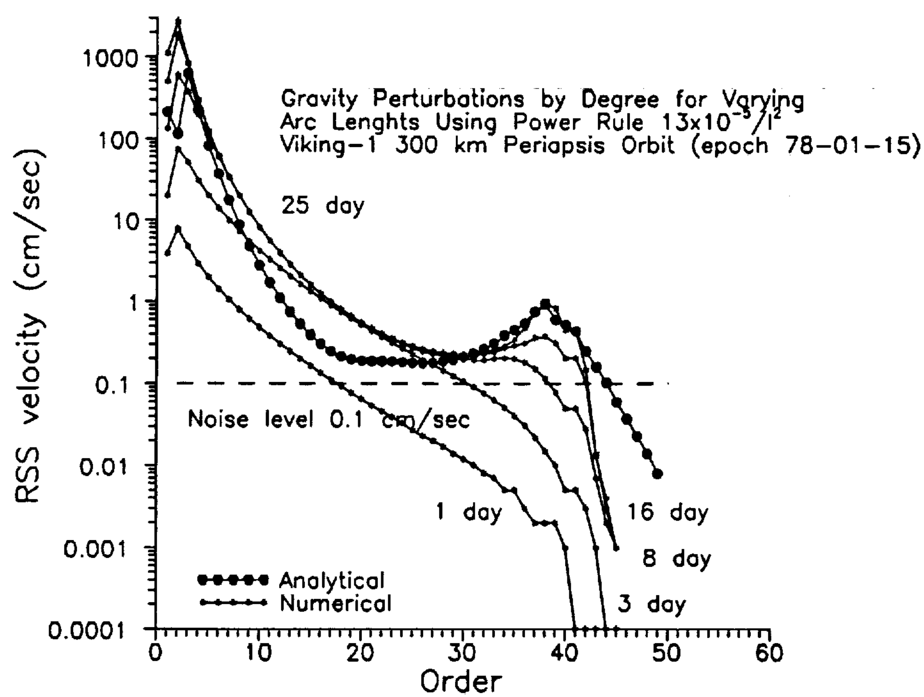


Figure 1b. Spectral Sensitivity of Gravity Signal (by Order).



3.4. Perturbations As a Function of Argument of Periapsis (ω)

The numerically integrated perturbations over 8-day arc lengths are employed in the present analysis for the VO1 and VO2 low orbits. For the VO1 low orbit, the spectrum of rms velocity perturbations sampled by degree and order are given in Figures 2a and 2b respectively for arc epochs where periapsis is near the equator with ω=175° and near maximum south latitude (-39°) with ω=261°. The high degree spectrum (degrees 31 to 50) for $\omega \approx$ 180° as in Figure 2a shows significant high order terms and for $\omega \approx$ 270° as in Figure 2b shows significant low order terms. For $\omega \approx$ 235° midway between 180° and 270° the mid-order terms as in Figure 2c dominate the sensitivity for high degree coefficients. Hence for a complete spectrum of sensitivity for all orders of the high degree terms we need at least half the coverage (180°) of the argument of periapsis, which corresponds to ±39° in latitude coverage for the VO1 low orbit. The signals for terms of lower degrees (less than 30), however, are not as dependent on the argument of periapsis.

The high order deep resonant effects in the 8-day orbital arcs are only seen in Figure 2a where the argument of the periapsis (ω) is near the equator but are not seen in Figures 2b and 2c where (ω) is removed from the equator. In Figure 2d the orbital period has changed and as noted in Table 1 the orbit has changed to a slow walk of 3° per day in longitude. Since ω=270° we see dominant perturbations again for the low order terms. This result gives a very significant velocity spectrum by degree for all degrees through degree 50.

As indicated in Section 3.1 from the analytical theory, the VO2 low orbit has a similar period with deep resonant effects of order 38 as in the case of VO1 low orbit. Similar spectral results to those in VO1 (Figures 2a, 2b and 2c) occur in the VO2 low orbit but the complete benefit of those in Figures 2b and 2c is not available because of lack of coverage in the argument of periapsis. In the case of the VO2 low orbit, the argument of periapsis (ω) coverage ranges from about 0° to 28° and hence yields significant sensitivities for only the high order terms in the high degree coefficients. However, because of its high inclination (i=80°), VO2 provides stronger sensitivity than VO1 to the sectorial and near sectoral terms (Figure 3a) as well as contributing significantly to greater resolution of gravity features because of the different ground track geometry from VO1 (i=39°).

Assuming a coverage of ω=-60° and -90° as in Figures 3b and 3c, although not available from the actual VO2 low periapsis orbit, the figures show similar spectral sensitivities by order for the high degree perturbations as in Figures 2b and 2c of the VO1 low orbit. The spectral pattern of sensitivity by order as a function of argument of periapsis is associated with the high eccentricity and the ground track geometry of the orbits and the way that the satellite samples the gravitational harmonics near periapsis in a somewhat regular fashion from day to day due to the commensurability of Mars rotational period and the orbital period of the satellite.

Figure 2a Signal Sensitivity by Degree and Order for VO1 Low Orbit ($\omega=175°$)

Velocity Perturbations In .001 cm/sec

```
   RSS*  DEG              VKG1: 78-01-15
 392339    2 ***          8 day arc length
 124383    6 ******
  28804   10 469******
   8027   14  54******156
   2909   18   6 52***843 28
   1297   22   3 10138592237  8
    675   26   1  2  7161289 76  3
    400   30   1  1  2 20137143 29  2
    268   34   0  0  1  2 33104 77 13  1
    229   36   0  0  0  1 13 69 86 29  2  1
    203   38   0  0  0  0  4 39 79 48  9  2  1
    187   40   0  0  0  0  1 20 63 62 22  7  1  0
    179   42   0  0  0  0  0  9 43 67 41 19  5  1  0
    186   46   0  0  0  0  0  1 14 49 73 59 29  9  1  0
    199   50   0  0  0  0  0  0  3 22 69 84 67 34 11  0  0

         ORD:  2  6 10 14 18 22 26 30 34 36 38 40 42 46 50
```

* RSS taken over all orders, not just sampled orders.

Figure 2b Signal Sensitivity by Degree and Order for VO1 Low Orbit ($\omega=261°$)

Velocity Perturbations In .001 cm/sec

```
   RSS*  DEG              VKG1: 78-12-20
 ******    2 ***          8 day arc length
  79846    6 ******
  20969   10 ******104
   7762   14 ******233  3
   3431   18 ******145 21  0
   1697   22 618657153 20  3  0
    902   26 159263112 19  4  1  0
    500   30  64 89 62 20  3  1  0  0
    283   34  85 51 29 15  4  1  1  0  0
    216   36  84 80 36 11  3  1  0  0  0  0
    164   38  75 47 17 10  4  1  0  0  0  0  0
    132   40  29 35 21  9  3  1  0  0  0  0  0  0
     97   42  54 37 14  6  3  1  0  0  0  0  0  0  0
     60   46  33 24 11  4  2  1  0  0  0  0  0  0  0  0
     39   50  17 14  7  4  2  1  0  0  0  0  0  0  0  0  0

         ORD:  2  6 10 14 18 22 26 30 34 36 38 40 42 46 50
```

* RSS taken over all orders, not just sampled orders.

Figure 2c Signal Sensitivity by Degree and Order for VO1 Low Orbit ($\omega=235°$)

Velocity Perturbations In .001 cm/sec

```
   RSS*  DEG              VKG1: 78-08-20
 609532    2 ***          8 day arc length
  91040    6 ******
  23718   10 ******379
   8044   14 ******992 25
   3142   18 514***709137  3
   1349   22 155483382162 22  1
    623   26  79191187123 40  4  0
    307   30  43 81 89 77 41 11  1  0
    161   34  20 35 42 44 33 15  3  0  0
    119   36   9 22 29 33 27 15  5  1  0  0
     89   38   8 15 20 24 22 15  6  1  0  0  0
     68   40   6 10 14 18 18 13  7  2  0  0  0  0
     53   42   3  7 10 13 14 12  7  3  1  0  0  0  0
     33   46   1  3  5  7  8  9  7  4  2  1  0  0  0  0
     23   50   1  1  2  3  5  6  6  4  3  2  1  0  0  0  0

         ORD:  2  6 10 14 18 22 26 30 34 36 38 40 42 46 50
```

* RSS taken over all orders, not just sampled orders.

Figure 2d Signal Sensitivity by Degree and Order for VO1 Low Orbit ($\omega=270°$)

Velocity Perturbations In .001 cm/sec

```
   RSS*  DEG              VKG1: 78-12-28
 819333    2 ***
 100379    6 ******
  31096   10 ***893485
  13222   14 ******974  4
   6598   18 ******349130  1
   3621   22 964***782 71  8  0
   2106   26 240665612 84 19  1  0
   1269   30  87131313135  6  2  0  0
    782   34 159104 91104 18  2  1  0  0
    620   36 166256216 60 12  4  1  0  0  0
    492   38 154152 39 48 24  1  0  0  0  0  0
    402   40  56107129 68  7  4  0  0  0  0  0  0
    317   42 116133 75 10 17  4  0  0  0  0  0  0  0
    209   46  75 92 72 23  6  4  0  0  0  0  0  0  0  0
    140   50  40 54 52 27  4  3  1  0  0  0  0  0  0  0  0

         ORD:  2  6 10 14 18 22 26 30 34 36 38 40 42 46 50
```

* RSS taken over all orders, not just sampled orders.

Figure 3a Signal Sensitivity by Degree and Order for VO2 Low Orbit (ω=28°)

```
Velocity Perturbations In .001 cm/sec

  RSS*  DEG
******   2 ***                    VKG2: 77-11-17
118572   6 ******                 Arg. of periapsis = 28 deg
 20864  10 *********              8 day arc length
  6006  14 153795******
  2430  18  24116302723***
  1280  22   4 19 54156391591
   855  26   1  3 10 33101263387
   743  30   0  1  2  7 24 78221346
   954  34   0  0  0  1  6 21 75245475
  1099  36   0  0  0  1  3 10 41156528471
  1147  38   0  0  0  0  1  5 22 93406552386
  1073  40   0  0  0  0  1  3 11 53272438476262
   908  42   0  0  0  0  0  1  6 29169301389339148
   514  46   0  0  0  0  0  0  2  9 58116178206173 32
   235  50   0  0  0  0  0  0  0  2 18 38 65 85 86 43  9

        ORD:  2  6 10 14 18 22 26 30 34 36 38 40 42 46 50

* RSS taken over all orders, not just sampled orders.
```

Figure 3b Signal Sensitivity by Degree and Order for VO2 Low Orbit (ω=-60°)

```
Velocity Perturbations In .001 cm/sec

  RSS*  DEG
******   2 ***                    VKG2: 77-11-17
 74956   6 ******                 Arg. of periapsis = -60 deg
 16051  10 *********              (not real coverage)
  5053  14 *********412            8 day arc length
  1999  18 582576539438133
   980  22 206210229253222 74
   651  26  56 76 99130163178 71
   620  30  18 31 44 65 99160231 21
   590  34  13 14 20 32 56113251 89  2
   546  36  11 10 14 23 42 91230107  6  1
   489  38   8  7  9 16 31 71202114 10  2  0
   427  40   6  4  6 11 22 55171112 13  4  1  0
   363  42   4  3  4  8 16 42141104 15  5  2  1  0
   250  46   1  1  2  4  9 24 91 80 15  7  3  1  1  0
   163  50   0  1  1  2  5 14 56 56 13  7  4  2  1  0  0

        ORD:  2  6 10 14 18 22 26 30 34 36 38 40 42 46 50

* RSS taken over all orders, not just sampled orders.
```

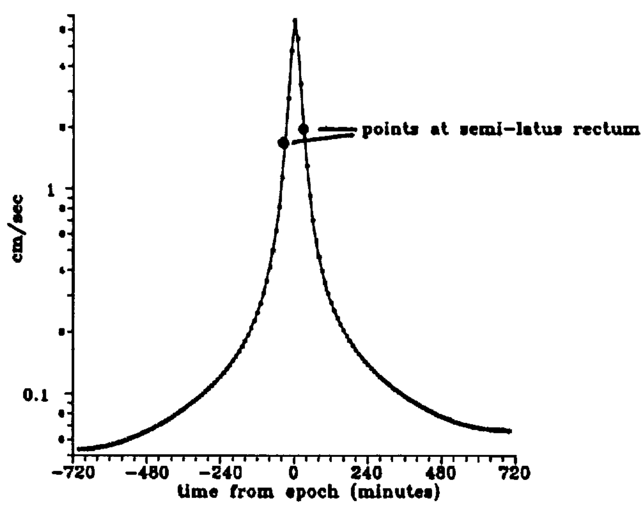
Figure 3c Signal Sensitivity by Degree and Order for VO2 Low Orbit (ω=-90°)

```
Velocity Perturbations In .001 cm/sec

  RSS*  DEG
******   2 ***                    VKG2: 77-11-17
 69876   6 ******                 Arg. of periapsis = -90 deg
 16426  10 742187 31              (not real coverage)
  5618  14 ******295 25            8 day arc length
  2278  18 ***759279 74  5
  1070  22 678351155 62 17  1
   576  26 216108 55 28 12  4  0
   325  30  25  9  4  3  2  1  0  0
   177  34  68 31 16 10  6  4  3  0  0
   129  36  71 31 17 11  7  5  5  1  0  0
    96  38  64 28 16 11  7  6  6  1  0  0  0
    72  40  52 22 13  9  7  6  6  1  0  0  0  0
    57  42  39 16 10  7  5  5  6  1  0  0  0  0  0
    39  46  15  6  4  3  2  3  3  1  0  0  0  0  0  0
    26  50   2  1  0  0  0  0  1  0  0  0  0  0  0  0  0

        ORD:  2  6 10 14 18 22 26 30 34 36 38 40 42 46 50

* RSS taken over all orders, not just sampled orders.
```

3.5. Range of Spectral Sensitivity Near Periapsis

We continue to use the numerically integrated perturbations as described above to examine the sensitivity around the orbit. In Figure 4 the velocity sensitivity (perturbations) of the terms from degrees 31 to 50 are plotted as a function of time at 10 minute intervals around one revolution of the VO1 low orbit (300 km periapsis altitude) using the 8-day orbital arc length with the same epoch as in Figures 1a and 2a.

Figure 4. Velocity Perturbation for Viking-1 300 km Periapsis Altitude Orbit Due to Signal from Harmonic Coefficients of Degrees 31-50 *

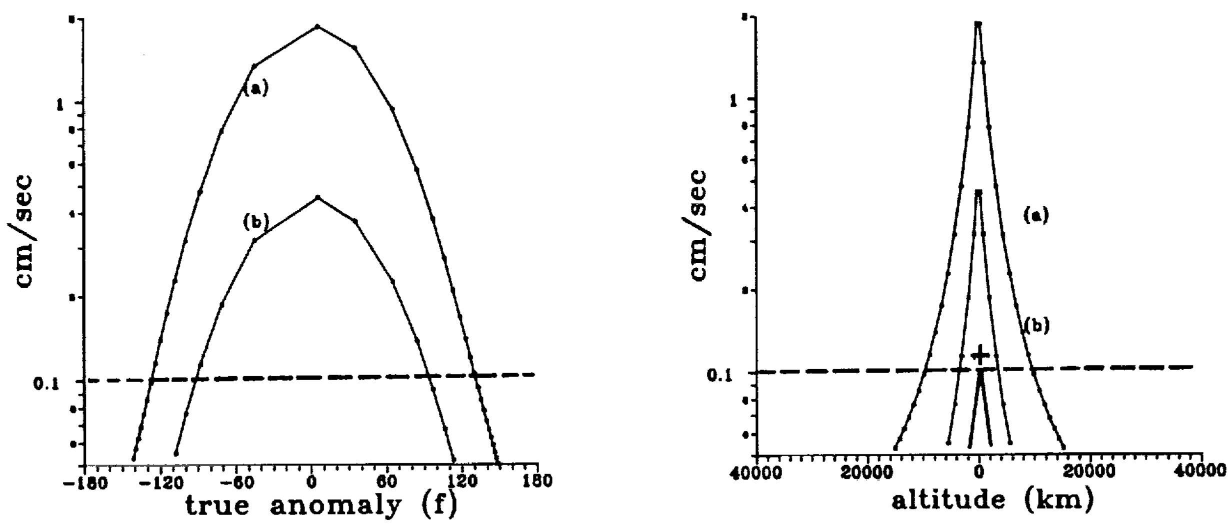


* based on 8 day arc from epoch 78-01-15

Sensitivity near periapsis is very strong at the 6 cm/sec level and does not fall off to the noise level (.1 cm/sec) of DSN Doppler until well beyond the semi-latus rectum as noted in the figure. The rms of the signal around the orbit is about 1 cm/sec well below the level at periapsis. It is the rms signal values which are presented in the previous figures for the harmonic components and hence they show a weaker signal for the velocity sensitivity than near periapsis.

In Figure 5 we plot for the same case just the terms of order 25 for the sensitivity from (a) degrees 31 to 50 and (b) degrees 41 to 50, both as a function of true anomaly and altitude. The signal is still above the noise level beyond the semi-latus rectum, over 180° around the planet, and the signal falls off at the noise level at altitudes of 10,000 km and 2,000 km respectively for cases (a) and (b).

Figure 5 Velocity Perturbation for Viking I 300 km Periapsis Altitude Orbit Due to 25^{the} Order Harmonic Coefficients for (a): Degrees 31-50, (b): Degrees 41-50 *



* based on 8 day arc from epoch 78-01-15
+ based on 1 day arc from epoch 78-01-15

For altitudes above 12,000 km, 10 minute normal points are employed in the solution and have a precision of about 0.03 cm/sec. This means significant sensitivity is seen over a wide span of the orbit which contributes toward separability of the terms. Sensitivity is also shown in the figure for a one day orbital arc length for case (a) which shows a greatly reduced signal. It should be pointed out that for the same altitude as the VO1 low orbit, orbits with 800 and 1500 km periapsis will have relatively smaller sensitivity for high degree terms (31-50) since the perturbations do not contain the integrated accelerations at the 300 km altitude level as is the case in the VO1 low orbit.

4. SUMMARY

Sensitivity analysis for the high degree (>30) harmonic spectrum of the gravity field was investigated for the high eccentricity satellite orbits employed in the GMM-1 model (50X50 field). The V01 and V02 300 km low periapsis altitude orbits show significant sensitivity for the velocity spectrum by degree for all degrees out through degree 50. A power law of $13 \times 10^{-5}/l^2$ for coefficients of degree l (Balmino et al., 1982) was employed for the gravity signal. The sensitivity was considered significant when the velocity perturbation of the orbit exceeded a precision level of the DSN Doppler of 1 mm/sec. Both the analytical perturbations (prorated to 8 days) and numerically integrated perturbations for 8-day arc lengths support these results for the V01 low orbits. Numerically integrated perturbations were used as the main method in our study for an 8-day orbital arc length. In Figure 1 it was shown how the numerically integrated perturbations vary using different arc lengths. Sensitivity for the velocity spectrum by degree in this figure was very small for 1-day arc lengths for the high degree field but significant sensitivity is seen for arc lengths of 3 days and greater. Using numerically integrated perturbations for 8-day arc lengths additional analysis has shown in Figures 2 and 3, respectively, for the V01 and V02 low orbits that the velocity spectrum by order varies with the argument of periapsis (ω). Again these results show significant sensitivity for the velocity spectrum by degree for the high degree field.

The range of spectral sensitivity from periapsis is analyzed for the V01 low orbit in Figures 4 and 5 for a set of harmonics of order 25 for the high degree (>30) spectrum and the results show significant sensitivity out beyond the semi-latus rectum over half-way around the planet. Although V02 with inclination of 80° has greater global coverage than V01 with inclination of 39° in their respective tracking periods, the V01 orbit has a greater span of periapsis subsatellite position covering ±39° in latitude. In all, the low altitude orbits show strong sensitivity to the velocity spectrum and indicate that the truncation level for exhausting the gravity signal in the Doppler tracking is beyond degree 50.

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13. ABSTRACT *(Maximum 200 words)*

Orbital sensitivity of the gravity field for high degree terms (> 30) is analyzed on satellites employed in a Goddard Mars Model GMM-1, complete in spherical harmonics through degree and order 50. The model is obtained from S-band Doppler data on Mariner 9 (M9), Viking Orbiter 1 (VO1), and Viking Orbiter 2 (VO2) spacecraft, which were tracked by the NASA Deep Space Network on 7 different highly eccentric orbits. The main sensitivity of the high degree terms is obtained from the VO1 and VO2 low orbits (300 km periapsis altitude), where significant spectral sensitivity is seen for all degrees out through degree 50. The velocity perturbations show a dominant effect at periapsis and significant effects out beyond the semi-latus rectum covering over 180° of the orbital groundtrack for the low altitude orbits. Because of the wideband of periapsis motion covering nearly 180° in w and +39° in latitude coverage, the VO1 300 km periapsis altitude orbit with inclination of 39° gave the dominant sensitivity in the GMM-1 solution for the high degree terms. Although the VO2 low periapsis orbit has a smaller band of periapsis mapping coverage, it strongly complements the VO1 orbit sensitivity for the GMM-1 solution with Doppler tracking coverage over a different inclination of 80°.

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