ADAPTIVE MODEL REDUCTION FOR CONTINUOUS SYSTEMS VIA RECURSIVE RATIONAL INTERPOLATION

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SUMMARY

A method for adaptive identification of reduced-order models for continuous stable SISO and MIMO plants is presented. The method recursively finds a model whose transfer function (matrix) matches that of the plant on a set of frequencies chosen by the designer. The algorithm utilizes the *Moving Discrete Fourier Transform (MDFT)* to continuously monitor the frequency-domain profile of the system input and output signals. The MDFT is an efficient method of monitoring discrete points in the frequency domain of an evolving function of time. The model parameters are estimated from MDFT data using standard recursive parameter estimation techniques. The algorithm has been shown in simulations to be quite robust to additive noise in the inputs and outputs. A significant advantage of the method is that it enables a type of on-line model validation. This is accomplished by simultaneously identifying a number of models and comparing each with the plant in the frequency domain. Simulations of the method applied to an 8th-order SISO plant and a 10-state 2-input 2-output plant are presented. An example of on-line model validation applied to the SISO plant is also presented.

1. INTRODUCTION

This paper presents a new method of robust adaptive identification for stable continuous SISO and MIMO systems via *recursive rational interpolation*. Interpolation, unlike many identification schemes, is frequency-domain based. It attempts to find a model, frequently a reduced-order model (ROM), whose frequency response (matrix) matches that of the plant at certain frequencies. In this case, the model is said to interpolate the plant data at these frequencies. The goal, of course, is to find a model whose transfer matrix closely matches that of the plant at *all* frequencies.

The interpolation approach has been studied by several researchers (e.g. [1]-[4]). The method presented in this paper is most similar to [4]. In that reference, the problem is cast in the form of a system of simultaneous linear algebraic equations in which the unknowns are the plant parameters. We improve upon the method of [4], however, in that the present method is recursive and therefore much more robust to noise.

The algorithm presented in this paper monitors a number of points in the frequency profiles of the plant inputs and outputs via a recursive version of the Discrete Fourier Transform, which in this paper we call the *Moving Discrete Fourier Transform*, or *MDFT*. This transform has been discussed in [4] in conjunction with SISO model reduction. The transform was also used in another application in [5], where it was called a *frequency sampling filter*. The MDFT has the advantage that it is recursive and therefore gives the evolving DFT at every time step. It has the further advantage that it is not necessary to calculate the DFT at all N frequencies (for an N-point DFT), as is necessary for standard DFTs and FFTs, but only at those frequencies desired, which are usually far fewer than N in number. This makes the algorithm numerically efficient.

The paper is organized as follows. Section 2 briefly discusses the MDFT algorithm. Section 3 describes the SISO identification algorithm and gives a numerical example in which a 2nd-order model is derived for an 8th-order plant. Section 4 describes a method of on-line model validation made possible by the algorithm and gives a numerical example. Section 5 describes the MIMO extension of the algorithm and gives a numerical example in which a 6-state model is derived for a 10-state, 2-input, 2-output plant. This ROM is compared to several others for the same plant. Conclusions are given in Section 6.

2. THE MOVING DISCRETE FOURIER TRANSFORM (MDFT)

Let $\{x_i\}$, $i=0, 1, 2, ..., N-1$ be a set of N consecutive samples of a continuous signal $x(t)$ sampled every Δt seconds. The well-known Discrete Fourier Transform (DFT) of $\{x_i\}$ is the set of complex numbers $\{X_k\}$, $k=0, 1, 2, ..., N-1$ defined by ([6]):

$$X_k = \sum_{i=0}^{N-1} x_i W^{ki} \ , \quad k=0, 1, 2, ..., N-1 \tag{2.1}$$

where $W = \exp(-j2\pi/N)$. Since there are N points in the calculation, this is called an N-point DFT. Under certain circumstances, the complex number X_k can be considered as a frequency component of $x(t)$ at the frequency $f_k=k/N\Delta t$ Hz. We will call the frequencies f_k *DFT frequencies.*

To make the process recursive, assume that a new sample of x becomes available, x_N. Our objective is to calculate a new DFT of x based on the *latest* N samples of x. That is, we will create a "moving" DFT by discarding the oldest sample, in this case x_0, and calculating the DFT of the new sequence $\{x_i\}$, $i=1, 2, ..., N$. In general, we will be interested in calculating the DFT of the $(n+1)$th sequence of x, i.e. $\{x_{n-N+2}, ..., x_{n+1}\}$, recursively in terms of the DFT of the nth sequence of x, i.e. $\{x_{n-N+1}, ..., x_n\}$. If $X_k^{\{n\}}$ is the kth frequency component of the DFT of the nth sequence of x, then it can be shown (see [4]) that $X_k^{\{n+1\}}$ is expressed recursively in terms of $X_k^{\{n\}}$ as follows:

$$X_k^{\{n+1\}} = [X_k^{\{n\}} - x_{n-N+1} + x_{n+1}]W^{-k} \tag{2.2}$$

Equation (2.2) gives the algorithm for the 1-step-ahead MDFT. Note that the algorithm is very efficient because it involves only the addition of a real number to $X_k^{\{n\}}$ and multiplication by a complex constant W^{-k}. Referring to (2.2), note that $X_k^{\{n+1\}}$ depends only on $X_k^{\{n\}}$ and not on any other frequency component. This means that *it is only necessary to calculate the MDFT at those frequencies which are desired, not all N frequencies*, as is necessary in standard FFTs. This can result in significant savings in calculations since usually derivation of a reduced-order model (see Section 3) requires the DFT at a relatively small number of frequencies, whereas N is usually large.

3. RECURSIVE SISO REDUCED-ORDER MODELING

Assume that a kth-order model for a given stable plant is desired. Then, the model can be described by a kth-order differential equation of the form:

$$\alpha(D)y(t) = \beta(D)u(t) \tag{3.1}$$

where $\alpha(D)$ is a kth-order monic Hurwitz polynomial in the differential operator $D=d/dt$, and $\beta(D)$ is a polynomial of order $k-1$ or less. That is,

$$\alpha(D) = D^k + \sum_{i=0}^{k-1} \alpha_i D^i \tag{3.2}$$

$$\beta(D) = \sum_{i=0}^{k-1} \beta_i D^i \tag{3.3}$$

Equation (3.1) can be rewritten in the form:

$$D^k y = -\alpha_{k-1} D^{k-1} y - \dots - \alpha_0 y + \beta_{k-1} D^{k-1} u + \dots + \beta_0 u \tag{3.4}$$

which can be rewritten as

$$Y(t) = \psi^{\mathrm{T}} Z(t) \tag{3.5}$$

where

$$Y(t) = D^k y \tag{3.6}$$

$$\psi^{\mathrm{T}} = \left[-\alpha_{k-1}, \dots, -\alpha_0, \beta_{k-1}, \dots, \beta_0\right] \tag{3.7}$$

$$Z(t) = \left[D^{k-1} y, \dots, y, D^{k-1} u, \dots, u\right]^{\mathrm{T}} \tag{3.8}$$

We now discuss a way by which the signals u, y, and all necessary derivatives of u and y in (3.5) may be parameterized using MDFT data. It is well-known that the DFT, hence the MDFT, gives an *exact parameterization* of a signal if the signal is sinusoidal and its frequency is exactly equal to one of the DFT frequencies. Specifically, it can be shown that, if n is an integer between 0 and $N/2$,

$$\mathrm{DFT}\left[A\sin\left(\frac{2\pi n}{N\Delta t}t+\theta\right)\right] = Me^{j\phi} = \frac{AN}{2}\exp\left[j\left(\theta-\frac{\pi}{2}\right)\right] \tag{3.9}$$

at the frequency $\omega_n = 2\pi n/N\Delta t$ radians/sec, and is zero at all other frequencies. In (3.9), A, θ, M, ϕ, N, and Δt are constants, with N and Δt defined as in Section 2. That is, for the sinusoidal signal defined in (3.9), the magnitude of the time-domain signal is given exactly by $2/N$ times the magnitude of the DFT, and the phase angle of the time-domain signal is given exactly by $\pi/2$ plus the angle of the DFT. Therefore, if the plant inputs are bandlimited and contain sinusoidal components at only the DFT frequencies, $u(t)$ and $y(t)$ can be exactly parameterized in terms of MDFT data.

For the ideal case assume $u(t)$ is of the form

$$u(t) = \sum_{n=1}^{N/2} U_n \sin\left(\omega_n t + \alpha_n\right) \tag{3.10}$$

where $\omega_n = 2\pi n/N\Delta t$ and U_n, α_n are constants. As stated in Section 2, the frequencies ω_n are the DFT frequencies. Note that, if N is large, $u(t)$ can consist of a large number of sinusoidal components. Assuming steady-state has been reached, the plant output corresponding to (3.10) is

$$y(t) = \sum_{n=1}^{N/2} Y_n \sin\left(\omega_n t + \beta_n\right) \tag{3.11}$$

where Y_n, β_n are constants.

Choose any desired set of k distinct DFT frequencies $\{\omega_{I1}, \omega_{I2}, ..., \omega_{Ik}\}$. We call these the *identification frequencies*. Taking MDFTs of $u(t)$ and $y(t)$ at the identification frequencies and utilizing the appropriate magnitude and phase information can be shown to yield the following parameterizations for u, y, Du, and Dy [4]:

$$u \doteq U_{I1}\sin(\alpha_{I1}) + U_{I2}\sin(\alpha_{I2}) + \ldots + U_{Ik}\sin(\alpha_{Ik}) \tag{3.12}$$

$$y \doteq Y_{I1}\sin(\beta_{I1}) + Y_{I2}\sin(\beta_{I2}) + \ldots + Y_{Ik}\sin(\beta_{Ik}) \tag{3.13}$$

$$Du \doteq \omega_{I1}U_{I1}\cos(\alpha_{I1}) + \omega_{I2}U_{I2}\cos(\alpha_{I2}) + \ldots + \omega_{Ik}U_{Ik}\cos(\alpha_{Ik}) \tag{3.14}$$

$$Dy \doteq \omega_{I1}Y_{I1}\cos(\beta_{I1}) + \omega_{I2}Y_{I2}\cos(\beta_{I2}) + \ldots + \omega_{Ik}Y_{Ik}\cos(\beta_{Ik}) \tag{3.15}$$

where U_{Ij}, α_{Ij} are the magnitude and phase of the sinusoidal component in $u(t)$ at identification frequency ω_{Ij}, and Y_{Ij}, β_{Ij} are the magnitude and phase of the sinusoidal component in $y(t)$ at identification frequency ω_{Ij}. These magnitudes and phases are derived from the MDFT as explained above. All higher derivatives of $u(t)$ and $y(t)$ may be parameterized in a like manner (see [4]). In this way, all entries in the vectors $Y(t)$ and $Z(t)$ in (3.5) can be parameterized with *constants* at every time step via the MDFT. Therefore, the parameters ψ can be estimated from (3.5) using standard parameter estimation techniques.

Incidentally, note that, although parameterizations for derivatives of u and y are obtained, *no derivatives of the signals are actually taken.* Noise in the signals introduces errors in $Y(t)$ and $Z(t)$, but at worst the errors remain in the same proportion to the signal magnitude with higher derivatives and do not increase in size relative to signal magnitude, as they would if noisy signals were differentiated.

Example

Consider the following 8th-order plant [4]:

$$G_P(s) = -\frac{3}{s+1}+\frac{4}{s+2}-\frac{6}{s+3}+\frac{8}{s+4}+\frac{5}{s+5}-\frac{12}{s+6}+\frac{14}{s+7}+\frac{8}{s+8} \qquad (3.16)$$

The plant input $u(t)$ is chosen to be a 0.01 Hz, ± 10 V square wave with an additive noise component which is uniformly distributed between ± 10 V. For the MDFT, we choose $N=5000$ and $\Delta t=0.02$. The identification frequencies are then chosen to be 0.01 Hz, 0.03 Hz, and 0.11 Hz, which correspond to DFT frequencies and are present in $u(t)$.

Figure 1 shows parameter estimates for a 2nd-order model found using the recursive rational interpolation method described above and applying the recursive least squares algorithm to find the parameters from (3.5). The model has the form

$$G_R(s) = \frac{c_1 s + c_0}{s^2 + a_1 s + a_0} \qquad (3.17)$$

where, from Figure 1, $c_1=15.021$, $c_0=4.798$, $a_1=5.958$, $a_0=4.795$.

A comparison of $G_P(s)$ and $G_R(s)$ is shown in Figure 2. There is good agreement at all frequencies. *It should be noted that $u(t)$ is not bandlimited, as specified in (3.10) for ideal inputs, and the noise level in $u(t)$ is comparable to the* *magnitude of $u(t)$ itself.* This indicates that the algorithm is robust to nonideal conditions.

4. ON-LINE MODEL VALIDATION

It is generally not automatically known whether or not an identified model is good, in the sense that its frequency response closely matches that of the plant over a range of frequencies. This usually necessitates a *model validation* stage following the identification of a candidate model by any method.

It has been found in practice that the identification frequencies for interpolation-based schemes must be chosen with care in order for the algorithm to find a good model. Obviously, choosing correct identification frequencies is difficult if the plant is unknown. This indicates the need for some type of model validation before a model identified by this method can be used with confidence.

The identification algorithm presented in the previous section has the built-in capability for *on-line* model validation due to the fact that frequency-domain plant data are available via the MDFT. Thus, the algorithm may be given a measure of "intelligence" by increasing the number of identification frequencies and *simultaneously* deriving several models using different combinations of identification frequencies. The frequency response of each model can then be compared with the plant at *all* identification frequencies. The model with the least overall fit error can be taken as the best model for the plant of those derived.

Example

Let us consider this strategy applied to the example of Section 3, i.e., we attempt to find a 2nd-order model for the 8th-order plant (3.16). Since we are unsure which identification frequencies to use for the identification, we specify a set of frequencies which span the system passband, e.g., choose $[\omega_{I1}, \omega_{I2}, \omega_{I3}, \omega_{I4}, \omega_{I5}, \omega_{I6}] = 2\pi[0.001, 0.01, 0.1, 1.0, 10.0, 100.0]$. Of course, this necessitates an input which is different from the square wave used in the previous example. It can be, for instance, a sum of six sinusoids for this example. A model is derived using each distinct pair of frequencies, i.e., we derive a model using $(\omega_{I1}, \omega_{I2})$ as the identification frequencies, another model using $(\omega_{I1}, \omega_{I3})$, etc. In all, 15 different models can be derived taking the above identification frequencies two at a time. *It* *is important to remember that the 15 models are derived simultaneously.*

If we take as an error measurement the *distance* between the plant and model responses in magnitude and phase summed over all identification frequencies, we get a *figure of merit* for each model. Note that these distances are easily calculated because plant magnitude and phase data at the identification frequencies are available via the MDFT. To illustrate, let us take the following as the fit error:

$$\text{Error} = \sum_{i=1}^{6}\Big\{ \,|\, 20\log_{10}(\text{mag}(G_P(j\omega_{Ii}))) - 20\log_{10}(\text{mag}(G_R(j\omega_{Ii}))) \,|\, + \,|\, \arg(G_P(j\omega_{Ii})) - \arg(G_R(j\omega_{Ii})) \,|\, \Big\} \tag{4.1}$$

where $|\cdot|$ stands for absolute value, mag stands for magnitude, and arg stands for argument in degrees. *Of course, this definition of fit error is completely arbitrary.* Whatever the error criterion, however, it must give *a single number* which quantifies the fit error between the plant and model *and be capable of being quickly calculated.*

The model which yields the lowest error can be taken as the best model of those derived. For the present example, we get the following errors corresponding to the above-defined identification frequencies and error criterion:

I.D. Freq.	Error	I.D. Freq.	Error	I.D. Freq.	Error
$(\omega_{I1}, \omega_{I2})$	15.585	$(\omega_{I2}, \omega_{I3})$	14.394	$(\omega_{I3}, \omega_{I5})$	10.164
$(\omega_{I1}, \omega_{I3})$	14.405	$(\omega_{I2}, \omega_{I4})$	3.6617	$(\omega_{I3}, \omega_{I6})$	10.242
$(\omega_{I1}, \omega_{I4})$	3.6938	$(\omega_{I2}, \omega_{I5})$	4.2390	$(\omega_{I4}, \omega_{I5})$	81.438
$(\omega_{I1}, \omega_{I5})$	4.2825	$(\omega_{I2}, \omega_{I6})$	4.2805	$(\omega_{I4}, \omega_{I6})$	81.370
$(\omega_{I1}, \omega_{I6})$	4.3242	$(\omega_{I3}, \omega_{I4})$	7.5107	$(\omega_{I5}, \omega_{I6})$	78.089

Obviously, the model derived using the frequency pair $(\omega_{I2}, \omega_{I4})$, or ROM #7, yields the lowest error and therefore the best fit. This model is given by (3.17) with $c_1 = 17.392$, $c_0 = 5.315$, $a_1 = 7.334$, $a_0 = 5.320$. A comparison of this model with the plant is given in Figure 3. Notice that this model is better than the one derived in Section 3, since we have "optimized" the identification frequencies via on-line model validation.

If no model can be found whose fit error is sufficiently small, the model order can be easily increased. This amounts to changing the number of entries in $Z(t)$ and correspondingly in ψ. *Note that this can be done without changing the inputs or the identification frequencies.* The order of the calculation (3.5) is simply incremented in the algorithm in real time.

Finally, since the input and output MDFTs are monitored in this method, it is possible to make decisions on how fast to update the model parameters or whether to update them at all. If the MDFTs are not changing and a suitable model has been determined, the parameter estimation part of the algorithm can be shut off. In this case, only the MDFTs would proceed as the plant signals evolve. Thus, the monitoring of the plant continues, but the parameter estimation part of the algorithm rests if there is no need for a parameter update.

5. AN EXTENSION TO MIMO SYSTEMS

The above scheme for SISO identification via recursive rational interpolation can be extended to MIMO plants as follows. Assume the plant has input vector $u(t)=[u_1, u_2, ..., u_q]^T$ and output vector $y(t)=[y_1, y_2, ..., y_p]^T$. The MIMO identification process finds constant matrices $P_i \sim (p \times p)$ and $Q_i \sim (p \times q)$, $i=0, 1, ..., m-1$ such that the model transfer matrix given by

$$T(s) = P^{-1}(s)Q(s) \tag{5.1}$$

where

$$P(D) = D^m I + \sum_{i=0}^{m-1} P_i D^i \tag{5.2}$$

$$Q(D) = \sum_{i=0}^{m-1} Q_i D^i \tag{5.3}$$

matches the plant transfer matrix on a given set of frequencies.

The model can be described by the matrix differential equation

$$P(D)y(t) = Q(D)u(t) \tag{5.4}$$

which can be rewritten in the form:

$$D^m y = -P_{m-1}D^{m-1}y - \ldots - P_0 y + Q_{m-1}D^{m-1}u + \ldots + Q_0 u \tag{5.5}$$

This can be put in a form analogous to (3.5):

$$Y(t) = \Psi^{\mathrm{T}} Z(t) \tag{5.6}$$

where now

$$Y(t) = D^m y \tag{5.7}$$

$$\Psi^{\mathrm{T}} = \left[-P_{m-1}, \ldots, -P_0, Q_{m-1}, \ldots, Q_0\right] \tag{5.8}$$

and

$$Z(t) = \left[D^{m-1}y^{\mathrm{T}}, \ldots, y^{\mathrm{T}}, D^{m-1}u^{\mathrm{T}}, \ldots, u^{\mathrm{T}}\right]^{\mathrm{T}} \tag{5.9}$$

Note that $Y(t) \sim p \times 1$, $\Psi^{\mathrm{T}} \sim p \times m(p+q)$, and $Z(t) \sim m(p+q) \times 1$.

Again, choose any set of k distinct DFT frequencies $\{\omega_{I1}, \omega_{I2}, \ldots, \omega_{Ik}\}$. Denote the set of identification frequencies as $S = \{\omega_{I1}, \omega_{I2}, \ldots, \omega_{Ik}\}$. Choose integers $l_i > 0$, $i = 1, \ldots, q$ such that

$$\sum_{i=1}^{q} l_i = k \tag{5.10}$$

Let S_i, $i = 1, 2, \ldots, q$ be subsets of S such that S_i contains l_i elements and $S_i \cap S_j = 0$, $i \neq j$. Now the input $u_i(t)$ is specified as containing frequency components at the frequencies contained in S_i. The input u_i may contain any other frequency components also, with the exception that the frequency components of S_i must be unique to u_i.

Again, MDFTs are taken of $u(t)$ and $y(t)$ at the appropriate identification frequencies. Note that, in the MIMO case, MDFTs are taken of the input $u_i(t)$ at only the identification frequencies contained is S_i, but the MDFTs of the output $y_i(t)$ must be taken at *all* identification frequencies.

As in the SISO case, the quantities in $Y(t)$ and $Z(t)$ in (5.6) are parameterized from MDFT data at each time instant. Therefore, standard parameter estimation techniques may be employed to solve for the parameter matrix Ψ in (5.6). The minimum number of identification frequencies necessary for unique identification and the minimum number of identification frequencies per input are topics of current research.

Example

As a numerical example, the 10-state, 2-input, 2-output plant TGEN given in [7] is considered. This is a linearized model of a large turbo-generator. The original nonlinear model is given in [8]. The A, B, and C matrices for the state-space representation of the linearized model are given in [9]. The corresponding plant transfer matrix is given by

$$G_P(s) = \begin{bmatrix} G_{P11}(s) & G_{P12}(s) \\ G_{P21}(s) & G_{P22}(s) \end{bmatrix} \tag{5.11}$$

$G_P(s)$ is given in the Appendix.

A 6-state model with observability indices equal to 3 is chosen for this system. Thus, for this example, $p=2$, $q=2$, $m=3$, and k is chosen as 6. The identification frequencies are chosen as $[\omega_{I1}, \omega_{I2}, \omega_{I3}, \omega_{I4}, \omega_{I5}, \omega_{I6}]=2\pi[0.01, 0.1, 0.2, 0.5, 1.0, 2.0]$. The plant inputs are specified as:

$$u_1(t) = \sum_{i=1}^{3} \sin\omega_{Ii}t \tag{5.12a}$$

$$u_2(t) = \sum_{i=4}^{6} \sin\omega_{Ii}t \tag{5.12b}$$

The model transfer matrix identified for this system using rational interpolation, $G_R(s)$, is given in the Appendix. Also considered for comparison are a 6-state model for this plant given in [9] (denoted as G_M), and a 6-state model derived from a balanced realization of the plant (denoted as G_B). These models are also given in the Appendix. Figures 4 - 7 show comparisons of the magnitude and phase characteristics of G_P with those of G_R, G_M, and G_B.

To get a numerical measure of the closeness of the models to the plant, the plant magnitude and phase responses are compared with those of the various models at 100 frequencies logarithmically spaced between 10^{-3} and 10^{1} cycles/sec. That is, the following fit error is calculated:

$$E_X = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{n=1}^{100}\Big\{ \,|\, 20\log_{10}(\mathrm{mag}(G_{P_{ij}}(j\omega_n))) - 20\log_{10}(\mathrm{mag}(G_{X_{ij}}(j\omega_n))) \,| \\ + \ |\, \arg(G_{P_{ij}}(j\omega_n)) - \arg(G_{X_{ij}}(j\omega_n)) \,|\Big\} \qquad (5.13)$$

where $X = R$, M, and B, and $\omega_n = 2\pi \times 10^{-3+4(n-1)/99}$. This measure is, of course, completely arbitrary. However, it is one that is easily and accurately calculated and measures ROM fit in the frequency range 10^{-3} - 10^{1} cycles/sec, which is the critical range for this plant. Note that this calculation has nothing to do with on-line model validation (Section 4), but is being done simply to compare the goodness of the various ROMs. The resulting fit errors are calculated to be $E_R = 3.0157 \times 10^3$, $E_M = 2.4135 \times 10^4$, and $E_B = 9.4244 \times 10^3$. Although G_R has the lowest error, it should be kept in mind that G_R is designed specifically to match the plant in the frequency range 10^{-3} - 10^{1} cycles/sec.

6. CONCLUSIONS

A method of adaptively identifying reduced-order models for SISO and MIMO plants has been presented. The method is based on matching the plant and model transfer matrices at a number of frequencies chosen by the designer. The method recursively monitors frequency components in the plant inputs and outputs via the Moving Discrete Fourier Transform (MDFT). This is a computationally efficient method of recursively calculating the DFT of an evolving function of time. The identification algorithm has the capability for on-line model validation.

There is no necessity in this method for the plant input and output signals to be purely sinusoidal because the MDFT filters out sinusoidal components in these signals. As always, better results are obtained if the system, whatever its order, has a good reduced-order model. Computer simulations indicate that the algorithm is robust to additive input and output noise and non-bandlimited inputs.

Areas of current research include error analysis of the algorithm for nonideal inputs and stochastic disturbances, and determination of meaningful on-line figures of merit for identified models.

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APPENDIX

The plant transfer matrix is given by (5.11) with

$$G_{P11}(s) = \frac{b_7 s^7 + b_6 s^6 + \ldots + b_0}{\Delta_P}$$

with $b_7 = -4.6075 \times 10^1$, $b_6 = -4.5705 \times 10^3$, $b_5 = -4.7153 \times 10^6$, $b_4 = -1.8619 \times 10^8$, $b_3 = -2.4976 \times 10^9$, $b_2 = -1.2507 \times 10^{10}$, $b_1 = -1.5146 \times 10^{10}$, $b_0 = -3.9506 \times 10^9$,

$$G_{P12}(s) = \frac{b_6 s^6 + b_5 s^5 + \ldots + b_0}{\Delta_P}$$

with $b_6 = -2.0048 \times 10^4$, $b_5 = -1.7916 \times 10^6$, $b_4 = -2.3909 \times 10^9$, $b_3 = -1.1727 \times 10^{11}$, $b_2 = -1.2373 \times 10^{12}$, $b_1 = -2.8374 \times 10^{12}$, $b_0 = -1.8166 \times 10^{12}$,

$$G_{P21}(s) = \frac{b_7 s^7 + b_6 s^6 + \ldots + b_0}{\Delta_P}$$

with $b_7 = -1.1590 \times 10^0$, $b_6 = -6.9327 \times 10^1$, $b_5 = 5.6807 \times 10^5$, $b_4 = 2.3299 \times 10^7$, $b_3 = 2.9400 \times 10^8$, $b_2 = 1.3118 \times 10^9$, $b_1 = 1.4696 \times 10^9$, $b_0 = 1.3869 \times 10^9$, and

$$G_{P22}(s) = \frac{b_8 s^8 + b_7 s^7 + \ldots + b_0}{\Delta_P}$$

with $b_8 = 8.1715 \times 10^2$, $b_7 = 6.5386 \times 10^4$, $b_6 = 7.9770 \times 10^7$, $b_5 = 3.9454 \times 10^9$, $b_4 = 4.6442 \times 10^{10}$, $b_3 = 2.8534 \times 10^{11}$, $b_2 = 1.8656 \times 10^{12}$, $b_1 = 3.7379 \times 10^{12}$, $b_0 = 2.0592 \times 10^{12}$.

In the above, $\Delta_P = s^{10} + a_9 s^9 + \ldots + a_0$ with $a_9 = 1.0098 \times 10^2$, $a_8 = 1.0256 \times 10^5$, $a_7 = 4.2252 \times 10^6$, $a_6 = 6.5835 \times 10^7$, $a_5 = 5.4549 \times 10^8$, $a_4 = 3.2781 \times 10^9$, $a_3 = 1.4535 \times 10^{10}$, $a_2 = 2.7412 \times 10^{10}$, $a_1 = 1.8901 \times 10^{10}$, $a_0 = 3.1027 \times 10^9$.

The ROM transfer matrix $G_R(s)$ is given by

$$G_{R11}(s) = \frac{b_5 s^5 + b_4 s^4 + \ldots + b_0}{\Delta_R}$$

with $b_5 = -7.7158 \times 10^{-3}$, $b_4 = 7.6781 \times 10^{-2}$, $b_3 = -4.6515 \times 10^1$, $b_2 = -4.3011 \times 10^2$, $b_1 = -5.8884 \times 10^2$, $b_0 = -1.5861 \times 10^2$,

$$G_{R12}(s) = \frac{b_5 s^5 + b_4 s^4 + \ldots + b_0}{\Delta_R}$$

with $b_5 = 3.9934 \times 10^0$, $b_4 = -2.6408 \times 10^0$, $b_3 = 1.1580 \times 10^3$, $b_2 = -3.6478 \times 10^4$, $b_1 = -9.6222 \times 10^4$, $b_0 = -6.4423 \times 10^4$,

$$G_{R21}(s) = \frac{b_5 s^5 + b_4 s^4 + \ldots + b_0}{\Delta_R}$$

with $b_5 = -5.9553 \times 10^{-3}$, $b_4 = 3.2304 \times 10^{-2}$, $b_3 = 6.0336 \times 10^0$, $b_2 = 4.6175 \times 10^1$, $b_1 = 5.2247 \times 10^1$, $b_0 = 5.5681 \times 10^1$, and

$$G_{R22}(s) = \frac{b_5 s^5 + b_4 s^4 + \ldots + b_0}{\Delta_R}$$

with $b_5 = -1.6462 \times 10^1$, $b_4 = 1.1952 \times 10^3$, $b_3 = 3.1988 \times 10^3$, $b_2 = 5.7420 \times 10^4$, $b_1 = 1.3469 \times 10^5$, $b_0 = 7.4649 \times 10^4$.

In the above, $\Delta_R = s^6 + a_5 s^5 + \ldots + a_0$ with $a_5 = 1.1105 \times 10^1$, $a_4 = 7.2366 \times 10^1$, $a_3 = 4.5892 \times 10^2$, $a_2 = 1.0098 \times 10^3$, $a_1 = 7.4376 \times 10^2$, $a_0 = 1.2457 \times 10^2$.

The ROM transfer matrix $G_M(s)$ is given by

$$G_{M11}(s) = \frac{b_5 s^5 + b_4 s^4 + \ldots + b_0}{\Delta_M}$$

with $b_5 = 4.8290 \times 10^{-2}$, $b_4 = 1.8757 \times 10^0$, $b_3 = 1.9494 \times 10^0$, $b_2 = -1.2283 \times 10^3$, $b_1 = -5.7616 \times 10^3$, $b_0 = -2.3848 \times 10^3$,

$$G_{M12}(s) = \frac{b_5 s^5 + b_4 s^4 + \ldots + b_0}{\Delta_M}$$

with $b_5 = 2.9153 \times 10^{-2}$, $b_4 = 1.2889 \times 10^0$, $b_3 = -3.0414 \times 10^{-1}$, $b_2 = -2.3880 \times 10^4$, $b_1 = -8.2902 \times 10^5$, $b_0 = -9.5445 \times 10^5$,

$$G_{M21}(s) = \frac{b_5 s^5 + b_4 s^4 + \ldots + b_0}{\Delta_M}$$

with $b_5 = -4.9194 \times 10^{-1}$, $b_4 = 9.4826 \times 10^0$, $b_3 = -4.9046 \times 10^1$, $b_2 = 5.5164 \times 10^2$, $b_1 = -9.3964 \times 10^2$, $b_0 = 9.0758 \times 10^2$, and

$$G_{M22}(s) = \frac{b_5 s^5 + b_4 s^4 + \ldots + b_0}{\Delta_M}$$

with $b_5 = -6.7159 \times 10^{-1}$, $b_4 = 8.0447 \times 10^2$, $b_3 = 2.7814 \times 10^4$, $b_2 = 8.0852 \times 10^4$, $b_1 = 1.2135 \times 10^6$, $b_0 = 1.0820 \times 10^6$.

In the above, $\Delta_M = s^6 + a_5 s^5 + \ldots + a_0$ with $a_5 = 2.8238 \times 10^1$, $a_4 = 2.5831 \times 10^2$, $a_3 = 1.4679 \times 10^3$, $a_2 = 8.2145 \times 10^3$, $a_1 = 8.8012 \times 10^3$, $a_0 = 1.6306 \times 10^3$.

The ROM transfer matrix $G_B(s)$ is given by

$$G_{B11}(s) = \frac{b_6 s^6 + b_5 s^5 + \ldots + b_0}{\Delta_B}$$

with $b_6 = -1.9102 \times 10^{-2}$, $b_5 = -6.0759 \times 10^{-1}$, $b_4 = -1.4699 \times 10^1$, $b_3 = -4.5056 \times 10^1$, $b_2 = -1.4480 \times 10^3$, $b_1 = -3.5314 \times 10^3$, $b_0 = -1.1386 \times 10^3$,

$$G_{B12}(s) = \frac{b_6 s^6 + b_5 s^5 + \ldots + b_0}{\Delta_B}$$

with $b_6 = -1.8202 \times 10^{-2}$, $b_5 = -4.3626 \times 10^{-3}$, $b_4 = -5.6174 \times 10^0$, $b_3 = 3.0908 \times 10^2$, $b_2 = -3.1770 \times 10^4$, $b_1 = -4.7812 \times 10^5$, $b_0 = -5.2357 \times 10^5$,

$$G_{B21}(s) = \frac{b_6 s^6 + b_5 s^5 + \ldots + b_0}{\Delta_B}$$

with $b_6 = 1.5093 \times 10^{-1}$, $b_5 = -2.9259 \times 10^0$, $b_4 = 1.6680 \times 10^1$, $b_3 = -1.1690 \times 10^2$, $b_2 = 6.2777 \times 10^2$, $b_1 = 1.0947 \times 10^2$, $b_0 = 3.9972 \times 10^2$, and

$$G_{B22}(s) = \frac{b_6 s^6 + b_5 s^5 + \ldots + b_0}{\Delta_B}$$

with $b_6 = 1.6164 \times 10^{-1}$, $b_5 = -9.4448 \times 10^0$, $b_4 = 1.0597 \times 10^3$, $b_3 = 1.5933 \times 10^4$, $b_2 = 7.2009 \times 10^4$, $b_1 = 6.9226 \times 10^5$, $b_0 = 5.9350 \times 10^5$.

In the above, $\Delta_B = s^6 + a_5 s^5 + \ldots + a_0$ with $a_5 = 2.0310 \times 10^1$, $a_4 = 1.6829 \times 10^2$, $a_3 = 9.9166 \times 10^2$, $a_2 = 4.7178 \times 10^3$, $a_1 = 4.8675 \times 10^3$, $a_0 = 8.9425 \times 10^2$.

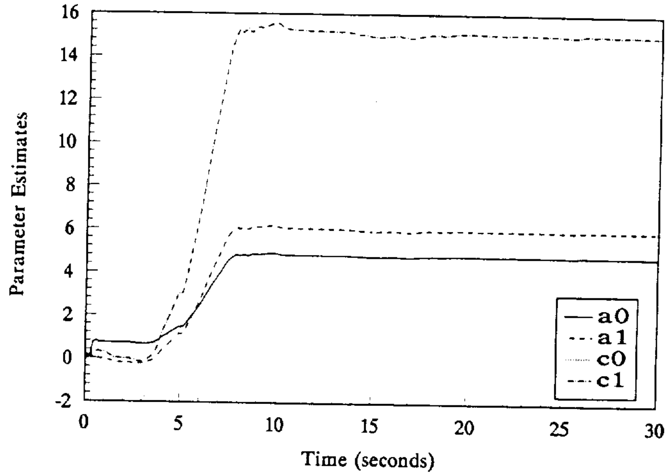

Figure 1 - Parameter estimates via recursive interpolation, ID frequencies = 0.01 Hz, 0.03 Hz, and 0.11 Hz. (a0 and c0 approximately coincide)

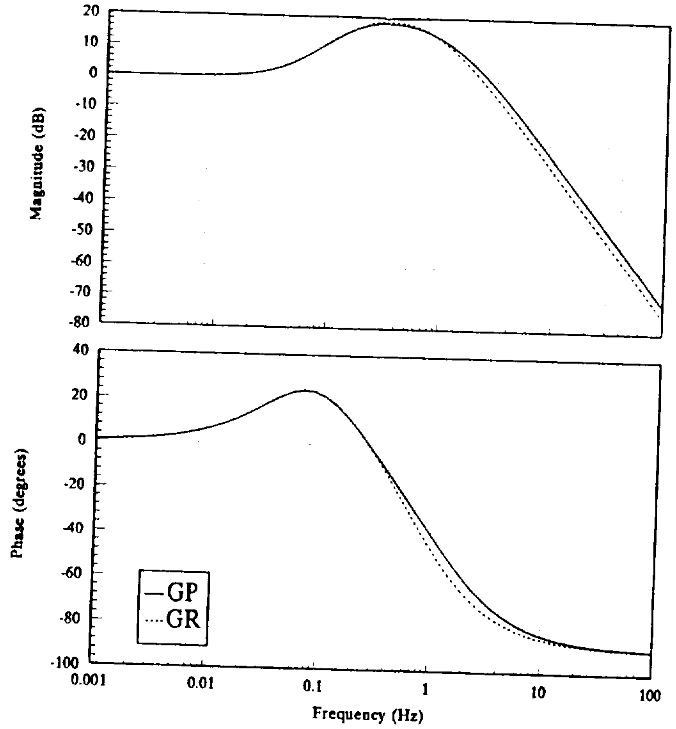

Figure 2 - Comparison of 8th-order plant (GP) with 2nd-order model (GR) derived via recursive interpolation, ID frequencies = 0.01 Hz, 0.03 Hz, and 0.11 Hz

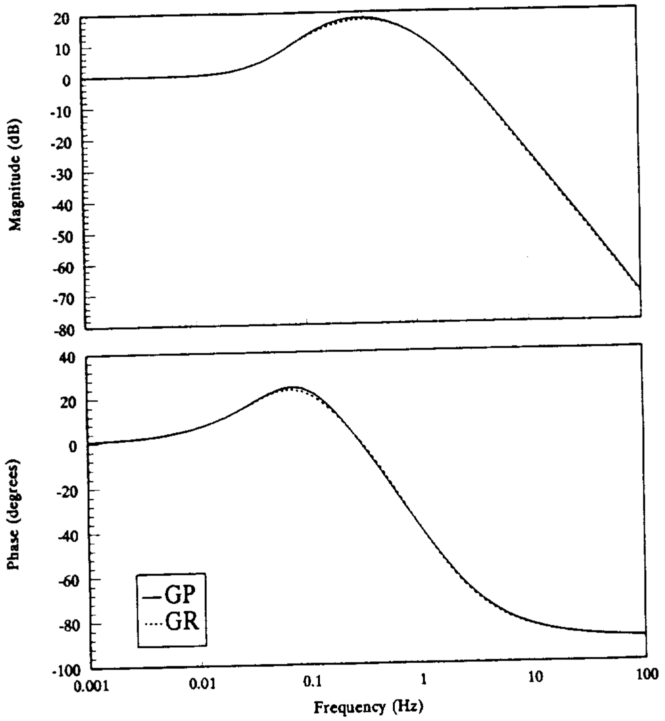

Figure 3 - Comparison of 8th-order plant (GP) with ROM #7 (GR), derived via recursive interpolation with on-line model validation

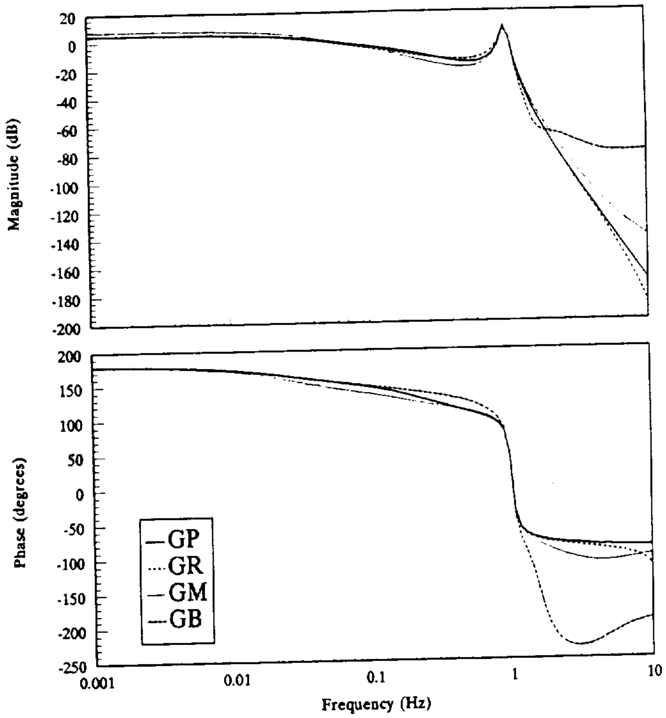

Figure 4 - Comparison of (1,1) elements of plant and model transfer matrices

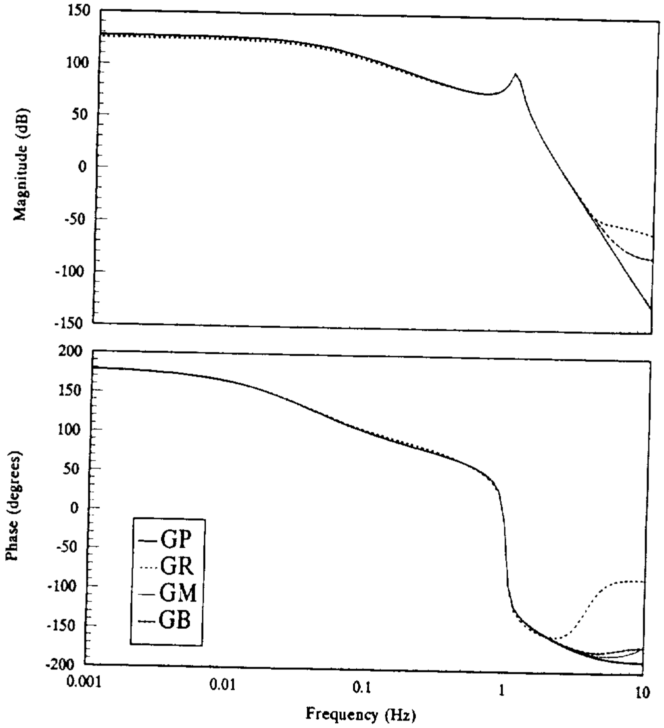

Figure 5 - Comparison of (1,2) elements of plant and model transfer matrices

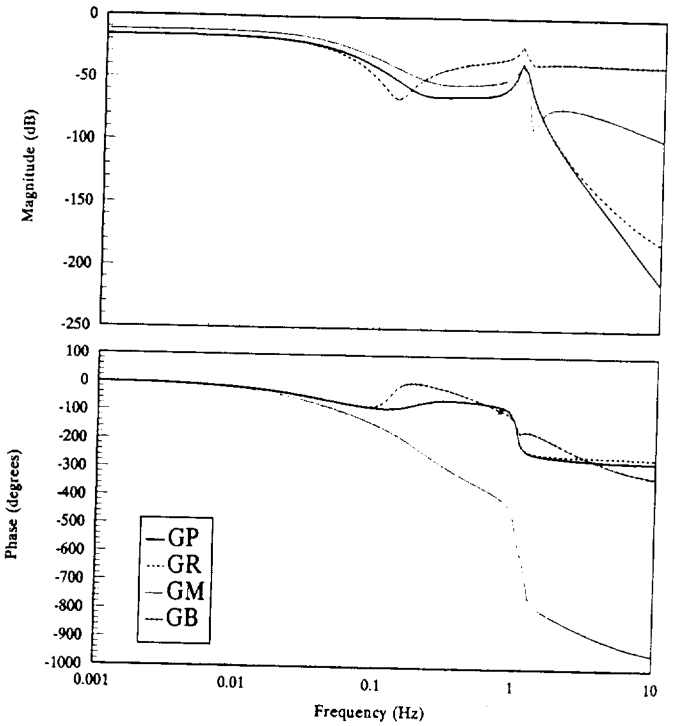

Figure 6 - Comparison of (2,1) elements of plant and model transfer matrices

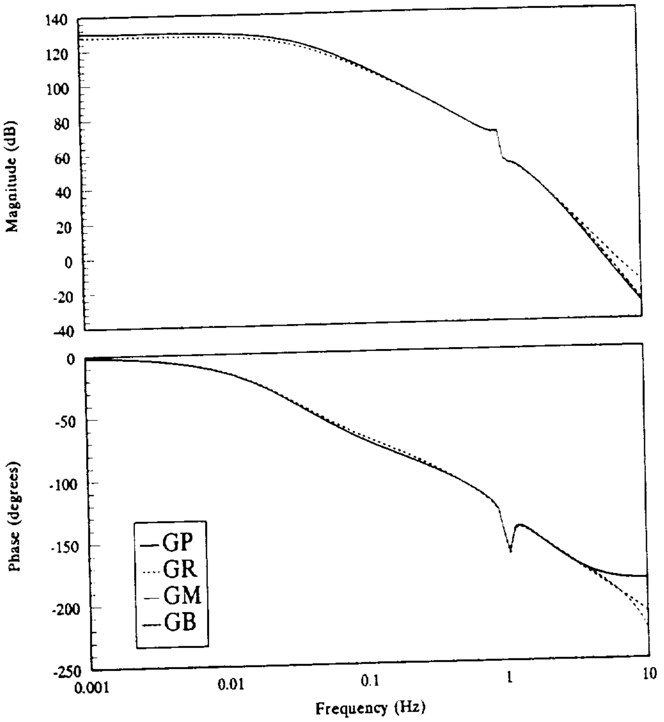

Figure 7 - Comparison of (2,2) elements of plant and model transfer matrices