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A Case Study in Nonlinear Dynamics and Control of Articulated Spacecraft: The Space Station Freedom with a Mobile Remote Manipulator System*

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Outline

1. Modeling of Articulated Spacecraft as Multi-Flex-Body Systems
2. Nonlinear Attitude Control by Adaptive Partial Feedback Linearizing (PFL) Control
3. Attitude Dynamics & Control for SSF/MRMS
4. Performance Analysis Results for Attitude Control of SSF/MRMS
5. Conclusions

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Nonlinear Control Design Approach for SSF/MRMS

- Modeling: Attitude Pointing Dynamics of Multi-Flex-Body Systems: Hamilton's Principle
- Attitude Control Decoupling/Linearizing Control by Nonlinear Feedback: Partial Feedback Linearization
- Adaptive Control: Modified Model Reference Adaptive Control (MRAC): Enhance Decoupling and PFL Robustness
- CSI: Keel flexure and MRMS motion results in nonlinear inertial couplings which effect attitude control on short time scale

This study addresses attitude control of the SSF with MRMS motion and considers nonlinear dynamic instabilities not previously considered in the work of Mah et al, Automatica 1989 and Wie et al, AIAA GNC 1990. Stability issues considered in these previous works concentrated on the slow time scale disturbance rejection of gravity gradient and cyclic aerodynamic torques on the time scale of the orbital period. This study addresses control of short time scale dynamic instability due to nonlinear inertia coupling which arises due to keel flexure and MRMS motion. The study addressed the following points:

1. Nonlinear inertia coupling due to keel flexure seriously constrains the stabilization of SSF attitude via linear control methods.
2. Feedback Linearization for Attitude Control and MRMS decoupling can achieve precision stabilization subject to limitations of: control authority, actuator bandwidth, and model uncertainty.
3. MRAC based on nonlinear design model with explicit parameter dependence can be effective for stabilization of SSF attitude with uncertain keel stiffness.

Work reported here in modeling and control design builds on previous work reported in:

1. Baillieul & Levi (1987) Physica D
2. Bennett, Kwatny, & Dwyer (1988) AFOSR Technical Report TSI-TR-88-07

Lagrangian Dynamics for Mixed LPS/DPS

1. identify configuration space (generalized coordinates)

$$q \in \mathcal{M} \quad \dot{q} \in T_q\mathcal{M}$$

□ choose DPS coordinates to eliminate $\mathcal{G}$ *geometric B.C.'s*

2. Hamilton's principle: motion is natural if:

$$\int_{t_1}^{t_2} (\delta L + Q^T \delta q)dt = 0$$

System Lagrangian: $L(q, \dot{q}) : \mathcal{M} \times T_q\mathcal{M} \to \Re$ obtains $\mathcal{N} = \{\text{natural B.C.'s}\}$, or

3. solve Euler-Lagrange Eqns.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i$$

subject to B.C.'s , $\mathcal{B} = \mathcal{G} \cup \mathcal{N}$.

The approach followed in this study for model construction utilizes quasi-coordinates and generalized Lagrange equations often referred to as Poincare equations. The method includes explicit construction of Finite Element Methods (FEM) for flexure and spatial recursive construction of Multi-Body systems introduced by Rodriguez and Jain (1991) AIAA GNC.

The formalism of Lagrangian dynamics proceeds by identification of configuration space in terms of generalized coordinates and their velocities. Hamilton's principle identifies the natural motion as the solution of a variational problem. If the coordinates are independent then the usual Euler-Lagrange equations result. If the coordinate variations are constrained (e.g. nonholonomic systems) then the d'Alembert-Lagrange equations apply (Neimark & Fufaev 1972).

Lagrangian Dynamics for Mixed LPS/DPS

we say $v \in H^p$ if $\|v\|_p^2 = \int_0^\ell |D^p v|^2 + \cdots + |v|^2 \, dz < \infty$
$v \in H_{\mathcal{G}}^p$ satisfies B.C.'s up to p^{th} order

1. Hamilton's Principle $\Rightarrow$ "weak" (distributional) solutions in $H_{\mathcal{G}}^p$

2. Euler-Lagrange Eqns. $\Rightarrow$ "strong" (pointwise) solutions in $H_{\mathcal{B}}^{2p}$

Finite Dimensional Modeling and FEM Approximations:

- Approximate weak solutions by discretization of Lagrangian and apply Hamilton's Principle

- We use collocation by splines for FEM approximation

The extension of the Lagrangian approach to mixed Lumped Parameter and Distributed Parameter Systems arising in Multi-Flex-Body systems involves reduction based on Finite Element Methods. Our approach utilizes splines for construction of the elements with continuity requirements at knots consistent with the variational problem.

Lagrange's Equations using Quasi-Velocities

Given configuration space $q \in \mathcal{M}$. Consider *quasi-velocities* p such that

$$
\begin{aligned}
\dot{q} &= V(q)p &= [v_1, v_2, \ldots, v_m]p \\
p &= U(q)\dot{q} &= [u_1, u_2, \ldots, u_m]\dot{q}
\end{aligned}
$$

□ generally not related to valid set of coordinates $\dot{\pi} = p$ unless

$$\delta\pi = U(q)dq$$

is an exact differential

□ Hamilton's principle applied to Lagrangian in quasi-velocities $\widetilde{L}(q,p) = L(q,\dot{q})$

Poincare equations (Arnold et al 1988)

$$\dot{p}^t \frac{\partial^2 \widetilde{L}}{\partial p^2} = -p^t V^t(q) \frac{\partial^2 \widetilde{L}}{\partial q^t \partial p} + \sum_{j=1}^{m} p_j \frac{\partial \widetilde{L}}{\partial p} U X_j + \frac{\partial \widetilde{L}}{\partial q} V + Q^t V$$

$$X_j = [[v_j, v_1][v_j, v_2] \ldots [v_j, v_m]] \qquad \text{w/ } j = 1, \ldots, m$$

$$\text{commutators: } [v_i, v_j] = \sum_{k=1}^{m} c_{ij}^k(q) v_k$$

□ v_j form right invariant vector field on of Lie group G associated with $\mathcal{M}$

□ If $\mathcal{M}$is Lie group and v_i are independent, then c_{ij}^k are independent of q

Poincare equations are related to Boltzman-Hamel equations and Caplygin's equations in quasi-coordinates. The use of quasi-velocities extends Lagrangian framework to nonholonomic systems. Poincare equations together with the quasi-velocity definition form a system of first order ODE's describing the equations of motion for the N-body model.

1. Quasi-velocities are not time derivatives of physically significant coordinates.

2. Formulation of Poincare equations considered here is also related to the constructive methods of Kane.

3. The modeling approach has also been applied to much simpler prototype spacecraft attitude slewing of the SCOLE model in Bennett, Kwatny, LaVigna 1991 ASME.

Structure of Poincare's Equations in Quasi-Velocities

Kinetic Energy: $\tilde{T}(q,p) = p^t \mathcal{M}(q) p$

$$\mathcal{M}(q)\dot{p} + \mathcal{C}(q,p)p + \mathcal{F}(q) = \mathcal{Q}_p$$

where

$$\mathcal{C}(q,p) := -\left[\frac{\partial[\mathcal{M}p]}{\partial q} V\right] p + \frac{1}{2}\left[\frac{\partial[\mathcal{M}p]}{\partial q} V\right]^t p + \sum_{j=1}^{m} p_j X_j^t U^t \mathcal{M}$$

$$\mathcal{F}(q) := V^t(q)\frac{\partial \mathcal{V}(q)}{\partial q^t}$$

Potential Energy: $\mathcal{V}(q)$

Generalized Forces in p-frame:
(often convenient when quasi-velocities are referenced to body frame)

$$\mathcal{Q}_p := V^t(q) Q$$

Lagrangian formalism provides an explicit construction of the system dynamic coefficients. The transformation of the generalized forces to the p-frame defined by the quasi-velocities is more convenient for the actuator command frame. The construction facilitates the definition of nonlinear control laws which include explicit model parameter dependence. This is useful for evaluating tradeoffs in gain scheduled vs. adaptive control implementations.

Conventional Linearization by Taylor Expansion

- conventional linearization by Taylor expansion is valid in the neighborhood of an equilibrium (when $\dot{q} = 0$ and $q = 0$)
- assume $p = 0$, then equilibrium configuration is: $\mathcal{F}(q) = \mathcal{Q}_p$

Linear Perturbation Dynamics:

$$\dot{q} = V(0)p$$

$$\mathcal{M}(0)\dot{p} + \mathcal{C}(0,0)p + \frac{\partial \mathcal{F}}{\partial q}(0) = \triangle \mathcal{Q}_p$$

Conventional Linear Control Design Methods:

- fixed gain control limited to neighborhood of equilibrium
- extension to gain scheduled designs is ad hoc

System equilibria can be identified for the case of constant generalized forces defined in the p-frame. Then conventional methods for identification of linear models proceed by Taylor expansion. Note that dynamical changes in configuration such as deployment of appendages, articulation of robot arms, etc. do not necessarily involve motions relative to a well defined equilibrium.

Partial Feedback Linearization attempts to impose an I/O linear with reference to a nominal system model. Explicit model construction for PFL provides explicit control dependence on parametric model uncertainty.

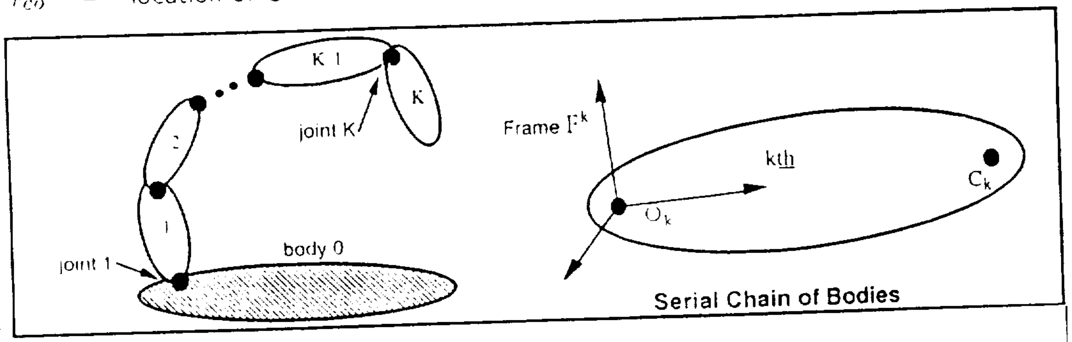
Recursive Formulation for Serial Chain of Bodies

Notation: velocity in body-frame at C

$V_C := \begin{pmatrix} \omega \\ v_C \end{pmatrix}$ angular velocity / translation

r_{CO} – location of C in frame at O

$$V_C = \phi(r_{CO})V_O$$
$$\phi(r_{CO}) = \begin{bmatrix} I & 0 \\ -\widetilde{r_{CO}} & I \end{bmatrix}$$



Serial Chain of Bodies

Joint parameters: σ_k of dimension n_k
quasi-velocities: $\beta(k)$

$$\dot{\sigma}_k = \Sigma_k(\sigma_k)\beta(k)$$
$$V_{O_k} - V_{C_{k-1}} = H(k)\beta(k)$$

Recursive Formulation for Serial Chain of Bodies

Notation
- $I_{cg}(k)$ -inertia tensor about CG
- $m(k)$ -mass
- $a(k)$ -location O in k-frame
- $M_{cg}(k)$-spatial inertia about CG
- $M_O(k)$ -spatial inertia tensor about O

$$M_{cg}(k) = \begin{bmatrix} I_{cg} & 0 \\ 0 & mI \end{bmatrix}$$
$$M_O(k) = \phi^*(a)M_{cg}\phi(a)$$

- Coordinate free recursion (Jain & Rodriguez 1990)

$$V(k) = \phi(r_{CO}(k-1))V(k-1) + H(k)\beta(k)$$

Chain model: (constructed from convenient choice of coordinates)
spatial velocity: $V := [V^t(1)\dots V^t(K)]^t$
joint quasi-velocity: $\beta := [\beta^t(1)\dots\beta^t(K)]^t$

$$V = \Phi H \beta$$

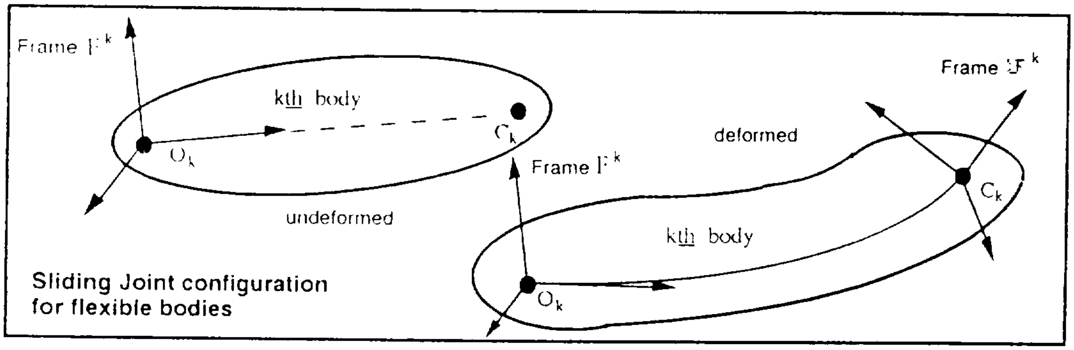
$$\Phi := \begin{bmatrix} I & & \cdots & 0 \\ \phi(2,1) & I & \cdots & 0 \\ \vdots & & \ddots & 0 \\ \phi(K,1) & \phi(K,2) & \cdots & I \end{bmatrix} \qquad H = \begin{bmatrix} H(1) & & \\ & \ddots & \\ & & H(K) \end{bmatrix}$$

Chain Kinetic Energy: K. E.$_{chain} = \frac{1}{2}\beta^t \mathcal{M}\beta$
in terms of
Chain inertia matrix: $\mathcal{M} = H^*\Phi^*\text{diag}\{M_O(1)\dots M_O(K)\}\Phi H$

Model Formulation with Sliding 1-DOF Joints with Elastic Bodies

Sliding Joint: 1 DOF relative motion along path $\mathcal{P}$ defined in $k-1$-frame

- $\mathcal{P}$ defined by map $\gamma : \Re \rightarrow \Re^3$ as image of $\epsilon \in [\epsilon_0, \epsilon_1]$



Model Formulation with Sliding 1-DOF Joints

- relative velocity of point P on path $\mathcal{P}$ wrt $(k-1)$-frame $\mathcal{F}^{k-1}$:

$$v_p^{k-1} = \frac{\partial \gamma}{\partial \epsilon} \dot{\epsilon}$$

inertial velocity of P

$$v_p^{k-1} = v_p^{k-1}(k-1) - \widetilde{[\gamma]}\, \omega^{k-1}(k-1) + \frac{\partial \gamma}{\partial \epsilon} \dot{\epsilon}$$

- $\dot{\epsilon}$ single DOF translational quasi-velocity $\beta(k)$ such that spatial velocity has the form:

$$V^k(k) = \phi(\gamma^{k-1}(\epsilon))V^{k-1}(k-1) + H^k(k)\beta^k(k)$$

with

$$H^k(k) := \begin{bmatrix} 0_{3\times 1} \\ \partial\gamma/\partial\epsilon \end{bmatrix} \qquad \beta^k(k) := \dot{\epsilon}$$

The recursive construction for chains of bodies with revolute joints can be extended to include sliding joints (such as the Mobile Remote Manipulator System) by defining the velocities relative to the joint path constraint defined in a local body fixed frame. The recursive construction for elastic bodies can be established by defining a local body frame fixed at the preceeding joint. Elastic deformations are assumed small in the local body frame but can contribute to large motions in the system inertia frame. Such dynamics can be highly nonlinear.

Dynamic Decoupling & PFL for Multi-Flex-Body Systems

Coordinate partitioning:

$$\eta = \begin{pmatrix} \xi \\ u \end{pmatrix} \quad \leftarrow \text{body attitude}$$

Quasi-velocities:

$$p = \begin{pmatrix} \omega \\ v \end{pmatrix} \quad \leftarrow \text{body rates}$$

τ – control torques applied to main body

Multi-Flex-Body Model: Poincare equations:

$$\begin{aligned} \dot{\xi} &= \Gamma(\xi)\omega \\ \dot{u} &= \Sigma(\xi, u)v \\ M_\omega \dot{\omega} &+ N\dot{v} + F_\omega = G_\omega \tau \\ N^t \dot{\omega} &+ M_v \dot{v} + F_v = G_v \tau \end{aligned}$$

$\exists$ *PFL (decoupling) Control Law:* $\tau = \mathcal{A}(\xi, \omega, u, v) + \mathcal{B}(\xi, \omega, u, v)\alpha$
such that in closed loop α is commanded attitude accelerations

$$\ddot{\xi} = \alpha$$

Fact: exact system attitude PFL using torques referenced to the principal body frame; i.e., $G_w = I, G_v = 0$

$$\begin{aligned} \mathcal{A} &= F_\omega - NM_v^{-1}F_v + [NM_v^{-1}B^t - M_\omega]\Gamma^{-1}\frac{\partial \Gamma \omega}{\partial \xi}\Gamma\omega \\ \mathcal{B} &= [M_\omega - NM_v^{-1}N^t]\Gamma^{-1} \end{aligned}$$

For multi-DOF revolute joints the angular coordinates can be expressed using Euler angles, quaternions, or Gibbs parameters. Choice is significant for computational complexity and numerical stability of inverse transformation for PFL.

PFL attitude control achieves decoupling of keel flexure, rigid body translational modes, and MRMS motions from attitude dynamics. Moreover this is achieved consistently with nonlinear large angle motions of multi-body articulation.

For application to decoupling of the MRMS motions from the SSF the PFL control is parametrized by the MRMS motions. Thus the attitude regulation includes direct feedforward of the MRMS motion. This is a form of gain scheduling using nonlinear models.

Construction of the inertia matrix using quasi-velocities based on the spatial chain recursion together with the assumption of small deformations in the local body frame simplifies the form inertia matrix to be inverted for PFL. The simplification for on-line PFL is related to the efficiency of the order-n recursions currently in use for efficient simulation of multi-body dynamics. Note the construction works for implementation of nonlinear PFL control laws.

Design Considerations for PFL Attitude Control for SSF/MRMS

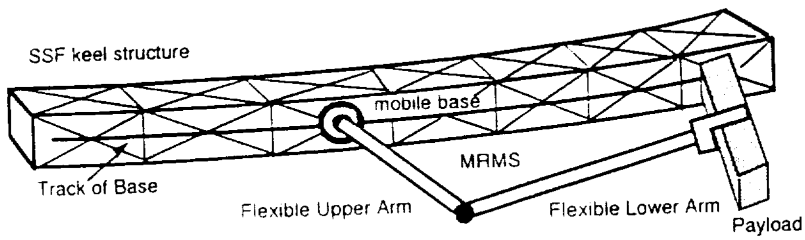
Performance Objectives:

- Achieve Decoupling of Independent Axis Attitude for Slewing/Pointing Control
- Achieve Decoupling of Flexible Interactions From Primary System Attitude Pointing
- Decouple sensitivity to MRMS motions
- Decouple design of active structural control (smart structures) from principal body attitude control

Practical Limitations

- Requires Additional Control Authority to Achieve Nonlinear Compensation
- Compensation Based on Nominal Design Model

SSF/MRMS System Configuration



System Physical Data

Body	length (m)	mass (kg)	inertia	cg location (m)
space station	110	211,258	J_x=2.13×10^8 J_y=2.13×10^8 J_z=880,241.6	x=55 y=55 z=55
mobile base	1.5	316.9	J_x=178.25 J_y=178.25 J_z=356.5	x=0 y=0 z=0
upper arm	14.3	3169	J_x=54,002 J_y=54,002 J_z=0	x=0 y=0 z=7.15
lower arm	14.3	3169	J_x=54,002 J_y=54,002 J_z=0	x=0 y=0 z=7.15

keel beam parameters	
material density	7.860·10^3 [kg/m^2]
elastic modulus	200·10^8 N/m^2
shear modulus	79·10^8 N/m^2

The simplified SSF/MRMS model includes four articulated bodies: 1) SSF keel, 2) MRMS base, 3) inner MRMS arm, 4) outer MRMS arm. Each joint is 1 DOF. Bodies 1 and 2 are connected by a sliding joint. The SSF/MRMS is modeled based on physical data taken from Mah, et al. Automatica. 1989. The SSF keel is modeled as a uniform beam with a 5m square cross section. The FEM model for the beam is reduced from Timoshenko assumptions with finite elements constructed from splines. Using 2 elements with 5 DOF. Simulations were conducted on a reduced model with 4 DOF to eliminate fast time scale effects beyond the control bandwidth. The joint velocities and SSF body rates are chosen as quasi-velocities.

Performance Evaluation of PFL Control for SSF/MRMS

Control Laws:

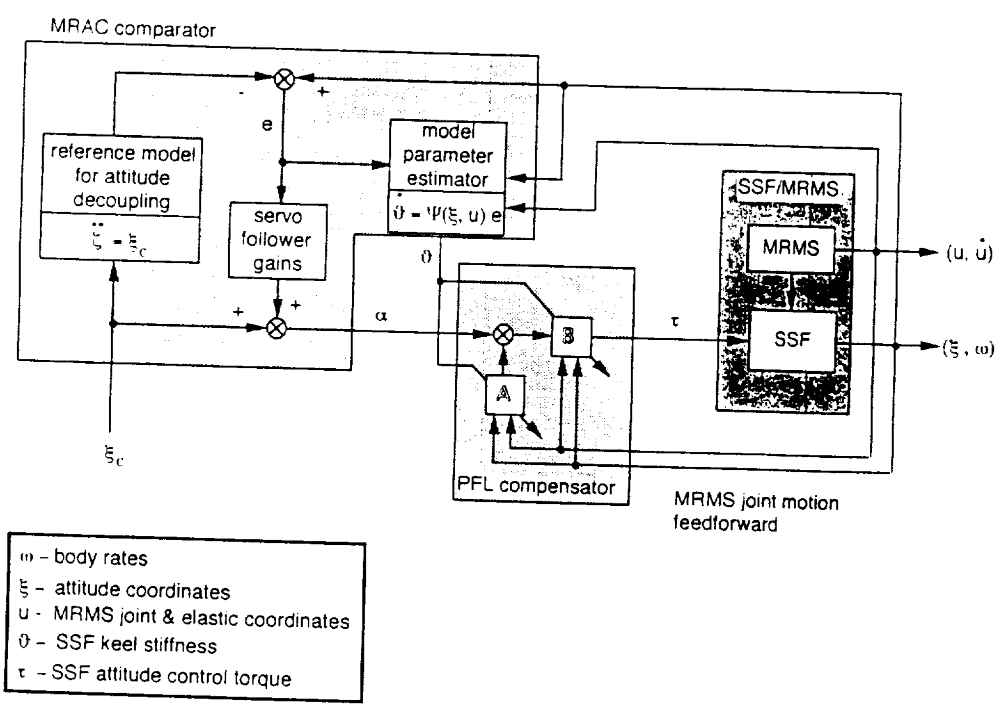
- PFL SSF Attitude Decoupling
- SSF Attitude Decoupling based on linearized equations of motion
- Parameter Adaptive PFL SSF Attitude Decoupling (SSF keel stiffness estimated)

Maneuvers:

- 3 axis simultaneous attitude maneuver of 0.5 rad (Euler angles)
- MRMS 3 simultaneous 1 DOF joint motions
  (translation equivalent to 18m in 60 sec)

Simulated performance evaluation of SSF attitude control with MRMS decoupling was performed for three control law variations including: nonlinear PFL, a linear decoupling control law, and MRAC modified nonlinear PFL control. The maneuvers considered were aggressive enough to differentiate the results.

MRAC PFL Attitude Decoupling Control for SSF/MRMS



Tradeoff Studies for PFL Attitude Control for SSF/MRMS

1. Nonlinear coupling due to keel flexure in attitude control: Nominal (precision model-based) PFL for attitude maneuver compared with Linear, fixed-gain, decoupling control
   - MRMS feedforward accounted in both designs
   - nonlinear inertia variations due to keel flexure limit domain of attraction in linear design
2. PFL robustness to SSF keel stiffness uncertainty
   - marginally stable slew response with 5% uncertainty (reduction) in keel stiffness
   - robustness limited by (active/passive) damping of keel flexure
3. Robustness of PFL attitude control w/ MRAC correction for keel stiffness
   - improved slew response with 10% uncertainty (reduction) in keel stiffness
   - marginal slew response with 20% reduction in keel stiffness
   - guarantee of stability margin in keel flexure response with MRAC is difficult without active structure control

SSF/MRMS Scaling of Control Gains

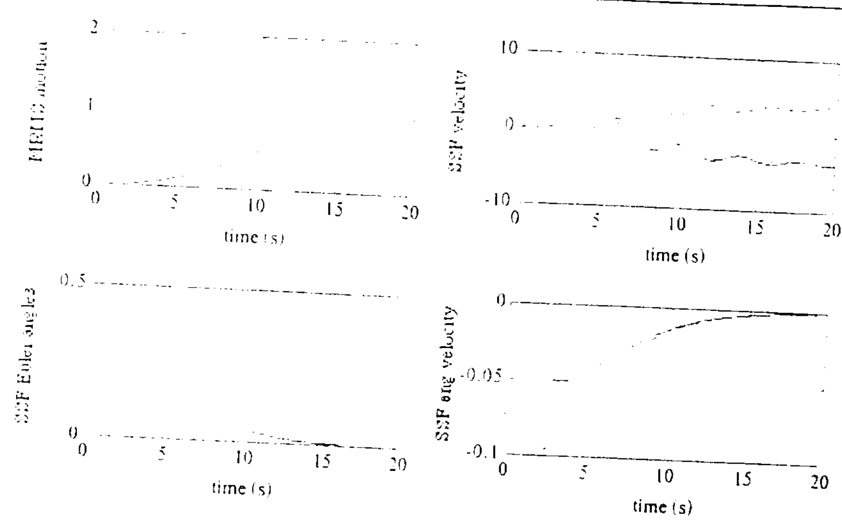
- Control gains chosen for Decoupled Attitude Linear Dynamics

System Eigenvalues

Open Loop	Closed Loop Nominal (k)	Closed Loop Detuned (k/8)
0 0 0 0 0 0 0^2 0^2 0^2 -10.4212 ±10.5963i -10.8762 ±10.5876i -0.2053 ± 3.3267i -0.2053 ± 3.3290i	0 0 0 0 0 0 -0.1763 ± 3.3205i -0.1763 ± 3.3205i -0.1364 ± 1.6505i -0.1364 ± 1.6505i -0.2000 ± 0.2040i -0.2000 ± 0.2040i -0.2000 ± 0.2040i	0 0 0 0 0 0 -0.1763 ± 3.3205i -0.1763 ± 3.3205i -0.1364 ± 1.6505i -0.1364 ± 1.6505i -0.0250 ± 0.0979i -0.0250 ± 0.0979i -0.0250 ± 0.0979i

- Linear controller effective for .01 rad slewing with Detuned Gains

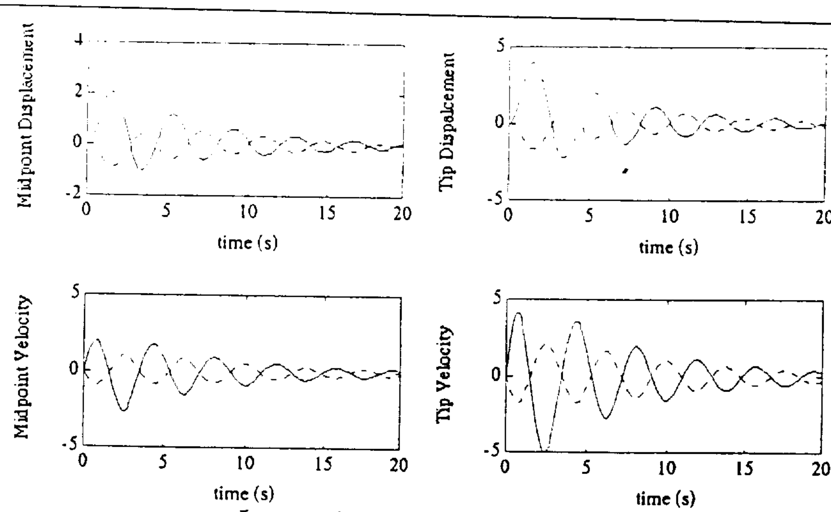
SSF/MRMS Attitude Slew with Precision PFL & MRMS Decoupling



PFL with MRMS motion - regulator gains: (-0.8,-0.1632)

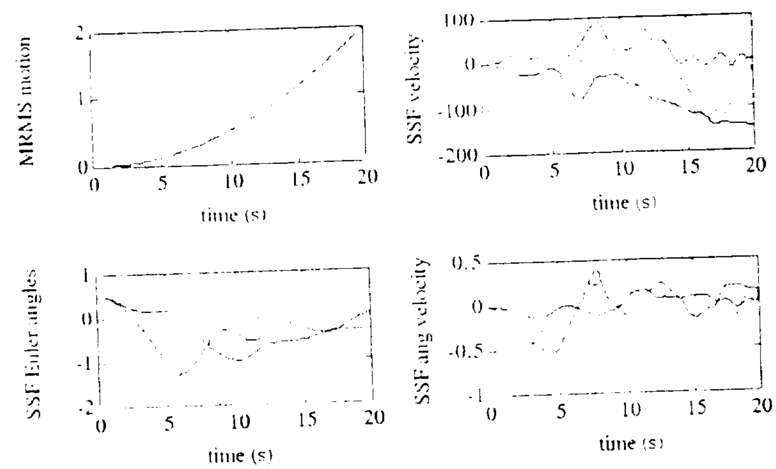
SSF/MRMS PFL attitude control nominal response for 3-body maneuver

SSF/MRMS Attitude Slew with Precision PFL & MRMS Decoupling



SSF Keel Flexure Response with nominal maneuver

SSF/MRMS PFL Attitude Slew with 5% Keel Stiffness Reduction

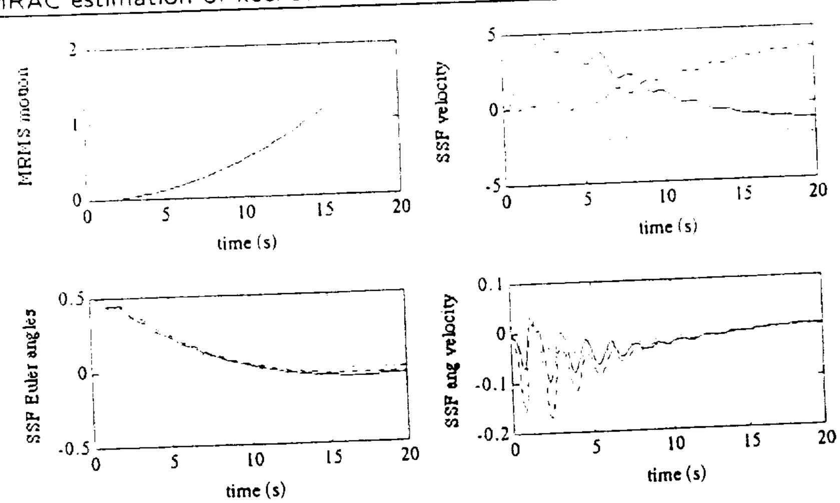


Nominal PFL slewing with 5% reduction in keel stiffness

SSF/MRMS response degradation is evident for nominal PFL attitude control

SSF/MRMS Attitude Slew with MRAC PFL & MRMS Decoupling

MRAC estimation of keel stiffness recovers nominal slew performance

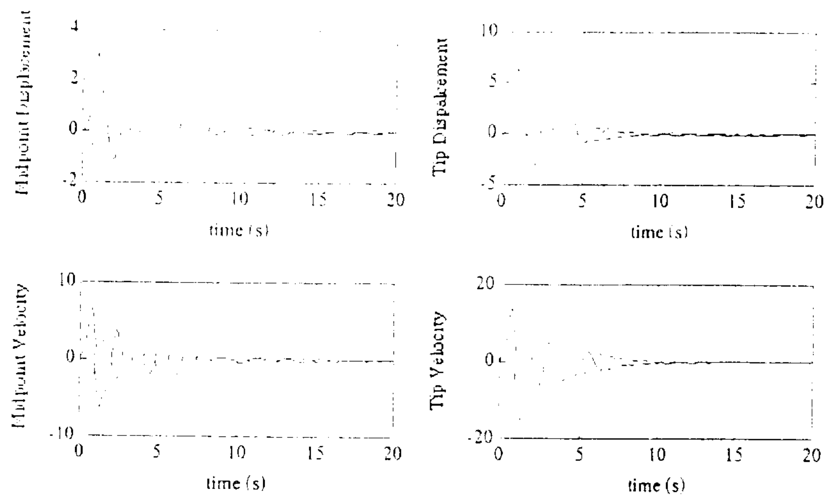


PFL with MRMS motion - regulator gains: (-0.8,-0.1632)

10% reduction in keel stiffness wrt nominal value for PFL t=0

SSF/MRMS Keel Flexure with MRAC PFL & MRMS Decoupling

PFL MRAC obtains stable response in keel flexure



keel flexure with MRAC

10% reduction in keel stiffness

Conclusions from Simulations

Observations from SSF/MPMS Modeling:

- Keel flexibility drastically alters nonlinear inertial coupling in attitude maneuvers
- MRMS motion exacerbates nonlinear inertial coupling

Control Law Comparsions:

- linear, fixed-gain control can achieve stabilization of small amplitude SSF attitude motions on a slow time scale
  - □ stability robustness limitation is not evident from linear model
  - □ domain of attraction is limited for fast time scale attitude regulation
- linear, fixed-gain, decoupling control demonstrates extreme sensitivity to MRMS motions & model uncertainty
- PFL stability sensitive to keel stiffness parameters
  5 % reduction in keel stiffness results in oscillations with magnitude on order of length of keel
- Adaptive PFL attitude maneuver control with extreme MRMS multi-DOF motions demonstrated tolerance to initial model uncertainty of up to 10% reduction in keel stiffness

The SSF/MRMS system model predicts a significant elastic deformation response of SSF keel during attitude slewing transients. This leads to large motions in the inertial frame although the beam model assumes small relative displacements in local body frames. The result is significant nonlinear cross axis coupling during attitude maneuvers. For short time scale attitude control of the SSF/MRMS system the significance of the nonlinear inertia coupling due to keel flexure appears more significant than MRMS motion sensitivity—even for drastic, worst-case maneuvers considered in this study.

The robustness and performance limits observed in the linear, decoupling attitude control law appear to arise from a vanishingly small domain of attraction for fast time scale attitude regulation. The tradeoff of attitude control gains vs. domain of attraction cannot be predicted from linear models alone.

PFL decoupling attitude control offers a direct design approach including feedforward of MRMS motions which compensates for predictable inertia changes due to keel deformation. PFL attitude control sensitivity to keel stiffness uncertainty is improved over linear decoupling control. Performance and bandwidth limits in PFL design are traded off against stability of the decoupled dynamics (keel flexure dissipation).

Parameter adaptive methods based on MRAC underlie practical application of nonlinear decoupling control designs where model uncertainty is due to unmeasureable parameter variation.