ACTIVE VIBRATION DAMPING USING SMART MATERIAL*

John S. Baras and Yan Zhuang
Electrical Engineering Department
and
Systems Research Center
The University of Maryland, College Park, MD 20742

SUMMARY

We consider the modeling and active damping of an elastic beam using distributed actuators and sensors. The piezoelectric ceramic material (PZT) is used to build the actuator. The sensor is made of the piezoelectric polymer polyvinylidene fluoride (PVDF). These materials are glued on both sides of the beam. For the simple clamped beam, the closed loop controller has been shown to be able to extract energy from the beam. The shape of the actuator and its influence on the closed loop system performance are discussed. It is shown that it is possible to suppress the selected mode by choosing the appropriate actuator layout. It is also shown that by properly installing the sensor and determining the sensor shape we can further extract and manipulate the sensor signal for our control need.

1 INTRODUCTION

There has been an increasing interest in the control of large space structures and flexible structures in recent years. These structural systems are usually large in size,light in mass and hence weakly damped. In order to achieve vibration suppression and precision pointing, it is necessary to introduce artificial damping to such systems. One approach is passive damping by adding the minimum weight of damping material to the effective locations on the structure. Another way is to use external mechanisms with feedback of the systems' state or output to counteract the undesired motion

modes. In modern structural engineering, active feedback controls to stabilize the structure are preferred.

Proper modeling is essential to control system design and to avoid spillover due to the infinite dimensional nature of these systems. Here we consider the beam model as part of the structure and study its modeling and active damping.

The actuator considered here is a distributed one made of piezoelectric ceramic material which is glued to the beam. Its constitutive property, i.e. its strain and stress relation, is influenced by the external voltage applied to it. Bonding or embedding segmented elements of this material in a structure would allow the application of the localized strain to be transferred to the structure whose deformation can be controlled. Under proper bonding conditions, the coupling between the actuator strain and the beam strain can be determined to implement the control mechanism. In [6] [2] [7] active vibration control is described using spatially distributed actuators. The PVDF sensor is bonded to the beam in a similar way. The output voltage is a functional of beam curvature. Unlike the conventional point sensor, this is a distributed one. Cudney [5] provides some detailed explanation of the nature of piezoelectric actuators.

We first discuss the modeling of the beam and the distributed actuator. A static model of the actuator coupled into the structure is developed. We have developed the beam model from the Euler-Bernoulli model with rotational inertia added. Next the sensor model is addressed. We then discuss the controller design using Lyapunov methods. We finally investigate the actuator and sensor shapes and their impact on the system elastic modes.

2 SYSTEM MODEL

One approach to build the desired actuator is to take advantage of the special constitutive properties of certain materials. The actuation is due to the property change under certain stimulation other than the external actuation force. Such materials are the so called smart materials. Once properly embedded into the structure the induced actuation will produce bending or stretching or both to control the structure deformation. One of the advantages of using smart materials as actuators and sensors is that the structure will not change much.

Piezoelectric actuators were used as elements of intelligent structures by Crawley and de Luis [4]. Bailey and Hubbard [1] have used PVDF actuators to control the vibration of a cantilever beam. The control voltage applied across the PVDF is the sign of the tip rotation velocity multiplied by a constant.

Figure 1 shows the structure of the beam with both the sensor and the actuator layers glued together. In this figure, h stands for the thickness of the different layers of the beam. The subscripts s, b and a denote sensor, beam and actuator respectively. The constitutive law for piezoelectric materials has several equivalent forms. The stress -strain relationship for the piezoelectric material is similar to that
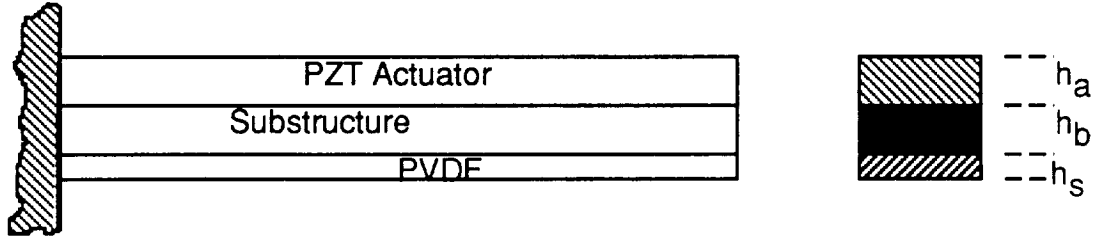
Figure 1: The composite beam

of thermoelastic materials, with the thermal strain term replaced by the piezoelectric strain Λ. The constitutive equation of the actuator is given by

$$\sigma = E_a(\epsilon - \Lambda) \tag{1}$$

where Λ is the actuation strain due to the external electric field, and ϵ is the strain without external electric field. E_a is the Young's modulus of the actuator, σ is the stress of the actuator. The actuation strain is given by

$$\Lambda(x,t) = \frac{d_{31}}{h_a}V(x,t) \tag{2}$$

where d_{31} is the piezoelectric field and strain field constant. $V(x,t)$ is the distributed voltage. The strain has two effects on the beam. One effect is that it induces a longitudinal strain ϵ_l to insure a force equilibrium along the axial direction. This steady state value of ε_l can be derived by solving a force equilibrium equation. The other effect is that the net force in each layer acts through the moment arm with the length from the midplane of the layer to the neutral plane of the beam. The resultant of the actions produces the bending moment. Taking a similar approach as in [1] the actuation moment can be expressed as

$$M_a = K_a\Lambda(x,t) \tag{3}$$

where K_a is a constant depending on the geometry and the materials of the beam.

We model the beam with linear bending, no shear but with the rotational inertia included. This is more accurate than the Euler-Bernoulli beam model. The Euler-Bernoulli beam model

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0 \tag{4}$$

is often used to describe the beam dynamics because of its simple form and ease for analysis. E stands for Young's modulus and I stands for moment of inertia. This equation can be rewritten as

$$\frac{\partial^2}{\partial x^2}[EI\frac{\partial^2 w(x,t)}{\partial x^2}] + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0 \tag{5}$$

where

$$EI = E_a I_a + E_b I_b + E_s I_s. \tag{6}$$

The bending moment of the composite beam without actuation is

$$M_b = EI\frac{\partial^2 w(x,t)}{\partial x^2}. \tag{7}$$

The Euler-Bernoulli model is a linear model without accounting the rotational inertia and the shear effect. It is easy to see that during vibration the beam elements perform not only a translational motion but also rotate. The variable angle of rotation which is equal to the slope of the deflection curve will be expressed by $\partial w/\partial x$ and the corresponding angular velocity and angular acceleration will be given by

$$\frac{\partial^2 w}{\partial x\partial t} \ and \ \frac{\partial^3 w}{\partial x\partial t^2}. \tag{8}$$

Therefore the moment of the inertia forces of the element about the longitudinal axis will be

$$-\rho I\frac{\partial^3 w}{\partial x\partial t^2}. \tag{9}$$

The equation with rotational inertia is [8]

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} - \rho I\frac{\partial^4 w(x,t)}{\partial x^2\partial t^2} = 0. \tag{10}$$

where

$$\rho A = \rho_a A_a + \rho_b A_b + \rho_s A_s.. \tag{11}$$

We take this equation as our beam model under consideration. It falls in between the Euler-Bernoulli beam and the Timoshenko beam.

The total bending moment with actuation is

$$M = M_b + M_a. \tag{12}$$

Substituting Equation (12) into (10), we have

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} - K_a\frac{\partial^2 \Lambda(x,t)}{\partial x^2} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} - \rho I\frac{\partial^4 w(x,t)}{\partial x^2 \partial t^2} = 0. \tag{13}$$

Considering the actuation strain and the applied voltage, we get

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} - c\frac{\partial^2 V(x,t)}{\partial x^2} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} - \rho I\frac{\partial^4 w(x,t)}{\partial x^2 \partial t^2} = 0 \tag{14}$$

with boundary conditions

$$
\begin{aligned}
w(0,t) &= 0 \\
\frac{\partial w(0,t)}{\partial x} &= 0 \\
EI\frac{\partial^2 w(L,t)}{\partial x^2} &= c.V(L,t) \\
EI\frac{\partial^3 w(L,t)}{\partial x^3} &= c.\frac{\partial V(L,t)}{\partial x}
\end{aligned}
\tag{15}
$$

where

$$c = \frac{d_{31}}{h_a}K_a.$$

The distributed voltage $V(x,t)$ is the control applied to the system. Equation (14) and the boundary conditions (15) form the control system model.

3   SENSOR MODEL

A distributed sensor is the one whose output is a function of structural responses at different locations. It can be a group of point sensors or a spatially continuous one. These responses are observed either discretely or continuously in space. Using the latter has the advantage that complicated computations based on point measurements can be reduced because the sensor geometry itself provides the processing. The spatial aliasing from an array of point sensors can be avoided. Typical noncausal sensor dynamics such as gain rolloff without phase shift is possible by using distributed sensors [3].

Figure 2 shows the sensor structure. PVDF is strain sensitive as it relies on the piezoelectric effect to produce the electric charge. The charge is proportional to the strain induced by the structure. This type of sensing is actually an inverse process of piezoelectric actuation. Based on the constitutive equation, the induced charge per unit length from the sensor strain is
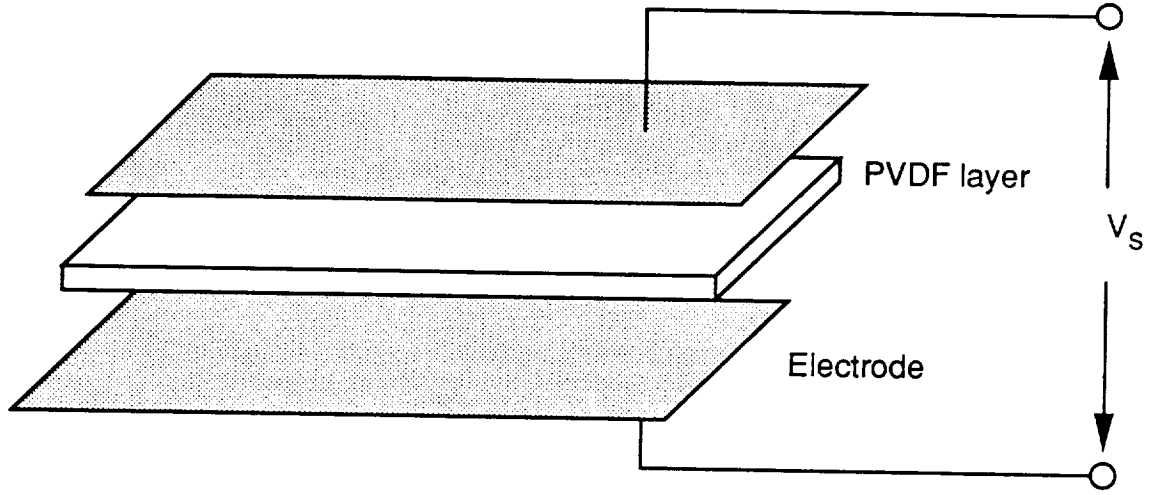
$$q(x,t) = -Esd_{31}\epsilon_s. \tag{16}$$

Figure 2: The sensor model

The sensor strain is related to the beam curvature by

$$\epsilon_s = \frac{h_b + h_s}{2}\frac{\partial^2 w}{\partial x^2}. \tag{17}$$

The electrical charge along the beam is

$$
\begin{aligned}
Q(x,t) &= \int_0^x q(x,t)F(x)dx \\
&= -E_s d_{31}\frac{h_b + h_s}{2}\int_0^x F(x)\frac{\partial^2 w}{\partial x^2}dx
\end{aligned} \tag{18}
$$

where $F(x)$ is the weight function or shape function of the sensor. It is the local width of the electrodes covering both sides of the sensor layer. The function $F(x)$ can be designed according to the need for interpreting the sensor signal. The capacitance between the electrodes of the sensor layer is

$$C = \frac{\varepsilon_0 \varepsilon_r A_s}{h_s} \tag{19}$$

where ε_0 and ε_r are the vacuum permittivity and relative permittivity constants respectively. The output voltage from the sensor is

$$
\begin{aligned}
V_s(x,t) &= \frac{Q(x,t)}{C} \\
&= -K_s \int_0^x F(x)\frac{\partial^2 w}{\partial x^2}dx
\end{aligned} \tag{20}
$$

where

$$K_s = \frac{E_s d_{31}(h_b + h_s)}{2C} \tag{21}$$

is a constant. Suppose the sensor covers the whole beam, then

$$V_s(t) = -K_s \int_0^L F(x)\frac{\partial^2 w}{\partial x^2}dx. \tag{22}$$

Equation (22) is the sensor output equation. The output voltage is the weighted integration of the beam curvature. Integrating the output voltage by parts twice in spatial variable, we have another form of the sensor output,

$$V_s(t) = -K_s \frac{\partial w(L,t)}{\partial x}F(L) + w(L,t)\frac{\partial F(L)}{\partial x} - K_s \int_0^L w(x,t)\frac{\partial^2 F(x)}{\partial x^2}dx. \tag{23}$$

We shall see later from Equation (23) that different measurement outputs can be formulated to meet our control needs by choosing the appropriate sensor shape function $F(x)$.

4 DISTRIBUTED CONTROL ALGORITHM

We design the control algorithm by Lyapunov's direct method. The energy function is used to measure the amount of vibration of the system. We need to find a control algorithm such that the closed loop system is asymptotically stable. One advantage of this method is that there is no need for model truncation.

Given the system (14) with boundary conditions (15) and an energy functional $E(t)$, we need to find a control $V(x,t)$ such that

$$\lim_{t \to \infty} E(t) = 0. \tag{24}$$

It suffices to find a control $V(x,t)$, such that

$$\frac{dE(t)}{dt} < 0, \ t > 0. \tag{25}$$

We define the energy function as follows:

$$E(t) = \frac{1}{2}\int_0^L [a(\frac{\partial^2 w}{\partial x^2})^2 + (\frac{\partial w}{\partial t})^2 + b(\frac{\partial^2 w}{\partial x \partial t})^2]dx \tag{26}$$

The first term is the stored energy due to bending. The second term is the kinetic energy due to the translation motion. The last term of the integrand is the kinetic energy from rotation of the beam element corresponding to Equation (14). a and b are positive constants.

Taking derivative of $E(t)$ with respect to time and incorporating the system equation (14) into it, we have

$$\frac{dE(t)}{dt} = \int_0^L [a\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^3 w}{\partial x^2 \partial t} + b\frac{\partial^2 w}{\partial x \partial t}\cdot\frac{\partial^3 w}{\partial x \partial t^2} + \frac{I}{A}\frac{\partial w}{\partial t}\cdot\frac{\partial^4 w}{\partial x^2 \partial t^2}$$
$$+ \frac{c}{\rho A}\frac{\partial w}{\partial t}\cdot\frac{\partial^2 V}{\partial x^2} - \frac{EI}{\rho A}\frac{\partial w}{\partial t}\frac{\partial^4 w}{\partial x^4}]dx. \tag{27}$$

Integrating by parts and setting

$$a = \frac{EI}{\rho A}, \ b = \frac{I}{A} \tag{28}$$

We obtain

$$\frac{dE(t)}{dt} = \frac{EI}{\rho A}\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial x \partial t}\Big|_{x=0}^{x=L} - \frac{EI}{\rho A}\frac{\partial^3 w}{\partial x^3}\cdot\frac{\partial w}{\partial t}\Big|_{x=0}^{x=L}$$
$$+ \frac{I}{A}\cdot\frac{\partial w}{\partial t}\cdot\frac{\partial^3 w}{\partial x \partial t^2}\Big|_{x=0}^{x=L} + \frac{c}{\rho A}\int_0^L \frac{\partial w}{\partial t}\cdot\frac{\partial^2 V}{\partial x^2}dx. \tag{29}$$

Introducing the boundary condition (15), we have

$$\frac{dE(t)}{dt} = \frac{c}{\rho A}V(L,t)\frac{\partial^2 w}{\partial x \partial t}\Big|_{x=L} - \frac{c}{\rho A}\frac{\partial V(L,t)}{\partial x}\cdot\frac{\partial w}{\partial t}\Big|_{x=L}$$
$$+ \frac{I}{A}\cdot\frac{\partial w}{\partial t}\cdot\frac{\partial^3 w}{\partial x \partial t^2}\Big|_{x=L} + \frac{c}{\rho A}\int_0^L \frac{\partial w}{\partial t}\cdot\frac{\partial^2 V}{\partial x^2}dx. \tag{30}$$

The first term in Equation (30) contains the rotational velocity of the beam at the end. The second term has the force applied by the actuator. The third term is the product of the velocity of the displacement and the angular acceleration at the end of the beam. There is a second partial derivative of $V(x,t)$ with respect to the spatial variable. We can design the appropriate modal controller by choosing the right $V(x,t)$. Our purpose here is to find the control such that the time derivative of the energy function is negative.

Let $V(x,t)$ be decomposed as the product of a spatial and a time function

$$V(x,t) = v(x)q(t) \tag{31}$$

where $v(x)$ is the actuator shape function, $q(t)$ is the coordinate function. We assume that the function $v(x)$ has continuous second derivative on the interval $(0,L)$ and has compact support over the interval; then the first two terms in Equation (30) vanish. Since the third term is negative from its physical meaning it will not cause energy

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increase. We need to analyze the influence of the integration term in the equation. Substituting the voltage function into the last term in Equation (30)

$$\frac{c}{\rho A} \int_0^L \frac{\partial w}{\partial t} \cdot \frac{\partial^2 V}{\partial x^2} dx = \frac{c}{\rho A} q(t) \int_0^L \frac{\partial w}{\partial t} \cdot \frac{\partial^2 v(x)}{\partial x^2} dx \qquad (32)$$

where $q(t)$ is the time coordinate of the controller.

We further introduce the feedback control by using the sensor output signal V_s,

$$
\begin{aligned}
q(t) &= \frac{dV_s(t)}{dt} \\
&= -K_s \int_0^L \frac{\partial w(x,t)}{\partial t} \frac{\partial^2 F(x)}{\partial x^2} dx.
\end{aligned} \qquad (33)
$$

Then (32) becomes

$$
\begin{aligned}
\frac{dE(t)}{dt} &\leq \frac{c}{\rho A} q(t) \int_0^L \frac{\partial w}{\partial t} \cdot \frac{\partial^2 v(x)}{\partial x^2} dx \\
&= -K_s \frac{c}{\rho A} \int_0^L \frac{\partial w}{\partial t} \cdot \frac{\partial^2 F(x)}{\partial x^2} dx \int_0^L \frac{\partial w}{\partial t} \cdot \frac{\partial^2 v(x)}{\partial x^2} dx \\
&< 0
\end{aligned} \qquad (34)
$$

Hence the system is asymptotically stable. The feedback control is given by

$$V(x,t) = -K_s v(x) \int_0^L \frac{\partial w(x,t)}{\partial t} \frac{\partial^2 F(x)}{\partial x^2} dx. \qquad (35)$$

The introduced control is velocity feedback control. It takes into account the bending rate along the beam and introduces damping to the system. Here there is no need for the model modal truncation.

When the control $V(x,t)$ is uniformly distributed in space, $\frac{\partial V}{\partial x} = 0$, if we further assume that there is no elastic bonding layer to be present between the piezoelectric and the substructure, that is, there is no shear lag between the two layers, the strain is transferred between the piezoelectric and the beam over an infinitesimal distance near the end of the actuator [4]. We then have the simplified equation,

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} - \rho I \frac{\partial^4 w(x,t)}{\partial x^2 \partial t^2} = 0 \qquad (36)$$

with boundary conditions

$$
\begin{aligned}
w(0,t) &= 0 \\
\frac{\partial w(0,t)}{\partial x} &= 0 \\
EI\frac{\partial^2 w(L,t)}{\partial x^2} &= c.V(t) \\
EI\frac{\partial^3 w(L,t)}{\partial x^3} &= 0.
\end{aligned} \qquad (37)
$$

V depends only on time in this case. This is a boundary control problem. We again use Equation (26) as the energy function. Repeating the procedure, we observe that only the first term in Equation (30) survives. This gives us

$$\frac{dE(t)}{dt} \leq \frac{c}{\rho A} V(L,t) \left. \frac{\partial^2 w}{\partial x \partial t} \right|_{x=L} \tag{38}$$

Notice that it is sufficient to choose

$$V(t) = -k \left. \frac{\partial^2 w}{\partial x \partial t} \right|_{x=L} \tag{39}$$

to make

$$\frac{dE(t)}{dt} < 0 \tag{40}$$

so as to asymptotically stable the system. The tip rotation speed is available from the sensor output Equation (23).

5 SENSOR AND ACTUATOR SHAPE CONSIDERATION

It is interesting to see that by introducing velocity feedback controller (35), the energy decay rate is given by (34) whose right hand side is a function of $F(x)$ and $v(x)$. Here $F(x)$ and $v(x)$ are the shape or weight functions of the actuator and the sensor. They add weight for the control and measurement at each cross section along the beam. If we consider the displacement of the beam as the sum of a series of products of modal function and its coordinate, we can further analyze the effect of sensor and actuator shapes to different vibration modes.

When the electric field is applied to the piezoelectric lamina, the actual piezoelectric actuation happens only in the region where both sides are covered by the electrodes. The same is true for collecting charge from the sensor layer. Hence, changing the width of the layout of the conductor is equivalent to varying the weighting functions. In this sense, it is possible to design the controller to suppress a particular mode or to design a distributed sensor to measure an interested mode.

Consider the sensor output (23). We can get different information from the system by tailoring the right weighting function $F(x)$. For example, we may select $F(x)$ in such a way that

$$\begin{aligned}
\frac{\partial^2 F(x)}{\partial x^2} &= 0, \ 0 \leq x \leq L \\
\frac{\partial F(L)}{\partial x} &= 0, \ x = L \\
F(L) &= -\frac{1}{K_s}
\end{aligned} \tag{41}$$

the integral term vanishes, so does the second term, and then the sensor output

$$V_s(t) = \frac{\partial w(L, t)}{\partial x} \tag{42}$$

represents the angular deflection at the tip of the beam. Similarly, we can measure the displacement of the tip. From (20), the sensor can be used to build strain gauge by setting $F(x)$ to be a spatial Dirac delta function $\delta(x)$.

We think that it is theoretically possible to use segmented sensors for the control of flexible structures. Digital control provides the ability to implement a sensing network with simple computation to rearrange the sensor layout and get different measurements with one sensor layer. Some measurements which are difficult to obtain in conventional way may be available by using distributed sensors. It may be feasible to implement full state feedback in relevant semigroup control formulations.

The effectiveness of the PZT controller in introducing structural damping and its influence to the system dynamics of realistic size is based on the control authority of the controller. Using velocity feedback shall increase the damping, but the control gain is limited to the electric field limit to avoid depolarization of the actuator. The actuator weighting function $v(x)$ also plays a role here. We know that the bending moment is concentrated mostly at the end of the actuator of the beam. Hence more weighting should be placed on the region with high average strain.

The feedback control (35) actually provides Voigt type damping since the rate of change of the bending curvature is used for feedback (22) and this rate is proportional to the rate of change of the structural strain. The augmented composite beam has an altered constitutive equation. The stress is no longer just proportional to the strain, but a linear combination of strain and the rate of strain change with respect to time.

We now analyze the effect of both sensor and actuator shape functions to the damping control of different vibration modes. We use a Ritz-Galerkin procedure to implement modal expansion. We write the beam displacement $w(x, t)$ as

$$w(x, t) = \sum_{k=1}^{n} \Phi_k(x) d_k(t) \tag{43}$$

where $\Phi_k(x)$ is the modal function and d_k is the time coordinate. We can choose the orthogonal modal functions. We rewrite here the control form of the previous section

$$V(x, t) = v(x)q(t). \tag{44}$$

Substituting the modal forms into Equation (14), multiplying each term with $\Phi_l(x)$ and then taking spatial integration along the beam, we get

$$\sum_{k=1}^{n} \left[\int \Phi_l(x)(\rho A \Phi_k(x) - \rho I \Phi_k^{(2)}(x)) \ddot{d}_k + EI \Phi_l(x) \Phi_k^{(4)}(x) d_k dx \right] = cq(t) \int \Phi_l(x) v^{(2)}(x) dx \tag{45}$$

where $\dot{d}$ and $\ddot{d}$ stand for the first and second time derivatives of the function $d(t)$; $v^{(i)}(x)$ stands for the ith spatial derivative of $v(x)$. We then have

$$\sum_{k=1}^{n} m_{lk}\ddot{d}_k + \sum_{k=1}^{n} k_{lk}d_k = Q_l \tag{46}$$

$$Q_l = -\sum_{k=1}^{n} c_{lk}\dot{d}_k(t) \tag{47}$$

where

$$m_{lk} = \sum_{k=1}^{n} \int \Phi_l(x)[\rho A\Phi_k(x) - \rho I\Phi_k^{(2)}(x)]dx \tag{48}$$

$$k_{lk} = \sum_{k=1}^{n} \int EI\Phi_l(x)\Phi_k^{(4)}(x)dx \tag{49}$$

$$c_{lk} = cq(t)\int \Phi_l(x)v^{(2)}(x)dx. \tag{50}$$

The compact modal form is

$$M\ddot{d}(t) + Kd(t) = Q(t) \tag{51}$$

where $d(t)$ and $Q(t)$ are nth order column vector functions. M is the inertial matrix, K is the stiffness matrix, $Q(t)$ is the modal control input. The damping of different elastic modes is influenced by

$$Q(t) = -C\dot{d}(t) \tag{52}$$

and

$$M\ddot{d}(t) + C\dot{d}(t) + Kd(t) = 0 \tag{53}$$

where C is a n by n damping coefficient matrix. Its elements are derived from the control law (35)

$$c_{lk} = cK_s\int_0^L \Phi_l\frac{d^2v(x)}{dx^2}dx \int_0^L \Phi_k\frac{d^2F(x)}{dx^2}dx. \tag{54}$$

Observing Equation (54), we notice that in addition to the control authority determined by the actuation and sensing constants c and K_a the added damping to a specific mode depends on the shape functions $F(x)$ and $v(x)$. The function $v(x)$ in the first integral decides the amount of control effort applied to the lth mode. Similarly, $F(x)$ provides the observation of the kth elastic mode. The coefficient c_{lk} can be viewed as a measure of the damping to the lth mode by control based on the information from the kth elastic mode. If we choose the sensor shape to be such a function that

$$\frac{d^2F(x)}{dx^2} = \Phi_k(x), \tag{55}$$

we can measure the kth mode completely. When the second spatial derivative contains several modes, we shall get the combined information from the sensor. The similarity holds for the actuator, too. Properly selecting $F(x)$ and $v(x)$, we can observe and suppress the vibration modes.

6   CONCLUSIONS

We have embedded a static PZT actuator model into the improved Euler-Bernoulli beam model to form a composite beam model with the rotational inertia effect considered. We further used a distributed PVDF sensor to measure the elastic bending modes. A closed loop controller has been designed by using Lyapunov's direct method. The closed loop system extracts energy from the system. The closed loop system is asymptotically stable. Finally, we have discussed the effects of different sensor and actuator shapes to the elastic modes. We point out that it is feasible to select suitable sensing and control weight to implement vibration control to some specified elastic modes.

Further research is needed regarding aspects of estimation of the energy decay rate and real time implementation of the control law. We also would like to consider modeling the substructure with the Timoshenko model or the geometric exact rod model. Also, the real impact of the modal controller needs to be verified and further explored by experiments.

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