CSI, OPTIMAL CONTROL, & ACCELEROMETERS: TRIALS AND TRIBULATIONS*

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SUMMARY

New results concerning optimal design with accelerometers are presented. These results show that the designer must be concerned with the stability properties of two Linear Quadratic Gaussian (LQG) compensators, one of which does not explicitly appear in the closed-loop system dynamics. The new concepts of *virtual* and *implemented* compensators are introduced to cope with these subtleties: The virtual compensator appears in the closed-loop system dynamics and the implemented compensator appears in control electronics. The stability of one compensator does not guarantee the stability of the other. For strongly stable (robust) systems, both compensators should be stable. The presence of controlled and uncontrolled modes in the system results in two additional forms of the compensator with corresponding terms that are of like form, but opposite sign, making simultaneous stabilization of both the virtual and implemented compensator difficult. A new design algorithm termed *sensor augmentation* is developed that aids stabilization of these compensator forms by incorporating a static augmentation term associated with the uncontrolled modes in the design process.

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1.0 INTRODUCTION

Dynamic systems that are not strictly proper complicate linear quadratic gaussian (LQG) control design. These dynamic systems are characterized by transfer functions where the order of the numerator equals the denominator. Sensors, such as accelerometers, whose transfer functions are not strictly proper can also generate such systems. Linear time invariant systems that employ these sensors may be represented in the time domain by state space equations characterized by the matrix quadruplet (A, B, C, D) where A is the plant matrix, B is the input (influence) matrix, C is the output (sensor) matrix, and D is a thru-put matrix representing the direct transmission properties associated with systems that are not strictly proper. The presence of the D matrix complicates LQG control design particularly in the area of compensator stability, and consequently closed-loop system robustness. The designer must consider two forms of the optimal compensator, one of which does not explicitly appear in the closed-loop system dynamics.

There is very little consideration of systems that are not strictly proper in the optimal control literature. Standard texts on optimal control (refs. 1-7) do not consider these systems in the context of LQG closed-loop control. A preliminary version of the material presented in this paper is contained in ref. (8).

This paper is organized as follows: Section 2 derives the two LQG compensator forms required for design and introduces the concepts of *implemented* and *virtual* compensators. Section 3 considers additional compensator forms caused by the presence of neglected known vibration modes (suppressed modes) which are not explicitly modeled in the control design process. Section 4 presents a design algorithm termed *sensor augmentation* that copes with the complexities introduced by the suppressed (neglected) vibration modes, and Section 5 presents our conclusions.

2.0 IMPLEMENTED AND VIRTUAL COMPENSATORS

The LQG compensator plays a significant role in the determination of closed-loop robustness properties. As shown in Figure 1, the compensator is that dynamic system that has the sensor vector as its input and the control vector as its output. Its dynamics are determined by the transfer function matrix between points "a" and "b" of Figure 1.

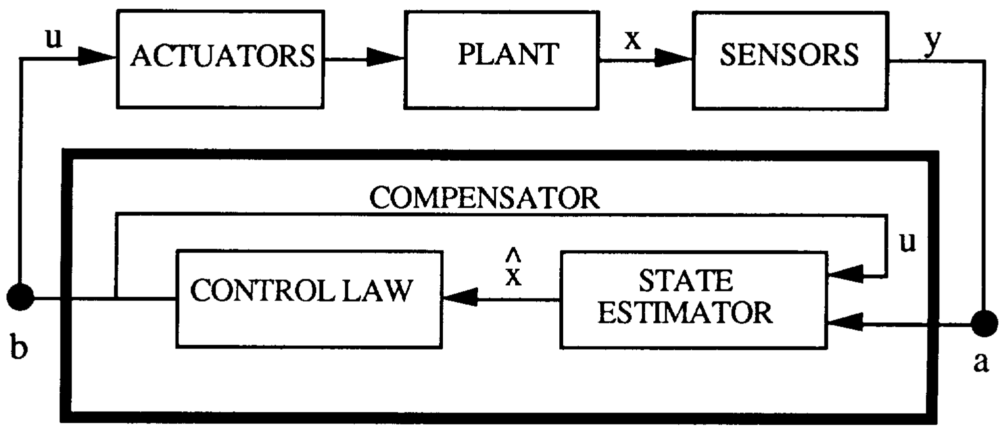

Figure 1. LQG compensator stability affects robustness.

In general, the stability properties of the compensator tend to influence the robustness properties of the closed-loop system. For strictly proper systems (no D matrix) the designer must consider only one compensator form; however, for systems incorporating a D matrix in their description, two compensator forms must be considered: an *implemented* and a *virtual* compensator. The implemented compensator has the sensor vector as its input, which drives the estimator-based dynamics. These dynamics, which are functions of the D matrix, do not explicitly appear in the matrix description of the closed-loop system. Conversely, the virtual compensator dynamics are not functions of the D matrix, but do appear in the closed-loop system matrix. For strictly proper systems (no D matrix) the implemented compensator dynamics and the virtual compensator dynamics are identical. The development of the two compensator forms is accomplished by direct substitution of the LQG control and estimation laws in the plant dynamics. The implemented compensator emerges by careful distinction between the sensed and computed variables of the closed-loop system.

Consider the following open-loop, dynamic system

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx + Du \tag{2}$$

where $x(n \times 1)$ is the state vector, $u(r \times 1)$ is the control vector, $y(s \times 1)$ is the output vector and (A, B, C, D) are matrices of appropriate dimension. For flexible structure control, the A matrix is composed of modal frequencies and damping factors, the B and C matrices are based on eigenvector solutions of the finite element model characterizing the structure. For such systems employing accelerometers, the D matrix has the following form

$$D = CB \tag{3}$$

The control law is

$$u = -K\hat{x} \tag{4}$$

where K(r x n) is the optimal feedback control matrix and $\hat{x}$(n x 1) is the estimated state vector.

The state estimator has the following form

$$\dot{\hat{x}} = A\hat{x} + Bu + G(y - \hat{y}) \tag{5}$$

where G(n x s) is the estimator gain matrix and $\hat{y}$(s x 1) is the estimated output vector.

Implemented Compensator Derivation

The implemented compensator dynamics are now derived. Substituting $u = -K\hat{x}$ in the estimator dynamics for the control law, and $\hat{y} = C\hat{x} + Du$ for the estimated sensor vector yields

$$\dot{\hat{x}} = (A - BK)\hat{x} - G(C\hat{x} + Du) + Gy \tag{6}$$

Substituting $u = -K\hat{x}$ for the control vector in equation (6) and collecting terms yields

$$\dot{\hat{x}} = (A - BK - GC + GDK)\hat{x} + Gy \tag{7}$$

Equation (7) characterizes the implemented compensator dynamics for the closed-loop system. The sensor vector is an input that drives the estimation-based dynamic system for the compensator.

In sequel, we shall show that although the implemented compensator is necessary to generate the required closed-loop dynamics, it does not explicitly appear in the closed-loop system matrix.

Virtual Compensator Derivation

The virtual compensator dynamics are derived by continued expansion of the dynamic expression for the implemented compensator. Substituting y = Cx + Du for the sensor vector in equation (7) yields

$$\dot{\hat{x}} = (A - BK - GC + GDK)\hat{x} + G(Cx + Du) \tag{8}$$

Substituting $u = -K\hat{x}$ in equation (8) yields

$$\dot{\hat{x}} = (A - BK - GC + GDK)\hat{x} + GCx - GDK\hat{x} \tag{9}$$

Collecting terms in $\hat{x}$ yields the virtual compensator dynamics

$$\dot{\hat{x}} = (A - BK - GC)\hat{x} + GCx \tag{10}$$

where we note that the D matrix has been eliminated from equation (10).

Inspection of the closed-loop dynamics matrix shows that the D matrix, which may influence robustness properties, has been eliminated from the closed-loop system description. Only the virtual compensator appears. Comparison of the implemented compensator dynamics (equation 7) and the virtual compensator dynamics (equation 10) shows that the two expressions are not identical and, in general, will not have the same eigenvalues. In fact, the stability of one of these compensator forms does not guarantee the stability of the other. Substituting $u = -K\hat{x}$ in equation (1) and assembling equations (1) and (10) in matrix form yields the closed-loop system matrix

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -BK \\ GC & A - BK - GC \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} \tag{11}$$

As the expression containing the D matrix does not appear in equation (11), the implemented compensator could be unstable and this fact would not be detected by a closed-loop eigenvalue analysis. Thus, both compensator forms must be checked in order to ensure the design of a strongly stable system in the sense of reference 9.

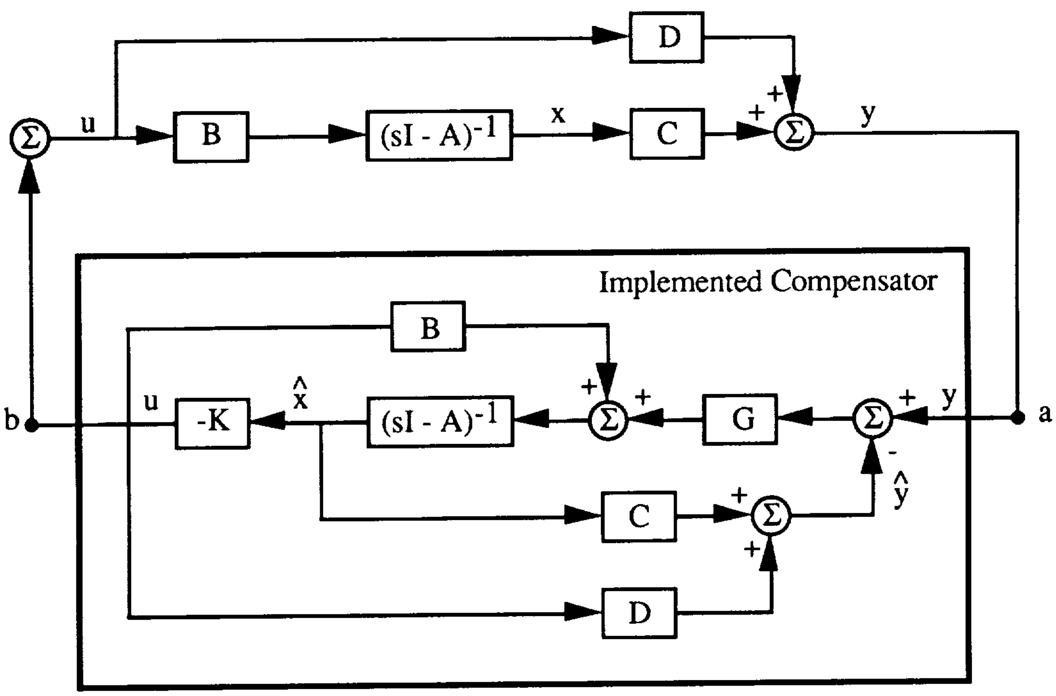

Figure 2. Implemented compensator detail shows the effect of the thru-put matrix

Figure 2 provides a detailed matrix block diagram of the implemented compensator for systems that are not strictly proper. Examination of this diagram provides insight to the compensator problem. The compensator dynamics are characterized by the transfer function matrix between points "a" and "b" of Figure 2. The control vector, u, is multiplied by the D matrix and summed with $C\hat{x}$ to form the estimated sensor vector, $\hat{y}$. However, the sensor vector, y, contains an identical term, Du, involving the control vector. As the two sensor vectors, y and $\hat{y}$, are subtracted at the compensator summing junction, any terms involving the matrix D are eliminated from the closed-loop system matrix, i.e., the compensator is uncontrollable, in u, at the sensor summing junction. This condition is analogous to that which occurs during estimator design using the separation principle with strictly proper systems, i.e., the separation principle holds (ref. 10) and the estimator is uncontrollable via the control vector. The separation principle also holds for systems that are not strictly proper; however, one must consider both the error space and implementation space during the design process: the component of the control vector transmitted by the D matrix is eliminated from the closed-loop dynamics in the implementation space. In this context, a partial separation principle can be said to hold, and the compensator dynamics *appear* to be determined solely by the (A, B, C) matrices.

The presence of unmodeled dynamic (suppressed) modes further complicates the design process. In this case the D matrix cancellation is incomplete in the implementation space, and the implemented and virtual compensators have differing dynamics that *are* functions of different modal thru-put matrices. This phenomenon is discussed in the following section.

3.0 LQG COMPENSATOR DYNAMICS AND SUPPRESSED MODES

The presence of uncontrolled vibration dynamics significantly complicates the compensator design process. The implemented and virtual compensator dynamic matrices contain corresponding terms of similar form, but opposite sign, that can severely constrain the compensator stabilization process. Consider the following open-loop dynamic system representing a flexible structure

$$\begin{bmatrix} \dot{x}_c \\ \dot{x}_s \end{bmatrix} = \begin{bmatrix} A_c & 0 \\ 0 & A_s \end{bmatrix} \begin{bmatrix} x_c \\ x_s \end{bmatrix} + \begin{bmatrix} B_c \\ B_s \end{bmatrix} u \qquad (12)$$

$$y = \begin{bmatrix} C_c & C_s \end{bmatrix} \begin{bmatrix} x_c \\ x_s \end{bmatrix} + \begin{bmatrix} D_c + D_s \end{bmatrix} u \tag{13}$$

where $x_c(n_c \times 1)$ is the controlled state vector, $x_s(n_s \times 1)$ is the suppressed state vector characterizing the uncontrolled but modeled modes (refs. 11, 12), $u(r \times 1)$ is the control vector, and $y(s \times 1)$ is the output vector. The plant submatrices, $A_c(n_c \times n_c)$ and $A_s(n_s \times n_s)$, are composed of modal frequencies and damping factors. The input matrices, $B_c(n_c \times r)$ and $B_s(n_s \times r)$, the output matrices, $C_c(s \times n_c)$ and $C_s(s \times n_s)$, and the thru-put matrices, $D_c(s \times r)$ and $D_s(s \times r)$, are based on eigenvector solutions of the finite element model characterizing the structure. For such systems employing accelerometers the submatrices comprising the D matrix are given by

$$D_t = D_c + D_s \tag{14}$$

$$D_c = C_c B_c \tag{15}$$

$$D_s = C_s B_s \tag{16}$$

The control law is

$$u = -K\hat{x}_c \tag{17}$$

where $K(r \times n_c)$ is the optimal feedback matrix and $\hat{x}_c(n_c \times 1)$ is the estimated state vector.

The state estimator has the following form

$$\dot{\hat{x}}_c = A_c\hat{x}_c + B_c u + G(y - \hat{y}_c) \tag{18}$$

where $G(n_c \times s)$ is the estimator gain matrix and $\hat{y}_c(s \times 1)$ is the estimated output vector for the controlled states.

Implemented Compensator With Suppressed Modes

The implemented compensator dynamics are now derived. Substituting $u = -K\hat{x}_c$ in the estimator dynamics for the control law, and $\hat{y}_c = C_c\hat{x}_c + D_c u$ for the estimated sensor vector yields

$$\dot{\hat{x}}_c = (A_c - B_c K)\hat{x}_c - G(C\hat{x}_c + D_c u) + Gy \tag{19}$$

Substituting $u = -K\hat{x}_c$ for the control law in equation (19) and collecting terms yields

$$\dot{\hat{x}}_c = (A_c - B_c K - GC_c + GD_c K)\hat{x}_c + Gy \tag{20}$$

Equation (20) characterizes the implemented compensator dynamics for the closed-loop system. We note that the implemented compensator is a function of D_c, the thru-put matrix for the controlled modes.

Virtual Compensator With Suppressed Modes

The virtual compensator dynamics are now derived. Substituting equation (13) for the sensor vector in equation (20) yields

$$\dot{\hat{x}}_c = (A_c - B_c K - GC_c + GD_c K)\hat{x}_c + G(C_c x_c + C_s x_s + D_c u + D_s u) \quad (21)$$

Substituting $u = -K\hat{x}_c$ in equation (21) yields

$$\dot{\hat{x}}_c = (A_c - B_c K - GC_c + GD_c K)\hat{x}_c + G(C_c x_c + C_s x_s) - G(D_c K + D_s K)\hat{x}_c \quad (22)$$

Collecting terms in $\hat{x}_c$ yields the virtual compensator dynamics

$$\dot{\hat{x}}_c = (A_c - B_c K - GC_c - GD_s K)\hat{x}_c + GC_c x_c + GC_s x_s \quad (23)$$

where the virtual compensator dynamics are a function of D_s, the thru-put matrix for the suppressed modes. A term by term examination of the submatrices comprising the dynamic matrix for the implemented compensator, equation (20), and the virtual compensator, equation (23), yields the interesting result: The dynamic matrices of the two compensators are composed of identical submatrices except for those terms arising from the modal thru-put matrices. These submatrices, $GD_c K$ and $-GD_s K$, are similar in form, but opposite in sign. Thus, in general, it will be difficult to simultaneously stabilize the implemented and virtual compensators. Conflicting constraints will tend to be placed on the gain matrices G and K.

The closed-loop dynamics in matrix form may be written as

$$
\begin{bmatrix} \dot{x}_c \\ \dot{x}_s \\ \dot{\hat{x}}_c \end{bmatrix} = \begin{bmatrix} A_c & 0 & -B_c K \\ 0 & A_s & -B_s K \\ GC_c & GC_s & A_c - B_c K - GC_c - GD_s K \end{bmatrix} \begin{bmatrix} x_c \\ x_s \\ \hat{x}_c \end{bmatrix} \tag{24}
$$

Examination of equation (24) shows that the implemented compensator dynamics do not appear in the closed-loop system matrix. Thus, an eigenvalue analysis of this closed-loop matrix would not reveal the stability properties of the implemented compensator. Both compensator forms must be checked for stability to design a strongly stable system.

Table 1 shows the dynamic matrices that occur during LQG control design of flexible structures that employ accelerometers. Included are matrices for the estimator, controller and various compensator forms. The number of matrix forms requiring stabilization or conditioning is five, and the number of gain matrices is two. This situation leads to difficulty in design, especially when one desires stable compensation matrices. A design algorithm is presented in Section 4 to cope with difficulties introduced by the suppressed modes.

Table 1. LQG Dynamic Matrices For Accelerometer Systems

Controller	$A_c - B_c K$
Estimator	$A_c - GC_c$
Virtual Compensator	$A_c' - B_c K - GC_c$
Implemented Compensator	$A_c - B_c K - GC_c + GD_c K$
Virtual Compensator (Suppressed Modes)	$A_c - B_c K - GC_c - GD_s K$
Thru-put Term (Controlled Modes)	$D_c = C_c B_c$
Thru-put Term (Suppressed Modes)	$D_s = C_s B_s$

4.0 SENSOR AUGMENTATION

We now develop an algorithm that addresses the problem caused by the suppressed mode contamination of the virtual compensator dynamics. As shown in the previous section, the dynamic matrices of the implemented compensator and the virtual compensator differ only in terms arising from the modal dynamics (compare equations 20 and 23). These modal terms, GD_cK and $-GD_sK$, which are similar in form but opposite in sign, create difficulties for stable compensator design. The difficulty arises because we are requiring two similar matrix forms of opposite sign to stabilize identical matrices, i.e., if we define A_{comp} as the standard LQG dynamic compensator matrix

$$A_{comp} = A - BK - GC \tag{25}$$

the dynamic matrix for the implemented compensator is

$$A_{comp} + GD_cK \tag{26}$$

and that for the virtual compensator is

$$A_{comp} - GD_sK \tag{27}$$

As D_c and D_s are of similar structure, the gain matrices G and K will tend to have opposing effects on the stability properties of the two compensator forms.

We can cope with this problem by developing an algorithm that eliminates the offending terms caused by the suppressed modes from one of the compensator forms. This is accomplished by augmenting the estimated sensor output vector with suppressed mode data, i.e., with reference to equation (14), D_c is replaced by D_t in the design process, where

$$D_t = D_c + D_s \tag{28}$$

It should be noted that the number of controlled modes remains constant, and that this procedure is analogous to incorporating a "d.c. gain," or static portion of the suppressed mode transfer function, into the design process. The design algorithm may also be interpreted as using a hybrid dynamic model, augmented with the static gains of the uncontrolled, but modeled modes. The effect of this procedure on the implemented and virtual compensators is easily derived. Consider the following open-loop dynamic system representing a flexible structure

$$\begin{bmatrix} \dot{x}_c \\ \dot{x}_s \end{bmatrix} = \begin{bmatrix} A_c & 0 \\ 0 & A_s \end{bmatrix} \begin{bmatrix} x_c \\ x_s \end{bmatrix} + \begin{bmatrix} B_c \\ B_s \end{bmatrix} u \tag{29}$$

$$y = \begin{bmatrix} C_c & C_s \end{bmatrix} \begin{bmatrix} x_c \\ x_s \end{bmatrix} + \begin{bmatrix} D_c + D_s \end{bmatrix} u \tag{30}$$

where $x_c(n_c \times 1)$ is the controlled state vector, $x_s(n_s \times 1)$ is the suppressed state vector characterizing the uncontrolled but modeled modes (refs. 11, 12), $u(r \times 1)$ is the control vector, and $y(s \times 1)$ is the output vector. The plant submatrices, $A_c(n_c \times n_c)$ and $A_s(n_s \times n_s)$, are composed of modal frequencies and damping factors. The input matrices, $B_c(n_c \times r)$ and $B_s(n_s \times r)$, the output matrices, $C_c(s \times n_c)$ and $C_s(s \times n_s)$, and the thru-put matrices, $D_c(s \times r)$ and $D_s(s \times r)$, are based on eigenvector solutions of the finite element model characterizing the structure. For such systems employing accelerometers the submatrices comprising the D matrix are given by

$$D_t = D_c + D_s \tag{31}$$

$$D_c = C_c B_c \tag{32}$$

$$D_s = C_s B_s \tag{33}$$

The control law is

$$u = -K\hat{x}_c \tag{34}$$

where $K(r \times n_c)$ is the optimal feedback matrix and $\hat{x}_c(n_c \times 1)$ is the estimated state vector.

The state estimator has the following form

$$\dot{\hat{x}}_c = A_c\hat{x}_c + B_c u + G(y - \hat{y}_c) \tag{35}$$

where $G(n_c \times s)$ is the estimator gain matrix and $\hat{y}_c(s \times 1)$ is the estimated output vector for the controlled states. The estimated sensor vector is now given by

$$\hat{y}_c = C_c\hat{x}_c + (D_c + D_s)u \tag{36}$$

where D_s has now been included in the design process, i.e., the sensor has been augmented.

Implemented Compensator Using Sensor Augmentation

The implemented compensator dynamics are now derived. Substituting $u = -K\hat{x}_c$ in the estimator dynamics (equation 35) for the control law, and $\hat{y}_c = C_c\hat{x}_c + (D_c + D_s)u$ for the estimated sensor vector yields

$$\dot{\hat{x}}_c = (A_c - B_c K)\hat{x}_c - G(C_c \hat{x}_c + D_c u + D_s u) + Gy \quad (37)$$

Substituting $u = -K\hat{x}_c$ in equation (37) yields

$$\dot{\hat{x}}_c = (A_c - B_c K - GC_c + GD_c K + GD_s K)\hat{x}_c + Gy \quad (38)$$

Substituting the relationship $D_t = D_c + D_s$ in equation (38) yields

$$\dot{\hat{x}}_c = (A_c - B_c K - GC_c + GD_t K)\hat{x}_c + Gy \quad (39)$$

which is the desired expression for the implemented compensator dynamics. Examination of the dynamics for this compensator, which uses augmented sensor data, and those of the unaugmented compensator of equation (20) shows that they differ by the term $GD_s K$ which appears in equation (38).

Virtual Compensator With Sensor Augmentation

The expression for the augmented virtual compensator dynamics may now be derived. Substituting equation (13) for the sensor vector in equation (39), and noting that $D_t = D_c + D_s$, yields

$$\dot{\hat{x}}_c = (A_c - B_c K - GC_c + GD_t K)\hat{x}_c + G(C_c x_c + C_s x_s + D_t u) \quad (40)$$

Substituting $u = -K\hat{x}_c$ in equation (40) yields

$$\dot{\hat{x}}_c = (A_c - B_cK - GC_c + GD_tK)\hat{x}_c + G(C_cx_c + C_sx_s) - GD_tK\hat{x}_c \quad (41)$$

Collecting terms in $\hat{x}_c$ yields the virtual compensator dynamics

$$\dot{\hat{x}}_c = (A_c - B_cK - GC_c)\hat{x}_c + G(C_cx_c + C_sx_s) \quad (42)$$

The closed-loop dynamics may be written in matrix form as

$$\begin{bmatrix} \dot{x}_c \\ \dot{x}_s \\ \dot{\hat{x}}_c \end{bmatrix} = \begin{bmatrix} A_c & 0 & -B_cK \\ 0 & A_s & -B_sK \\ GC_c & GC_s & A_c - B_cK - GC_c \end{bmatrix} \begin{bmatrix} x_c \\ x_s \\ \hat{x}_c \end{bmatrix} \quad (43)$$

Examination of the virtual compensator dynamics, equation (42), or the closed-loop dynamics, equation (43), shows that optimal design using augmented sensor data allows the virtual LQG compensator dynamics to revert to the simpler form of the standard optimal compensator. However, the implemented compensator, equation (39), *does* contain the augmented thru-put matrix, D_t, and must be checked for stability independently of the closed-loop system matrix. Thus, the use of sensor augmentation has eliminated the conflicting sign conditions present in the implemented and virtual compensator dynamics, equations (20) and (23) respectively, that can cause stabilization difficulties.

The system matrices requiring stabilization, or stability verification, using augmented sensor design for accelerometers on flexible structures are shown in Table 2.

Table 2. LQG Dynamic Matrices For Sensor Augmentation

• Controller	$A_c - B_c K$
• Estimator	$A_c - G C_c$
• Virtual Compensator	$A_c - B_c K - G C_c$
• Implemented Compensator	$A_c - B_c K - G C_c + G D_t K$
• Total Thru-Put Term	$D_t = C_c^T B_c + C_s^T B_s$
	$D_t = D_c + D_s$

In summary, system matrices must be checked for stability, namely, those of the controller, the estimator, the virtual compensator, and the implemented compensator. The poles of the controller, estimator, and virtual compensator appear in the closed-loop system dynamics and may be checked for stability in the usual closed-loop stability analyses. The implemented compensator does not explicitly appear in the closed-loop dynamics and must be checked for stability independently of the closed-loop analysis.

5.0 CONCLUSION

Our analysis of LQG optimal control design involving systems that are not strictly proper has shown that such systems generate control complexities: Two different LQG compensator forms must be considered, namely, an *implemented* compensator and a *virtual* compensator. The implemented compensator resides in the control electronics and generates the estimator-based control signals. The virtual compensator appears in the closed-loop dynamics. The dynamic properties of both forms strongly affect the robustness of the closed-loop system.

With regard to flexible structure control, the direct feedback of accelerometer signals results in systems that are not strictly proper. The additional problems generated by uncontrolled modes cause conflicting stability constraints in the implemented and virtual compensators that makes simultaneous stabilization of both forms difficult to achieve. A new algorithm, *Sensor Augmentation*, has been developed that copes with this situation by incorporating a static augmentation term in the design process that eliminates conflicting the stability constraints.

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