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Software for Continuum Modeling of Controls-Structures Interactions

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ABSTRACT

It is clear that computer software is needed to assist in the generation of the equations of motion for complex, flexible spacecraft. Daniel Poelaert of ESTEC has developed the software DISTEL with which he has modeled the structural dynamics for different satellites. He is interested in expanding the capabilities of DISTEL to include structural damping and control systems. Unfortunately, the software has not been released. The author has developed similar software, PDEMOD, which has been used to model the Spacecraft control Laboratory Experiment (SCOLE), the Solar Array Flight Experiment (SAFE), the Mini-MAST truss, and the LACE satellite. PDEMOD has been used also for optimal parameter estimation and integrated control-structures design. PDEMOD is also being extended to include structural damping and control systems which are imbedded into the same equations for the structural dynamics.

This paper will address the formulation of the equations for the structural dynamics of spacecraft structures which are constructed of a 3-dimensional arrangement of rigid bodies and flexible beam elements. Control system dynamics are imbedded into the same equations so that model order reduction approximations are not necessary. The input data consists of the physical data of the elements and the topological information describing how the elements are connected. PDEMOD (1) automatically assembles the equations of motion for the entire structural model, (2) calculates the modal frequencies, (3) calculates the mode shapes, (4) generates perspective views of the mode shapes, and (5) forms selected transfer functions.

The software PDEMOD continues to be developed to provide additional features to assist in analyzing and synthesizing control and structural systems for flexible spacecraft.

Issues in Modeling Composite Structure

Finite Element Modeling

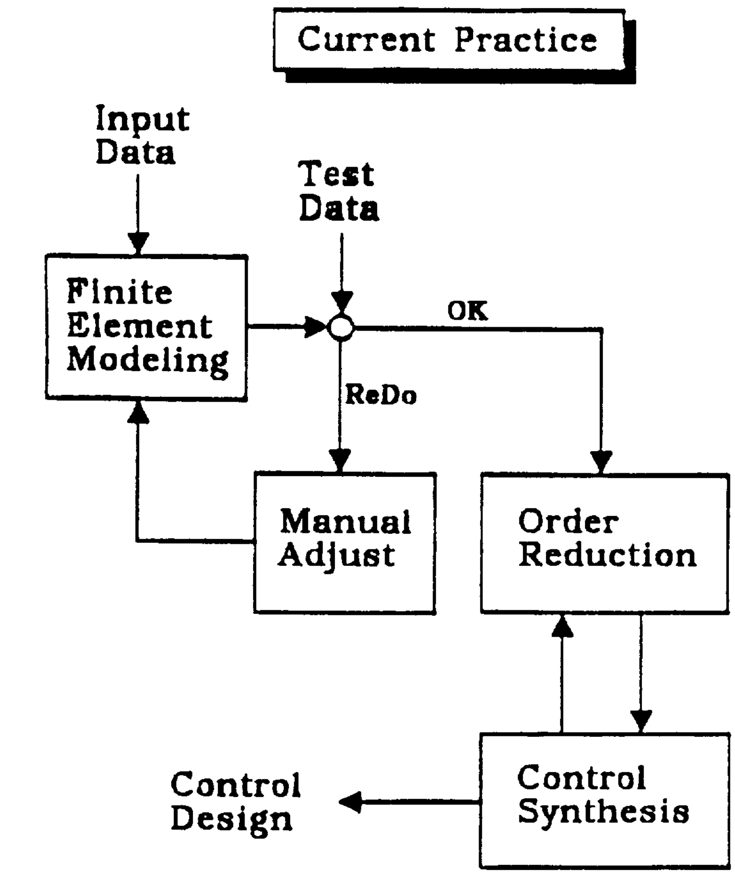
- Excessive Complexity
- Parameter Estimation is Difficult
- Model Order Reduction Required for Control Analysis

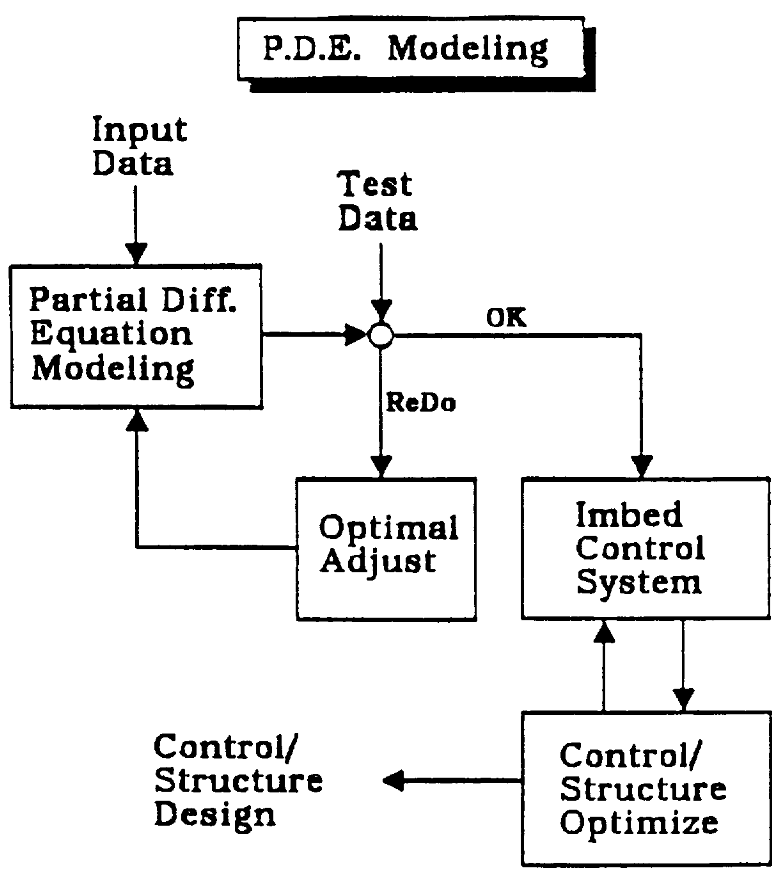
Distributed Parameter Modeling

- Fewer Model Parameters
- Parameter Estimation Straightforward
- Closed-Loop Stability Analysis does not Require Order Reduction

The current practice of modeling flexible structures is to use finite element modeling. It is then necessary to dispose of most of the modal characteristics because of their inaccuracy. Damping is also defined in an ad hoc manner. When designing a control law for such a model it is necessary to iterate because of the order reduction process. Also the number of model parameters is too great to allow optimal parameter estimation.

The recommended alternative is to use distributed parameter modeling. It is not necessary to reduce the order of the model since the control system dynamics can be imbedded into the same equation which represent the structural dynamics. Damping can be included more accurately into the structural equations. The reduced number of model parameters enables optimum parameter estimation.


Current Practice
Input Data
Test Data
Finite Element Modeling
OK
ReDo
Manual Adjust
Order Reduction
Control Design
Control Synthesis


P.D.E. Modeling
Input Data
Test Data
Partial Diff. Equation Modeling
OK
ReDo
Optimal Adjust
Imbed Control System
Control/ Structure Design
Control/ Structure Optimize

Hurdles for P.D.E. Modeling

- Ability to Generate P.D.E. Models of Complex Structures

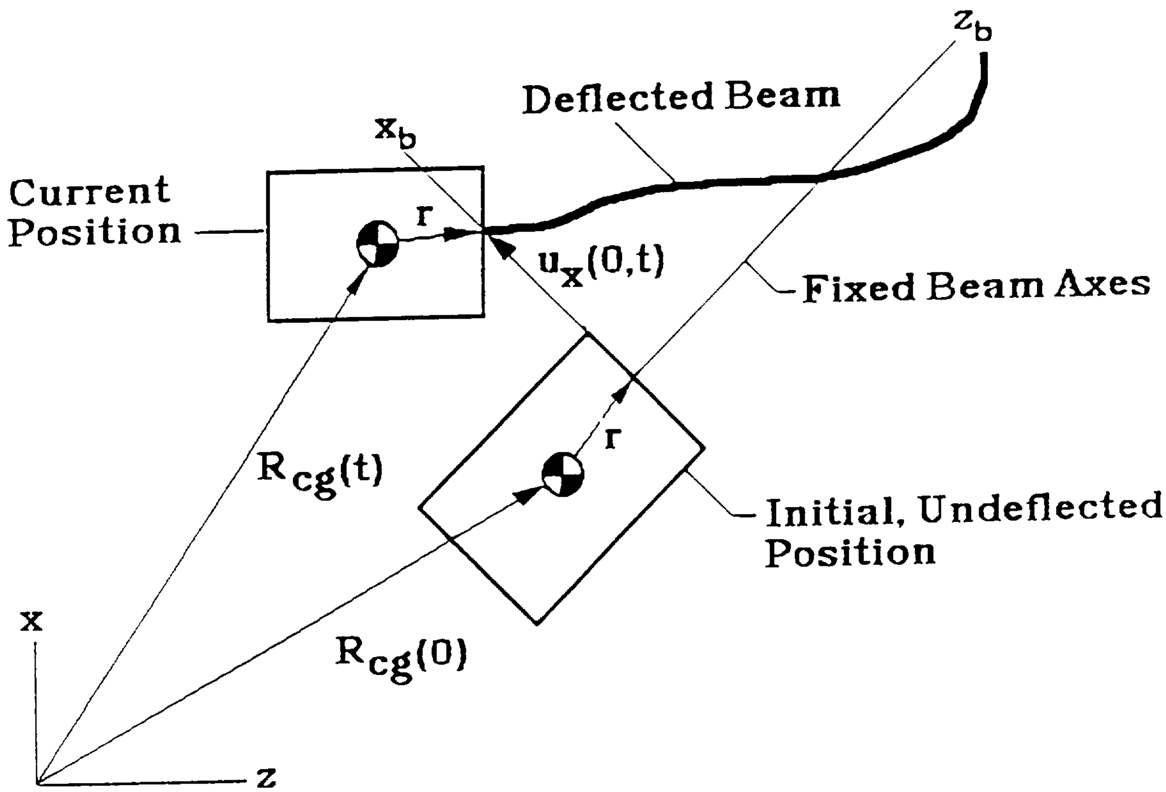
- Accuracy of P.D.E. Models for Different Types of Structure

- Ability to Imbed Control/Structural Dynamics

Before continuum or distributed parameter modeling can become a viable alternative to finite element modeling, it is necessary to develop software which will enable the modeling of complex structures. The software, PDEMOD, can provide that capability. The software continues to be developed to provide additional features.

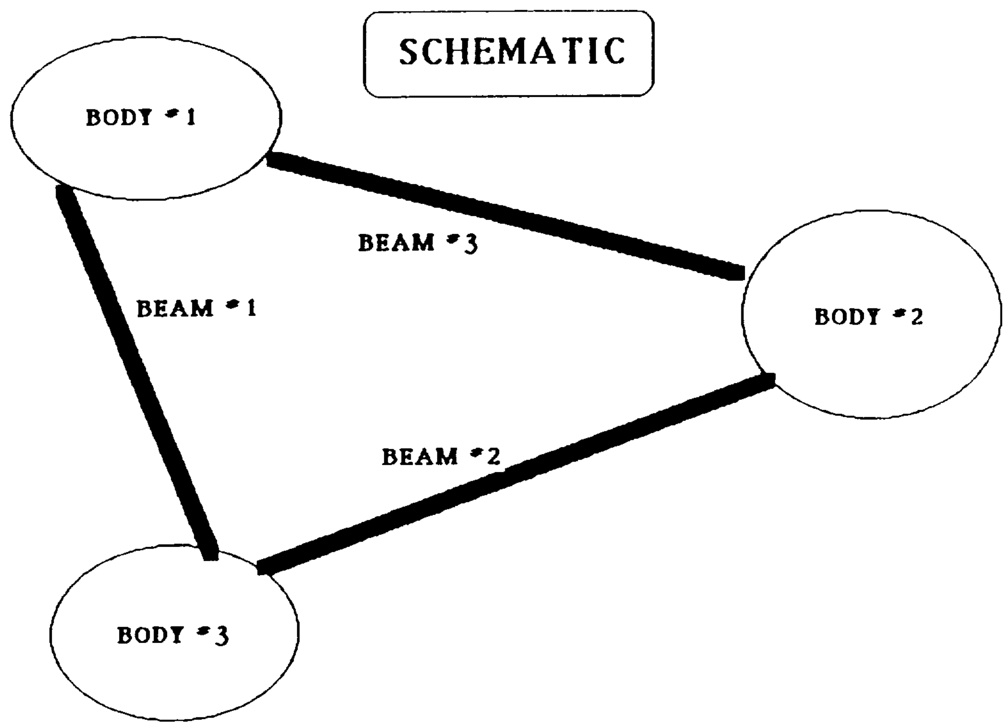
It is also necessary to examine the accuracy of continuum models. The number of example configurations continues to grow. The accuracy can be equal to or better than that of finite element models. Eventually, it will be possible to use both approaches in the same software, thereby taking advantage of the features of both approaches.

It is valuable to control applications to imbed the control system dynamics into the same equations for the structural dynamics. The inaccuracies due to order reduction can then be avoided.



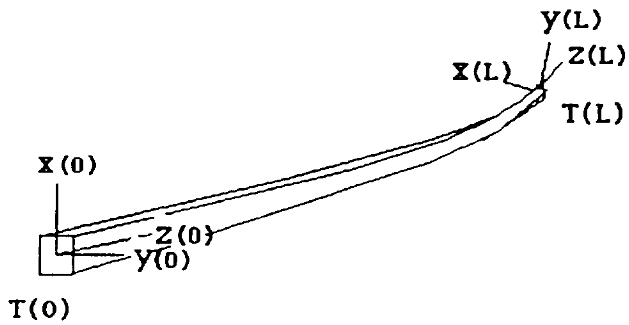
The equations of motion are formulated in terms of the motion of bodies attached to the ends of flexible beam elements. The coordinates of a body are chosen to be those of one of the beams to which it is attached. The reference beam axes remain fixed. When the beam element deflects the body moves accordingly. Account must be taken of both linear and angular deflection, however.

The acceleration of the body is then related to the sum of the forces and moments that result from the attached beam elements.



Three-dimensional configurations can be modeled which are comprised of rigid bodies and beams which deflect laterally (two directions), longitudinally, and twist.

Beam Model



The Moments and Forces at (0) in Beam Axes are:

$M_x = EI_y u_y''(0)$ $\quad$ $F_x = EI_y u_y'''(0)$

$M_y = -EI_x u_x''(0)$ $\quad$ $F_y = -EI_x u_x'''(0)$

$M_z = EI_\psi u_\psi'(0)$ $\quad$ $F_z = EA_z u_z'(0)$

The force and moment vectors are first expressed in terms of spatial derivatives of the deflection of the beam element. After noting that the beam deflections are functions of sinusoidal and hyperbolic functions and their coefficients, the linear deflection, angular deflection, and force and moment vectors are expressed in terms of a vector of the beam deflection coefficients.

Beam Deflection Function

$$u_x(z)=a_x+b_xz+A_x\sin(b_xz)+B_x\cos(b_xz)$$
$$+C_x\sinh(b_xz)+D_x\cosh(b_xz)$$

$$u_y(z)=a_y+b_yz+A_y\sin(b_yz)+B_y\cos(b_yz)$$
$$+C_y\sinh(b_yz)+D_y\cosh(b_yz)$$

$$u_\psi(z)=a_\psi+A_\psi\sin(b_\psi z)+B_\psi\cos(b_\psi z)$$

$$u_z(z)=a_z+A_z\sin(b_zz)+B_z\cos(b_zz)$$

The shape of the beam super element can be expressed in terms of sinusoidal and hyperbolic functions for lateral bending. The axial elongation and torsion deformations require only sinusoidal terms. This is true for general configurations which are comprised of such super elements and rigid bodies as well. The introduction of slight damping and dissipative control effects causes only slight errors, so that sinusoidal and hyperbolic functions remain useful approximations to the actual deformations.

Beam Deflection Matrices

$$u(z) = \begin{bmatrix} u_x(z) \\ u_y(z) \\ u_z(z) \end{bmatrix} \qquad u'(z) = \begin{bmatrix} -u_y'(z) \\ u_x'(z) \\ u_\psi(z) \end{bmatrix}$$

$$= Q_u(z) \begin{bmatrix} A_x \\ B_x \\ C_x \\ D_x \\ A_y \\ B_y \\ C_y \\ D_y \\ A_z \\ B_z \\ A_\psi \\ B_\psi \end{bmatrix} = Q_u(z)\Theta \qquad = Q_{u'}(z) \begin{bmatrix} A_x \\ B_x \\ C_x \\ D_x \\ A_y \\ B_y \\ C_y \\ D_y \\ A_z \\ B_z \\ A_\psi \\ B_\psi \end{bmatrix}$$

Forces and Moments

The forces and moments in body axes are:

$$F_{beam} = P_F \begin{bmatrix} A_x \\ B_x \\ C_x \\ D_x \\ A_y \\ B_y \\ C_y \\ D_y \\ A_z \\ B_z \\ A_\psi \\ B_\psi \end{bmatrix} \qquad M_{beam} = P_M \begin{bmatrix} A_x \\ B_x \\ C_x \\ D_x \\ A_y \\ B_y \\ C_y \\ D_y \\ A_z \\ B_z \\ A_\psi \\ B_\psi \end{bmatrix}$$

It is useful to express the linear and angular deflections, force and moment as matrices multiplying a vector of the coeffecients of the sinusoidal and hyperbolic finctions. The equations of motion, transfer matrix, or the dynamic stiffness matrix can then be expressed in terms of these matrices.

Partial Differential Equations

A similar result is obtained for the other bending equation.

$$m\ddot{u}_y + EI_y u_y'''' = 0$$

$$(\beta_y L)^2 = \frac{\omega}{\sqrt{\dfrac{EI_y}{mL^4}}}$$

For the elongation equation:

$$m\ddot{u}_z + EA_z u_z'' = 0$$

$$\beta_z L = \frac{\omega}{\sqrt{\dfrac{EA_z}{mL^2}}}$$

Similarly for the torsion equation:

$$\rho I_\psi \ddot{u}_\psi + EI_\psi u_\psi'' = 0$$

$$\beta_\psi L = \frac{\omega}{\sqrt{\dfrac{EI_\psi}{\rho I_\psi L^2}}}$$

All of the "b" parameters have been related to the frequency, ω.

The beam equation relates the frequency to the β coefficients that appear in the sinusoidal and hyperbolic beam deflection functions. There are different relationships for bending in the x-z plane, bending in the y-z plane, elongation along the z axis, and twisting about the z axis.

The relationships are more complicated for the Timoshenko beam equation, for a constant axial force, and for attached, smeared appendages.

Structural Damping

Small levels of structural damping would not affect the mode shapes for zero damping. It should be possible to handle small levels of damping. The mode shapes would become complex and the eigen values would have both real and complex parts.

The beam equation might be:

$$m\ddot{u} - C\dot{u}'' + EIu'''' = 0$$

The string equation might be:

$$m\ddot{u} + C\dot{u}' - EAu'' = 0$$

The undamped mode shapes will be used as Galerkin approximate damped mode shapes.

PDEMOD

INPUT

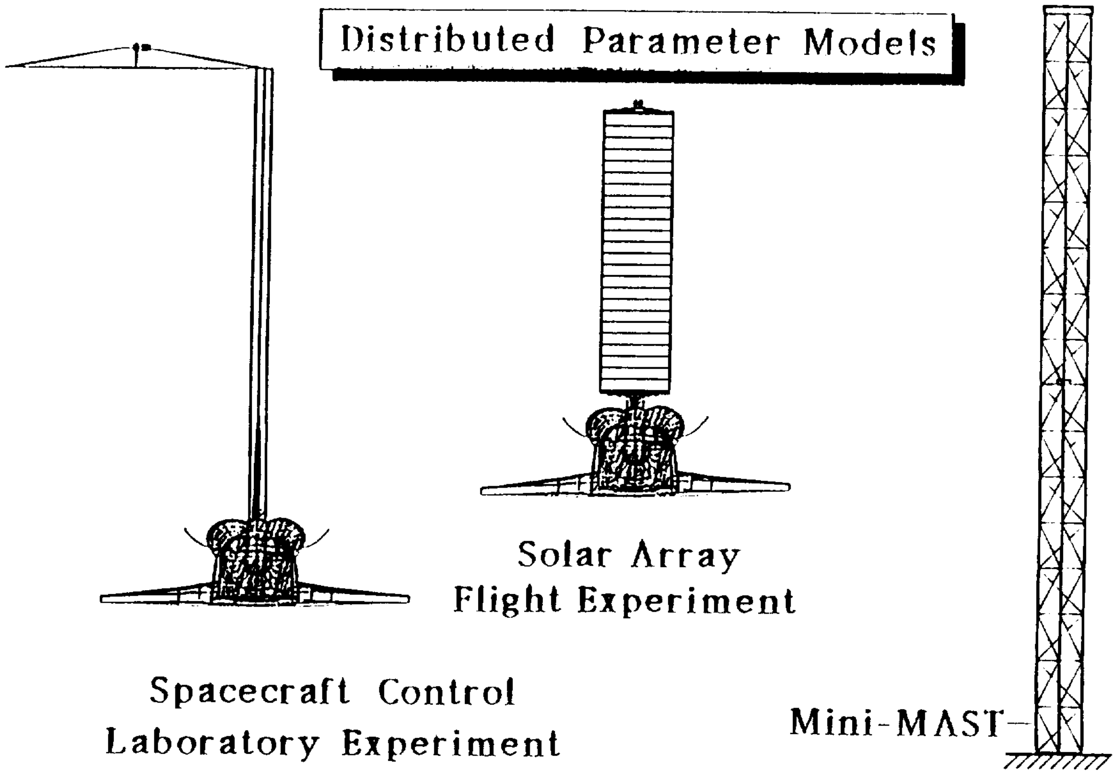
- MASS + INERTIA
- STIFFNESS + DAMPING + CONTROL
- DIMENSIONS + TOPOLOGY

OUTPUT

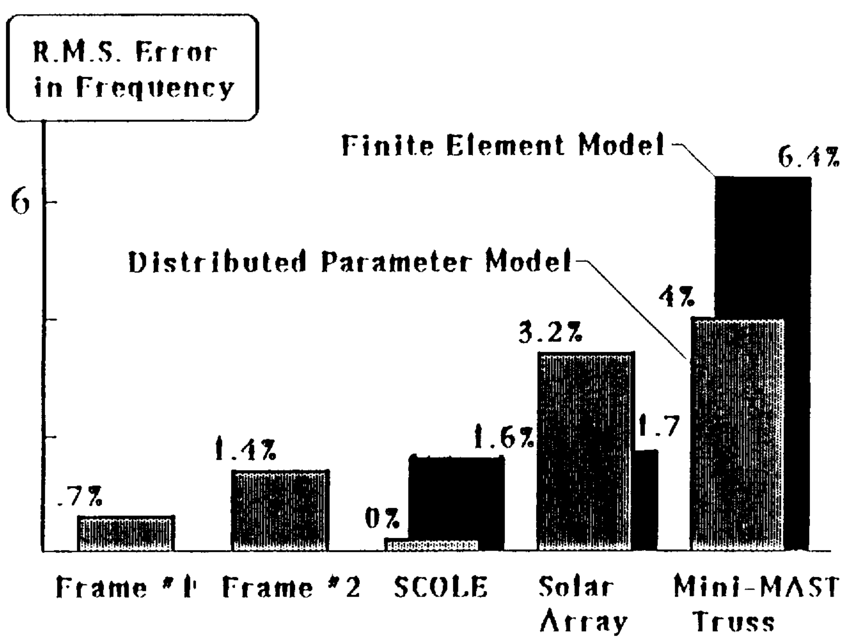
- MODAL FREQUENCIES
- MODE SHAPES
- GRAPHICS
- TRANSFER FUNCTIONS
- SENSITIVITY FUNCTIONS
- MODAL PARTICIPATION
- OPTIMIZATION

The continuum modeling software PDEMOD forms the total system equations from the input data of the mass, stiffness, damping, control and geometrical information. The dynamics of the total system is analyzed and particular responses and functional relationships can then be generated.

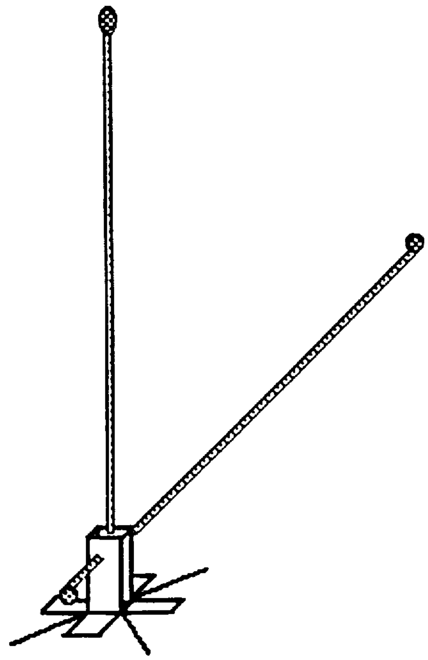
Distributed Parameter Models



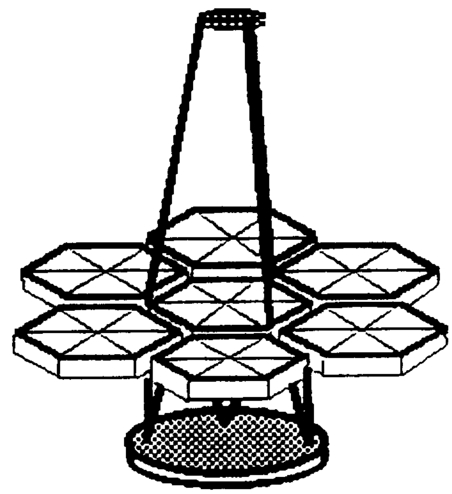
Model Accuracy



Distributed Parameter Models

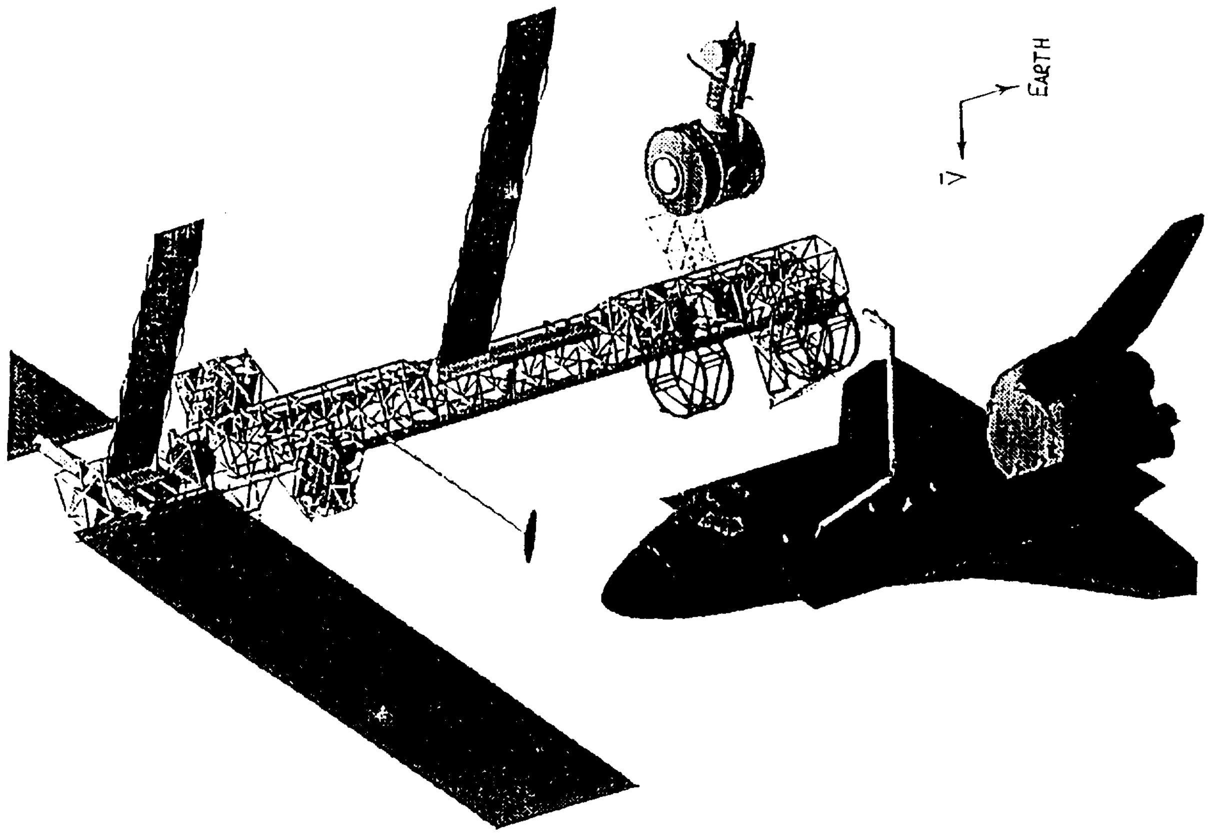


LACE Satellite

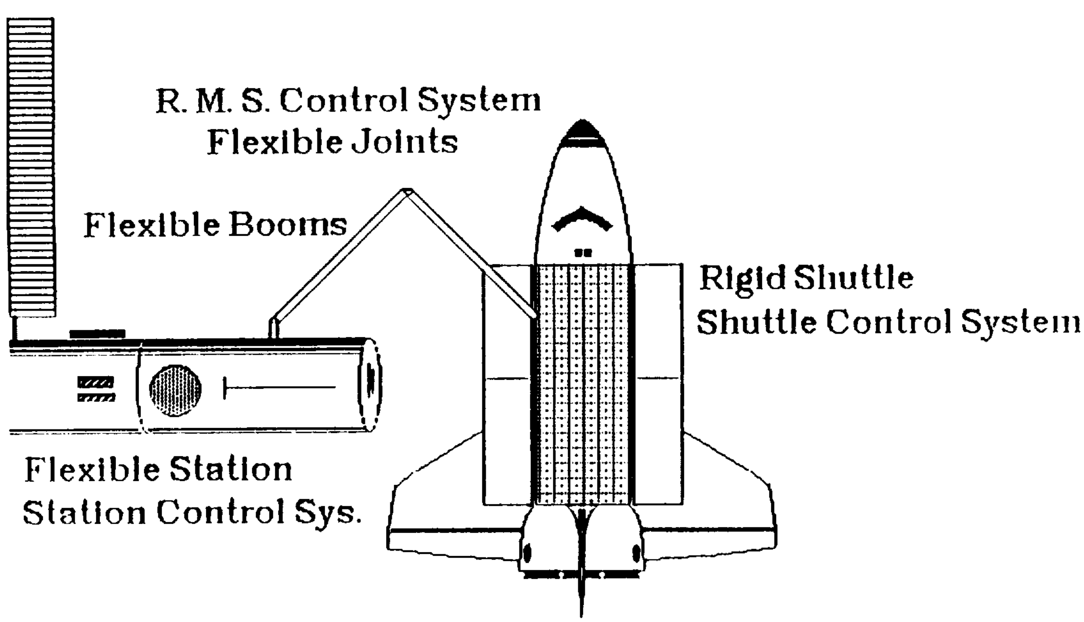


Multi-Hex
Prototype Experiment

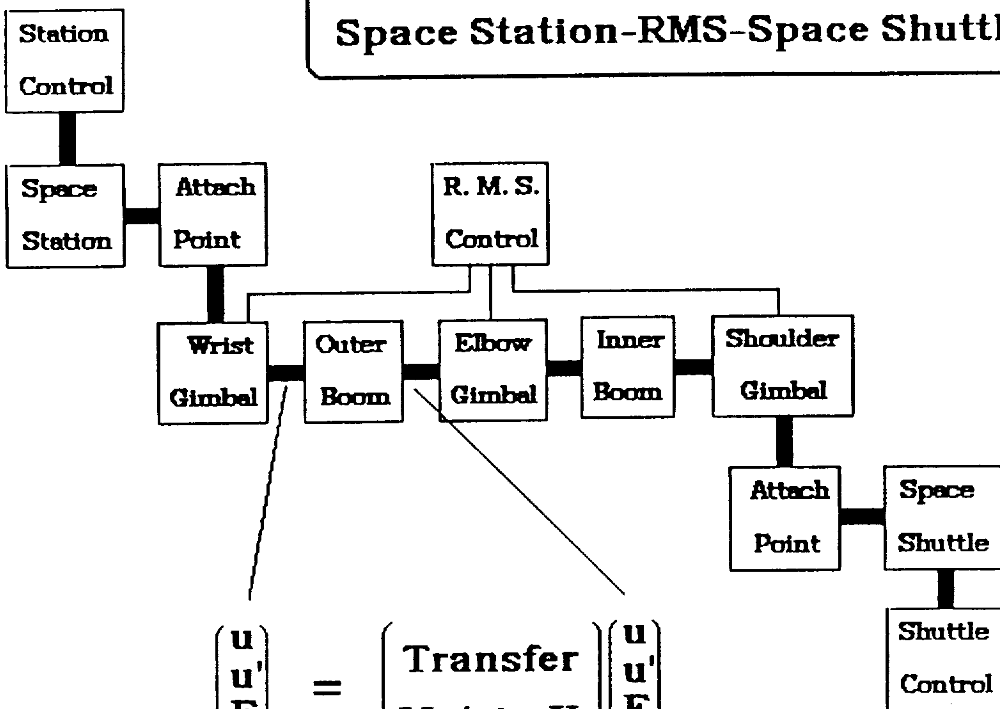
Although a number of flexible spacecraft configurations have been successfully modeled, additional models of the LACE Satellite, the Multiple Hex Prototype Experiment and the Shuttle Remote Manipulating System are being generated. By modeling more complex configurations, the experience of continuum modeling and the capabilities of the PDEMOD software will continue to grow.



Distributed Parameter Model



Continuum Model of Space Station-RMS-Space Shuttle



$$\begin{pmatrix}u\\u'\\F\\M\end{pmatrix}_{i+1} = \begin{pmatrix}\text{Transfer}\\\text{Matrix}, U_i\end{pmatrix}\begin{pmatrix}u\\u'\\F\\M\end{pmatrix}_i$$

$$\begin{pmatrix}u\\u'\\F\\M\end{pmatrix}_{10} = U_9 U_8 U_7 U_6 U_5 U_4 U_3 U_2 U_1 \begin{pmatrix}u\\u'\\F\\M\end{pmatrix}_1$$

The task of developing a continuum model of the Space Shuttle-RMS-Space Station Freedom assembly configurations brings together all of the modeling experience to date. Previous models of the Mini-MAST truss, the Spacecraft Control Laboratory Experiment, and the Solar Array Flight Experiment models will contribute to the complete model of Station assembly. Similarly, the tasks of estimating the model parameters are steps toward estimating the total model parameters of the Station assembly model. The success of this task should serve as an example of the power and usefulness of the distributed parameter modeling approach.

Concluding Remarks

- The use of Finite Element Modeling presents Obstacles to Parameter Estimation and Optimization

- Partial Differential Equation Modeling Facilitates Control/Structure Optimization

- P.D.E. Models have been Successfully Generated for
  1. Spacecraft Control Laboratory Experiment
  2. Solar Array Flight Experiment
  3. Mini-MAST Truss

- P.D.E. Model Accuracy is Competitive with Finite Element Models

- The Software PDEMOD Enables Modeling Complex, Flexible Spacecraft. PDEMOD Continues to be Developed, is being Applied to:
  1. Evolutionary Model Experiment
  2. Space Station Scaled Model
  3. LACE Satellite

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Although significant advances have been made in modeling and controlling flexible systems, there remains a need for improvements in model accuracy and in control performance. The finite element models of flexible systems are unduly complex and are almost intractable to optimum parameter estimation for refinement using experimental data.

Distributed parameter or continuum modeling offers some advantages and some challenges in both modeling and control. Continuum models often result in a significantly reduced number of model parameters, thereby enabling optimum parameter estimation. The dynamic equations of motion of continuum models provide the advantage of allowing the embedding of the control system dynamics, thus forming a complete set of system dynamics. There is also increased insight provided by the continuum model approach.

The challenges of distributed parameter modeling include (1) overcoming the burden of the complexity of partial derivative equations, (2) developing software for model making and analysis, and (3) overcoming complacency.

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