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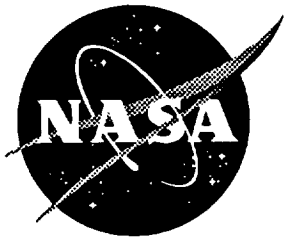

Issues Concerning the Updating of the Finite-Element Models From Experimental Data

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Abstract

Some issues concerning the updating of dynamic finite-element models by incorporation of experimental data are examined here. It is demonstrated how the number of unknowns can be greatly reduced if the physical nature of the model is maintained. The issue of uniqueness is also examined and it is shown that a number of previous workers have been mistaken in their attempts to define both sufficient and necessary measurement requirements for the updating problem to be solved uniquely. The relative merits of modal and frequency-response-function data are discussed and it is shown that for measurements at fewer degrees-of-freedom than are present in the model, frf data will be unlikely to converge easily to a solution. It is then examined how such problems may become more tractable by using new experimental techniques which would allow measurements at all degrees-of-freedom present in the mathematical model.

Introduction

The issues investigated in this report concern the modification, or development, of finite-element-models such that the dynamic behaviour of that model correlates well with experimental data. The significance attached to the updating of finite-element-models by the use of experimental data is an implicit recognition that dynamic FEM's are often a poor representation of the actual structure. Such updating of FEM's enables the effects of structural modifications to be assessed with much greater confidence by computer modelling rather than full-scale testing, thereby reducing costs. Another potential advantage to model updating is that it can help to highlight where errors in the initial modelling process are being made and, therefore, increase the efficiency of future FEM development.

Static stress analysis, the other main engineering application of finite-element-modelling, is typically subjected to far less scrutiny than dynamic modelling for two reasons: firstly, it is considerably more difficult to rigorously compare FEM stress analysis data with experiment than is the case for dynamic data; and secondly, many more approximations are typically

made in dynamics models. Approximations involved in dynamics modelling arise from trying to keep the model simple, i.e. to as few degrees-of-freedom as possible. Another main approximation in dynamic FEM's concerns damping; the most common approximation being that it is zero. Zero damping is obviously never the case, but the mechanisms of damping in built-up structures are so poorly understood and variable that it is often practically impossible to model. Throughout this report, it will be assumed that damping is negligible.

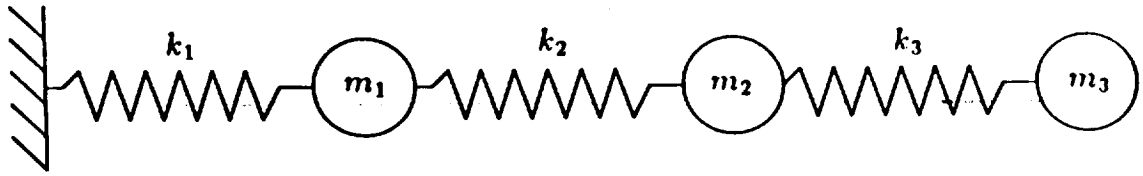
In this report, issues such as uniqueness of an updated model, and the relative merits of modal data and frequency response function data will be explored with reference to the current literature. Some concepts will be investigated with simulations using simple mass-spring systems.

Uniqueness

Given a set of experimental data and an analytical model, is there more than one set of analytical model parameters which will match the experimental data? This question describes the issue of uniqueness.

Su and Juang (1992) carried out a study on the uniqueness of updated stiffness parameters for the general case of updating the complete stiffness matrix. It was shown that either the number of actuators, or the number of sensors, must be greater than or equal to the number of degrees-of-freedom to enable the identification of the stiffness matrix in physical coordinates. For the case where matrix symmetry was enforced, it was shown that the sum of the number of sensors and actuators must be greater than or equal to the number of degrees-of-freedom. In Su and Juang (1992), it was assumed that the number of measured modes was equal to the number of analytical degrees of freedom. The issue under investigation here is: what are the measurement requirements for uniqueness when the physical nature of the stiffness matrix is enforced? Maintaining the 'physical nature' of the stiffness matrix means that actual element stiffness properties are updated. These updated properties are then used to reform the stiffness matrix. For example: consider

the three degree of freedom model shown in Figure(1).



Figure(1) simple three degree-of-freedom mass-spring system

Techniques which operate on the stiffness matrix ($[K]_{N\times N}$) where N is the number of degrees of freedom of the model (in this case, $N = 3$), as a whole, would require the determination of N^2 (here, $N^2 = 9$) independent parameters. If matrix symmetry is enforced, the number of terms required reduces to $\sum_{i=1}^{N} i$ (here, $\sum_{i=1}^{N} i = 6$. These are the two cases studied in Su and Juang (1992). Not surprisingly, it was found that where the symmetry of $[K]$ was enforced, the corresponding uniqueness criterion was relaxed, ie. fewer measurements are required to determine fewer unknown parameters.

An option to reduce the number of parameters required to be determined even further is to enforce physical connectivity and symmetry. This means that the stiffness matrix would appear as

$$\begin{bmatrix} a & b & 0 \\ b & c & d \\ 0 & d & e \end{bmatrix}$$

where the 0's mean that there is no direct connection between the first and the third degree of freedom. This leaves $2N - 1$ (here, $2N - 1 = 5$) parameters to be determined. Now the number of parameters to be determined varies linearly, rather than quadraticaly, with

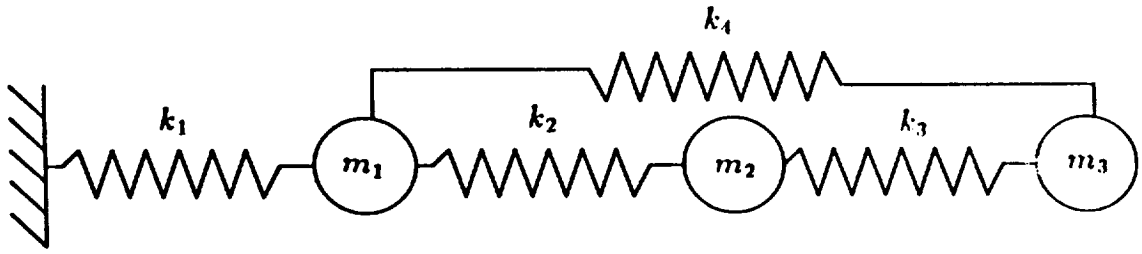
the number of degrees of freedom of the model. Note: there can be physical connectivity between the first and third degrees of freedom as shown in Figure(2) meaning that the $K(1,3)$ and $K(3,1)$ term in the above matrix will no longer be 0; for a real srtucture, it is unlikely that every modelled point will be directly connected to every other point, so it can still be expected that the number of unknowns will grow linearly with the number of degrees of freedom.

Further constraints can be placed on the model by updating only the physical parameters. In this case, our stiffness matrix will look like

$$\begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}$$

which requires the determination of only the spring stiffnesses k_1, k_2 and k_3. For the case shown in Figure(2), the stiffness matrix would be of the form

$$\begin{bmatrix} k_1 + k_2 + k_4 & -k_2 & -k_4 \\ -k_2 & k_2 + k_3 & -k_3 \\ -k_4 & -k_3 & k_3 + k_4 \end{bmatrix}$$



Figure(2) Example showing three degree-of-freedom system with a direct connection between the first and third degrees-of-freedom.

If the desired analytical model is as shown in Figures(1) or (2) (i.e. the desired analytical degrees-of-freedom are as represented in these Figures), then the above representation reduces the number of unknown parameters to a minimum (See Appendix 1 for a similar treatment of the stiffness matrix of a beam element). In directly updating the physical parameters of the stiffness matrix, we are accepting the implicit approximations involved in the finite-element process. If we were to allow the updated model to have greater freedom, then the resulting system matrix may be a better representation of the actual behaviour of the structure, but the modifications may not be physically intrepretable. Also, in allowing the updated model too much freedom; i.e. allowing too many parameters to be updated, there is a possibility that the updating process will begin to model the experimental noise in addition to the real dynamic behaviour. In addition, if we update our finite-element models with respect to physical parameters, it may help us to gain an understanding of where we are going wrong in the initial modelling and, therefore, help to improve initial modelling in the future. For these reasons, this study will be primarily concerned with updating physical parameters within the finite-element model directly.

The nature of the data, with respect to the model, broadly defines the method of solution for the system matrix which can be used. If measurements are carried out at all of the degrees-of-freedom represented in the model, then a direct solution may be carried out. If all degrees-of-freedom present in the model are not measured, then an iterative solution technique may be required.

First: look at the case where all degrees-of-freedom are measured. Working with eigenvalues and eigenvectors, the following eigenvalue equation is of interest

$$[K - \lambda_i M]\{\phi_i\} = 0 \tag{1}$$

or

$$K\{\phi_i\} = \lambda_i M\{\phi_i\} \tag{2}$$

For the 3 dof mass spring system shown in Figure(1)

$$K\{\phi_i\} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} \phi_{1,i} \\ \phi_{2,i} \\ \phi_{3,i} \end{Bmatrix} \tag{3}$$

Rearranging gives

$$K\{\phi_i\} = \Phi_i\{k\} = \begin{bmatrix} \phi_{1,i} & \phi_{1,i} - \phi_{2,i} & 0 \\ 0 & -\phi_{1,i} + \phi_{2,i} & \phi_{2,i} - \phi_{3,i} \\ 0 & 0 & -\phi_{2,i} + \phi_{3,i} \end{bmatrix} \begin{Bmatrix} k_1 \\ k_2 \\ k_3 \end{Bmatrix} \tag{4}$$

substituting back and rearranging, allows the k's to be solved directly

$$\{k\} = \lambda_i \Phi_i^{-1} M\{\phi_i\} \tag{5}$$

Given the structure of the matrix Φ_i, it will typically be of full-rank and, therefore sufficient to determine the three unknown parameters. This is for measurements performed at the i'th mode and shows that measurements at all degrees-of-freedom for only one mode are sufficient to determine the stiffness parameters for this case. This is not surprising when the analogous case of static deflections is considered; if the system is statically deformed by a known force at the third degree of freedom, then, using the principles of equilibrium, knowledge of the displacements at each degree-of-freedom is obviously sufficient to determine each spring stiffness. Static deformation data has been used to update the stiffness data for a large scale finite-element problem by Lallement *et al* (1992). For the dynamic case, the knowledge of the force is embedded in the mass matrix in that it defines the inertial forces.

Additional measurements give redundancy and, therefore, greater robustness to the analysis. If both the i'th and j'th modes are measured, we can write

$$\left\{ \begin{array}{c} K\{\phi_i\} \\ --- \\ K\{\phi_j\} \end{array} \right\} = \left\{ \begin{array}{c} \lambda_i M\{\phi_i\} \\ ---- \\ \lambda_j M\{\phi_j\} \end{array} \right\} \tag{6}$$

A similar rearrangement to that carried out previously gives

$$\left\{ \begin{array}{c} \Phi_i \\ -- \\ \Phi_j \end{array} \right\} \left\{ \begin{array}{c} k_1 \\ k_2 \\ k_3 \end{array} \right\} = \left\{ \begin{array}{c} \lambda_i M\{\phi_i\} \\ ---- \\ \lambda_j M\{\phi_j\} \end{array} \right\} \tag{7}$$

Which we will write as

$$\Phi_{i,j}\{k\} = \mathcal{M}_{i,j} \tag{8}$$

where

$$\Phi_{i,j} = \left\{ \begin{array}{c} \Phi_i \\ -- \\ \Phi_j \end{array} \right\} \quad \text{and} \quad \mathcal{M}_{i,j} = \left\{ \begin{array}{c} \lambda_i M\{\phi_i\} \\ ---- \\ \lambda_j M\{\phi_j\} \end{array} \right\} \tag{9}$$

The stiffness parameters may then be solved by

$$\{k\} = \Phi^{+}_{i,j}\mathcal{M}_{i,j} \tag{10}$$

where $\Phi^{+}_{i,j}$ is the pseudo inverse of $\Phi_{i,j}$.

The addition of further measured modes is trivial.

From the previous analysis, it is clear that, where the measured degrees-of-freedom match those of the analytical model, the parameters defining the stiffness matrix may be readily

identified with measurements of one mode only. Now to examine the case where there are fewer measured degrees-of-freedom than exist in an analytical model; in this situation, iterative techniques may have to be employed and the issue of uniqueness is far less straightforward. Consider the case of the two degree of freedom system shown in figure (3), where the mass and stiffness matrices are given by

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \quad \text{and} \quad K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

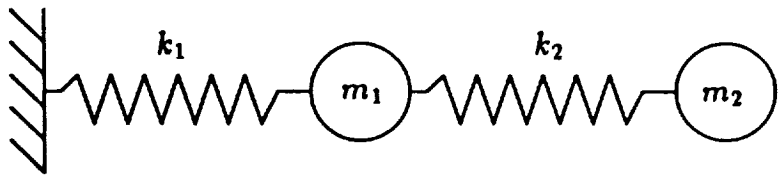


Figure (3) Simple two degree-of-freedom system

The characteristic equation for the system represented by the above mass and stiffness matrices may be written as

$$\lambda^2(m_1 m_2) - \lambda(m_1 k_2 + m_2 k_1 + m_2 k_2) + k_1 k_2 = 0 \tag{11}$$

which may be rewritten as

$$\lambda^2 a_2 - \lambda a_1 + a_0 = 0 \tag{12}$$

where the roots of this equation give the eigenvalues λ_1 and λ_2. Now, if we assume that only

the measured frequencies are used in the updating procedure, we can use the characteristic equation to attempt to determine the stiffness parameters. It is not unreasonable to employ only the measured frequencies, as many model updating procedures used in practice use only a comparison of modal frequencies as a criterion in assessing the validity of an updated model; it is also the data available in the case where only one sensor is used. Allowing only the stiffness parameters, k_1 and k_2, to change, it is clear from the above equation that the coefficients of λ^1 and λ^0, a_1 and a_0 respectively, will define the uniqueness of the problem. If there is more than one set of k_1 and k_2 which will give the same a_1 and a_0, then there is more than one set of stiffness parameters which will give rise to identical eigenvalues and the system cannot be uniquely identified from frequencies alone. This is easily investigated by a numerical example. If we let $k_1 = 8$, $k_2 = 6$, $m_1 = 2$ and $m_2 = 4$ we have the coefficients $a_1 = 68$ and $a_0 = 48$. Turning the problem around and determining k_1 and k_2 given m_1, m_2, a_1 and a_0, we find that k_1 and k_2 may equal 8 and 6 respectively (as previously given) or 9 and 16/3. So, even for this simple example, it can be seen that two measured modal frequencies will not give a unique identification of the two stiffness parameters.

Jung and Ewins (1992), Lin (1993) and Botto *et al* (1993) have investigated the issue of the number of measurements required to update a number of parameters for models where the physical nature of the system matrices is maintained. Jung and Ewins (1992) and Lin (1993) state that

$$m\,(n+1) \geq L \tag{13}$$

where n is the number of measured degrees-of-freedom, m is the number of measured modes and L is the number of unknown parameters. Botto *et al* (1993) states that both

$$mn > L \tag{14}$$

and

$$\frac{m}{2}\{2N-(m+1)\} > L \tag{15}$$

must be satisfied. N is the number of analytic degrees of freedom.

These criteria for uniqueness obviously conflict. From our previous development of the direct solution, it was shown that the stiffness parameters may be determined from having sensors at each degree of freedom and having measurements for only one mode. According to the criterion of Jung and Ewins (1992) and Lin (1993), we have $4 \geq 3$ which agrees with our statement that the stiffness parameters are identifiable. Examining the criteria of Botto *et al* (1993), however, we have $3 \not> 3$ and $2 \not> 3$ respectively. Therefore, by the criterion of Botto *et al* (1993), the stiffness parameters should not be identifiable whereas by that of Jung and Ewins (1992) and Lin (1993), they should be. Let us investigate another example, that of the two degree of freedom system with measurements at only one degree-of-freedom for two modes. We have shown previously, that the stiffness parameters are not uniquely identifiable for this case. By the criterion of $m(n+1) \geq L$, we have $4 \geq 3$ so the parameters should be uniquely identifiable, but clearly are not.

An obvious criterion for uniqueness is that the number of independent measurements be greater than or equal to the number of parameters required to be updated. Whilst this is clearly a necessary condition of uniqueness, from the results of the two degree of freedom system already shown, it would not appear to be sufficient.

Jung and Ewins (1992) and Lin (1993) determined the criterion for uniqueness by investigating the rank of the sensitivity matrix. The sensitivity matrix is given by the first term of a Taylor series expansion to the changes in eigenvalues and eigenvectors experienced due to a change in the relevant parameters. Use of sensitivity matrices is a common means of solving non-linear identification problems and have been used by many workers in the field of model updating. The basis behind sensitivity methods is given in the following equation:

$$\{\Delta\} = [S]\{\Delta D\} \tag{16}$$

where $\{\Delta\}$ is a vector of differences between experimentally and analytically derived data, $[S]$ is the sensitivity matrix. $\{\Delta D\}$ is the change in variable required to reconcile the differences expressed in the vector $\{\Delta\}$ if the model were linear; for non-linear models, $\{\Delta D\}$ is an approximation to the required changes. For model updating from modal analysis data, Eq.(16) can be written as:

$$\begin{Bmatrix} \Delta\lambda_1 \\ \{\Delta\phi_1\} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \Delta\lambda_m \\ \{\Delta\phi_1\} \end{Bmatrix} = \begin{bmatrix} \frac{\partial\lambda_1}{\partial D_1} & \frac{\partial\lambda_1}{\partial D_2} & \cdots & \frac{\partial\lambda_1}{\partial D_L} \\ \frac{\partial\{\phi_1\}}{\partial D_1} & \frac{\partial\{\phi_1\}}{\partial D_2} & \cdots & \frac{\partial\{\phi_1\}}{\partial D_L} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \frac{\partial\lambda_m}{\partial D_1} & \frac{\partial\lambda_m}{\partial D_2} & \cdots & \frac{\partial\lambda_m}{\partial D_L} \\ \frac{\partial\{\phi_m\}}{\partial D_1} & \frac{\partial\{\phi_m\}}{\partial D_2} & \cdots & \frac{\partial\{\phi_m\}}{\partial D_L} \end{bmatrix} \begin{Bmatrix} \Delta D_1 \\ \Delta D_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \Delta D_L \end{Bmatrix} \tag{17}$$

where the dimensions of the matrices are

$$\{\Delta\}_{m(n+1)\times 1} = [S]_{m(n+1)\times L}\,\{\Delta D\}_{L\times 1} \tag{18}$$

As noted by Jung and Ewins (1992), such a sensitivity matrix is likely to be ill-conditioned because the magnitudes of the eigenvalue derivatives are likely to be much greater than that of the scaled eigenvector derivatives. This problem is solved by normalizing the eigenvalue derivatives by their respective eigenvalues giving

$$\begin{Bmatrix} \Delta\lambda_1/\lambda_1 \\ \{\Delta\phi_1\} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \Delta\lambda_m/\lambda_m \\ \{\Delta\phi_1\} \end{Bmatrix} = \begin{bmatrix} \frac{\partial\lambda_1}{\partial D_1}/\lambda_1 & \frac{\partial\lambda_1}{\partial D_2}/\lambda_1 & \cdots & \frac{\partial\lambda_1}{\partial D_L}/\lambda_1 \\ \frac{\partial\{\phi_1\}}{\partial D_1} & \frac{\partial\{\phi_1\}}{\partial D_2} & \cdots & \frac{\partial\{\phi_1\}}{\partial D_L} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \frac{\partial\lambda_m}{\partial D_1}/\lambda_m & \frac{\partial\lambda_m}{\partial D_2}/\lambda_m & \cdots & \frac{\partial\lambda_m}{\partial D_L}/\lambda_m \\ \frac{\partial\{\phi_m\}}{\partial D_1} & \frac{\partial\{\phi_m\}}{\partial D_2} & \cdots & \frac{\partial\{\phi_m\}}{\partial D_L} \end{bmatrix} \begin{Bmatrix} \Delta D_1 \\ \Delta D_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \Delta D_L \end{Bmatrix} \tag{19}$$

If the sensitivities consisted of no higher order terms in the Taylor series than the first term represented in $[S]$, then the solution to the required change in parameters could be immediately achieved. The actual eigenvalue and eigenvector sensitivities, however, are highly non-linear, so an iterative scheme based on the following equation must be used to attempt to find the parameters required to minimize the vector $\{\Delta\}$:

$$D_{j+1} = D_j + \Delta D_j = D_j + S_j^{\dagger} \Delta_j \tag{20}$$

where j is the iteration number.

One considerable difficulty surrounds determining $[S]$. The sensitivity matrix may be most simply approximated by perturbation of the desired parameters (ie. solving the eigenvalue problem for slightly varied parameters thereby giving an approximation to the derivative). Given that this requires solving the eigenvalue problem for each design variable at each iteration, it is clearly not an efficient means of determining the sensitivity matrix for larger problems. Fox and Kapoor (1968), give the derivative of an eigenvalue with respect to a parameter as

$$\frac{\partial \lambda_1}{\partial D_j} = \phi_i^T \left[\frac{\partial K}{\partial D_j} - \lambda_i \frac{\partial M}{\partial D_j} \right] \phi_i \tag{21}$$

The approximation for the eigenvalue derivative is easily derived, that for the eigenvector derivative, however, is not so easy. A number of workers have given approximations to eigenvector derivatives based on three approaches: i) direct differentiation, ii) modal synthesis and iii) iterative techniques. Methods based on both the direct and modal approach were first presented by Fox and Kapoor (1968). The direct method is based on differentiating both the eigenvalue equilibrium equation and the mass orthonormalization equation. The eigenvector derivative of the i'th mode is based only on the components of that mode, thus, the full eigensolution is not required. The direct method of Fox and Kapoor (1968) was improved by Nelson (1976) whose solution method maintains the bandedness of the original problem and gives well-conditioned matrices, thereby increasing the accuracy of solution. The main computational cost involved in Nelson's method is that it requires the

inversion of a matrix of order $N-1$, where N is the order of the full analytic system.

The Modal method is based on a modal synthesis approach and uses all of the analytical modes of the system. This can be expensive in computational effort for solution of the full eigen problem for large systems. Fox and Kapoor (1968) postulated that a reduced system of eigenvectors could be used to approximate the eigenvector derivative, but did not pursue this investigation. Wang (1991) developed an accurate truncated modal synthesis method by introducing a vector defined as a static mode which helps to include some of the effects of the truncated modes. As with all modal synthesis techniques, great care must still be taken to ensure there is sufficient freedom to characterize the desired properties.

Iterative techniques for determining eigenvector derivatives based on the subspace iteration techniques used to solve eigenproblems of large systems, were first proposed by Rudisill and Chu (1975). The early techniques suffered from poor convergence rates and accuracy and a direct method, such as Nelson's method was considered superior. Ting (1992) developed an accelerated subspace iteration technique for determining eigenvector derivatives based on Bathe's accelerationtechnique (Bathe, 1977). For cases involving a small number of parameters, Ting (1992) claims decreases in computational cost, when compared to Nelson's method, of order 70%. As the number of parameters to be considered increases though, the accelerated iterative technique becomes less efficient and may be surpassed by Nelson's method.

It is clear that the method used to optimally determine eigenvector derivatives is very dependent upon the nature of the problem in question.

It would appear from Eq.(18) that a requirement to solve for Δ is that $m(n+1) \geq L$. It is important to remember, however, that the eigenvectors must be scaled and that they have no absolute value, therefore, some information is lost in this scaling process. Examining the case where there is only one sensor shows that, for this case, an eigenvector derivative is meaningless; all we are left with are eigenvalue derivatives in our sensitivity matrix. This would lead to the assumption that because information is lost in the requirement to scale eigenvectors, the requirement to solve Eq.(18) is that $mn \geq L$, which for the case of only one sensor, becomes $m \geq L$. This assumption will be examined by example.

Going back to the original 2 dof problem, with $k_1 = 8$ and $k_2 = 6$, the sensitivity matrix, using Eq.(21) is:

$$[S] = \begin{bmatrix} \frac{\partial \lambda_1}{\partial k_1}/\lambda_1 & \frac{\partial \lambda_1}{\partial k_2}/\lambda_1 \\ \frac{\partial \lambda_2}{\partial k_1}/\lambda_2 & \frac{\partial \lambda_2}{\partial k_2}/\lambda_2 \end{bmatrix} = \begin{bmatrix} 0.2427 & 0.2802 \\ 0.1224 & 0.1885 \end{bmatrix} \tag{22}$$

which is of full rank with a condition number of 16.3. As has already been shown, however, there is not a unique answer to this problem. Examining the rank of the sensitivity matrix, however, is how some workers have attempted to define the conditions for uniqueness. The difficulty in using the rank of the sensitivity matrix to define the uniqueness or otherwise of such a problem arises because of the inherent non-linearity of the problem in that the process involves iterating from an initial condition to a final answer. A full rank sensitivity matrix tells us that, given a set of initial conditions, there is only one solution that can finally be arrived at. This is because, if the matrix is of full rank, each iterative step will be unique. Given another set of initial conditions, however, another solution may be found which will be unique for the initial conditions, but not for the overall problem.

The solution process for a non-linear sensitivity analysis was performed on a six degree of freedom mass spring system using noise-free simulated data with randomly generated initial conditions.

It was assumed that the six spring stiffnesses were unknown and solutions were attempted for a number of combinations of measured degrees-of-freedom and number of measured modes. Each case used 100 different initial conditions. From this simulation, it would seem that the uniqueness condition for this system is given by $mn \geq 6$ with the additional condition that $n > 1$. Additional care would have to be exercised with noisy data for some of the cases due to poor conditioning. For the case with two measured degrees-of-freedom and 3 modes ($mn = 6$), the condition number for the matrix is of order 10^{13} and for the case of three measured degrees-of-freedom and two modes, the condition number is of order 10^5. Even though these cases satisfy the criterion for uniqueness, with 'real' data, they would be almost impossible to solve. As with any identification problem, great care should be exercised in ensuring that the experimental data has as little as possible noise and no bias, and, given unbiased data, the more data, the better.

Frequency Response Functions

A method of analysis based upon frequency response functions, promises a greater amount of data to work with than for techniques based on modal properties. As well as providing more data, an advantage of working with frequency response functions is that the process of determining modal properties is avoided; hence, the errors that would be introduced in this process are also avoided. In other words, the frequency response function data is more 'pure' than modal data. The only way to work with data that is even more 'pure' would be to work with the time response data directly; the data storage requirements for this would probably be prohibitive.

Consider the undamped, steady-state, forced response equation of motion with the forcing function being sinusoidal at ω rad/sec:

$$\left[-M\omega^2 + K\right]\{x\} = \{f\} \tag{23}$$

where M is the mass matrix, K is the stiffness matrix, $\{x\}$ is a vector of response amplitudes and $\{f\}$ a vector of input force amplitudes. As was done previously, it will be assumed that the masses are known and that the unknowns are the actual stiffness properties (as opposed to the stiffness matrix as a whole). Equation(23) can be re-written as

$$K\{x_i\} = \{f_i\} + M\omega_i^2\{x_i\} \tag{24}$$

For the two degree-of-freedom system represented in figure (3), the left-hand-side of Eq.(24) would be written as

$$\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_{1i} \\ x_{2i} \end{Bmatrix}$$

which may be re-written as

$$\begin{bmatrix} x_{1i} & x_{1i} - x_{2i} \\ 0 & -x_{1i} + x_{2i} \end{bmatrix} \begin{Bmatrix} k_1 \\ k_2 \end{Bmatrix} = X_i \{k\} \tag{25}$$

leading to the solution for $\{k\}$ being given by

$$\{k\} = X_i^{-1} \left(\{f_i\} + M\omega_i^2 \{x_i\} \right) \tag{26}$$

Measurements for additional frequencies can be incorporated as follows:

$$\begin{bmatrix} x_{1,1} & x_{1,1} - x_{2,1} \\ 0 & -x_{1,1} + x_{2,1} \\ -- & --- \\ x_{1,2} & x_{1,2} - x_{2,2} \\ 0 & -x_{1,2} + x_{2,2} \\ -- & --- \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ -- & --- \\ x_{1,2} & x_{1,2} - x_{2,2} \\ 0 & -x_{1,2} + x_{2,2} \end{bmatrix} \begin{Bmatrix} k_1 \\ k_2 \end{Bmatrix} = \begin{Bmatrix} f_{1,1} \\ f_{2,1} \\ -- \\ f_{1,2} \\ f_{2,2} \\ -- \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ -- \\ f_{1,N} \\ f_{2,N} \end{Bmatrix} + \begin{bmatrix} m_1\omega_1^2 & 0 \\ 0 & m_2\omega_1^2 \\ -- & -- \\ m_1\omega_2^2 & 0 \\ 0 & m_2\omega_2^2 \\ -- & -- \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ -- & -- \\ m_1\omega_N^2 & 0 \\ 0 & m_2\omega_N^2 \end{bmatrix} \begin{Bmatrix} x_{1,1} \\ x_{2,1} \\ -- \\ x_{1,2} \\ x_{2,2} \\ -- \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ -- \\ x_{1,N} \\ x_{2,N} \end{Bmatrix} \tag{27}$$

$$X \{k\} = \{f\} + \left[M\omega^2\right] \{x\} \tag{28}$$

giving

$$\{k\} = X^+ \left(\{f\} + \left[M\omega^2\right] \{x\} \right) \tag{29}$$

The above example is easily extended to incorporate additional degrees of freedom.

An important consideration when dealing with frequency response function data is that data collected at different frequencies cannot necessarily be considered independent. The concept of linear independence of frequency response function data and its effect on the condition number of the matrix X, as defined in Eq.(28), is illustrated in figure (4).

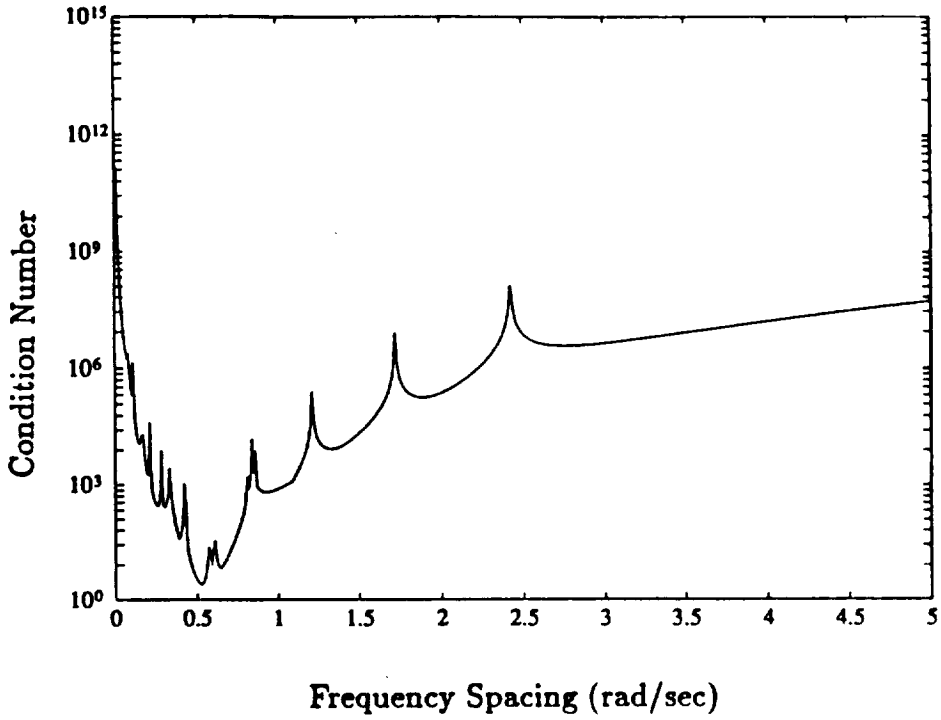


Figure (4) Condition number for the matrix X, as defined in Eq.(28), for simulated frequency response function data collected at five frequencies where both the lowest frequency sampled and the frequency spacing, are given on the horizontal axis.

The results presented in figure (4) are the condition numbers of the matrix X for simulated, noise-free, data for five sampled frequencies spaced by the amounts given on the x-axis. As can be seen, when the data are closely spaced, the condition number is very high, it then reduces as the data become more linearly independent. There is no requirement that all of the sampled data be linearly independent, but, the more linearly independent measurements taken, the more robust the solution should be. It should be noted that the most informative frequency spacing will be very problem dependent.

The previous examples of analysis using frequency response functions were based on the assumption that measurements were taken at all the degrees of freedom present in the analytic model; now to examine the analysis involved when this is not the case, ie. the case involving non-linear solution. As for the modal case, a sensitivity approach to the non-linear solution of the design parameters may be written as

$$\{\Delta\} = [S]\{\Delta D\} \tag{30}$$

where $\{\Delta\}$ is still a vector of differences between experimental and analytically derived data, $[S]$ is a sensitivity matrix, and $\{\Delta D\}$ is a vector of approximate changes in stiffness parameters to account for the differences in experimental and analytic data. For frequency response functions, eqn(30) takes the form

$$\begin{Bmatrix} \{\Delta x_1\} \\ \{\Delta x_2\} \\ \cdot \\ \cdot \\ \cdot \\ \{\Delta x_n\} \end{Bmatrix} = \begin{bmatrix} \frac{\partial\{x_1\}}{\partial D_1} & \frac{\partial\{x_1\}}{\partial D_2} & \cdots \frac{\partial\{x_1\}}{\partial D_L} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \frac{\partial\{x_n\}}{\partial D_1} & \cdots & \cdots \frac{\partial\{x_n\}}{\partial D_L} \end{bmatrix} \begin{Bmatrix} \{\Delta D_1\} \\ \{\Delta D_2\} \\ \cdot \\ \cdot \\ \cdot \\ \{\Delta D_n\} \end{Bmatrix} \tag{31}$$

Determining the sensitivity matrix for frequency response functions is a more difficult proposition than that for modal parameters. Once again, the sensitivities may be approximated by perturbation, but this would require the inversion of a $n \times n$ matrix (where n is the order of the analytic model) $L \times n$ times for each iteration (where L is the number of parameters to be determined and n is the number of sensors); impractical for most cases. An approximation to the receptance sensitivity has been developed by Brandon (1987) who showed that

$$\frac{\partial[\alpha(\omega_i)]}{\partial D_j} = -[\alpha(\omega_i)][\Delta Z(\omega_i)][\alpha(\omega_i)] \tag{32}$$

where $[\alpha(\omega_i)]$ is the inverse of the dynamic stiffness matrix $[K - \omega_i^2 M]$ and $[\Delta Z(\omega_i)]$ is called the 'dynamic stiffness error matrix' $[(\Delta Z(\omega_i)] = [\Delta K] - \omega^2[\Delta M]$. The response

vector sensitivities may then simply be determined by multiplying $\partial[\alpha(\omega)]/\partial D_j$ by the force input vector. The advantage of this method over the perturbation method, is that the inversion required to create $[\alpha(\omega)]$ only need be done once for each frequency step rather than for every design variable for every frequency step. He (1993) shows that the accuracy of Eq.(32) reduces as the frequency approaches a resonance and concludes that sensitivity analysis using such an approach will be more successful if data around anti-resonances are used and data near resonances discarded.

From the perspective of measurement noise, rather than analytical errors, Ren and Beards (1993) state that experimental data in the vicinity of resonances should not be used because of higher measurement sensitivities to small changes in frequency in these regions.

Now, to compare the non-linear solution process using modal, and then frequency response function data and examine how worthwhile it is to attempt to solve for parameters using sensitivity methods, let us investigate simulated data for the two degree-of-freedom system shown in figure (3). For the case where there are two unknowns to be determined by a non-linear method of solution, a visual understanding of how sensitivity techniques work, can be gained by plotting the residuals against variation in each of the two parameters to be determined. First, let us examine the residuals for modal data being given by

$$r2 = \sum_{i=1}^{2} \left\{ \left(\frac{\lambda_{Ei} - \lambda_{Ai}}{\lambda_{Ai}} \right)^2 + (\phi_{E2,i} - \phi_{A2,i})^2 \right\} \tag{33}$$

where the subscript E denotes experimental data (here, noise-free simulated experimental data) and A denotes analytically determined data (note: the differences in the eigenvectors only use data from the second degree-of-freedom because the eigenvector data has been scaled with the amplitude of the first freedom being one). The residuals over the range of $k_1 = 3$ to 13Nm and $k_2 = 1$ to 11Nm are shown in figure (5).

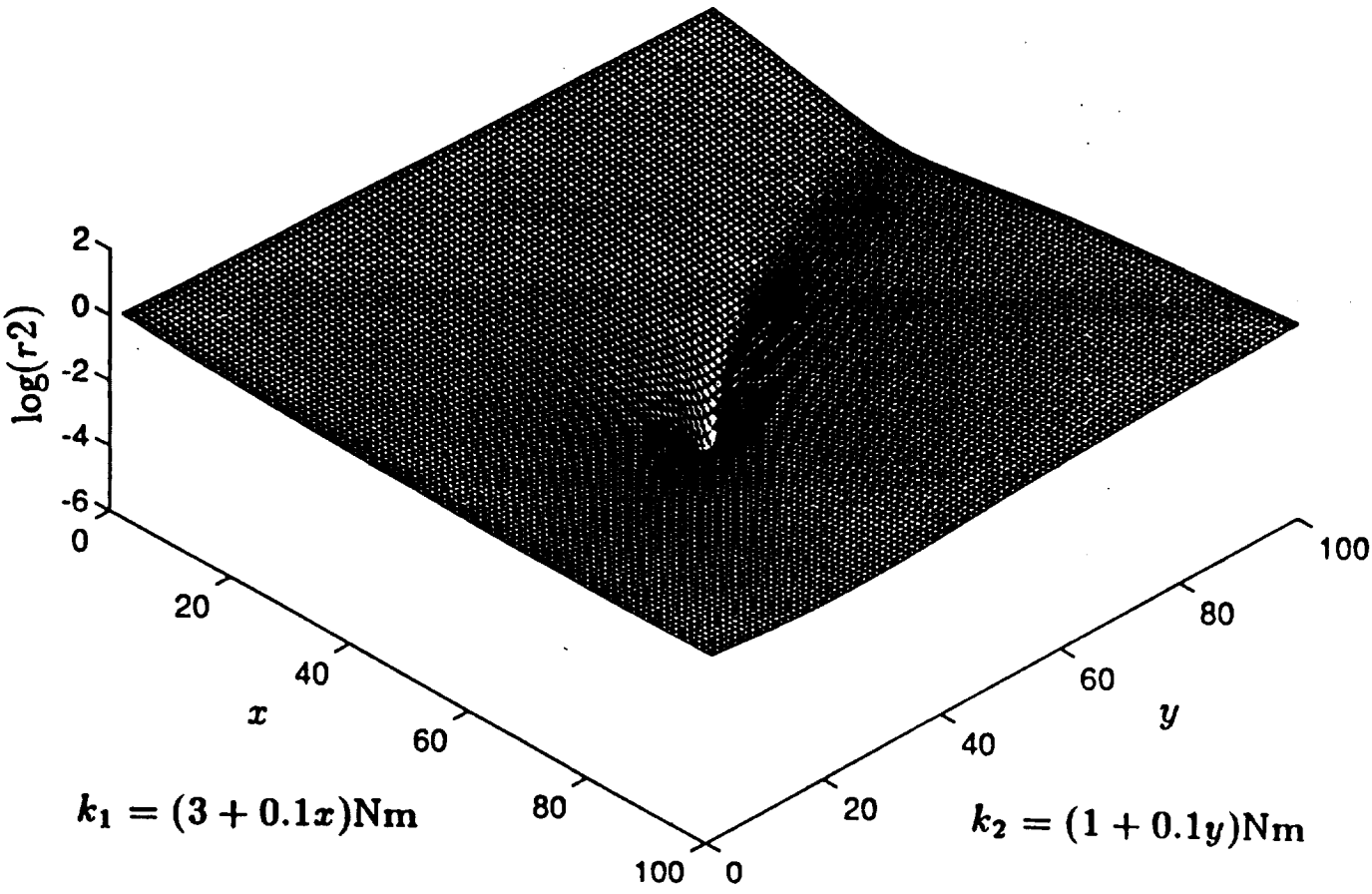


Figure (5) Surface plot of the log of $r2$, as defined in eqn(33), for k_1 ranging from 3 to 13 Nm and k_2 from 1 to 11 Nm with the actual solution being at $k_1 = 8$ Nm and $k_2 = 6$ Nm.

A sensitivity method of solution can be thought of as placing a ball on the surface shown

in figure (5) at the co-ordinates of the initial estimates for k_1 and k_2, and letting the ball roll; the ball will stop rolling when it falls into a low region (if this region is the lowest on the surface, then it has found the global minimum and, hence, the solution to the problem). The surface shown in figure (5) would seem to be ideal for such a solution process; a ball placed almost anywhere on this surface is likely to end up in the global minimum. Therefore, with such data, the solution is likely to be quite robust and relatively independent of initial conditions.

Now, we examine the same surface for the frequency response function with measurements at both degrees-of-freedom for five frequency points spaced 0.5 rad/sec apart. Here, the residuals are given by

$$r2 = \sum_{j=1}^{5} \left\{ \sum_{i=1}^{2} (x_{Ei,j} - x_{Ai,j})^2 \right\} \tag{34}$$

The plot of this surface over the same range of k's as that shown for the modal residuals, is presented in figure (6). Clearly, this system will not be as insensitive to initial conditions as the case for modal analysis. The situation is even worse if we take measurements at more frequencies c.f. figure (7) where data was simulated as being collected at seven frequencies spaced 0.4 rad/sec apart. It must be remembered that the example shown here is for a two degree-of-freedom system; for higher order models, the situation would be much worse. These results show that for frequency response function data, sensitivity methods will not be robust with respect to initial estimates; this is unfortunate because such methods can be particulary efficient in solving non-linear problems. For the types of surfaces shown in figures (6) & (7), other non-linear approaches such as random walks or genetic algorithms (Larson and Zimmerman (1993)) would have to be applied; such methods, for a significant number of unknowns, can become unwieldy. Ren and Beards (1993) found similar problems with convergence and concluded that frequency selection and the imposition of weightings is of the utmost importance in attempting to gain convergence, it is also suggested that random selection of frequencies can aid convergence; this suggests that sensitivity approaches lose much of their appeal and the techniques suggested in Ren and Beards (1993) are not far removed from approaches such as random walks. Link and Zhang (1992), working with incomplete experimental data, found that a non-linear

technique based on frequency response functions was much more sensitive to measurement noise than was a method based on measured modal parameters.

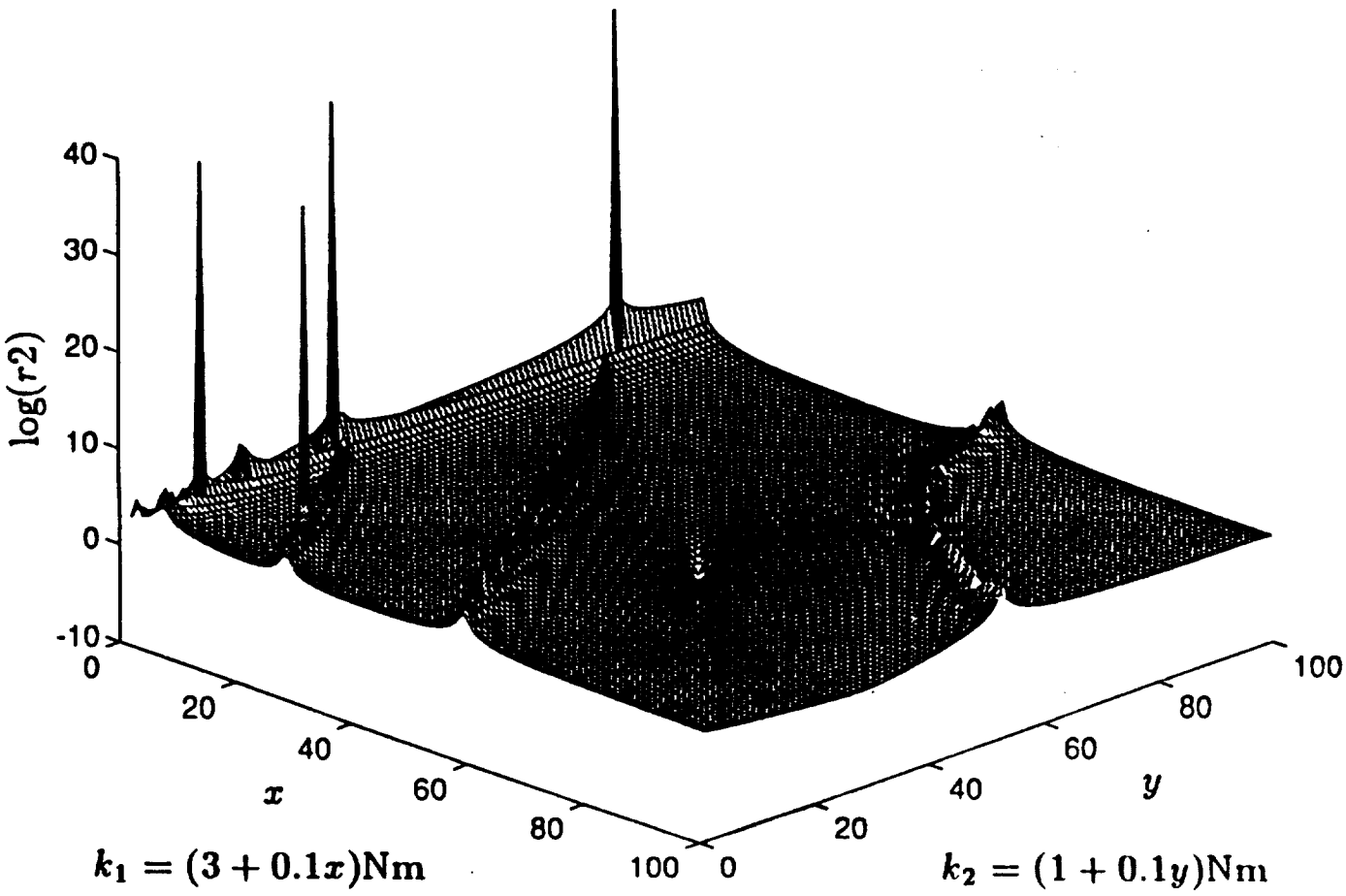


Figure (6) Surface plot of the log of $r2$, as defined in Eq.(34), for 5 frequency points spaced 0.5 rad/sec apart and for k_1 ranging from 3 to 13 Nm and k_2 from 1 to 11 Nm with the actual solution being at $k_1 = 8$ Nm and $k_2 = 6$ Nm (note: the large spikes arise from a measurement point in the simulation being almost exactly on a resonance)

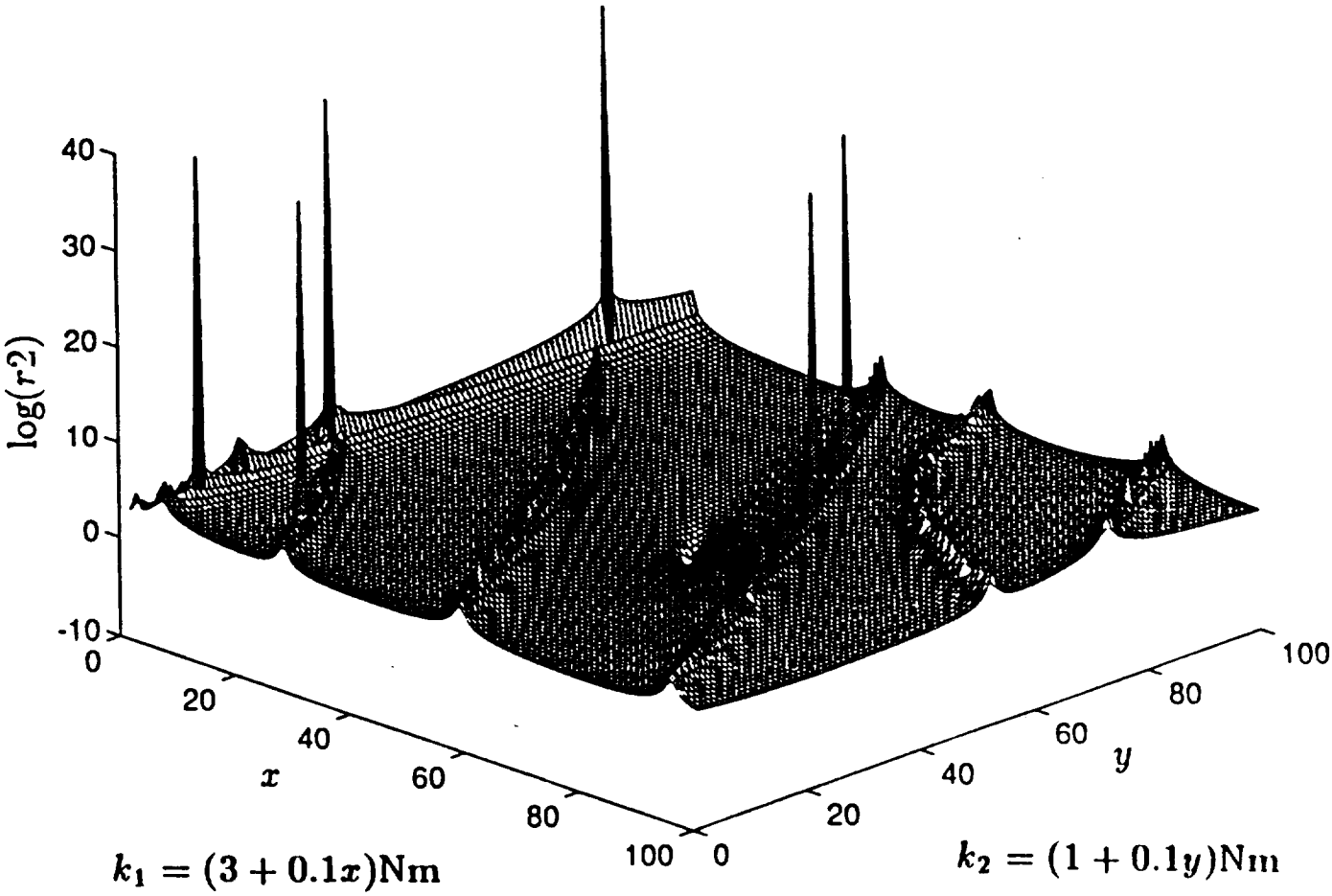


Figure (7) Surface plot of the log of $r2$, as defined in Eq.(34), for 7 frequency points spaced 0.4 rad/sec apart and for k_1 ranging from 3 to 13 Nm and k_2 from 1 to 11 Nm with the actual solution being at $k_1 = 8$ Nm and $k_2 = 6$ Nm (note: the large spikes arise from a measurement point in the simulation being almost exactly on a resonance)

These results suggest that non-linear techniques will not be very successful when frequency response function data is used; for this reason, it is probably best to further explore the possibilities of linear methods of solution when using such data. The reason for attempting non-linear techniques is because the measured degrees-of-freedom are incomplete with respect to the analytical model, in order to attempt a linear solution, either the experimental data must be expanded to encompass all of the analytic degrees-of-freedom, or, the analytic model reduced to the measured degrees-of-freedom.

Model Reduction Techniques

The aim of any model reduction process is to, as closely as possible, represent the behaviour of the more complete model in the reduced model. With fewer degrees-of-freedom in the reduced model, there will naturally be fewer natural frequencies, but the aim is that these represented frequencies be as close as possible to the corresponding frequencies in the full model. There are four commonly used techniques used to reduce an analytic model to the measured degrees-of-freedom; these are:

i. Guyan Reduction (Guyan, 1965)

ii. Improved Reduction System (IRS) (O'Callahan, 1989)

iii. Modal (Kammer, 1987)

iv. Hybrid (Kammer, 1987)

The most commonly used (and oldest) process is Guyan reduction. For such a reduction process, the stiffness matrix is re-ordered and partitioned such that the desired degrees-of-freedom (in this case, those degrees-of-freedom where measurements are carried out) are partitioned from the omitted degrees-of-freedom, as follows for the stiffness matrix:

$$[K]\{x\} = \begin{bmatrix} K_{aa} & K_{ao} \\ K_{ao}^T & K_{oo} \end{bmatrix} \begin{Bmatrix} x_a \\ x_o \end{Bmatrix} \tag{35}$$

The reduction transformation can be written as

$$\begin{Bmatrix} x_a \\ x_o \end{Bmatrix} = [T]\{x_a\} \tag{36}$$

For Guyan reduction, the transformation matrix is given by

$$[T]_{\mathrm{G}} = \begin{bmatrix} I \\ -K_{oo}K_{ao}^T \end{bmatrix} \tag{37}$$

(Note: Guyan reduction is also known as static condensation because the transformation matrix $[T]_{\mathrm{G}}$ is a function only of the stiffness matrix). Applying this transformation to the stiffness matrix gives $[K_{\mathrm{G}}]$, the Guyan reduced stiffness matrix as

$$[K_G] = [K_{aa}] - [K_{ao}][K_{oo}]^{-1}[K_{ao}]^T \tag{38}$$

Applying Guyan reduction to the mass matrix, where the mass matrix is partitioned as

$$[M] = \begin{bmatrix} M_{aa} & M_{ao} \\ M_{ao}^T & M_{oo} \end{bmatrix} \tag{39}$$

gives

$$\begin{aligned}[M_{\mathrm{G}}] = [M_{aa}] - [K_{ao}][K_{oo}]^{-1}[M_{ao}]^T - [M_{ao}][K_{oo}]^{-1}[K_{ao}]^T + \\ + [K_{ao}]\left[K_{oo}^{-1}\right][M_{oo}][K_{oo}]^{-1}[K_{ao}]^T\end{aligned} \tag{40}$$

Guyan reduction is based on the assumption that there are no forces on the omitted degrees-of-freedom. When the situation involves dynamics, forces in the form of inertias

will arise at the omitted degrees-of-freedom if there is any inertia associated with these degrees-of-freedom. As can be seen in Eq.(40), if the omitted degrees-of-freedom have no mass, then the reduced matrix is exact.

O'Callahan (1989) developed the Improved Reduced System (IRS) to include terms which approximate the effects of the removed masses. The IRS transformation matrix is given by

$$[T]_{\text{IRS}} = [T]_{\text{G}} + [T]_{\text{inrt}} \tag{41}$$

where the inertial transformation matrix $[T]_{\text{inrt}}$ is given by

$$[T]_{\text{inrt}} = -[K_{oo}]^{-1}\left([M_{oa}] + [M_{oo}][T]_{\text{G}}\right)[M_{aa}]^{-1}[K_{aa}] \tag{42}$$

Kammer (1987) developed both the Modal and Hybrid methods which use the full analytical mode shapes to estimate the motion of the omitted degrees-of-freedom. The transformation matrix for the modal method is given by

$$[T]_{\text{Mod}} = \phi_o\left[\phi_a^T\phi_a\right]^{-1}\phi_a^T \tag{43}$$

and that for the Hybrid method is given by

$$[T]_{\text{Hyb}} = [T]_{\text{G}} + \left([T]_{\text{Mod}} - [T]_{\text{G}}\right)[\phi]\left[\phi\right]^T[T]_{\text{Mod}}^T[M][T]_{\text{Mod}} \tag{44}$$

where $[M]$ is the full analytic mass matrix and $[\phi]$ is the full order eigenvector matrix.

In an attempt to assess both the accuracy and robustness of these reduction techniques,

Tan (1993) carried out an analysis on a 10-bay truss which had been used for substructure modal testing at NASA LaRC. This structure was initially reduced using Guyan reduction, but, given that it was a truss structure with largely evenly distributed mass, this did not provide satisfactory results (the retained degrees-of-freedom accounted for only 5% of the total mass). The accuracy of the method was defined by the ability of the reduced model to match the full FEM mode shapes and frequencies. The robustness was defined by the ability to provide reliable cross-orthogonality (analytical vs test) and self orthogonality (test vs test). The Guyan reduced system showed poor accuracy, but acceptable robustness. The overall accuracy of the IRS was a great improvement over that for the Guyan reduced results; the robustness also improved. Both the Modal and Hybrid methods gave highly accurate results, which should not be surprising given that they are effectively based on a modal synthesis of the full FEM. Both methods, however, performed considerably worse, where robustness was concerned, than either the Guyan or IRS methods. Tan (1993) makes the point that the robustness of the Modal and Hybrid methods is very dependent upon the accuracy of the original FEM which *a-priori* is an unknown and that as a result, use of such reduction methods for model verification may produce misleading results. The same reasoning applies to methods which expand the experimental data to the analytical degrees of freedom based upon the analytical model (Larsson and Sas (1992) and Nalitocla (1993)) (note: this is not necessarily the case for problems involving damage detection, as will be discussed later).

It has been demonstrated by Gordis (1992) that a condition for IRS to be accurate and robust is that there be minimum overlap between the eigenvalues of the included and omitted degrees-of-freedom, where the omitted eigenvalues are found from the system $[M_{oo}]^{-1}[K_{oo}]$. Comparing this to the requirement for Guyan reduction, it is likely that this criterion would be met by including as much of the mass as possible in the included degrees-of-freedom. In an analogous study to that of Gordis (1992), Larsson and Sas (1992), showed that, when working with frequency response functions, the useful frequency range for measurements is for frequencies below the lowest eigenvalue found when all of the retained degrees-of-freedom are grounded.

It is important to note that once a model has undergone one of these reduction processes, the system matrices become fully populated, thereby requiring that, for the stiffness matrix

for example, $\nu \times \nu$ parameters be determined in any identification process (where ν is the order of the reduced model). Lin (1993), argues that degree-of-freedom reductions should be carried out in such a way that the bandedness (hence, the physical connectivity) of the system matrices be maintained. Such a reduction may be carried out in a number of ways: the first being that standard finite-elements be used to replace a larger group of elements, effectively continuing the type of approximation made in creating the original FEM; a second method, would be to apply one of the previously described reduction processes to a group of elements creating a 'super-element', such 'super-elements' may then be combined to create an over-all reduced model. The first method will obviously have greater sparsity than the second, but the second will still be considerably more sparse than the fully reduced model. The appeal of this, is that there are far fewer unknown parameters to be determined and that some physical meaning may be associated with the updated parameters. Lin (1993) shows that such a reduction process is successful for the fairly simple case of a 102 degree-of-freedom beam model reduced to a 52 degree-of-freedom model by examining the first 10 modes. This is clearly not a very rigorous test. It would seem that considerable analysis would have to be carried out on any structure in question before an assessment could be made of how successful such techniques would be.

Improving Experimental Data

As has already been mentioned, one of the largest problems when working with modal experimental data, is the scarcity of data. This can be partly overcome by collecting data with perturbed boundary conditions (Barney *et al* (1992), Li and Brown (1992) and Lammens *et al*(1993)). For perturbed boundary condition testing, data is collected for different configurations of essentially the same structure; each configuration has either known mass or stiffness changes made to the structure. Each set of experimental data will, therefore, be different, but the number of unknowns will remain the same. As frequency response function data at adjacent spectral lines will not be very linearly independent, perturbed boundary condition data where the changes to the structure do not result in sufficiently significant changes to the measured modes will not give very independent data, thereby giving a solution that is not as robust as one where the data were independent. For

such testing, pre-test finite-element analysis would be of the utmost value when determining optimum structural changes.

Another possible means of improving data collection is to use a full-field optical technique to perform measurements. Recent availability of tools for full-field laser-doppler imaging has made this a possibility (Oliver (1988) and Sriram *et al* (1991)). Such devices scan over the structure being tested and give an area plot of the normal velocities. The velocity data is typically correlated such that only the velocities at the frequency of loading are displayed. This would present the problem of too much data, in that there would be likely to be experimental data for more points than there are degrees-of-freedom in the model. Thereby, offering the choice of using only the data at the points of interest, or, using interpolation routines to smooth the data prior to analysis. There is likely to be sufficient data such that the potential for bias by a well chosen interpolation routine would be small. Another significant advantage to such a technique is that it uses only one sensor, so different calibrations are not an issue and the likelihood of sensor malfunction during the course of testing is greatly reduced. Also, being non-contacting, it is suitable for very light structures where the placement of sensors may significantly alter the experimental results. Such a technique could be potentially very useful for orbiting structures.

Damage Detection

The potential for dynamic data to detect and quantify structural damage has been a driving force for much of the FEM updating work. Most of the work in this field has been carried out in the field of space structures (eg. Kashangaki *et al* (1992) and Kaouk and Zimmerman (1993)). For such structures, damage detection would be of immense importance in that dynamic measurements could, potentially, be used to locate and quantify damage before a hazardous and expensive repair process is instituted.

The problem of damage detection is, in many ways, the same as the model updating problem; however, there are some important differences. When examining the model updating problem for a truss structure, assumptions may be made about identical elements,

thereby greatly reducing the number of unknowns to be found. For the damage detection case, the properties of each element must be treated as being unknown. It has been said previously that experimental data expansion using the analytic model is dangerous because the assumptions made in the modelling can greatly bias the results; when working on a damage detection problem, this may not be the case. Before carrying out a damage detection analysis, it is assumed that a 'valid' finite-element model exists. The aim of the damage detection process is, essentially, to determine where differences from this original model occur. Given that such an original model must be assumed to be 'valid', data expansion based on this model should be acceptable.

An important consideration for damage detection in truss structures has been highlighted by Doebling *et al* (1993). It was found that a damage detection algorithm which worked satisfactorily for the case of a cantilevered truss, had much less success in identifying damage in a free-free truss. The problem seems to stem from the increased significance of local modes in the lower modal frequencies for the free-free case as opposed to the cantilevered case due to the effects of localized masses in the structure examined. Earlier studies with cantilevered structures by Kashangaki *et al* (1992) and Smith and McGowan (1989) have been able to consider the changes brought about by damage to the global modes as 'frequency shifts'; for the free-free case, however, the changes to the local modes cannot be viewed as a 'shift' in the sense that the effect on the local modes is large. It has been mentioned before that the damage detection case involves looking for change from a previously validated model and that that model may be used for experimental data expansion purposes; when the damage is too much, from the perspective of the change in the dynamic behaviour, then such methods will not be valid. It would implicitly be possible to show that damage has occurred, but to identify that damage would require that the model identification process be started from scratch. This would be a very difficult identification if there was no *a-priori* knowledge about the location of the damage.

Summary

It has been demonstrated how the number of unknowns in a model updating process can be reduced to a minimum if the actual finite-element properties, rather than the mass or stiffness matrices as a whole, are treated as unknowns. This implicitly accepts the assumptions made in the finite-element process, but should allow the updated model to be modified with greater confidence in the results. It should also indicate where errors are being made in the initial modelling process. For example, if the updated model still does not adequately reflect the behaviour of the true structure, it may indicate insufficient finite-element density in an important region.

The issue of uniqueness for non-linear solution methods involving modal data has been investigated. By example, it has been demonstrated that the criteria for uniquenéss given by a number of previous workers is incorrect. A numerical simulation of the non-linear solution process with randomly generated initial conditions, suggests that, for the case studied, the criteria for uniqueness are that $mn \geq L$ and $n > 1$ (where m is the number of measured modes, n is the number of measured degrees-of-freedom and L is the number of unknown parameters). It must be noted that this is not intended to be a rigorous proof and was simply an exercise in attempting to determine where the truth may lie.

For linear problems, the amount of data offered by frequency response function data should make the use of such data superior to modal data for model updating. The appeal of the amount of data offered by frequency response functions, however, is greatly diminished if a non-linear method of solution is required. It is suggested that, rather than attempt such a non-linear solution, either the modal properties be extracted and a non-linear solution be then attempted with these new data, or that the model be reduced to the number of measured degrees-of-freedom. The availability of new measurement technologies (such as laser-doppler vibrometry) should allow the collection of data at more locations than has previously been routinely available. This opens the possibility for measurements to be carried out at all degrees-of-freedom represented in the finite-element model (rotational degrees-of-freedom would have to be inferred from translational measurements). With complete measurements, direct, linear, solution techniques could be used to create the

physical parameters of a finite-element model given the geometry. Measurements for a range of frequencies should give considerable robustness to the solution. New measurement technologies would seem to offer a very powerful tool in the finite-element updating problem.

On the subject of limited modal data: literature has been reviewed concerning how the amount of such data can be increased by testing a structure in different configurations, where the difference between each configuration is accurately known.

Model updating for the purposes of damage detection in truss structures has been, very briefly, discussed.

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Appendix: Reducing the number of unknowns for a 12 dof beam element

$$
\begin{bmatrix}
\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI_y}{l^3} & 0 & 0 & 0 & \frac{6EI_y}{l^2} & 0 & -\frac{12EI_y}{l^3} & 0 & 0 & 0 & \frac{6EI_y}{l^2} \\
0 & 0 & \frac{12EI_z}{l^3} & 0 & -\frac{6EI_z}{l^2} & 0 & 0 & 0 & -\frac{12EI_z}{l^3} & 0 & -\frac{6EI_z}{l^2} & 0 \\
0 & 0 & 0 & \frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{GJ}{l} & 0 & 0 \\
0 & 0 & -\frac{6EI_z}{l^2} & 0 & \frac{4EI_z}{l} & 0 & 0 & 0 & \frac{6EI_z}{l^2} & 0 & \frac{2EI_z}{l} & 0 \\
0 & \frac{6EI_y}{l^2} & 0 & 0 & 0 & \frac{4EI_y}{l} & 0 & -\frac{6EI_y}{l^2} & 0 & 0 & 0 & \frac{2EI_y}{l} \\
-\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{l} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{12EI_y}{l^3} & 0 & 0 & 0 & -\frac{6EI_y}{l^2} & 0 & \frac{12EI_y}{l^3} & 0 & 0 & 0 & -\frac{6EI_y}{l^2} \\
0 & 0 & -\frac{12EI_z}{l^3} & 0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{12EI_z}{l^3} & 0 & \frac{6EI_z}{l^2} & 0 \\
0 & 0 & 0 & -\frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{l} & 0 & 0 \\
0 & 0 & -\frac{6EI_z}{l^2} & 0 & \frac{2EI_z}{l} & 0 & 0 & 0 & \frac{6EI_z}{l^2} & 0 & \frac{4EI_z}{l} & 0 \\
0 & \frac{6EI_y}{l^2} & 0 & 0 & 0 & \frac{2EI_y}{l} & 0 & -\frac{6EI_y}{l^2} & 0 & 0 & 0 & \frac{4EI_y}{l}
\end{bmatrix}
\begin{Bmatrix}
x_1 \\ y_1 \\ z_1 \\ \theta_1 \\ \phi_1 \\ \psi_1 \\ x_2 \\ y_2 \\ z_2 \\ \theta_2 \\ \phi_2 \\ \psi_2
\end{Bmatrix}
$$

This is the stiffness matrix for a beam element with six degrees-of-freedom at each end. If the stiffness matrix as a whole were to be treated as being unknown, there would be 144 unknown parameters. If the matrix was treated as being symmetric, there would be 78 unknown parameters. If symmetry and the bandedness of the matrix were maintained, there would be 26 unknown parameters.

$$\begin{bmatrix} x_1 - x_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & y_1 - y_2 & \psi_1 + \psi_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & z_1 - z_2 & -\phi_1 - \phi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_1 - \theta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -z_1 + z_2 & 0 & 2\phi_1 + \phi_2 & 0 \\ 0 & 0 & y_1 - y_2 & 0 & 0 & 0 & 0 & 2\psi_1 + \psi_2 \\ -x_1 + x_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -y_1 + y_2 & -\psi_1 - \psi_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -z_1 + z_2 & \phi_1 + \phi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\theta_1 + \theta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -z_1 + z_2 & 0 & \phi_1 + 2\phi_2 & 0 \\ 0 & 0 & y_1 - y_2 & 0 & 0 & 0 & 0 & \psi_1 + 2\psi_2 \end{bmatrix} \begin{Bmatrix} EA/l \\ 12EI_y/l^3 \\ 6EI_y/l^2 \\ 12EI_z/l^3 \\ 6EI_z/l^2 \\ GJ/l \\ 2EI_z/l \\ 2EI_y/l \end{Bmatrix}$$

Above is a rearrangement of the previous matrix with the unknown physical properties placed in the vector. As can be seen, there are now only 8 unknown parameters.

$$\begin{bmatrix} x_1 - x_2 & 0 & 0 & 0 & 0 \\ 0 & y_1 - y_2 & \psi_1 + \psi_2 & 0 & 0 \\ 0 & z_1 - z_2 & -\phi_1 - \phi_2 & 0 & 0 \\ 0 & 0 & 0 & \theta_1 - \theta_2 & 0 \\ 0 & 0 & -z_1 + z_2 & 0 & 2\phi_1 + \phi_2 \\ 0 & 0 & y_1 - y_2 & 0 & 2\psi_1 + \psi_2 \\ -x_1 - x_2 & 0 & 0 & 0 & 0 \\ 0 & -y_1 + y_2 & -\psi_1 - \psi_2 & 0 & 0 \\ 0 & -z_1 + z_2 & \phi_1 + \phi_2 & 0 & 0 \\ 0 & 0 & 0 & -\theta_1 + \theta_2 & 0 \\ 0 & 0 & -z_1 + z_2 & 0 & \phi_1 + 2\phi_2 \\ 0 & 0 & y_1 - y_2 & 0 & \psi_1 + 2\psi_2 \end{bmatrix} \begin{Bmatrix} EA/l \\ 12EI/l^3 \\ 6EI/l^2 \\ GJ/l \\ 2EI/l \end{Bmatrix}$$

Here is a further reduction of the previous case where it is assumed that $I_y = I_z$, as would be the case for a beam that had a square or round cross section. If this were the case, there would be only 5 unknown parameters. The purpose of this exercise has been to show how, if we are willing to accept the fundamental assumptions made in finite-element modelling, we can use our knowledge of the stiffness matrix structure and the physical structure of the actual element, to reduce the number of unknowns to a minimum.

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Some issues concerning the updating of dynamic finite-element models by incorporation of experimental data are examined here. It is demonstrated how the number of unknowns can be greatly reduced if the physical nature of the model is maintained. The issue of uniqueness is also examined and it is shown that a number of previous workers have been mistaken in their attempts to define both sufficient and necessary measurement requirements for the updating problem to be solved uniquely. The relative merits of modal and frequency-response-function data are discussed and it is shown that for measurements at fewer degrees-of-freedom than are present in the model, frf data will be unlikely to converge easily to a solution. It is then examined how such problems may become more tractable by using new experimental techniques which would allow measurements at all degrees-of-freedom present in the mathematical model.

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