# A Method for Thermal Analysis of Spiral Bevel Gears

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# A Method for Thermal Analysis of Spiral Bevel Gears

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**SYNOPSIS** A modelling method for analyzing the three-dimensional thermal behavior of spiral bevel gears has been developed. The model surfaces are generated through application of differential geometry to the manufacturing process for face-milled spiral bevel gears. Contact on the gear surface is found by combining tooth contact analysis with three-dimensional Hertzian theory. The tooth contact analysis provides the principle curvatures and orientations of the two surfaces. This information is then used directly in the Hertzian analysis to find the contact size and maximum pressure. Heat generation during meshing is determined as a function of the applied load, sliding velocity, and coefficient of friction. Each of these factors change as the point of contact changes during meshing. A nonlinear finite element program was used to conduct the heat transfer analysis. This program permitted the time- and position-varying boundary conditions, found in operation, to be applied to a one-tooth model. An example model and analytical results are presented.

## 1 INTRODUCTION

During the design process for spiral bevel gears the most important initial calculations are for the bending and contact stress expected at the design conditions. Typically the process requires many iterations whereby a satisfactory gear configuration emerges. The thermal behavior may be neglected or not even considered if the operational conditions do not warrant its consideration.

In aviation applications, however, the thermal behavior is usually important. These gear meshes can operate at sufficiently high speed and load such that temperatures and scoring can be of concern. A typical aviation spiral bevel design application [1] is shown in the main rotor transmission depicted in Figure (1).

Operational conditions of a given design and the likelihood of a particular failure mode [2,3] are shown in Figure (2). Depending on the operating conditions imposed a region of safe operation can be found for a design. However, if the gear system is loaded sufficiently high or if the operational speed is increased the region of thermal concern can be reached.

The basic understanding of the heat generating mechanism, with respect to gears, began with the early analytical and experimental work of References [4,5]. The first thermal analysis of spiral bevel and hypoid gears was contained in the pioneering work of Reference [6]. The procedures of References [6,7] are still in use today and are very useful in evaluating several design candidates in a quick manner.

The finite element analysis technique has the capability to model extremely complex structures and boundary conditions and has gained popularity in recent years. The first finite-element-based thermal study of spiral bevel gears was developed by Reference [8]. An influence coefficient methodology was used to determine tooth stiffness and bulk temperature. The resulting formulation was used to predict dynamic load and gear temperature. The results found were consistent with those of References [6,7].

The modelling procedure to be described herein is based on combining gear design/gear geometry, 3-D Hertzian contact theory, and operating conditions with a finite element analysis procedure [9]. Therefore the objective of this report is to describe the procedure necessary to build the model, configure the boundary conditions, and conduct the analysis to assess the thermal behavior of spiral bevel gears.

## 2 SPIRAL BEVEL GEAR SURFACE DETERMINATION

The spiral bevel gear manufacturing process that will be briefly described in this section is that of the face-milled type. The main components used in this type of manufacture, shown in Figure (3), are the head cutter, the cradle, and the workpiece.

The head cutter contains either cutting blades or a grinding wheel that have a conical shape. The conical shape describes the so-called blade angle. The rotational speed of the head cutter is controlled to produce the most efficient metal removal and is not related to the produced gear geometry.

The head cutter is mounted on the cradle, at a distance from the cradle center and orbits this center during

manufacture. Location of the cutter axis with respect to the cradle axis depends on the cutter diameter used and on the required mean spiral angle of the workpiece.

The cradle and workpiece are rotated in a predetermined relationship called the ratio of roll. In older machine tools this motion was produced through index gears, cams, and shafts that mechanically linked the cradle and workpiece. Machines produced today use computer numerical controlled motion that duplicates the generation motion of the older machine tools.

The numerical simulation of the manufacturing process uses the procedure developed in References [10-13]. In these references the gear tooth surfaces are described using a differential geometry approach. Homogeneous coordinates are used and permit the coordinate transformations to be performed for rotations and translations simultaneously. An "Equation of Meshing" is used to relate the machine settings and cutter geometry to the surface being generated. This equation numerically states that the normal vector of the generating surface must be perpendicular to the relative velocity between cutter and the gear tooth surface at the point in question (the scalar product between these vectors is zero).

Taking the procedure one step further permits the description of the surface coordinates of the gear under study [14]. Using the design data of the gear along with the equation of meshing permits the calculation of tooth surface locations of interest. The objective of this process is to find the unknown parameters of the cutter and orientation of the cutter that satisfy the equation of meshing for radial and axial locations on the gear profile.

To develop the finite element model, needed for the analysis, the procedure described above is repeated for a grid of locations on the concave and convex sides of the tooth surface. The present analysis uses a grid of 10 by 10 points as shown in Figure (4).

From the rest of the gear design information a three-dimensional model can be generated with a geometric modelling package [15]. Now the model needs to have the finite element mesh and the boundary conditions applied.

## 3 TOOTH CONTACT ANALYSIS, THREE-DIMENSIONAL CONTACT ANALYSIS, AND HEAT GENERATION

Spiral bevel gears are manufactured with an intentional mismatch between surfaces. This is done so that the gear mesh can perform in a satisfactory manner even when misalignment, manufacturing errors, or high load cause the gears to deflect to locations different than what was called for in their design. While conjugate motion would provide the ideal transmission of motion, this is not practical in spiral bevel gears.

Therefore spiral bevel gears operate with a certain amount of transmission error, defined as the deviation from the intended gear ratio, to maintain the ability to perform successfully over a range of conditions. The basis of tooth contact analysis is described in Reference [16]. Many others since Reference [16] also have investigated the kinematics of spiral bevel gears [11-13,17,18].

In the study described herein, the curvatures of the meshing surfaces as the contact moves across the gear tooth surface, the velocities of the meshing surfaces, and the load based on one tooth pair carrying the entire load are also important. This information was determined using the computer program of Reference [19].

Therefore at this point the transmission error as a function of mesh position is known and is shown in Figure (5) for the example data of Table 1. The data of Table 1 is for the test specimen of the NASA Spiral Bevel Gear Rig [20]. The no-load transmission error curve is shown for three adjacent teeth in this figure. Where the curves overlap, two pairs of teeth will be assumed to be in contact; otherwise only one pair will carry the entire load. The parabolic shape of the error curve (see Figure (5)) tends to absorb the transmission errors caused by misalignment and other defects [21].

From the analysis of Reference [19], the velocities of the meshing members are known with respect to a plane perpendicular to the common normal at the point of contact. The curvatures of the surfaces at the point of contact under the load will produce an elliptical hertzian contact. The velocities of the meshing gears are described in this plane with respect to the major-minor directions of the contact ellipse. These parameters will be used to find the hertzian contact and heat generated as a function of mesh position.

The contact between the surfaces must be determined next. Using the results of Reference [19] and the load sharing assumed, as shown in Figure (5), three-dimensional contact analysis is conducted [22,23]. The curvatures found at the points of contact from the no-load tooth contact analysis are assumed to exist over both contacting bodies. Therefore at each point of contact the ellipse size and maximum contact pressure are found based on the principal curvatures, their orientation, load, and material properties of the gears.

Finally the heat generation as a function of mesh position must be found. Only the heat generated due to surface relative sliding was considered. The heat generated due to sliding is the product of the coefficient of friction, load, and sliding velocity. At this point only the friction coefficient still needs to be determined.

Friction characteristics of several lubricants, over a range of operating conditions and gear surface finishing has been reported in References [24,25]. The data of Reference [25], for a turbine engine oil such as that used in aerospace power transmission, will be utilized in this report. A typical set of results for friction as a function of sliding-to-rolling velocity ratio and three different rolling velocities is shown in Figure (6).

Therefore the combination of the assumed load sharing, the calculated velocities, coefficient of friction and heat generation are plotted in Figures (7) and (8) for fifteen contact locations, 2.5° of pinion rotation apart, as the gear tooth pair goes through mesh. This rotation amount represents the entire meshing cycle of a single tooth pair.

## 4 FINITE ELEMENT MODEL, BOUNDARY CONDITIONS, AND EXAMPLE ANALYSIS

A finite element model was created as described above for a one tooth sector of the spiral bevel pinion (for basic design data see Table 1). Eight-node isoparametric heat transfer elements were used. A total of 20,250 elements and 22,586 grid points were contained in the model. The resultant model is shown in Figure (9). A total of 874 grid points were located on the drive-side profile (concave side). These points could be exposed to the time- and position-varying heat flux that gear meshing causes.

Locations on the active profile where the contact ellipse, based on the three-dimensional contact analysis conducted, overlapped the finite element grid needed to be found. The active profile grid points were put into a radial-axial plane. Next the ellipse size and orientation were projected onto a similar plane. Then the intersection of active profile grid points contained within the ellipse were found. The result of ellipse #10 of the 15 used in this example for the analysis conducted herein is shown in Figure (10). Shown are the major-minor ellipse dimensions and the resultant grid points contained within the ellipse. This procedure is repeated for each ellipse. In this manner the time-varying flux locations are determined.

Next the heat flux was distributed to the grid points contained within a particular ellipse. In the present analysis the heat flux was distributed based on the distance from the center of the contact. A fairly flat elliptical shape was chosen and the heat flux was then distributed.

Heat transfer coefficients for the model were determined using applicable data for some classic geometries from heat transfer texts. A listing of the heat transfer coefficients for the various surfaces of the finite element model is given in Table 2.

The implementation of the boundary conditions was controlled using subroutines appended to the bulk data of the finite element model [26]. The analysis uses two sets of subroutines. First, the time-averaged heat flux, which is the heat flux totaled at each active profile grid point, averaged over an entire revolution, is used. This is done to reduce the computation time necessary for the solution to reach a predetermined elapsed time. Then the subroutine is substituted with the time- and position-varying boundary conditions.

The example temperature field results are shown in Figures (11) and (12). In Figure (11) a single grid point on the active profile is followed for several revolutions after the subroutines were switched from the time-averaged to the time- and position-varying boundary conditions. The grid point temperature fluctuates between the coolest just before entering mesh to the highest as the contact ellipse moves over the grid point. For the grid point chosen a temperature fluctuation of 35°C is predicted.

Another way to look at the results is through looking at the three-dimensional temperature field on the model at various times during the solution. An example of the temperature field at the 13th of 15 contact ellipses is shown in Figure (12). The temperature field of the model ranges from 75°C far away from the contact to approximately 200°C on the drive-side profile where contact occurs.

## 5 CONCLUSIONS

A method for the determination of the thermal behavior of spiral bevel gears has been presented. The method can be used to determine the steady state behavior as well as the time- and position-varying temperature field. The finite element method was used to model and analyze the thermal behavior. A one tooth sector model of a spiral bevel pinion was used as an example of the technique. The results of the example analysis demonstrated that locations on the active profile can experience temperature "flashes" of over 40°C. Locations not on the active profile tended to be unaffected by the moving heat flux.

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| Number of teeth pinion/gear | 12/36 |
|---|---|
| Diametrial pitch | 5.141 |
| Mean spiral angle, deg. | 35 |
| Mean cone distance, mm (in.) | 81.1(3.191) |
| Face width, mm (in.) | 25.4(1.0) |
| Nominal pressure angle, deg. | 22.5 |
| Shaft angle, deg. | 90 |
| Pinion rotational speed, rpm | 14400 |
| Pinion torque, N·m (in·lb) | 356(3150) |
| Oil inlet temperature, deg. C (deg. F) | 38(100) |

Table 1 - Gear design and operating conditions for the example analysis.

| Surface | h (W/(m²·°C)) |
|---|---|
| Drive side of tooth | 4450 |
| Coast side of tooth | 50 |
| Tooth top land | 50 |
| Tooth toe | 160 |
| Tooth heel | 160 |
| Axial sections of gear | 0 |
| Gear body inside diameter | 0 |

Table 2 - Heat transfer coefficients used for the example analysis.
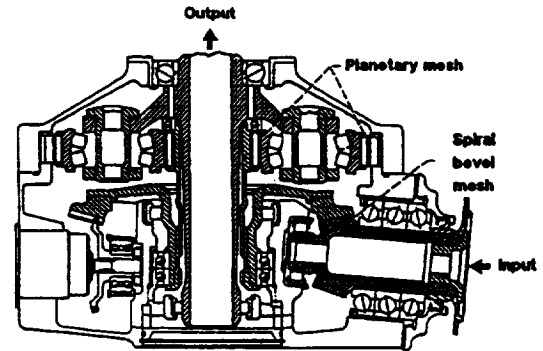


Figure 1 - OH-58A helicopter main rotor transmission.
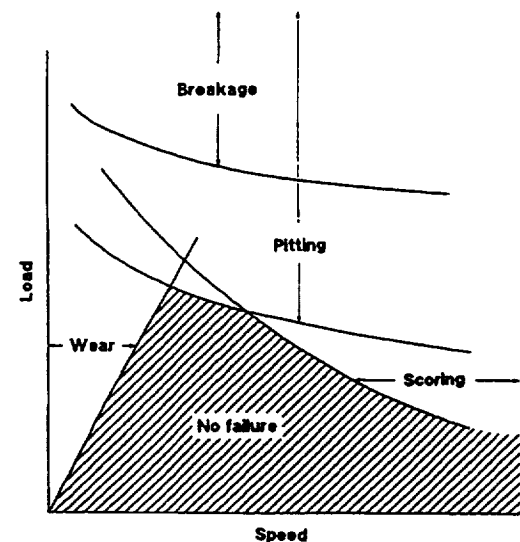


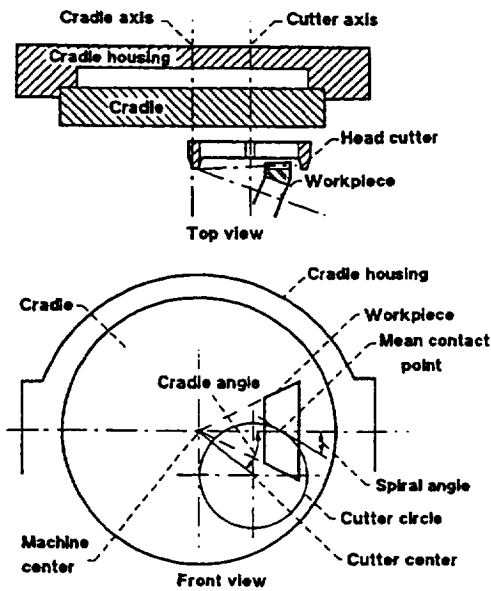Figure 2 - Gear design parameters of concern based on operating conditions.

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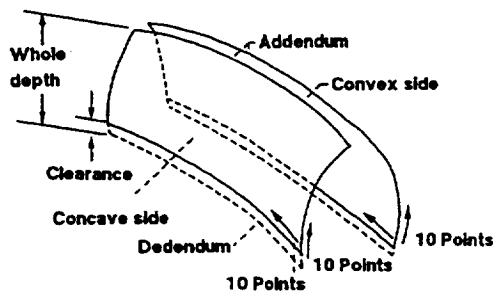Figure 3 - Orientation of workpiece to generation machinery.



Figure 4 - Calculation points (10 by 10 grids, i.e., 100 points each side) for concave and convex sides of tooth surface.
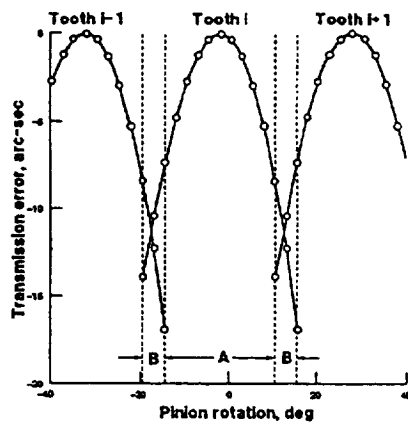


Figure 5 - Transmission error of output gear rotation as a function of pinion rotation angle (region A - 1 tooth pair in contact; B - 2 pairs in contact).
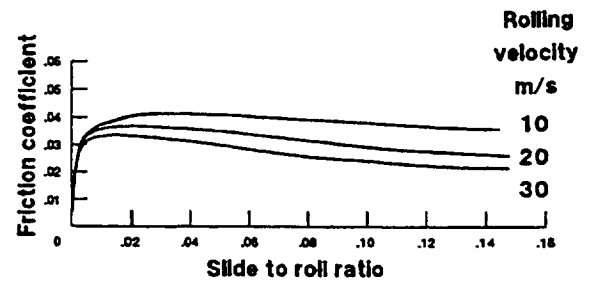


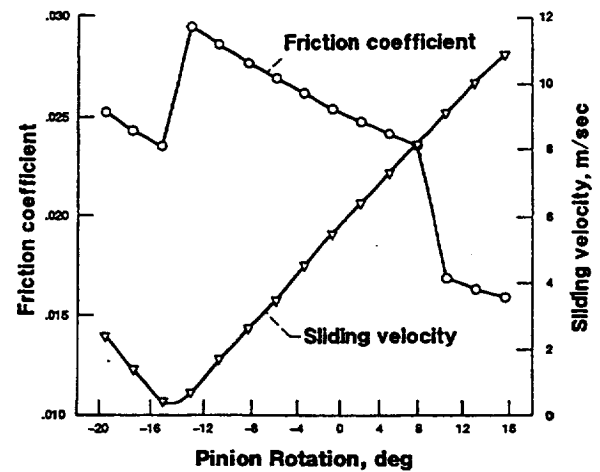Figure 6 - Typical friction data as taken from Ref.[24] for a turbine engine lubricant at 37°C and a maximum contact pressure of 1.92 GPa.



Figure 7 - Friction coefficient and sliding velocity as a function of pinion rotation angle.
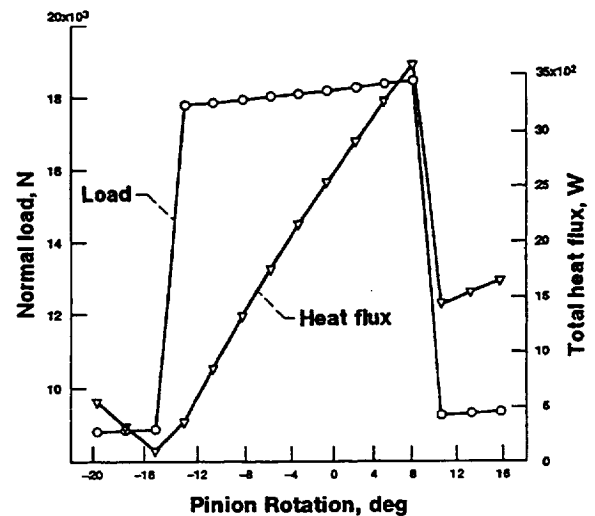


Figure 8 - Load and heat flux as a function of pinion rotation angle.
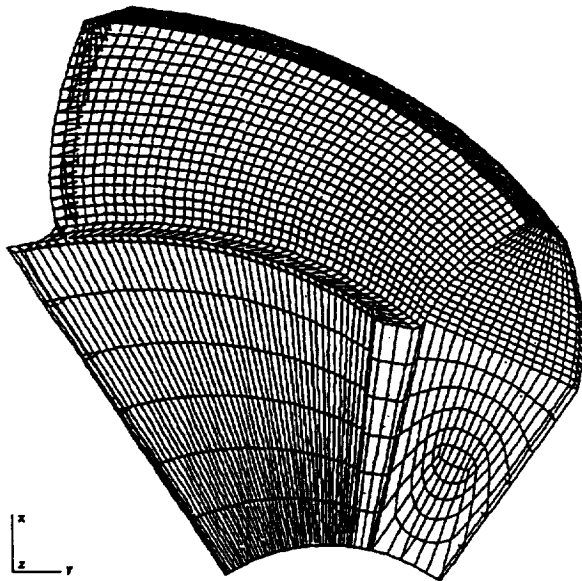
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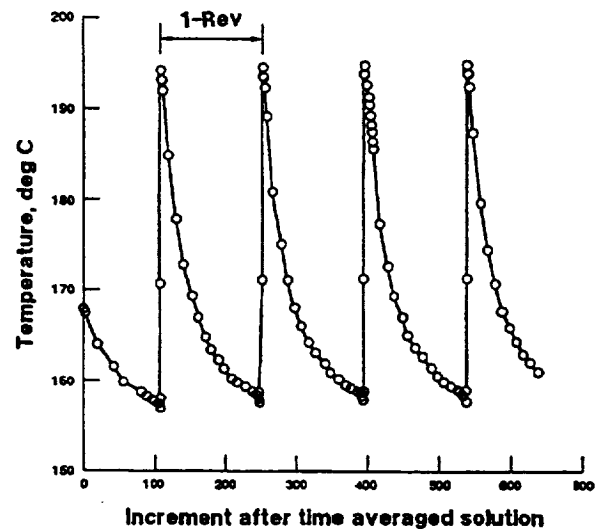Figure 9 - Finite element model used for heat transfer study.



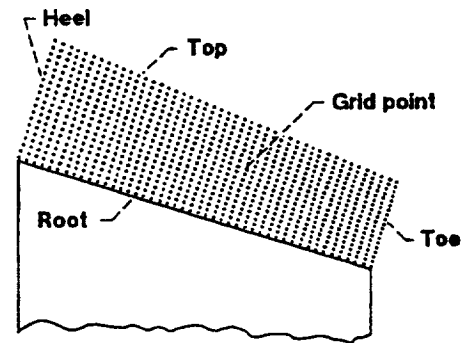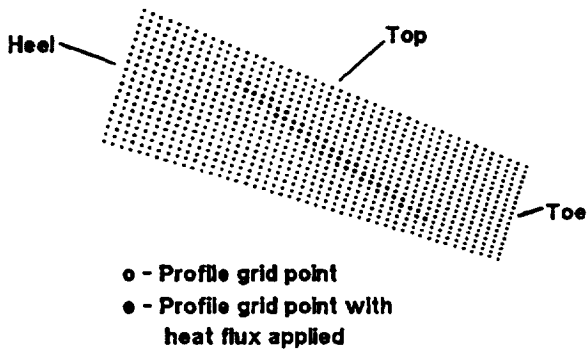Figure 11 - Active profile grid point location and temperature transient.



o - Profile grid point

● - Profile grid point with
    heat flux applied

Figure 10 - Contact ellipse number 9 of 15, shown plotted on the entire concave profile grid.



☐ 175 - 200
■ 150 - 175
▨ 125 - 150
▨ 100 - 125
▥ 75 - 100
▨ 50 - 75
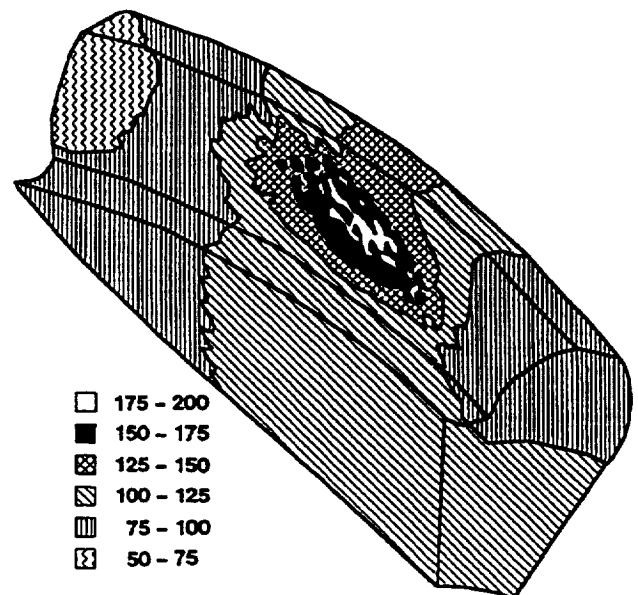
Figure 12 - Temperature field for increment number thirteen heat flux using finite element model (°C).

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**13. ABSTRACT (Maximum 200 words)**

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