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FRACTAL CHARACTERISTICS OF OZONOMETRIC NETWORK

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ABSTRACT

The fractal (correlation) dimensions are calculated which characterize the distribution of stations in the ground-based total ozone measuring network and the distribution of nodes in a latitude-longitude grid. The dimension of the ground-based ozonometric network equals 1.67±0.1 with an appropriate scaling in the 60 - 400 km range. For the latitude-longitude grid two scaling regimes are revealed. One regime, with the dimension somewhat greater than 1, is peculiar to smaller scales and limited from a larger scale by the latitudinal resolution of the grid. Another scaling regime, with the dimension equaled 1.84, ranges up to 15,000 km scale. The fact that the dimension of a measuring network is less than 2, possesses problems in observing sparse phenomena. This has to have important consequences for ozone statistics.

1. INTRODUCTION

A geometrical set of points can be characterized by a dimension. There are different methods for estimating the dimension (Eckman and Ruelle, 1985). For example, the so-called correlation dimension, ν, can be calculated through:

$$\nu = \lim_{r \to 0} \frac{\langle \ln n(r) \rangle}{\ln r}, \qquad (1)$$

where $n(r)$ is the number of points within the r-neighborhood of each point, r is the size (e.g. diameter) of the neighborhood, the angle brackets denote averaging over all points of a set. The existence of ν implies the existence of scaling (at small r):

$$\langle n(r) \rangle \quad r^{\nu}. \qquad (2)$$

If the dimension is fractional it is called the fractal dimension. In practice, r and the number of points in a set are limited, and dimension ν is determined as an angle coefficient in graph of $\ln \langle n(r) \rangle$ versus $\ln r$, and scaling (1) can exist at finite intervals of r.

If D is the dimension of a space including a point set, the density of a point set in this space is proportional to $r^{\nu}/r^{D} = r^{-c}$. Here $c=D-\nu$ is the co-dimension of a point set (Lovejoy and Schertzer, 1990). If $c=0$, the density of a point set does not depend (on the average) upon the scale of r (in the scale range where (2) holds). If $c>0$, the density of points decreases with a scale, i.e. points are concentrated on diminishing relative part of a space. Hence, the fractal dimension of a point set is a measure of sparseness of a set (Lovejoy and Schertzer, 1990).

2. FRACTAL STRUCTURE OF THE GROUND-BASED OZONOMETRIC NETWORK

Any ground-based measuring network is mainly distributed on continents, concentrating in densely populated regions. Hence, its surface distribution is highly inhomogeneous and has to have a fractal dimension. So, the global meteorological network has the fractal (correlation) dimension equaled 1.75 (Lovejoy et al, 1985; Lovejoy and Schertzer, 1990). The ground-based total ozone measuring network includes more than a hundred stations. Fig.1 shows the location of the 137 stations which worked during the last decade. Calculating the fractal dimension of the network, one should take into account the sphericity of the Earth's surface. Let r be the radius of the covering sphere, R be the radius of the Earth (Fig.2). Then $r = 2R \sin(\theta/2)$.

Fig.3 shows the graph of $\langle n(L) \rangle$ versus $L=2r$. In the 60-400 km range the scaling (2) holds with $\nu=1.67\pm0.1$. The

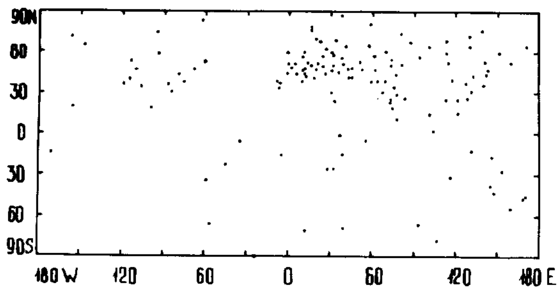

Fig.1. The location of 137 stations in the ground-based total ozone measuring network.

function $L^{1.67}$ is shown in Fig.3 by the straight line. Following Lovejoy et al (1986) one can estimate the greatest possible scale range, where the found scaling holds. Proceeding from the limited number of stations, one finds the minimum value, $\langle n(L)\rangle_{min} \sim 1/137 \approx 7.3\cdot10^{-3}$, and the maximum value, $\langle n(L)\rangle_{max} \sim 137/2 \approx 69$. The assumption that the scaling of $L^{1.67}$ holds in this range, gives $L_{min} \sim 50$ km and $L_{max} \sim$ 10,000 km. Fig.3 shows that the lower limit of the scaling of the $L^{1.67}$ function coincides approximately L_{min}. However, this scaling is broken at L>400 km. Perhaps, there is another scaling at L>1000 km, but an appropriate reliable ν value cannot be determined because of insufficient number of stations and saturation of the $\langle n(L)\rangle$ function at large scales.

3. FRACTAL CHARACTERISTICS OF A GLOBAL LATITUDE-LONGITUDE GRID

Satellite measurements give more full and detailed information about the ozone global distribution. For different practical purposes, the information collected by a satellite during a lot of circuits may be considered as related to a single time moment. Necessarily, data of measurements can be brought to the standard times by interpolating in time and space, if one has wind data. In any case, satellite data are inhomogeneous, because satellite orbits are nearer each other over polar regions than over tropics. In order to estimate the fractal dimension of such a "network", let us consider it as a regular latitude-longitude grid with nodes denoting the sites of satellite measurements. Note that in different problems data of measurements are interpolated to such a grid.

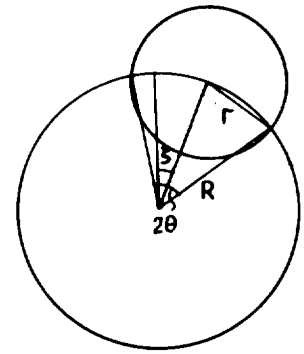

Fig.2. The geometry for calculating the correlation dimension of a point set on a sphere.

3.1. *Analytical consideration*

Let $N_\varphi+1$ be the number of nodes along meridian (including the poles), N_λ be the number of nodes along a circle of latitude. Then the density of nodes along meridian, $n_\varphi = N_\varphi/\pi R$, is constant, and the density of nodes along a circle, $n_\lambda = N_\lambda/(2\pi R\ \sin\varphi)$, depends upon latitude, φ. To simplify the analytical analysis let the grid be dense enough to replace summation by integration. The number of nodes within the θ-neighborhood of any node at the polar angle, ζ (see Fig.2), is

$$n = \int_{\varphi_1}^{\varphi_2}\int_{\lambda_1}^{\lambda_2} n_\varphi n_\lambda R^2\ \sin\varphi\ d\varphi\ d\lambda, \qquad (3)$$

where the integration limits satisfy the equation for the intersection of the two spheres. Calculating the integral (3), one should distinguish two cases: $\zeta\leqslant\theta$ and $\zeta>\theta$.

a) Case 1, the near-pole neighborhood: $\zeta\leqslant\theta$. Besides this, let $\theta\ll1$ and, hence, $\zeta\ll1$. In this case the integration of (3) gives:

$$n \approx 2R/\pi\ n_\varphi N_\lambda\ \theta\ E(\zeta/\theta) \qquad (4)$$

where E is the full normal elliptical integral of the second type.

b) Case 2, the out-polar neighborhood: $\zeta>\theta$. Let also $\zeta\ll1$ and, hence, $\varphi\ll1$. Then the integration of (3) gives:

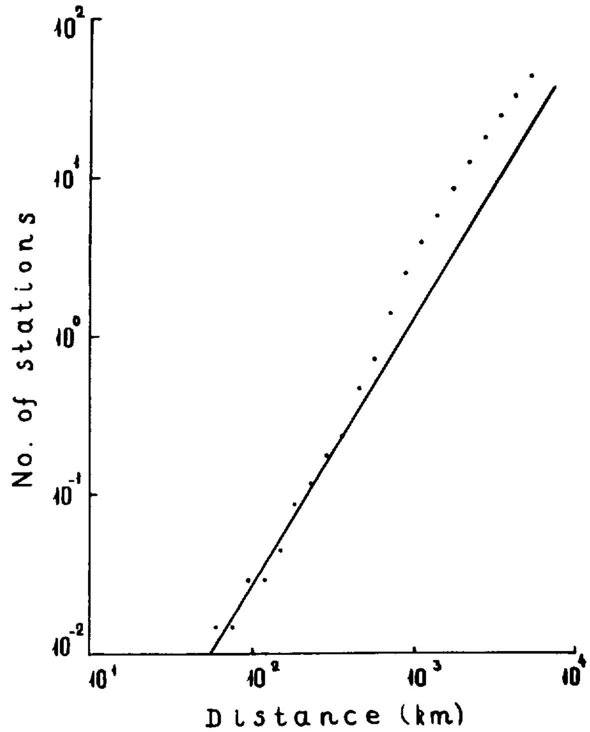

Fig.3. The average number of stations in the ground-based ozonometric network, $\langle n(L)\rangle$, within annuli of geometrically increasing radii. The straight line corresponds to scaling with an index equaled 1.67.

$$n \approx 2R/\pi\; n_{\varphi} N_{\lambda}\; \zeta\; \{E(k)-(1-k^2)K(k)\}. \quad (5)$$

Here K is the full normal elliptical integral of the first type, and $k=\sin\lambda_m$, where λ_m is determined through:

$$\cos\lambda_m = 1/\sin\zeta \sqrt{2\cos\zeta\,(\cos\theta-\cos\zeta)}.$$

Now let ζ be arbitrary, but such that $\zeta \gg \theta$. Then

$$n \approx R/2\sin\zeta\; n_{\varphi} N_{\lambda}\; \theta^2. \quad (6)$$

Let the local scaling of type (2) exist: $n(\theta) \sim \theta^{\nu}$. Transition from r to θ is correct, as $r \approx R\theta$ when $\theta \ll 1$. Then $\nu \approx (\theta/n)(dn/d\theta)$, if ν does not depend upon θ or ν is a slow function of θ. Under this condition one gets from (4) that for the near-polar region

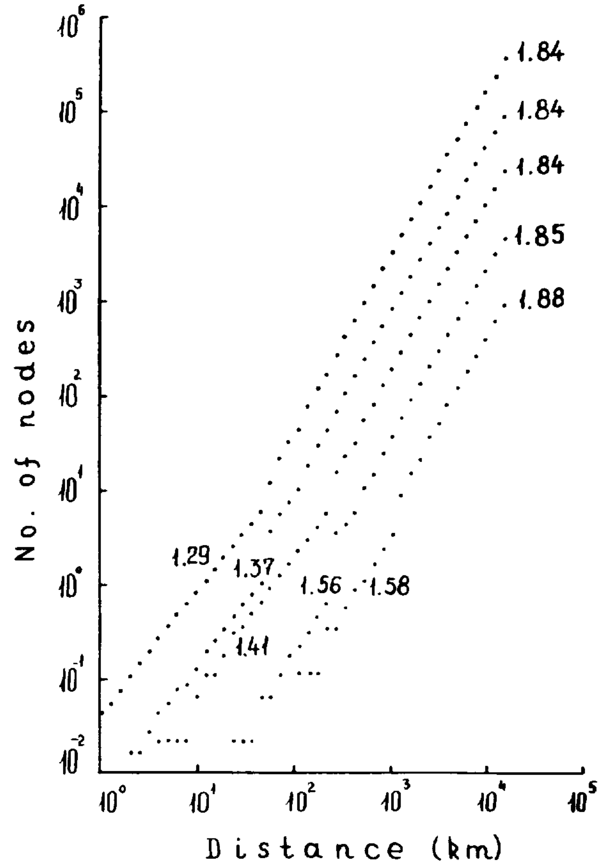

Fig.4. The average number of nodes in a latitude-longitude grid, $\langle n(L)\rangle$, within annuli of geometrically increasing radii, at different values of latitudinal, $\Delta\varphi$, and longitudinal, $\Delta\lambda$, resolutions. Five series of points correspond (from top to bottom) to: 1) $\Delta\varphi=\Delta\lambda=0.25^\circ$; 2) $\Delta\varphi=0.25^\circ$, $\Delta\lambda=1^\circ$; 3) $\Delta\varphi=\Delta\lambda=1^\circ$; 4) $\Delta\varphi=1^\circ$, $\Delta\lambda=5^\circ$; 5) $\Delta\varphi=\Delta\lambda=5^\circ$. The numbers at series denote correlation dimensions.

$$\nu \approx 1 + 1/2\,(\zeta/\theta)^2,$$

while for out-polar region (but at $\zeta \ll 1$)

$$\nu \approx 2 - 1/2\,(\theta/\zeta)^2.$$

If ζ is large one gets from (6) that $\nu \approx 2$ with a high precision. Thus, the local fractal dimension of a latitude-longitude grid is a monotonous function of latitude and changes from 1 at the poles to 2 at the equator. It is evident that the correlation dimension (1) which characterizes the global distribution of nodes of the grid, has to have an intermediate value.

3.2. *Empirical fractal characteristics*

In practice the distribution of the grid nodes is discrete. Fig.4 shows the calculated mean value, $\langle n \rangle$, as a function of L at different values of latitudinal and longitudinal resolutions. This function has two different scaling regimes. One regime is peculiar to smaller scales and limited from a larger scale by the latitudinal grid resolution. This regime is characterized by the small correlation dimension which decreases if the grid resolution improves. One can show that $\nu \to 1$ then. Another scaling regime ranges up to a global scale and is characterized by a correlation dimension equaled to 1.84 at sufficiently good resolution.

4. CONCLUDING REMARKS

The fact that the fractal dimension of ozone measuring network is less than the Earth's surface dimension (equaled 2), can have important consequences for ozone statistics. According to Lovejoy et al (1986), such a network is not able to detect phenomena with dimension $\nu_p < D-\nu$. The detectability limits are: ν_p= 2-1.67=0.33 for the ground-based ozonometric network, ν_p=0.16 for the latitude-longitude network at large scales, and $\nu_p \sim$ 0.4÷0.7 for the latitude-longitude network at small scales. If one suppose the fractal dimension of ozone field to be decreasing function of ozone content (similar to the case of some meteorological fields, see e.g. Lovejoy et al., 1986; Lovejoy and Schertzer, 1990), then insufficiently large fractal dimension of ozone measuring network has to lead to biases in ozone statistics. In particular, this can lead to biases in the spatial averages.

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