

AN ASYMPTOTIC METHOD FOR ESTIMATING THE VERTICAL OZONE DISTRIBUTION IN THE EARTH'S ATMOSPHERE FROM SATELLITE MEASUREMENTS OF BACKSCATTERED SOLAR UV-RADIATION

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ABSTRACT

An asymptotic approach to solution of the inverse problems of remote sensing is presented. It consists in changing integral operators characteristic of outgoing radiation into their asymptotic analogues. Such approach does not add new principal uncertainties into the problem and significantly reduces computation time that allows to develop the real (or about) time algorithms for interpretation of satellite measurements. The asymptotic approach has been realized for estimating vertical ozone distribution from satellite measurements of backscattered solar UV radiation in the Earth's atmosphere.

1. INTRODUCTION

The backscattered solar UV radiation flux  $F_i$  in a certain spectral range  $\Delta\nu$  centered at the wavelength  $n_i^0$  is formed by a relatively thin effective scattering atmospheric layer of the depth  $\Delta z_i$ . For this reason the flux  $F_i$  registered from space in the  $i$ -th spectral channel of a satellite instrument is connected with the integrated ozone content  $\Delta X_i$  in the atmospheric layer column by integral relation  $F_i = f(\Delta X_i)$  where  $f$  is a given integral operator. When measuring outgoing radiation in various spectral channels we obtain information about integrated ozone content in various atmospheric layers, i.e. about vertical ozone distribution. Such are heuristic grounds for remote ozonosphere sensing from space (Twomey, 1977). There are principal limitations for sensing accuracy specified by the effective scattering layer depth: the thinner is the layer, the higher is the height resolution. The integral operator  $f$  is such that its value for given ozone profile is defined by the integrand function behaviour in the narrow interval  $\Delta z$  only where it has evident maximum. It is well known (Olver, 1974; Nayfen, 1981) that for such integrals are available the asymptotic representations which connect their values with the integrand function values at the maximum point. In this case, other conditions being equal the error is specified by the interval  $z$  only. Thus, without new principal uncertainties being added, the remote sensing problem can be formulated by changing integral operators into their asymptotic analogues. It allows to develop the real time algorithms for interpretation of the satellite measurements. This is the idea of an asymptotic approach to solution of the remote ozonosphere sensing problem (as well as any integral operator inversion problem). Let us attempt to realize the asymptotic idea for estimating vertical

ozone profiles above the ozone maximum in the Earth's atmosphere from satellite measurements of backscattered solar UV radiation. As a first step, suitable inverse problems are necessary to be formulated as a problem with a large parameter.

2. ESTIMATING THE OZONE PROFILE AS A PROBLEM WITH LARGE PARAMETER

An interaction of solar radiation with the Earth's atmosphere depends on several factors (Liou, 1980; Brasseur et al., 1984). Apparently, the methods for inferring the vertical ozone distribution, which should be called into action for expediting the processing of large volumes of experimental data, must be based on choosing of main decisive factors. From the mathematical point of view it may be realized by introducing the large dimensionless parameters with following using of asymptotic expansions. In particular, such method of approach in the case of horizontally homogeneous plane-stratified Earth's atmosphere (the solar zenith angles  $\theta_0$  and the zenith angles of the receiver  $\theta \leq 75$ ) allows to receive for values  $Q_i$  (Thomas et al., 1977) which are dimensionless analogues of the backscattered solar UV radiation fluxes  $F_i$  registered from the space, the following expansion (Ishov, 1989):

$$Q_i = \int_0^1 dn \exp\{-y_i [M_i X(n) / X(1) + n]\}, \quad (1)$$

where  $n$  is dimensionless height,  $n = n(z) = Y(z)/Y(0)$ ,  $n=1$  on surface at  $z=0$ ,  $z$  is height,  $Y(z)$  is number of air molecules of atmospheric column above the height  $z$ ,  $X(n)$  is total ozone content of an atmospheric column above the level  $n$ ,  $X(1)$  is, obviously, total ozone content,  $y = a\beta_i Y(0)$ ,  $a = \sec\theta_0 + \sec\theta$  is geometrical factor of the experiment,  $\alpha_i$ ,  $\beta_i$  are the  $i$ -th wavelength band averaged ozone absorption and molecular scattering coefficients respectively (Klenk, 1980; Ishov et al., 1990) and values  $M_i$  of the dimensionless parameter  $M$  are given by

$$M_i = a\alpha_i X(1) / y_i. \quad (2)$$

Accurate calculations with taking account of multiscattering and number of other UV solar radiation transformation processes in the Earth's atmosphere (Liou, 1980) show that relation (1) is valid for spectral range of  $250 \leq \nu \leq 300$  nm with the relative error 1% and can be used for interpretation of the SBUV measurements aboard the Nimbus-7 (Fleig et al., 1985).

It is easily noted that the dimensionless parameter  $M$  is equal to the ratio of the optical depth of atmospheric column due to ozone absorption to the optical depth due to molecular scattering and can serve a measure of mutual significance of two above attenuation processes of UV solar radiation in the Earth's atmosphere in  $i$ -th spectral interval. The values of parameter  $M$  for various model atmospheres (Ippolitov et al., 1985) for spectral channels of SBUV are listed in Table 1. One can see that the parameter  $M$  is about 2 for channel 302nm and continuously increases up to nearly 40 for channel 256nm. This fact gives ground to accounting the parameter  $M$  as large and the problem of estimating vertical ozone distribution as a problem with large parameter.

### 3. THOMAS'S AND HOLLAND'S SCHEME

Thomas and Holland have propounded a simple approach for expediting the processing of large volumes of experimental data for the purpose of evaluating the vertical ozone profiles above the ozone maximum (Thomas et al., 1977). Therefore, it seems natural at first to illustrate efficiency of the asymptotical method of approach to the problems of the remote atmosphere sensing within the framework of this scheme.

Assuming that  $X(n)/X(1)=g(n)$  where  $g(n)$  is a given analytic function the inverse function  $G$  of which exists so that  $G(g(n))=n$  it is obtained that the equation (1) may be rewritten as

$$Q_i = \int_0^1 dn \exp\{-y_i[M_i g(n) + n]\}. \quad (3)$$

Further, let  $\tau_i=T$  for all spectral channels with central wavelength  $\nu_i^0, i=1, 2, \dots, L$ , where  $\tau_i$  is the ozone absorption optical thickness of an atmospheric layer with upper level at the top of the atmosphere. The lower level of this layer, it is obvious, will vary for various spectral channels. Having chosen  $L$  different channels, we have fixed  $L$  different layers of the atmosphere. Apparently, the problem of estimating vertical ozone distribution will be solved if lower levels  $z_i$  (or  $n_i$ ) and values  $X(z_i)$  (or  $X(n_i)$ ) have been indicated.

As following from our notation  $\tau_i=y_i M_i g(n)$  and we have:

$$n_i = G(T/y_i M_i), \quad (4)$$

$$X(n_i) = y_i g(n_i) / a \alpha_i = T / a \alpha_i. \quad (5)$$

Finally, it is necessary to determine the values of dimensionless parameter  $M$  only corresponding to experimentally obtained values  $Q_i$  for all  $L$  spectral channels.

### 4. ASYMPTOTIC ANALOGUE OF INTEGRAL EQUATION (3) AND ITS INVERSION

Recently, exact analytical methods of inversion of integral equation (3) are absent. So that we will use its asymptotic analogue for  $M \rightarrow \infty$ . From the mathematical point of view, this is the idea of the asymptotic method of approach to inverse problem. The asymptotic analogue is significantly simpler than initial equation since it includes linear combination of elementary functions with respect to  $M$  only. The error due to the substitution will approximate

to zero when  $M \rightarrow \infty$ . There are also foundations to suppose that, when using finite values of  $M$  in practical problem, the error will be not more than initial uncertainty originally inherent in the inverse problem. The thing is half-width of integrand of (3) (resolution with respect to height in given inverse problem) as well as error of asymptotic series expansion are determined by value of  $M$  only.

Making substitution of integrating variable in equation (3)  $t=y_i g(n)$ , finally, we have:

$$Q_i = \int_0^{y_i g(1)} dt H(t) \exp(-M_i t), \quad (6)$$

where  $H(t) = G(t/y_i) \exp(-t/y_i) G'(t/y_i) / y_i$ .

Table 1. The values  $M_i(2)$  for large dimensionless parameter  $M$ : 1-polar region, 2-middle latitudes winter, 3-middle latitudes summer, 4-tropics.

No.	$\lambda_i, nm$	1	2	3	4
1	255.7	41.45	44.55	40.04	31.63
2	273.6	30.23	32.47	29.21	23.08
3	283.1	16.51	17.65	16.46	12.56
4	287.7	10.58	11.24	10.52	8.01
5	292.3	6.47	6.89	6.45	4.91
6	297.6	3.46	3.67	3.44	2.62
7	302.0	2.05	2.17	2.04	1.56
8	312.6	0.52	0.54	0.51	0.39
9	317.6	0.34	0.36	0.34	0.26

Table 2. Estimates of  $z_i$  (km) and  $X_i(z_i)$  ( $cm^2$ ) for middle latitudes summer by the asymptotical method: 1-asymptotical, 2-exact, 3-error (%).

No.	$z_i$	1	2	3
1	46.1	.681(+17)	.669(+17)	-1.8
2	43.3	.126(+18)	.125(+18)	-1.3
3	39.9	.270(+18)	.272(+18)	-1.0
4	37.6	.453(+18)	.463(+18)	2.2
5	35.0	.793(+18)	.831(+18)	4.6
6	30.8	.161(+19)	.172(+19)	6.6
7	25.2	.290(+19)	.368(+19)	21.0
8	8.7	.135(+20)	.820(+19)	-64.0
9	5.6	.216(+20)	.845(+19)	-160.0

$z_m=24 \text{ km}, \delta=0.577$

Limiting oneself to consideration such functions  $g(n)$  only for which at  $t \rightarrow 0$  next expansion takes place

$$H(t) \sim \sum_{k=0}^{\infty} c_k t^{q_k}, \quad -1 < q_0 < q_1, \dots, \quad (7)$$

and using Laplas's method we can obtain following asymptotic at  $M \rightarrow \infty$  expansion instead of integral relation (3):

$$Q \sim \sum_{k=0}^{\infty} c_k \Gamma(q_k + 1) M^{-q_k - 1} \quad (8)$$

where  $\Gamma(x)$  is Gamma function. On the basis of (8) we can define  $M$  as function of  $Q$  by asymptotic at  $M \rightarrow \infty$  iteration method (Olver, 1974) using the next iteration function

$$M^{(n+1)} \sim M^{(n)} \left[ 1 + S_n(M^{(n)}) \right]^{1/(q_0 + 1)} \quad (9)$$

with the initial approximation  $M^{(0)}$  for  $M$  given by

$$M^{(0)} = [c_0 \Gamma(q_0 + 1) / Q]^{1/(q_0 + 1)} \quad (10)$$

where

$$S_n(x) = \sum_{k=1}^n c_k \Gamma(q_k + 1) x^{(q_0 - q_k)} / c_0 \Gamma(q_0 + 1).$$

Then stopping on the second iteration and limiting oneself to the three first terms in the expressions obtained, we have followed calculation formula:

$$M_i \approx M_i^{(0)} \{ 1 + c_1 \Gamma(q_1 + 1) [M_i^{(0)}]^{q_0 - q_1} / c_0 \Gamma(q_0 + 1) + c_2 \Gamma(q_2 + 1) [M_i^{(0)}]^{q_0 - q_2} / c_0 \Gamma(q_0 + 1) \} \quad (11)$$

which will be utilized below for expressing  $M_i$  through  $Q_i$ . Thus, the problem of determination of values  $M_i$  for large dimensionless parameter  $M$ , corresponding experimentally obtained values  $Q_i$  in the framework of the asymptotic method, is solved.

Table 3. The errors of the asymptotical estimates of  $M_i$  and  $z_i$  for tropics. 1-asymptotical  $M_i$  (exact  $M_i$  in Table 1), 2-error (%), 3-exact  $z_i$ , 4-asymptotical  $z_i$ , 5-error (%).

No.	1	2	3	4	5
1	31.63	0.00	47.17	47.29	-0.26
2	23.08	0.02	44.32	44.38	-0.16
3	12.55	0.08	40.90	40.81	-0.22
4	7.99	0.26	38.57	38.30	0.70
5	4.86	1.00	35.71	35.37	0.95
6	2.36	9.90	31.83	30.76	3.40
7	1.17	25.0	28.44	23.93	16.0

## 5. ASYMPTOTIC ESTIMATING THE VERTICAL OZONE DISTRIBUTION

Let us take as  $g(n)$  function, given by

$$g(n) = n^{1/\delta} / (n^{1/\delta} + s), \quad (12)$$

which accords with a complete Green's representation of integrated ozone content of vertical atmospheric column above the

level  $n$  (Green, 1964). It can be easily shown, in this case, that

$$G(t) = (st/(1-t))^\delta, G'(t) = \delta s^\delta t^{\delta-1} (1-t)^{-\delta-1},$$

$$H(t) \sim \delta s^\delta t^{\delta-1} \left\{ 1 - (st)^\delta / y_i^{\delta-1} + t(\delta+1)y_i + \dots \right\} / y_i^\delta$$

as  $t \rightarrow 0$ , so that

$$c_0 = \delta (s/y_i)^\delta, c_1 = \delta s^{2\delta} / y_i^{2\delta-1}, c_3 = \delta(\delta+1)s^\delta / y_i^{\delta+1}, q_0 = \delta - 1, q_1 = 2\delta - 1, q_3 = \delta.$$

By use equation (11) we obtain, finally, the next calculation formula for determining  $n_i$ :

$$n_i = \left[ T^{\delta} Q_i / \Gamma(\delta + 1) \right] \left\{ 1 + y_i \Gamma(2\delta + 1) / 2\Gamma^2(\delta + 1) + \delta(T - \delta - 1) / s [Q_i / \Gamma(\delta + 1)]^{1/\delta} \right\}. \quad (14)$$

At last, it is necessary to estimate values of the parameters  $\delta$  and  $s$  in (12) and to give  $T$ , having the experimentally obtained values  $Q_i$  only. Within shortwavelength range of spectrum, when parameter  $M$  is most large, we limit oneself to the principal term of asymptotic expansion (8) only. In consequence we obtain the next asymptotic relation:

$$\ln Q \sim \ln \Gamma(\delta + 1) / X^\delta(1) - \delta \ln \alpha \quad (15)$$

which is satisfied more accurately for shortwavelength channels. Following (Thomas et al., 1979) let us determine  $\delta$  from inclination of approximately linear dependence  $\ln Q$  of  $\ln(\alpha X)$  in shortwavelength range. The estimations show that the values  $\delta$  are contained between 0.5 and 0.7, so that  $\delta + 1$  is near by position of Gamma function minimum. So that the values  $n_i$  obtained by formula (14) will be not sensitive to the errors of value  $\delta$ . Parameter  $s$  is excluded from calculation formulas if let

$$T = \delta + 1. \quad (16)$$

The results of estimating the vertical ozone distribution obtained with the asymptotic method of approach for tropics and middle latitudes summer are presented in Table 2 and Figure. Apparently, the method allows estimating  $X(z_i)$  with relative error not more than 20% in the upper ozonosphere stretched down to the ozone concentration maximum height  $z_m$ . It appears that the values  $X(z_i)$  obtained with the asymptotic method near ozone concentration maximum height are systematically underestimated, underestimation incessantly increasing as approximating from top to  $z_m$ . It is due to asymptotic character of procedure (11) used for determining values  $M_i$  of dimensionless parameter  $M$ . Analyzing Table 3, one can receive this result. In Table 3 are listed relative errors of values  $M_i$  calculated by (11) and values  $z_i$  calculated by (14) in the most unfavourable case of tropic atmosphere. It denotes that the relative error in the case of sought for ozone profile depend on value  $M_i$  only. Therefore it can be easily correct, that allows estimating the vertical ozone distribution some below of the ozone concentration maximum.

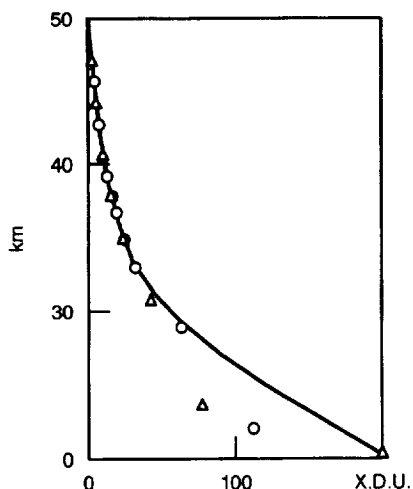


Fig. The integrated ozone content is as the asymptotic solution of integral equation (3) or (1):  $\Delta - g(n) = n^{1/\delta}$ ,  $o - g(n) = n^{1/\delta} / (n^{1/\delta} + 1)$ , solid curve - model.

## 5. CONCLUDING REMARKS

Asymptotic mathematical technique proposed above permits using as  $g(n)$  other distinguished from equation (12) functions which are able more adequately describe a state of ozonosphere. In the particular case of  $g(n) = n^{1/\delta}$  we have the result which specifies calculation formula for  $n_i$  obtained by Thomas et al. Although in this case the asymptotic method of approach does not give, in the main, new results, it give correct mathematical interpretation of them and increases the calculation accuracy.

Other problems of remote sensing of the atmosphere also have an asymptotic character, so far as all of which have the same mathematical and physical properties. A common mathematical property is that integrand of the operator, which forms outgoing radiation, has pronounced maximum, the position of which changes with changing spectral channel (when scanning by wavelength) or direction of vision (when scanning by zenith angle and when tangent sounding). A common physical property is that, when forming outgoing radiation, registered by satellite instrument, two processes are concurrent. The first is contribution to outgoing radiation by emitting (scattering) layer of the atmosphere and the second is attenuation of the outgoing radiation by upper layer on the path to registering satellite instrument. A ratio of characteristic of these concurrent processes will play a role of large parameter in the problem.

## REFERENCES

- Brasseur, G., S. Solomon, 1984: *Aeronomy of the Middle Atmosphere*. D. Reidel Publishing Company.
- Fleig, A.J., W.G. Planet, P.K. Bhartia, 1985: Backscattering ultraviolet measurements of ozone 1970-2000 an overview. *Atmospheric ozone*. D. Reidel Publishing Company.
- Green, A.E.S., 1964: Attenuation by ozone and the Earth's albedo

- in the middle ultraviolet. *Applied Optics*, vol. 3, No. 2, 203-208.
- Ishov, A.G., 1989: Asymptotic method for estimating state of Earth's ozonosphere from satellite measurements of back-scattered solar UV radiation. *Atmospheric Optics*, vol. 20(140).
- Thomas, R.W.L., A.C. Holland, 1977: Simple relationship between the UV radiation backscattered by the Earth's atmosphere and the vertical ozone profile. *Applied Optics*, vol. 16, 10.
- Liou, K.N., 1980: *An Introduction to Atmospheric Radiation*. Academic Press.
- Nayfen, A.H., 1981: *Introduction to Perturbation Techniques*. A. Wiley- Interscience Publication.
- Olver, F.W.J., 1974: *Asymptotics and Special Functions*. Academic Press.
- Twomey, S., 1977: *Introduction to the mathematics of inversion in remote sensing and indirect measurements*. Elsevier Scientific Publishing Company.