/N- 64

ICASE

A NOTE ON THE ACCURACY OF SPECTRAL METHOD APPLIED TO NONLINEAR CONSERVATION LAWS

Chi-Wang Shu
Peter S. Wong

(NASA-CR-194959) A NOTE ON THE
ACCURACY OF SPECTRAL METHOD APPLIED
TO NONLINEAR CONSERVATION LAWS
Final Report (ICASE)   15 p

N95-11226

Unclas

G3 /64 0019859

Institute for Computer Applications in Science and Engineering
NASA Langley Research Center
Hampton, VA 23681-0001

A NOTE ON THE ACCURACY OF SPECTRAL METHOD APPLIED TO NONLINEAR CONSERVATION LAWS [1]

Chi-Wang Shu and Peter S. Wong

Division of Applied Mathematics

Brown University

Providence, RI 02912

ABSTRACT

Fourier spectral method can achieve exponential accuracy both on the approximation level and for solving partial differential equations if the solutions are analytic. For a linear partial differential equation with a discontinuous solution, Fourier spectral method produces poor point-wise accuracy without post-processing, but still maintains exponential accuracy for all moments against analytic functions. In this note we assess the accuracy of Fourier spectral method applied to nonlinear conservation laws through a numerical case study. We find that the moments with respect to analytic functions are no longer very accurate. However the numerical solution does contain accurate information which can be extracted by a post-processing based on Gegenbauer polynomials.

i

1 Introduction

In this note we are concerned with the accuracy of Fourier spectral method for the solution of a nonlinear conservation law

$$\partial_t u + \partial_x f(u) = 0, \qquad -1 \leq x < 1$$
$$u(x,0) = u^0(x) \tag{1.1}$$

where the initial condition $u^0(x)$ is 2-periodic. As is well known, solutions to (1.1) typically contain discontinuities even if the initial condition $u^0(x)$ is analytic (In this paper, for simplicity of presentations, we will use analytic functions to represent general smooth functions; similar results can also be obtained for C^k or C^∞ functions). The purpose of this note is to assess accuracy under such a situation through a numerical study.

We start by recalling the Fourier approximation operator S_N to an L^2 function $u(x)$:

$$S_N u(x) = \sum_{k=-N}^{N} \hat{u}_k e^{ik\pi x} \tag{1.2}$$

where the Fourier coefficients $\hat{u}_k$ are defined by

$$\hat{u}_k = \frac{1}{2} \int_{-1}^{1} u(x) e^{-ik\pi x} dx \tag{1.3}$$

for Fourier Galerkin, and by

$$\hat{u}_k = \frac{1}{2N+1} \sum_{j=-N}^{N} u(x_j) e^{-ik\pi x_j}, \qquad x_j = \frac{2j}{2N+1} \tag{1.4}$$

for Fourier collocation. We will also use the notation $u_N(x) = S_N u(x)$. To solve the partial differential equation (1.1), the standard Fourier spectral algorithm is

$$S_N(\partial_t v_N + \partial_x f(v_N)) = 0, \qquad -1 \leq x < 1$$
$$v_N(x,0) = S_N u^0(x) \tag{1.5}$$

where $v_N(x,t) = \sum_{k=-N}^{N} \hat{v}_k(t) e^{ik\pi x}$ is supposed to approximate the exact solution $u(x,t)$ of (1.1), and S_N is either the Galerkin or the collocation Fourier approximation operator defined by (1.2)-(1.3) or by (1.2)-(1.4).

The approximation error

$$u(x) - S_N u(x) \tag{1.6}$$

is well known to be exponentially small (i.e., it is $O(r^N)$ for $0 < r < 1$) if $u(x)$ is analytic. However, if $u(x)$ is only piecewise analytic and discontinuous, the approximation error (1.6) is $O(1)$ near the discontinuity and $O(\frac{1}{N})$ elsewhere. This is known as the Gibbs phenomenon (see, e.g., [4] and [3]). Fortunately, even if the accuracy is poor in the point-wise sense, it is still excellent for the moments against any analytic functions. For any L^2 function $u(x)$ and any analytic function $w(x)$, we have [5]:

$$\left| \int_{-1}^{1} (u(x) - u_N(x))w(x)dx \right| \leq C r^N \tag{1.7}$$

for some constant C and $0 < r < 1$. This property is the basis of all the "reconstruction" or "post-processing" techniques. These techniques try to recover exponential or at least high order accuracy for point values based on the Fourier approximation $S_N u(x)$ of a piecewise analytic function. In other words, one tries to obtain a small post-processed approximation error

$$u(x) - P_N S_N u(x) \tag{1.8}$$

where P_N is some post-processing operator. Examples of P_N include various high frequency filters [14], [11], [16], [2], which are of the form

$$P_N S_N u(x) = \sum_{k=-N}^{N} \sigma\left(\frac{k}{N}\right) \hat{u}_k e^{ik\pi x} \tag{1.9}$$

with $S_N u(x)$ given by (1.2). The function $\sigma(\xi)$ in (1.9) is even (or satisfies $\sigma(-\xi) = \overline{\sigma(\xi)}$ if it is complex valued as in [2]), smooth (the accuracy of the filter depends upon its smoothness), supported in $(-1, 1)$ and satisfies $\sigma(0) = 1$ and $\sigma^{(k)}(0) = 0$ for $1 \leq k \leq K$ (with accuracy of the filter again depending on K). These filters can recover high order or even exponential accuracy in the smooth regions away from the discontinuities (the filter in [2] can also recover high order accuracy up to the discontinuity from one side). A more recent example of P_N is the Gegenbauer polynomial based procedure discussed in [6], [7], [8], [9] and [10], which can give uniform exponential accuracy for all x right up to the discontinuity for piecewise analytic functions. In this sense spectral

Fourier approximation is also exponentially accurate for piecewise analytic functions—one only has to extract the hidden information from the poor approximation $S_N(x)$ using the post-processor P_N.

When spectral method is used to solve the PDE (1.1), we can consider the following different types of errors. The strongest is the point-wise error from the exact solution $u(x,t)$:

$$u(x,t) - v_N(x,t), \tag{1.10}$$

which cannot be small even for $t = 0$ due to the Gibbs phenomenon. A more reasonable error is the point-wise error of the numerical solution $v_N(x,t)$ from the Fourier approximation of the exact solution $u_N(x,t)$:

$$u_N(x,t) - v_N(x,t). \tag{1.11}$$

If this error is exponentially small, we claim the spectral method for (1.1) is exponentially accurate because of the post-processor (1.8) for the exact solution $u(x,t)$. An even weaker error is defined by the error in the first few Fourier coefficients, i.e.

$$\hat{u}_k(t) - \hat{v}_k(t) \tag{1.12}$$

for the first few k, where $\hat{u}_k(t)$ are the Fourier coefficients of the exact solution $u(x,t)$ of (1.1). This is actually an example of the more general definition of error in moments with respect to any analytic function $w(x)$:

$$\int_{-1}^{1} (u_N(x) - v_N(x))w(x)dx \tag{1.13}$$

In fact, as long as this error in moments is exponentially small, we claim that the spectral method is exponentially accurate in solving (1.1) by using property (1.7) for the exact solution $u(x,t)$ and the post-processing (1.8).

If the PDE (1.1) is linear (i.e. $f(u) = a(x,t)u$), it is proven in [5], [1] that spectral Fourier method is exponentially accurate in the sense that (1.13) is exponentially small. A post-processing (1.8) applied to $v_N(x,t)$ would then yield an exponentially accurate point-wise approximation to the exact solution $u(x,t)$. However, if (1.1) is nonlinear, it is still a theoretically open problem whether

3

spectral Fourier method, equipped with either high frequency filtering or vanishing viscosity [15], [12], is exponentially (or high order) accurate in the sense of (1.13). Computational evidence in [13] seems to suggest that, even in this nonlinear case, highly accurate information is still implicitly contained in the numerical solution and can be extracted (at least away from the discontinuity) by a post-processing using high frequency filtering. In the next section we will perform a detailed numerical case study about this accuracy issue for Burgers' equation ($f(u) = \frac{u^2}{2}$). We use a high frequency solution filter to stablize the algorithm, and post process the numerical result using the procedure based Gegenbauer polynomials [6], [10]. We observe that the spectral Fourier method is not very accurate in the sense of moments against analytic functions (1.13). However, numerical evidence does indicate the possibility of very high accuracy under some weaker definition of accuracy, perhaps some average of Fourier coefficients, since the post-processed result $P_N v_N(x, t)$ is much more accurate than the Fourier coefficients themselves, and accurate Fourier coefficients can be "reconstructed" from this post-processed solution $P_N v_N(x, t)$.

2   A Numerical Case Study about Accuracy

In the numerical solution reported in this section, time discretization is by a third order Runge-Kutta method, with a time step Δt sufficiently small such that the spatial error is dominant in all cases. We compute the exact solutions of the PDE by Newton iterations on the implicit characteristic equations, and compute the Fourier coefficients of a function (if not analytically given) by using a sufficiently accurate numerical quadrature.

We first solve a linear equation

$$
\begin{aligned}
\partial_t u + \frac{3}{5 - 4\cos(\pi x)}\, \partial_x u &= 0, & -1 \leq x < 1 \\
u(x, 0) &= x
\end{aligned}
\tag{2.1}
$$

with periodic boundary conditions, up to t=1, using the Fourier Galerkin method:

$$
S_N \left(\partial_t v_N + \frac{3}{5 - 4\cos(\pi x)}\, \partial_x v_N \right) = 0,
$$

4

$$v_N(x,0) \;=\; S_N x \;=\; \sum_{\substack{k=-N \\ k \neq 0}}^{N} \frac{(-1)^k\, i}{k\pi} e^{ik\pi x} \tag{2.2}$$

Standard Galerkin method is stable for this linear problem but produces poor point value accuracy (Figure 1, left). However, the accuracy in the first few Fourier coefficients, as representatives of moments against analytic functions, are computed more accurately (Figure 1, right).
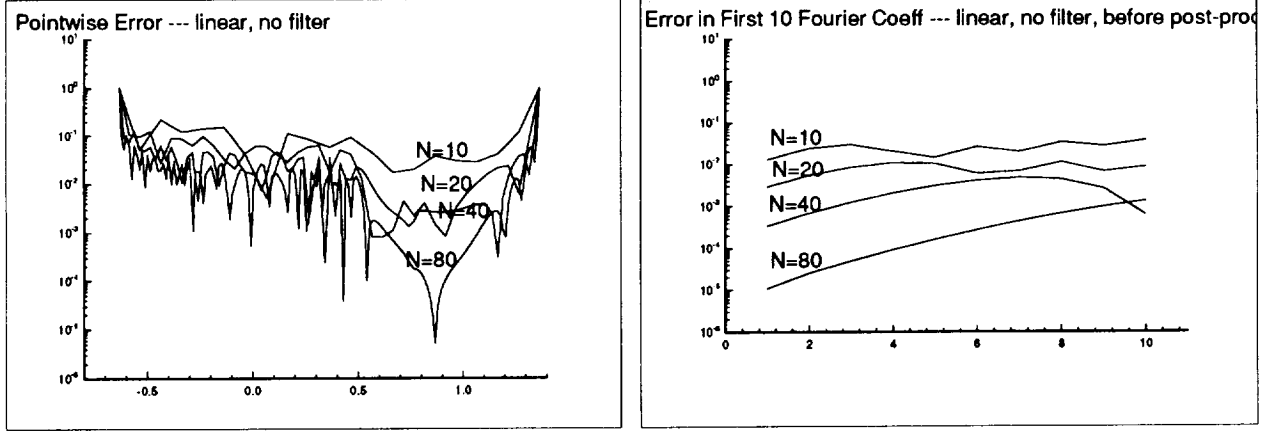

Fig. 1: Errors in log scale, linear PDE (2.1). Fourier Galerkin using $2N + 1$ modes, for $N = 10$; $N = 20$; $N = 40$ and $N = 80$. Left: point-wise errors; Right: errors in the first 10 Fourier coefficients.

In order to compare with the nonlinear case reported later, we solve the same linear equation (2.1) using the filtered Fourier method, i.e., after each Runge-Kutta time step, the numerical solution is filtered by (1.9) with the exponential filter:

$$\sigma(\xi) = e^{-\alpha|\xi|^r} \tag{2.3}$$

where r is increasing with N and is related to the order of the filter, and α is chosen such that $e^{-\alpha}$ equals machine zero (10^{-16} for double precision). The exponential filter (2.3) has the advantage of simplicity, and usually it works equally well as more complicated filters [16]. For this linear problem, as well as for the nonlinear Burgers' equation later, we will use the Fourier method with the following choice of filter orders: $r = 4$ for $N = 10$; $r = 6$ for $N = 20$; $r = 8$ for $N = 40$ and $r = 12$ for $N = 80$. The result is shown in Figure 2. Comparing with Figure 1, we can see better

point value accuracy in the smooth region because of the filters, and similar (good) accuracy for the first few Fourier coefficients.
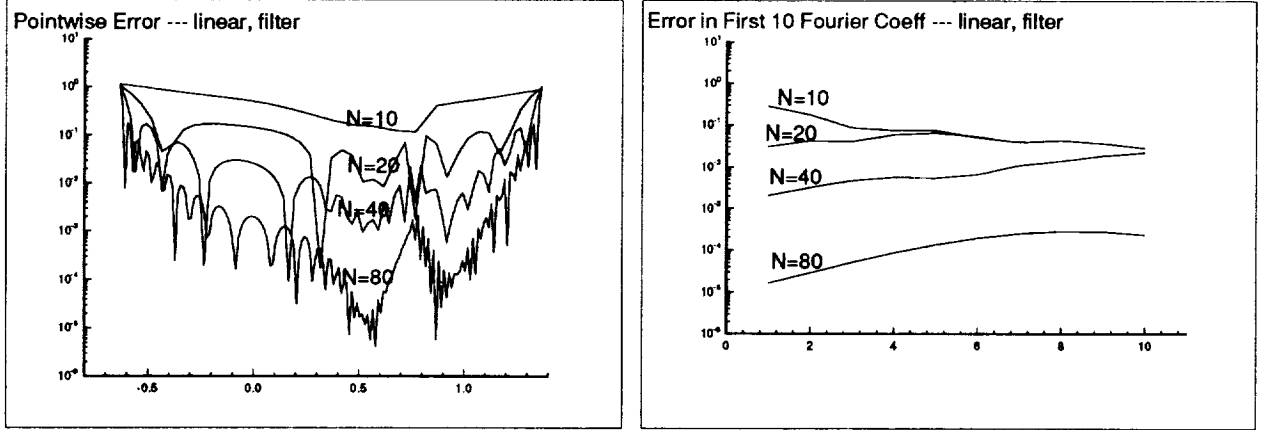

Fig. 2: Errors in log scale, linear PDE (2.1). Fourier Galerkin using $2N + 1$ modes with exponential solution filters of order r. $r = 4$ for $N = 10$; $r = 6$ for $N = 20$; $r = 8$ for $N = 40$ and $r = 12$ for $N = 80$. Left: point-wise errors; Right: errors in the first 10 Fourier coefficients.

The computational result for the linear equation is not surprising since it just shows the proven fact [5], [1] that Fourier coefficients, as representatives of moments against analytic functions, are computed with exponential accuracy by the spectral Fourier method, and filtering will recover exponential point value accuracy in smooth regions away from the discontinuity. It should be noticed that, for the same N, the accuracy for the first few Fourier coefficients is at the same level or better than the best point value accuracy in the smooth region after filtering. This is again not surprising since point value accuracy is obtained from the Fourier coefficients through filtering.

We now consider the nonlinear problem we are really interested in: we solve the nonlinear Burgers' equation

$$\partial_t u + \partial_x \left(\frac{u^2}{2} \right) = 0, \qquad -1 \le x < 1$$
$$u(x,0) = 0.3 + 0.7 \sin(\pi x). \qquad (2.4)$$

The solution develops a shock at $t = \frac{1}{0.7\pi}$ and we compute the solution up to $t = 1$. The initial condition is chosen such that the shock is moving with time. For this nonlinear PDE, the standard

Galerkin method cannot converge to the entropy solution [15]. One would need to add dissipations either by the high frequency solution filtering (1.9) or by the spectral vanishing viscosity [15], [12], [13]. Numerical results for the Burgers' equation with the vanishing viscosity method can be found in, e.g., [13]. Here we will only report the results obtained by solution filtering, using the same r as in the previous linear case (2.1). We have also computed with the vanishing viscosity methods and have obtained similar results.

In Figure 3 we plot the point-wise error $u(x,t) - v_N(x,t)$ (left), and the error for the first 10 Fourier coefficients (right). While the pattern of the point-wise errors are similar to the linear case in Figure 2, the errors for the Fourier coefficients are clearly much worse in comparison. As a matter of fact, for the same N, the errors for the first few Fourier coefficients are a few magnitudes larger than the smallest point value error in the smooth region. This is clearly different from what we observe in the linear case in Figure 2, and suggests that the first few Fourier coefficients, again as representatives of moments against analytical functions, are no longer computed with exponential or high order accuracy. It is sort of puzzling that each difference in the Fourier coefficients $\hat{u}_k(t) - \hat{v}_k(t)$ is relatively large (Figure 3 right), but the point-wise error $u(x,t) - v_N(x,t)$, which is just an average of $\hat{u}_k(t) - \hat{v}_k(t)$ (against an $O(1)$ function $e^{ik\pi x}$), is much smaller in the smooth region (Figure 3 left). Clearly some cancellation is present.
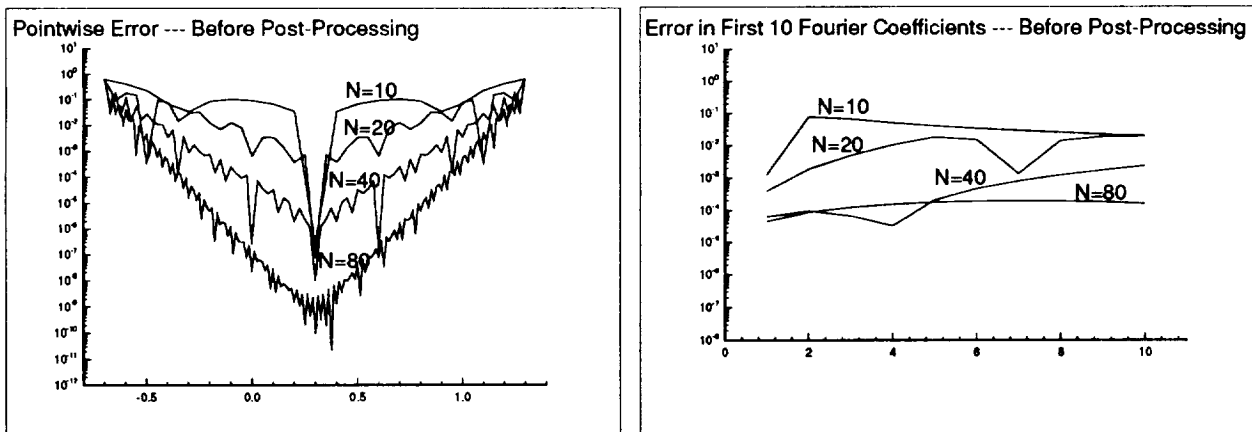

Fig. 3: Errors in log scale, Burgers equation (2.4). Fourier Galerkin using $2N + 1$ modes with exponential solution filters of order r. $r = 4$ for $N = 10$; $r = 6$ for $N = 20$; $r = 8$ for $N = 40$ and $r = 12$ for $N = 80$. Left: point-wise errors; Right: errors in the first 10 Fourier coefficients.

Next, we apply the Gegenbauer post-processor [6] to $v_N(x,t)$. This procedure can be roughly described as follows: given the Fourier partial sum $u_N(x)$ of an analytic but not periodic function $u(x)$, one first finds the approximations to the first m Gegenbauer expansion coefficients of the function $u(x)$. Here Gegenbauer polynomials are orthogonal polynomials in $[-1,1]$ under the weight function $(1-x^2)^{\lambda-\frac{1}{2}}$. One then uses this Gegenbauer series with the first m terms to approximate $u(x)$ everywhere in $[-1,1]$. To use this procedure, one must know the location of the discontinuity (however, the procedures in [8] allows one to handle the case where the location is not known exactly), and to choose the parameters λ and m. It is proven in [6] that when m and λ are both chosen proportional to (but less than) N, the reconstructed point values are exponentially accurate everywhere inside $[-1,1]$. Thus Gibbs phenomenon is completely removed. The details can be found in [6], [7], [8], [9] and [10].

We would like to point out that there is no theoretical justification in doing this post-processing for the current nonlinear case, since the post-processing procedure assumes that the Fourier coefficients are accurate, which is not true any more. However, the post-processed result is surprisingly good (Figure 4). We can observe good accuracy everywhere including at the discontinuity $x = \pm 1 + 0.3$ as in the approximation test case discussed [6]. The reconstructed Fourier coefficients, namely the Fourier coefficients of $P_N v_N(x,t)$, are much more accurate than before the post-processing (compare Figure 4 right with Figure 3 right).
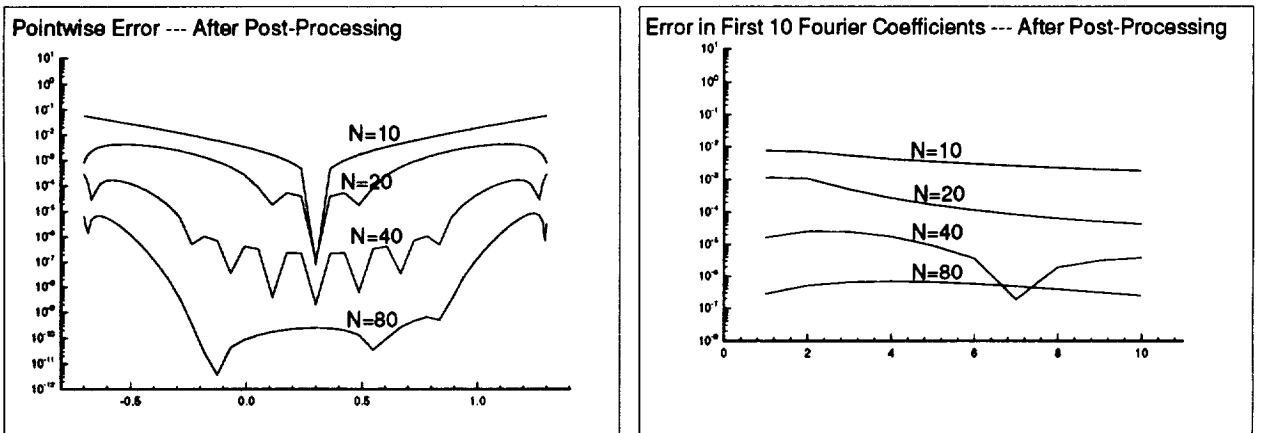

Fig. 4: Errors in log scale, Burgers equation (2.4). Fourier Galerkin using $2N + 1$ modes with exponential solution filters of order r. $r = 4$ for $N = 10$; $r = 6$ for $N = 20$; $r = 8$ for $N = 40$

and $r = 12$ for $N = 80$. Gegenbauer post-processed, with parameters $\lambda = 2, m = 1$ for $N = 10$; $\lambda = 3, m = 3$ for $N = 20$; $\lambda = 12, m = 7$ for $N = 40$ and $\lambda = 62, m = 15$, for $N = 80$. Left: point-wise errors; Right: errors in the first 10 Fourier coefficients.

This suggests that, even if $v_N(x, t)$ or its Fourier coefficients $\hat{v}_k(t)$ are not very accurate, it contains accurate information which is extracted in this case by the Gegenbauer polynomial based post-processor P_N. This numerical evidence suggests that in the nonlinear PDE case, Fourier coefficients $\hat{v}_k(t)$, just like point-wise values in the linear (or nonlinear) PDE case, are no longer good indicators of accuracy. They themselves are not very accurate, but they implicitly contain accurate information which can be extracted by adequate post-processors P_N. This accurate information might be contained in some averages of the Fourier coefficients (since the post-processing procedure based on Gegenbauer polynomials [6] uses certain averages of Fourier coefficients rather than the coefficients themselves).

We finally make two remarks:

Remark 2.1. In the Gegenbauer reconstruction procedure above we have used the exact shock location. The procedure in [8] allows us to use an approximate shock location, determined from the Fourier coefficients themselves (e.g., [2]). Similarly good results can be obtained when the reconstruction is performed in a slightly smaller sub-interval inside which the solution is guaranteed to be analytic. For example, we use the shock detector in [2], which in this case detects the shock location to within 0.0000025 for all the N values used, and a reconstruction inside the sub-interval $[-0.999997, 0.999997]$, which is just slightly smaller than $[-1, 1]$ (when numerically detected shock is shifted to $x = -1$) and guarantees that the true shock is outside this region. The result is shown in Figure 5 (left). It is clearly as good as the case where the exact shock location is used (compare with Figure 4 left).

Remark 2.2. If we use collocation (1.4) instead of Galerkin, (for the reconstruction procedure, see [10]), the result is almost identically good: Compare Figure 5 (right) with Figure 4 (left).
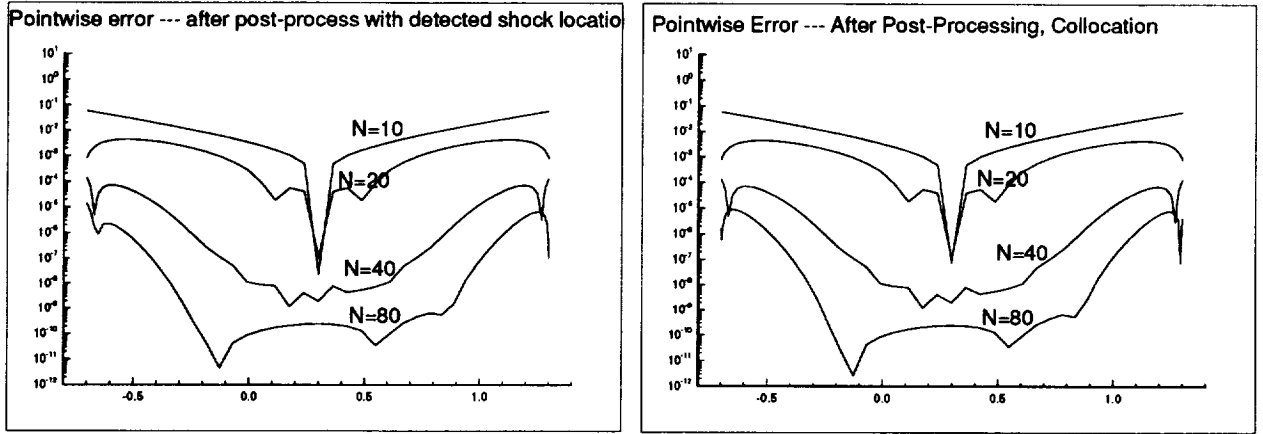
9

Fig. 5: Point-wise Errors in log scale, Burgers equation (2.4). Fourier method using $2N + 1$ modes with exponential solution filters of order r. $r = 4$ for $N = 10$; $r = 6$ for $N = 20$; $r = 8$ for $N = 40$ and $r = 12$ for $N = 80$. Left: Galerkin, Gegenbauer post-processed with a numerically determined shock location using the techniques in [2], which for this problem produce shock locations to within 0.0000025 for all the N used. The reconstruction sub-interval is $[-0.999997, 0.999997]$ when the numerical shock is shifted to $x = -1$. Parameters: $\lambda = 2, m = 1$ for $N = 10$; $\lambda = 3, m = 3$ for $N = 20$; $\lambda = 26, m = 9$ for $N = 40$ and $\lambda = 52, m = 17$, for $N = 80$. Right: collocation. Gegenbauer post-processed, with parameters $\lambda = 2, m = 1$ for $N = 10$; $\lambda = 3, m = 3$ for $N = 20$; $\lambda = 26, m = 9$ for $N = 40$ and $\lambda = 60, m = 15$, for $N = 80$.

3   Concluding Remarks

Through a careful numerical case study for the Burgers' equation, we have found that the Fourier spectral method, equipped with spectrally small dissipations in the form of high frequency filters or vanishing viscosities, are not accurate in the first few Fourier coefficients, or in moments against smooth functions. However, accurate information is indeed contained in the numerical solution, and can be extracted by using the Gegenbauer polynomial based post-processor in [6]-[10].

References

[1] A. Abarbanel, D. Gottlieb and E. Tadmor, *Spectral methods for discontinuous problems*, in *Numerical Methods for Fluid Dynamics II*, W. Morton and M.J. Baines, eds., Oxford University Press, London, 1986, pp.129-153.

[2] W. Cai, D. Gottlieb and C.-W. Shu, *On One-Sided Filters for Spectral Fourier Approximations of Discontinuous Functions*, SIAM J. Numer. Anal., v29 (1992), pp.905-916.

[3] C. Canuto, M.Y. Hussaini, A. Quarternoni and T. A. Zang, *Spectral Methods in Fluid Dynamics*, Springer-Verlag, 1988.

[4] D. Gottlieb and S. Orszag, *Numerical Analysis of Spectral Methods: Theory and Applications*, SIAM-CBMS, Philadelphia, 1977.

[5] D. Gottlieb and E. Tadmor, *Recovering Pointwise Values of Discontinuous Data Within Spectral Accuracy*, in *Progress and Supercomputing in Computational Fluid Dynamics*, E.M. Murman and S.S. Abarbanel, eds., Birkhaeser, Boston, 1985, pp.357-375.

[6] D. Gottlieb, C.-W. Shu, A. Solomonoff and H. Vandeven, *On The Gibbs Phenomenon I: recovering exponential accuracy from the Fourier partial sum of a nonperiodic analytic function*, J. Comput. Appl. Math., v43 (1992), pp.81-92.

[7] D. Gottlieb and C.-W. Shu, *Resolution properties of the Fourier method for discontinuous waves*, ICASE Report No. 92-27. June 1992. Meth. Appl. Mech. Engin., to appear.

[8] D. Gottlieb and C.-W. Shu, *On The Gibbs Phenomenon III: recovering exponential accuracy in a sub-interval from the spectral partial sum of a piecewise analytic function*, ICASE Report No. 93-82, NASA Langley Research Center, 1993. SIAM J. Numer. Anal., to appear.

[9] D. Gottlieb and C.-W. Shu, *On The Gibbs Phenomenon IV: recovering exponential accuracy in a sub-interval from the Gegenbauer partial sum of a piecewise analytic function*, ICASE Report No. 94-33, NASA Langley Research Center, 1994. Math. Comp., to appear.

11

[10] D. Gottlieb and C.-W. Shu, *On The Gibbs Phenomenon V: recovering exponential accuracy from collocation point values of a piecewise analytic function*, ICASE Report No. 94-61, NASA Langley Research Center, 1994. Numer. Math., submitted.

[11] H. Kreiss and J. Oliger, *Stability of the Fourier Method*, SIAM J. Numer. Anal., v16 (1979), pp.421-433.

[12] Y. Maday and E. Tadmor, *Analysis of the spectral vanishing viscosity method for periodic conservation laws*, SIAM J. Numer. Anal., v26 (1989), pp.854-870.

[13] Y. Maday, S. Ould Kaber and E. Tadmor, *Legendre pseudospectral viscosity method for nonlinear conservation laws*, SIAM J. Numer. Anal., v30 (1993), pp.321-342.

[14] A. Madja, J. McDonough and S. Osher, *The Fourier Method for Nonsmooth Initial Data*, Math. Comput., v32 (1978), pp.1041-1081.

[15] E. Tadmor, *Convergence of spectral methods for nonlinear conservation laws*, SIAM J. Numer. Anal., v26 (1989), pp.30-44.

[16] H. Vandeven, *Family of Spectral Filters For Discontinuous Problems*, J. Sci. Comput., v8 (1991), pp.159-192.

REPORT DOCUMENTATION PAGE

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE July 1994	3. REPORT TYPE AND DATES COVERED Contractor Report

4. TITLE AND SUBTITLE
A NOTE ON THE ACCURACY OF SPECTRAL METHOD APPLIED TO NONLINEAR CONSERVATION LAWS

5. FUNDING NUMBERS

C NAS1-19480
WU 505-90-52-01

6. AUTHOR(S)
Chi-Wang Shu
Peter S. Wong

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Institute for Computer Applications in Science
and Engineering
Mail Stop 132C, NASA Langley Research Center
Hampton, VA 23681-0001

8. PERFORMING ORGANIZATION REPORT NUMBER

ICASE Report No. 94-67

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23681-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA CR-194959
ICASE Report No. 94-67

11. SUPPLEMENTARY NOTES
Langley Technical Monitor: Michael F. Card
Final Report
Submitted to Journal of Scientific Computing

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified–Unlimited

Subject Category 64

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
Fourier spectral method can achieve exponential accuracy both on the approximation level and for solving partial differential equations if the solutions are analytic. For a linear partial differential equation with a discontinuous solution, Fourier spectral method produces poor point-wise accuracy without post-processing, but still maintains exponential accuracy for all moments against analytic functions. In this note we assess the accuracy of Fourier spectral method applied to nonlinear conservation laws through a numerical case study. We find that the moments with respect to analytic functions are no longer very accurate. However the numerical solution does contain accurate information which can be extracted by a post-processing based on Gegenbauer polynomials.

14. SUBJECT TERMS spectral method, accuracy, Gibbs phenomenon, nonlinear conservation laws	15. NUMBER OF PAGES 14
	16. PRICE CODE A03

17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT