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A Nonlinear Strategy for Sensor Based Vehicle Path Control

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Abstract

A new method to stabilize a vehicle, which is autonomously guided on the road by an optical sensor, is presented. Since the dynamical behavior of a vehicle is nonlinear and contains mutual couplings in the state variables, the design especially of algorithms for transverse control is based on the universal valid principle of nonlinear decoupling and control. This method works independent from any operating point and thus a high degree of accuracy and free pole assignment for the overall system is provided.

1 Introduction

In the last years many efforts have been achieved in realizing and integrating automation components in motor vehicles. In this context the task is followed to support the driver by efficient kinds of control and safety systems [1]. In this way anti block systems, methods to prevent a vehicle from skidding on icy roads [2] as well as the application of collision avoidance strategies are described [3]. Besides a very important aspect to increase safety in traffic is to direct a vehicle equipped with an on-board system automatically on the road [4]. Here a method is described, where the geometry of the street is detected by an optical system mounted on the car. Based on the information coming from this sensor the control system provides adequate control signals to stabilize the vehicle on the right lane. As the street has to be followed exactly, a high quality control system for the vehicle is required. In this paper a very efficient strategy of transverse control based on nonlinear formulations is presented. Furthermore a longitudinal controller to influence the velocity of the vehicle is described.

2 Mathematical model of the vehicle

As already mentioned, the task is followed to develop a control system, which directs the vehicle automatically along the road under consideration of a nominal speed. The right edge of the street is traced by an optical sensor, e.g. a video camera. In this way the distance l_{tr} between the point P on the optical axis and the point E on the edge of the road can be determined. By further consideration of distance l_{lo} between point P and the center of gravity of the vehicle a proportion of the car's direction of movement in dependence of the course of the road can be indicated. This structure is illustrated by Fig. 1, where especially in the lower sketch the kinematics of the vehicle including the optical sensor is shown. Please note that l_{lo} represents a constant parameter and, in contrast, l_{tr} is a dynamic variable.

To develop an efficient control system a mathematical model describing the dynamical behavior of the vehicle is a very important supposition. Besides the requirements of high accuracy this model, which consists of several differential equations, should be of certain clearness. A single track model describing transverse and longitudinal dynamics, neglecting roll and pitch angles and comprising front and rear wheels to one fictitious wheel respectively is well suited [5]. It is transferred into state space description, which will be

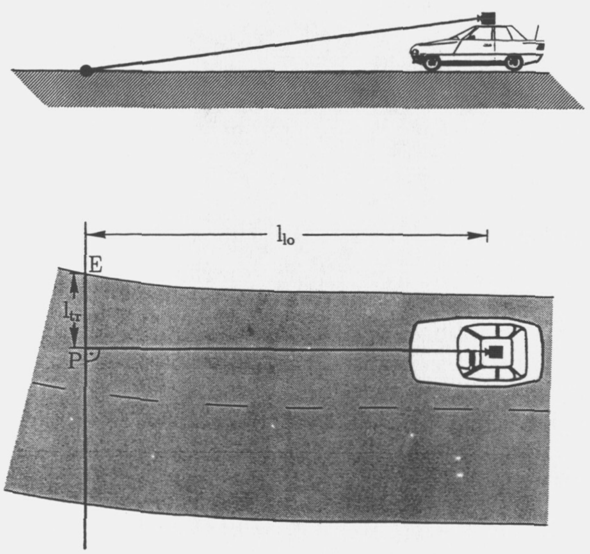

Figure 1: Mode of operation of the sensor

the starting point for the design of the path control system:

$$
\dot{\underline{x}} = \begin{bmatrix} x_3 + \frac{1}{m}\frac{T(\underline{x})\,x_1}{x_4} - \frac{1}{m}\frac{S_h(\underline{x})}{x_4} \\ x_3 \\ -\frac{l_h}{\theta}\,S_h(\underline{x}) \\ -\frac{1}{m}\,T(\underline{x}) \end{bmatrix}
+ \begin{bmatrix} -\frac{1}{m\,x_4} & -\frac{1}{m}\frac{x_1}{x_4} \\ 0 & 0 \\ \frac{l_v}{\theta} & 0 \\ 0 & \frac{1}{m} \end{bmatrix} \underline{u}(t) \qquad (1)
$$

$$
\underline{y} = \begin{bmatrix} x_2 \\ x_4 \end{bmatrix}.
$$

The 4-dimensional state vector $\underline{x}(t)$, which consists of the dynamical variables, as well as the 2-dimensional input and output vectors $\underline{u}(t)$ and $\underline{y}(t)$ are introduced as follows:

$$
\begin{aligned}
\underline{x} &= \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^{\mathrm{T}} \\
&= \begin{bmatrix} \beta & \psi & \psi' & v \end{bmatrix}^{\mathrm{T}} \\
\underline{u} &= \begin{bmatrix} u_1 & u_2 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} S_v & H \end{bmatrix}^{\mathrm{T}} \\
\underline{y} &= \begin{bmatrix} y_1 & y_2 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \psi & v \end{bmatrix}^{\mathrm{T}}.
\end{aligned} \qquad (2)
$$

Inputs are front side force S_v of the vehicle depending on the steering angle δ_v by

$$
S_v(\underline{x}) = c_v\left(x_1 - \frac{l_v}{x_4}\,x_3 + \delta_v\right) \qquad (3)
$$

and rear driving or braking force H, while output variables are the yaw angle ψ describing the orientation of the vehicle and the velocity v of the car. As shown in Fig. 2, important state variables are the yaw angle ψ, the yaw velocity ψ',

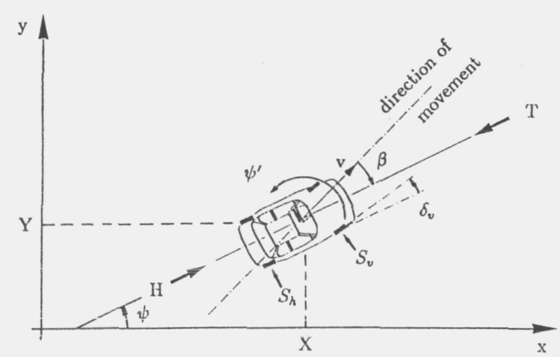

Figure 2: Dynamical variables of the vehicle

which consists its first derivation, the longitudinal velocity v and the sideslip angle β. The vehicle is of mass m and of moment of inertia θ, while c_v, c_h, l_v and l_h are further constant parameters. The variables $S_h(\underline{x})$ and $T(\underline{x})$, which consist of the rear side force and the air resistance respectively, depend on their turn on the state variables and can be computed by

$$
\begin{aligned}
S_h(\underline{x}) &= c_h\left(x_1 + \frac{l_h}{x_4}\,x_3\right) \\
T(\underline{x}) &= c_w\frac{\varrho_L}{2}A\,x_4^2.
\end{aligned} \qquad (4)
$$

Aerodynamic resistance coefficient, atmospheric density and surface of the vehicle's cross section are represented by the parameters c_w, ϱ_L and A. To keep clearness of the model equations for the derivation of the control laws these variables are not inserted explicitly. In this way also the front side force is stipulated as an input variable of the model.

Furthermore the actual position coordinates X and Y of the center of gravity in the stationary system is evaluated with respect to the differential equations

$$\dot{X} = v\,\cos(\psi - \beta)$$
$$\dot{Y} = v\,\sin(\psi - \beta). \qquad (5)$$

However, the coordinates X_P and Y_P of the point P on the optical axis, which is of distance l_{lo} to the center of gravity are determined by

$$X_P = X + l_{lo}\,\cos\psi$$
$$Y_P = X + l_{lo}\,\sin\psi. \qquad (6)$$

Please note that equations (5) and (6) are only required for the simulation. As shown in section 3, however, they are not required to develop the control algorithms.

3 Nonlinear Path Control

3.1 Design of the control algorithms:

The dynamical behavior of a vehicle is extremely nonlinear and consists of mutual couplings. Control laws, which are based on a linearized model, would have the disadvantage of dependence from an operating point. However, if the design of the control laws is based on the universal method of nonlinear decoupling and control, all couplings and nonlinearities are compensated in a direct way and the controlled vehicle will be impressed a linear behavior with free pole assignment [6]. Based on state space description (1) the task is to find algorithms to control the vehicle's yaw angle ψ and the velocity v by influencing front side force S_v and rear longitudinal force H.

The elements $C_1(\underline{x})$ and $C_2(\underline{x})$ of the matrix $\underline{C}(\underline{x})$, representing the coordinates of the car's center of gravity, are related to a subsystem each [7]. For both subsystems first the differential orders, which determine the order of the output variables' derivations connected in a direct way to an input variable, have to be found out respectively. To determine the value greater zero for the subsystem i the differential operator $N_A^k C_i(\underline{x})$ with $k = 0, 1, 2, \ldots$, which can be recurrently evaluated by

$$N_A^k C_i\,(\underline{x}) = \left[\frac{\partial}{\partial \underline{x}}\,N_A^{k-1} C_i(\underline{x})\right] \underline{A}\,(\underline{x}) \qquad (7)$$

under consideration of the initial value

$$N_A^0 C_i\,(\underline{x}) = C_i(\underline{x}), \qquad (8)$$

is introduced. Hence the differential orders of the subsystems are evaluated by the following formulation:

$$d_i = \min\left\{ j : \left[\frac{\partial}{\partial \underline{x}}\,N_A^{j-1} C_i\,(\underline{x})\right] \underline{B}(\underline{x}) \neq \underline{0}^{\mathrm{T}} \quad j = 1, 2, \ldots, n \right\} \qquad (9)$$

with $\underline{0}$ as the zero vector. With respect to state space description (1) the differential operators $N_A^k C_i(\underline{x})$ as well as the parts $\left[\frac{\partial}{\partial \underline{x}}\,N_A^k C_i(\underline{x})\right] \underline{B}\,(\underline{x})$ are evaluated for subsystem $i = 1$ to

$$N_A^0 C_1(\underline{x}) = x_2$$
$$\left[\frac{\partial}{\partial \underline{x}}\,N_A^0 C_1(\underline{x})\right] \underline{B}\,(\underline{x}) = \begin{bmatrix} 0 & 0 \end{bmatrix} \qquad (10)$$

and

$$N_A^1 C_1(\underline{x}) = x_3$$
$$\left[\frac{\partial}{\partial \underline{x}}\,N_A^1 C_1(\underline{x})\right] \underline{B}\,(\underline{x}) = \begin{bmatrix} \frac{l_v}{\theta} & 0 \end{bmatrix}. \qquad (11)$$

Furthermore for subsystem $i = 2$ these parts yield

$$N_A^0 C_2(\underline{x}) = x_4$$
$$\left[\frac{\partial}{\partial \underline{x}}\,N_A^0 C_2(\underline{x})\right] \underline{B}\,(\underline{x}) = \begin{bmatrix} 0 & \frac{1}{m} \end{bmatrix}. \qquad (12)$$

As for subsystem $i = 1$ part $\left[\frac{\partial}{\partial \underline{x}}\,N_A^0 C_1(\underline{x})\right] \underline{B}\,(\underline{x}) = \underline{0}^{\mathrm{T}}$ but part $\left[\frac{\partial}{\partial \underline{x}}\,N_A^1 C_1(\underline{x})\right] \underline{B}\,(\underline{x}) \neq \underline{0}^{\mathrm{T}}$ and for subsystem $i = 2$ already part $\left[\frac{\partial}{\partial \underline{x}}\,N_A^0 C_2(\underline{x})\right] \underline{B}\,(\underline{x}) \neq \underline{0}^{\mathrm{T}}$ the differential orders d_i for both subsystems result in

$$d_1 = 2$$
$$d_2 = 1\,. \qquad (13)$$

These values for the differential orders signify the immediate effect of the system inputs on the second derivation of the yaw angle and the first derivation of the car's velocity. As the output variables consist of the yaw angle, which is of kinematic character, and the velocity representing a kinetic variable, one receives different values for the differential orders of each subsystem.

For further consideration also the operators

$$N_A^2 C_1(\underline{x}) = -\frac{S_h\,l_h}{\theta}$$
$$N_A^1 C_2(\underline{x}) = -\frac{c_5}{m} x_4^2 \qquad (14)$$

have to be calculated. Thus the 2x1- and 2x2-matrices result in

$$\underline{C}^*(\underline{x}) = \begin{bmatrix} N_A^{d_1} C_1\,(\underline{x}) \\ N_A^{d_2} C_2\,(\underline{x}) \end{bmatrix} = \begin{bmatrix} -\frac{S_h\,l_h}{\theta} \\ -\frac{c_5}{m} x_4^2 \end{bmatrix} \qquad (15)$$

and

$$\underline{D}^*(\underline{x}) = \begin{bmatrix} \frac{\partial}{\partial \underline{x}}\,N_A^{d_1-1} C_1\,(\underline{x}) \\ \frac{\partial}{\partial \underline{x}}\,N_A^{d_2-1} C_2\,(\underline{x}) \end{bmatrix} \underline{B}(\underline{x}) = \begin{bmatrix} \frac{l_v}{\theta} & 0 \\ 0 & \frac{1}{m} \end{bmatrix} \qquad (16)$$

respectively. These matrices are basic requirements to derive the nonlinear control algorithms which satisfy the universal equation

$$\underline{u}(t) = \underline{D}^*(\underline{x})^{-1}\left\{ -\underline{C}^*(\underline{x}) + \underline{\Lambda}\,\underline{w}(t) - \underline{M}^*(\underline{x}) \right\}, \qquad (17)$$

where the control signals $\underline{u}(t)$ are generated in dependence from the state variables $\underline{x}(t)$ and the nominal values $\underline{w}(t)$. As the elements of $\underline{w}(t)$ correspond to the elements of $\underline{y}(t)$, the nominal values for yaw angle and velocity are determined by $w_1(t)$ and $w_2(t)$ respectively. By the 2x2-matrix $\underline{\Lambda}$, composed only by elements on the main diagonal, the nominal values are amplified, while by the 2x1-matrix $\underline{M}^*(\underline{x})$ pole assignment for the overall systems is carried out.

The existence of the inverse of $\underline{D}^*(\underline{x})$ is a necessary and sufficient condition for the decoupling of the system of differential equations (1). With respect to (16) the determinant of $\underline{D}^*(\underline{x})$ results in

$$\det\Big(\underline{D}^*(\underline{x})\Big) = \frac{l_v}{\theta\, m}, \tag{18}$$

whereby the inverse is calculated to

$$\underline{D}^*(\underline{x})^{-1} = \begin{bmatrix} \frac{\theta}{l_v} & 0 \\ 0 & m \end{bmatrix}. \tag{19}$$

To complete the control laws the matrices $\underline{\Lambda}$ and $\underline{M}^*(\underline{x})$ have to be specified explicitly. By

$$\underline{\Lambda} = \mathrm{diag}\Big\{\lambda_1, \lambda_2\Big\} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \tag{20}$$

the nominal values are weighted with respect to the amplifications λ_1 and λ_2, while by

$$\underline{M}^*(\underline{x}) = \begin{bmatrix} \sum_{k=0}^{d_1-1} \alpha_{k1}\, N_A^k C_1(\underline{x}) \\ \sum_{k=0}^{d_2-1} \alpha_{k2}\, N_A^k C_2(\underline{x}) \end{bmatrix} = \begin{bmatrix} \alpha_{01}\, x_2 + \alpha_{11}\, x_3 \\ \alpha_{02}\, x_4 \end{bmatrix} \tag{21}$$

the dynamical behavior of the decoupled subsystems can be adjusted by the constant parameters α_{01}, α_{02} and α_{11}.

With respect to the universal formulation (17) and the corresponding matrices in (19), (20) and (21) the control laws are evaluated to

$$\begin{aligned} u_1(t) &= \frac{l_h}{l_v}\, S_h(\underline{x}) + \frac{\theta}{l_v}\,(\lambda_1\, w_1(t) - \alpha_{01}\, x_2 - \alpha_{11}\, x_3) \\ u_2(t) &= T(\underline{x}) + m\; (\lambda_2\, w_2(t) - \alpha_{02}\, x_4)\,. \end{aligned} \tag{22}$$

As the control algorithms require the actual values for $S_h(\underline{x})$ and $T(\underline{x})$ of rear side force and air resistance, these variables are provided with respect to equations (4) in advance.

3.2 Dynamical behavior of the overall system:

With respect to the control algorithms (22) the dynamical behavior of the overall system is examined. Considering (1) and (13) these derivations yield

$$\begin{aligned} \dot{y}_1 &= \dot{x}_3 \\ \ddot{y}_2 &= \dot{x}_4. \end{aligned} \tag{23}$$

In this context d_1 and d_2 indicate, how often the output variables y_1 and y_2 have to be derived respectively. Replacing the first derivations $\dot{x}_3$ and $\dot{x}_4$ by the corresponding right sides of (1) and by further substitution of the control variables $u_1(t)$ and $u_2(t)$ by the right sides of (22) the overall behavior of the controlled vehicle results in

$$\begin{aligned} \ddot{y}_1 + \alpha_{11}\, \dot{y}_1 + \alpha_{01}\, y_1 &= \lambda_1\, w_1(t) \\ \dot{y}_2 + \alpha_{02}\, y_2 &= \lambda_2\, w_2(t). \end{aligned} \tag{24}$$

By these equations the overall dynamics of the vehicle describing in transverse direction a behavior of second order and describing in longitudinal direction a behavior of first order is determined. Desired variables are $w_1(t)$ for the nominal yaw angle and $w_2(t)$ for the desired velocity of the car.

With the free choosable parameters $\lambda_1, \lambda_2, \alpha_{01}, \alpha_{11}$ and α_{02} the dynamics of the controlled vehicle can be adjusted. It is sensible to introduce the following restrictions:

$$\begin{aligned} \alpha_{01} &= \lambda_1 \\ \alpha_{11} &= 2\,\sqrt{\lambda_1} \\ \alpha_{02} &= \lambda_2. \end{aligned} \tag{25}$$

In this way considering the stationary case the nominal values and the corresponding output variables are weighted by the same parameters. Furthermore the aperiodic borderline case is adjusted in transverse direction.

With the remaining degrees of freedom consisting of the parameters λ_1 and λ_2 rapidity of control circuits can be influenced by pole assignment. Here technical constraints of the vehicle have to be taken into consideration.

3.3 Modification of the nonlinear control laws:

As the value of the yaw angle, represented by variable x_2 in (22), is not measured and also a nominal value $w_1(t)$ for the yaw angle does not exist explicitly, an adequate modification of the relevant control algorithm is necessary. Rather the task is followed to find a control system, which adjusts automatically the car's direction of movement considering the course of the road. In this context the edge of the street is realized by the optical sensor mounted on the car. Orientations of the car and of the sensor are identical. So a control method must be found to adjust the yaw angle in dependence of the signal coming from the optical system. Due to these requirements in the control law for the yaw angle, which is represented by the first equation of (22), the variables x_2 and $w_1(t)$ have to be eliminated. In this way first the parameters λ_1 and α_{01} are equated to

$$\lambda_1 = \alpha_{01}, \tag{26}$$

so that the control error $w_1(t) - x_2$ can be factored out. The task is to substitute the control error in an adequate way. The deviation between the optical axis of the sensor and the edge of the street is determined by

$$\phi = \arctan\frac{l_{tr}}{l_{lo}}. \tag{27}$$

As the control system has to direct the vehicle on the right lane with nominal distance w_{ltr} to the right edge of the road, an offset

$$w_\phi(t) \;=\; \arctan\frac{w_{ltr}(t)}{l_{lo}} \tag{28}$$

has to be taken into account. By equation

$$w_1(t) - x_2 \;=\; w_\phi(t) - \phi \tag{29}$$

the control error is substituted by the difference $w_\phi(t) - \phi$. This relation is illustrated by Fig. 3. If further the parameters λ_2 and α_{02} are equated the control laws (22) considering (29) are transformed into

$$\begin{aligned} u_1(t) &= \frac{l_h}{l_v}\; S_h(\underline{x}) \;+\frac{\theta}{l_v}\Big(\lambda_1\;(w_\phi(t)-\phi)-\alpha_{11}\;x_3\Big) \\ u_2(t) &= \qquad T(\underline{x}) \;+m\;\Big(\lambda_2\;(w_2(t)-x_4)\Big). \end{aligned} \tag{30}$$

Herewith based on the present kinematic structure and the available sensors transverse and longitudinal dynamics are controlled. The desired values are represented by the nominal distance of the path of the vehicle to the right edge of the street and by the nominal velocity.

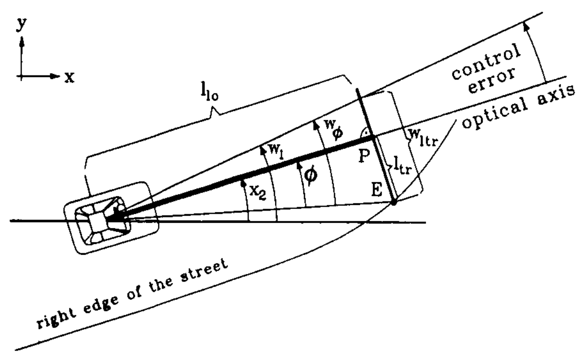

Figure 3: Determination of the transverse control error

3.4 Processing of original control signals:

By equations (30) the required values of the control variables for front side force S_v and rear driving or braking force H are computed. As the course of a vehicle is determined by the steering system, an algorithm is required to generate subsequently the corresponding value for the steering angle δ_v representing an original control signal in dependence of front side force S_v. In contrast control signal H for driving or braking force is correlated directly to the vehicle and thus it exists already as original control signal. For better distinction the original control signals $\bar{u}_1$ and $\bar{u}_2$ are marked by a bar.

Based on equation (3) and with respect to (2) the original control signal $\bar{u}_1$ is generated in dependence of the value S_v for front side force, which is represented by u_1 and evaluated by control law (30). In contrast the original control signal $\bar{u}_2$, which is identical to the desired value H for the rear driving or braking force, is represented by control variable u_2:

$$\begin{aligned} \bar{u}_1 &= \delta_v \;=\; \frac{u_1}{c_v} - x_1 + l_v\frac{x_3}{x_4} \\ \bar{u}_2 &= H \;=\; u_2. \end{aligned} \tag{31}$$

In this way the original control signals $\underline{\bar{u}}(t)$ are generated in a tandem arranged unit in dependence of vector $\underline{u}(t)$ and state variables $\underline{x}$ and subsequently they are transferred to the actuators of the vehicle.

3.5 Simulation results:

To demonstrate the quality of the control system, the dynamical behavior of the controlled vehicle was examined by computer simulations. Vicarious in Fig. 4 the resulting manœvre based on a sudden 45°-bend of the right edge of the street, which represents a hard test for the control system, is shown. This bend can be recognized by the graph marked with 'E', which consists of the fictitious path of the point E (see also Figs. 1 and 3). In contrast by the graph marked with 'P' the in the same way fictitious path of the point P, which is located on the optical axis of the sensor system and which is here of distance $l_{lo} = 5$ m to the car's center of gravity, is shown. It can be clearly recognized, how the vehicle, which drives a speed of 50 km/h, turns in this way that the controlled condition to keep the nominal distance of $w_{ltr} = 2$ m between the points P and E is followed. With respect to (25) the path of the vehicle's center of gravity drives through the bend with an aperiodic behavior and follows the street in

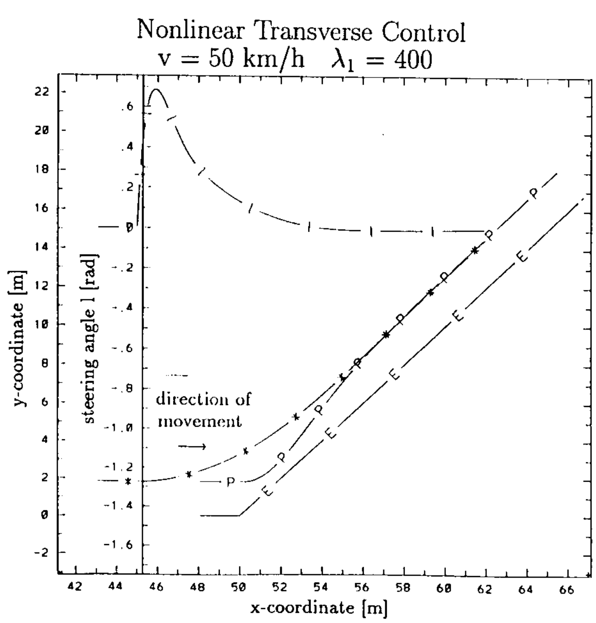

Figure 4: Dynamical behavior of the controlled vehicle

an exact way. Furthermore the course of the original control signal δ_v for the steering angle is described by the graph marked with '1' in dependence of the actual position of the car on the x-coordinate.

4 Conclusion

An on-board system for transverse and longitudinal control of an automatically guided vehicle based on nonlinear decoupling strategies is presented. With respect to the transverse dynamics of the car the control signal for the steering system is generated in dependence of the information recorded by an optical sensor, which detects the edge of the road. In contrast the drive and the brakes are influenced by the longitudinal controller taking the nominal and the actual speed into consideration. Control signals for the steering system as well as for the drive and and the brakes of the vehicle are generated by nonlinear algorithms. This method is especially distinguished by its efficiency and its preknowledgement about the dynamical behavior of the overall system. Herewith an important requirement to increase safety in future traffic is realized.

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